ON A COUPLED ELASTO-PLASTIC DAMAGE MODEL FOR FIBER REINFORCED LAMINATES

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Abstract
The modeling of non-linear response in fiber reinforced laminates must account for the coupled phenomena at the ply-level namely, matrix cracking and plasticity. In this work, a coupled elastoplastic damage model is proposed to capture the non-linearity at the length scale of individual plies(laminae). Non-linearity prior to damage onset is assumed to be purely plastic. We start with a modified von-Mises plasticity with a non-linear saturation type hardening that ensures a linear elastic response in fiber direction until damage. A modified formulation is then used to describe the onset and evolution of damage as a function of thermodynamic dual driving force. For a quantitative assessment of the model, predictions are evaluated in comparison with experimental data and some existing damage models.

1. Introduction

Prediction of damage onset and accumulation is of great interest in laminated composites. Matrix cracking is normally the first mode of damage and if left unattended can often lead to other failure modes such as delamination or fiber failure. The modeling of non-linearities caused by matrix has been very much focused on continuum damage mechanics, where it is assumed that accumulation of matrix cracks accounts for observed non-linearity. Recent findings however suggest that under shear dominated loading conditions, nonlinearity may also arise due to plasticity type effects.
The key aspect of this work is to formulate a coupled elastoplastic damage model using a thermodynamic formulation, conceptually in line with [1]. Several approaches for coupling damage and plasticity theory are present in the literature. For an overview see for example [2] or [3]. The concept of effective stress is used here and plasticity is restricted only to effective stress space. An orthotropic continuum damage formulation is employed to describe the anisotropic effect of damage observed in composite materials. While the constitutive framework is generally applicable, a plane stress state with homogeneous stress distribution inside the laminate plane is considered in the current work.

2. Constitutive Framework

2.1. State variables and energetic response

As a basic requirement of coupled elastoplastic damage theory, the effective stress $\bar{\boldsymbol{\sigma}}$ is defined as

$$\bar{\boldsymbol{\sigma}} = \left[(\sqrt{\mathbf{1} - \boldsymbol{d}})^{-1} \otimes (\sqrt{\mathbf{1} - \boldsymbol{d}})^{-1} \right] : \boldsymbol{\sigma} = \mathbb{M}^{\mathbf{-1}} : \boldsymbol{\sigma} \tag{1}$$

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where $\mathbb{M}$ denotes the damage effect tensor. Within the context of infinitesimal strain space, the elasto-plastic theory allows an additive split of total strain tensor $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^p$ into elastic and inelastic parts. The constitutive state that describes the model problem is given by primary fields $\{\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^p, \boldsymbol{d}, \alpha\}$ and the decoupled strain energy Ψ is defined as

$$\Psi = \Psi(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^p, \boldsymbol{d}, \alpha) = \frac{1}{2}\,(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p) : \boldsymbol{E}(\boldsymbol{d}) : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p) + \Psi^p(\alpha) \tag{2}$$

where $\boldsymbol{E}(\boldsymbol{d})$ expresses the stiffness of the damaged material defined in terms of effective stiffness as $\boldsymbol{E}(\boldsymbol{d}) = \mathbb{M} : \boldsymbol{E} : \mathbb{M}$, quadratic in terms of 2^{nd} order damage tensor $\boldsymbol{d}$. The plastic part $\Psi^p(\alpha)$ which describes a non linear saturation type isotropic hardening response is given by

$$\Psi^p(\alpha) = (y_\infty - y_0)\Big[-\frac{1}{\omega} + \alpha + \frac{1}{\omega}\exp(-\omega\alpha)\Big] \tag{3}$$

with y_0 and y_∞ being initial and final yield stress respectively. ω is the saturation parameter and α is the isotropic hardening variable. With the structure of free energy, it can be seen that damage is only associated with the elastic part.

2.2. Stresses and thermodynamic dual driving forces

The conjugate thermodynamic driving forces corresponding to the constitutive state are given by

$$\begin{aligned} \boldsymbol{\sigma} &:= \frac{\partial \Psi}{\partial \boldsymbol{\varepsilon}} = \boldsymbol{E}(\boldsymbol{d}) : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p) \\ \boldsymbol{Y} &:= -\frac{\partial \Psi}{\partial \boldsymbol{d}} = -\frac{1}{2}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p) : \frac{\partial}{\partial \boldsymbol{d}}[\boldsymbol{E}(\boldsymbol{d})] : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p) \\ \beta &:= -\frac{\partial \Psi}{\partial \alpha} = -(y_\infty - y_0)\big[1 - \exp(-\alpha\omega)\big] \end{aligned} \tag{4}$$

where $\boldsymbol{Y}$ represents the strain energy of the undamaged material which is also the work conjugate force to $\boldsymbol{d}$, and the driving force β is conjugate to the isotropic hardening variable α obtained from the plastic part of the free energy.

2.3. Irreversible plastic domain

The plasticity model is based on two assumptions, a) the incompressibility constraint as a result of which the formulation is in the deviatoric stress space and b) stresses in the fiber direction (σ_{11}) do not contribute to plasticity. A typical von-Mises yield criterion that specifies the elastic domain and its boundary is given by

$$\chi^p(\bar{\boldsymbol{\sigma}}, \beta) = ||\bar{\boldsymbol{\sigma}}'|| - \sqrt{\frac{2}{3}}[y_0 + \beta] \leq 0 \tag{5}$$

in terms of the effective deviatoric stress $\bar{\boldsymbol{\sigma}}'$ computed as the deviator of the effective stress tensor where σ_{11} has been subtracted. As the von-Mises plasticity is an associative type of plasticity model, the yield function is identical to the flow potential. The associated evolution of the plastic flow and hardening variable are given by

$$\begin{aligned} \dot{\boldsymbol{\varepsilon}^p} &= \dot{\lambda}^p \frac{\partial \chi^p}{\partial \bar{\boldsymbol{\sigma}}'} \\ \dot{\alpha} &= \dot{\lambda}^p \frac{\partial \chi^p}{\partial \alpha} \end{aligned} \tag{6}$$

While plastic deformations of composite materials are more evident for shear loading than longitudanal and transverse loading, plasticity in transverse direction is also taken into account by this formulation.

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2.4. Damage domain

The damage initiation criterion defined in terms of thermodynamic driving force $\boldsymbol{Y}$ is characterized by a damage threshold in terms of energy i.e., damage accumulation takes place if the thermodynamic force reaches the constitutive threshold

$$\chi^{d}(\boldsymbol{Y}) = \boldsymbol{Y} - \boldsymbol{\varphi}_c \ \leq \boldsymbol{0} \tag{7}$$

where $\boldsymbol{\varphi}_c$ represents failure energies obtained from experimentally determined strength values. Upon using a Lagrange type minimization principle to solve the dissipation potential(based on Eq. 7) we obtain

$$\dot{\boldsymbol{d}} = \dot{\lambda}^{d} \frac{\partial \chi^{d}}{\partial \boldsymbol{Y}} \tag{8}$$

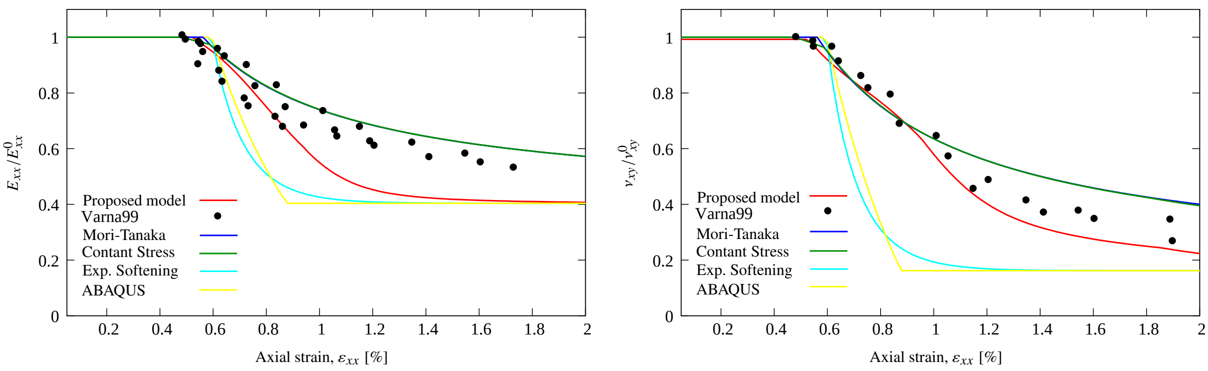
For a plane stress case a contracted notation for $\mathbb{M}$ is expressed below. Due to the symmetry of $\boldsymbol{\sigma}$ and $\bar{\boldsymbol{\sigma}}$, $\mathbb{M}$ is doubly symmetric.

$$\mathbb{M} = \begin{bmatrix} 1-d_1 & 0 & 0 \\ 0 & 1-d_2 & 0 \\ 0 & 0 & \dfrac{\sqrt{(1-d_1)(1-d_2)}}{2} \end{bmatrix} \tag{9}$$

The current model predicts localized damage i.e., the stresses and the moduli are gradually degraded to zero following the degradation law as per Eq. 8. For an individual ply, it is clear that in such models, the stresses have to be reduced to zero in the ultimate damage state, whereas for a ply embedded in a laminate, ply stresses will be zero at the crack but not further away from it as long as the ply interfaces are intact. This can be overcome by taking into account damage accumulation behavior with increasing strain. An extension to simulate accumulation of multiple ply cracks is planned.

3. Comparison to Experimental Data

For a quantitative assessment of the model, predictions are evaluated in comparison with experimental data from two series of uni-axial tests ([4, 5]) and some existing damage models as per [6]. Fig. 1 shows the variation of axial modulus and poisson's ratio for a $(0_2/90_8/0_{1/2})_s$ laminate. The change of E_{xx} and ν_{xy} at low strains is caused by plasticity which occurs due to σ_{22} stresses in the 90° ply and is in good agreement with experimental data. The degradation at higher strains is overpredicted due to the current damage formulation where ply stresses are reduced to zero (as explained in Sec. 2.4).



a) b)

Figure 1. Single element test-tensile loading. Results for a $(0_2/90_8/0_{1/2})_s$ lay-up; variation of a) laminate's axial modulus and b) poisson's ratio normalized by their intialized value.

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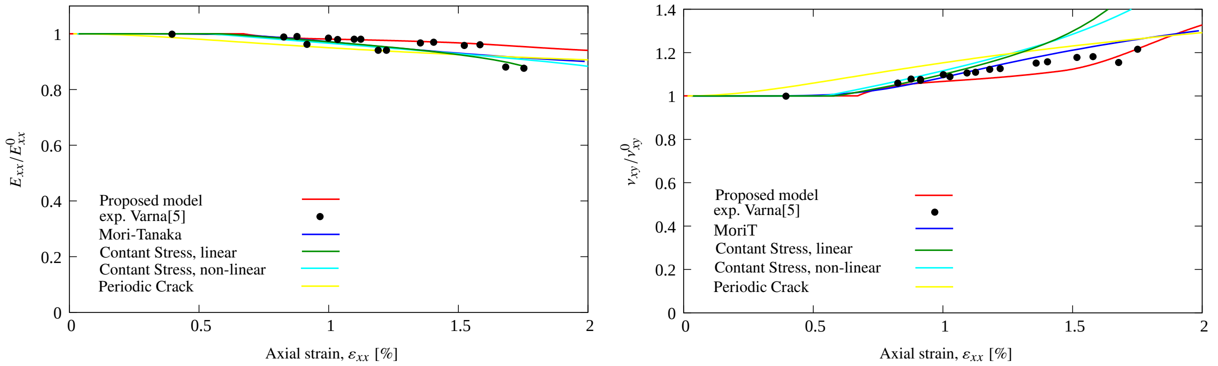

a) b)

Figure 2. Single element test-tensile loading. Results for a $(0/\pm25_4/0_{1/2})_s$ lay-up; variation of a) laminate's axial modulus and b) poisson's ratio normalized by their intialized value.

Fig. 2 shows the variation of axial modulus and poisson's ratio for a $(0_2/\pm25_4/0_{1/2})_s$ laminate. Due to a very small stress ratio of σ_{22}/σ_{12}, no damage can be seen in the strain range considered. As a result the moduli degradation can be attributed to plasticity, which is captured well by the current model. As shown in Fig. 2-b), the current plasticity formulation remidies some of the problems observed with plasticity formulations evaluated in [6] which is a result of the exponential hardening function.

4. Conclusion

The proposed model predicts damage and irreversible deformations phenomena of laminated composites. A limited number of internal variables representing the evolution of damage and plastic variables ensure a formulation simple enough for practical application. Comparisons between experimental data and model predictions are good in terms of plasticity and constitutes an improvement over previous plasticity formulations. Some further developments such as taking into account damage saturation effects and formulation of an anisotropic yield criterion are envisioned in the future.

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