AVOIDING SPURIOUS TRANSVERSE MODES IN 3D TEXTILE COMPOSITE REINFORCEMENT FORMING SIMULATION

P. Boisse, S. Mathieu, N. Hamila

Université de Lyon, LaMCoS, INSA Lyon, F-69621 Villeurbanne, France
Email: Philippe.Boisse@insa-lyon.fr, Web Page: http://lamcos.insa-lyon.fr/

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Abstract
Specific spurious transverse modes in bending-dominated simulations of thick 3D composite reinforcement forming are highlighted. The particular anisotropic material behavior due to the possible slippage between fibers induces these phenomena. To obtain coherent finite element responses, two solutions are proposed. The first one uses a simple assumed strain formulation usually prescribed to prevent volumetric locking. This solution avoids spurious transverse modes by stiffening of the hourglass modes. Nevertheless the deformation obtained by this approach still suffers from the inability of the standard continuum mechanics of Cauchy to describe fibrous material deformation. The second proposed approach is based on the introduction of a bending stiffness which both avoids the spurious transverse modes and also improves the global behavior of the element formulation by enriching the underlying continuum.

1. Introduction

Because of their important thickness, the 3D woven reinforcements need to be modeled with 3D finite elements and described with 3D constitutive law [1-4]. They display additional deformation modes in transverse shearing thanks to the ability of the yarns that constitutes the preform to slide transversely against each other. However, thick reinforcements are highly anisotropic materials due to large ratios between the tensile rigidities and the others. Numerical simulation involving these materials highlight spurious phenomena and limitations related to this specificity. One other problem arises in bending-dominated situations: transverse hourglassing displacement mode overdevelopment that leads to incoherent simulations. This phenomenon will be highlighted in a three point bending case. The reasons for the shown transverse hourglass development will be analyzed. Two solutions will then be proposed. The first one is based on strain assumed strain method averaging the dilatation in the element. It is simple to implement. It will be shown that this approach avoid spurious transverse hourglassing modes and the reasons for this [5]. The second proposed approach aims at avoiding both transverse hourglassing modes and taking into account the local rigidity of the fibres. It is based on the calculation of the curvature in a 3D finite element using the position of the neighbouring elements [6]. This second approach relies on the second gradient type mechanical behavior of textile reinforcements. However both the two proposed approaches can be implemented in standard FEM codes.

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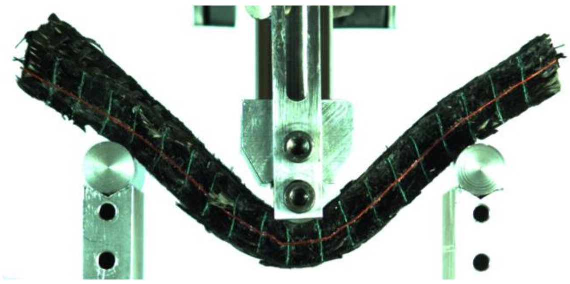
2. Anisotropic Hyperelastic Constitutive Equation

The deformation modes of interlock reinforcements are the summation of the deformation modes induced by the thickness of the reinforcement with those of a 2D woven fabric. Six deformation modes are thus considered: extensions in warp and weft directions, transverse compaction, in-plane shear and transverse shear in warp and weft directions. For each deformation mode, a physical strain invariant which is a combination of the invariants defined in [7] is defined. The model is detailed in [1, 6]

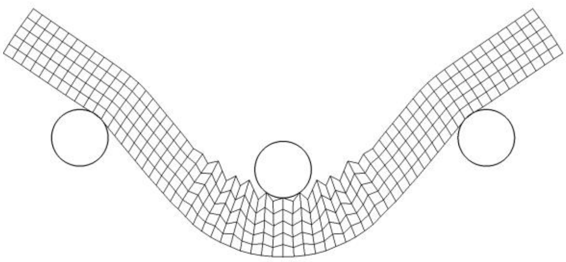
3. Flexural Parasitic Modes

3.1 Three Point Flexural Test Simulation

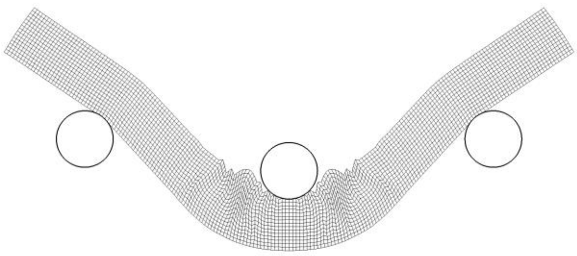
Unlike 2D textile reinforcements, due to their thickness, interlocks are suitable for three point bending tests. This testing method allows the study of kinematics characterized by a large curvature. A three point bending force is applied to a 200mm x 30mm x 15mm interlock sample. The sample is oriented in the warp direction. Figure 1 shows the deformed shape for a 40mm displacement of the central point. Because of the large tensile stiffness of the yarns, the deformation of the sample is mainly due to transverse shear. The sections, initially perpendicular to the midplane, remain almost vertical during the deformation (contrary to Kirchhoff's assumption). The result of a simulation based on the hyperelastic model presented [1,6] is displayed in figure 1. Two meshes with linearly interpolated, hexahedral elements whose sides are 3mm and 1mm long are used (figures 1b and 1c). The resulting simulations are incoherent with the experimental results. Peaks are produced and in the case of the finer mesh, evolve into smoother bulges. These are the result of the emergence of parasitic patterns due to the high anisotropy of dry thick woven preforms. The observed deformation cannot be credited to the inherent shear locking in bending [8] because the highly refined or coarser meshes lead to identical overall shapes.



a. Experiment



b. Coarse simulation



c. Refined simulation

Figure 1 : Three point bending - 40mm deflection

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3.2 Stiffening of the Hourglass Modes with the Mean Dilatation Method F-bar

The dilatation in hourglass pattern is nonzero everywhere except at the center of the element even though the overall element deformation is isochoric. With incompressible or quasi-incompressible behavior laws, a locking arises from the inability of the element to represent the isochoric field. One of the method to solve locking problems is to modify the interpolation in an appropriate manner: the so-called assumed strain methods. The mean dilation formulation has been extended to finite deformation, the so-called $\overline{\mathbf{F}}$ methods [9, 10], and is widely implemented in some commercial finite element code thanks to its simplicity and efficiency.
Although the $\overline{\mathbf{F}}$ method was initially proposed to reduce the hourglass mode stiffnesses of incompressible material, which is far from our preoccupation, its implementation can solve the spurious mode issue presented here.

3.3 Implementation

The goal of the $\overline{\mathbf{F}}$ method is to have a constant dilatation throughout the element, preventing the volumetric locking. The main idea is to divide the deformation gradient, which is the relevant quantity in finite deformation, into two parts: the deviatoric part (isochoric) and the volumetric part. A multiplicative decomposition is used, derived from the Hu-Washizu principle [9, 10]. The deformation gradient is defined along with its determinant, the volume variation:

$$\underline{\underline{F}} = \frac{\partial \underline{x}}{\partial \underline{X}} \quad , \quad \mathrm{Det}\left(\underline{\underline{F}}\left(\underline{\xi}\right)\right) = j\left(\underline{\xi}\right) \tag{1}$$

where $\underline{X}$ and $\underline{x}$ are the coordinates in the reference and current configuration of the physical domain, and $\underline{\xi}$ are the coordinates in the isoparametric domain. The constant dilatation to be applied must be chosen judiciously. In order for the patch test to be satisfied, the assumed volume variation $\overline{j}$ chosen is:

$$\overline{j} = \frac{1}{V_0}\int_{V_0} \mathrm{Det}\left(\underline{\underline{F}}\left(\underline{\xi}\right)\right) \mathrm{d}V_0 \tag{2}$$

where V_0 is the initial volume of the element. The new deformation gradient becomes:

$$\overline{\underline{\underline{F}}}\left(\underline{\xi}\right) = \underline{\underline{F}}\left(\underline{\xi}\right)\left(\frac{\overline{j}}{j\left(\underline{\xi}\right)}\right)^{\frac{1}{3}} \tag{3}$$

At every integration point, the dilatation is then fixed at $\overline{j}$. The resulting deformation state is modified [5]. This formulation is equivalent to a selective integration solution with a reduced integration of the volumetric part and a full integration of the deviatoric part.

3.4 Numerical Result Using the F-bar Method

As for the three point bending, new simulations are run in the exact same condition as previously, the only difference being the use of the $\overline{\mathbf{F}}$ method. The result for a 40mm deflection is given in figure . Contrary to the parasitic bulges seen
figure , the hourglass shapes are controlled and the new kinematics are more consistent with the experiment.

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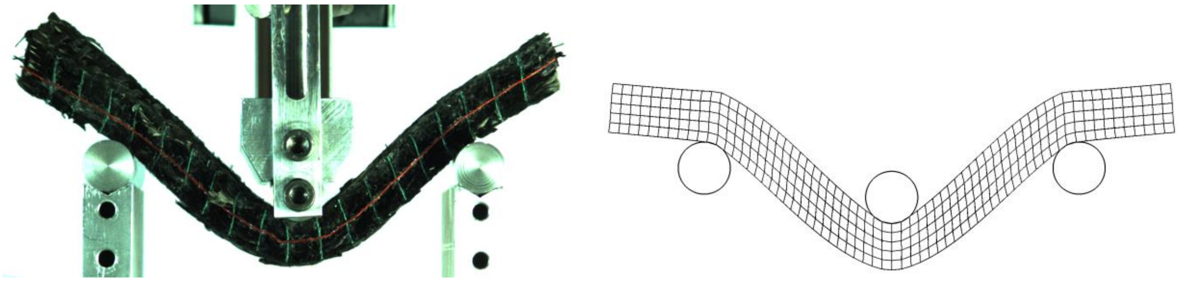

Figure 2 : Three point bending – 40mm deflection – F-bar method

The F-bar method is effective in preventing non-physical hourglass displacement mode development in bending dominated simulations of 3D fibrous reinforcements. Its implementation is fast and simple. But this technique does not come without drawbacks:

- The new hourglass stiffnesses directly depends on the large yarn stiffnesses E_1 and E_2. The quality and quantity of the hourglass stabilization relies directly on the magnitudes of those stiffnesses.
- The global shape of the three point bending (
- figure) is not entirely satisfactory: the central curvature is not consistent and both ends, despite supporting no load, remain horizontal.

This last issue is due to the fibrous nature of the preforms which induces a bending stiffness at the microscale (the scale of the fiber), impacting the macroscale behavior. A first gradient modeling, in that case, poorly represents the intrinsic physics at play. Second gradient approaches have been considered for the simulation of 2D fibrous material deformations [11] and recently of 3D reinforcements [12]. For reasons of efficiency, the introduction of a stiffness related to the curvature in 3D finite elements is proposed to account for the fiber's local bending stiffness and to stabilize the aforementioned spurious modes.

4. Additional Local Bending Stiffness to 3D element

4.1 Bending Virtual Work and Local Bending Moment

2D textile composite reinforcements (thin reinforcements) are usually modeled by shell finite elements. The bending stiffness of the fibers is taken into account and the problems highlighted in section 3 for 3D finite element models do not occur. In the present approach, a stiffness related to the local curvature is introduced in 3D finite elements in order to take into account the bending stiffness of the fibers. A hexahedral 3D finite element is considered. For clarity, this element is considered to be parallelepipedal and oriented by $\underline{M}_1$, $\underline{M}_2$, and $\underline{M}_3$. In the plane perpendicular to $\underline{M}_3$ and at position s along this direction, a curvature in the main yarn directions is introduced representing the fiber's curvature. This curvature χ of the fibers creates a local bending moment, $M(\chi)$. For a 3D finite element, the internal virtual bending work corresponding to the virtual curvature χ^* is:

$$W_{\text{int}}^{bend} = \int_{A(s)} M(\chi(s))\, \chi^*(s)\, dA \qquad (4)$$

where A(s) represents the area defined by the intersection of a plane orthogonal to the direction $\underline{M}_3$ and the finite element at global coordinate s. The bending moment is directly linked to the thickness of a structure. A bending moment per unit length M_l, is introduced:

$$M = \int_S M_l ds \qquad (5)$$

Since the curvature is supposed to be constant at coordinate s, the virtual work becomes:

$$W_{\text{int}}^{bend} = \int_S M_l(\chi(s))\ \chi^* A(s) ds \tag{6}$$

And in an 8 node finite element:

$$W_{\text{int}}^{bend} = \sum_{\alpha=1}^{2} \omega_\alpha M(\chi(\psi_\alpha))\ \chi^*(\psi_\alpha)\ A(\psi_\alpha)\ J_{s\alpha} \qquad \text{where} \quad J_{s\alpha} = \frac{\partial s}{\partial \psi} \tag{7}$$

For the two integration points, ψ_α, $J_{s\alpha}$ is the jacobian of the mapping from the isoparametric domain of bi-unit length [-1,1] to the physical domain.

4.2 Calculation of the Curvature

The curvature χ must be assessed for every area linked to its respective integration point. The curvature of the considered plane is obtained from the position of the neighboring elements. This approach is similar to those used in rotation free shell finite elements [13, 14]. The bending curvatures are computed using triangles based on the position of the neighboring elements (
figure).

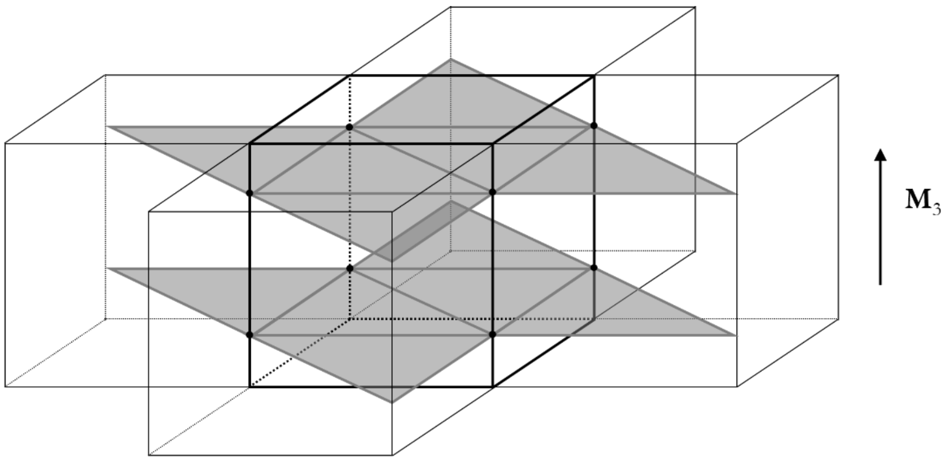

Figure 3 : Triangular elements patch for the curvature evaluation in a brick element

The purpose is to evaluate the curvature. The approach can be summarized as follows:

- Two triangles in a horizontal section of the hexahedral element under consideration are considered (figure 3).
- One of these triangles and its three neighboring triangles are considered. The displacements at each node of these triangles are retrieved from the hexahedral displacements;
- The bending angles are computed for each side from the displacements of the neighboring nodes and the rigid rotations;
- A constant curvature is computed inside the element and projected in the fiber directions to get the curvature in warp and weft directions [14].

Details on these curvature computations are given in [5, 14].

4.3 Numerical Result Adding a Local Bending Stiffness

The stabilization is demonstrated on the three point bending simulation. A non-linear constitutive law linking the per unit length moment to the curvature in warp and weft directions is chosen of the form:

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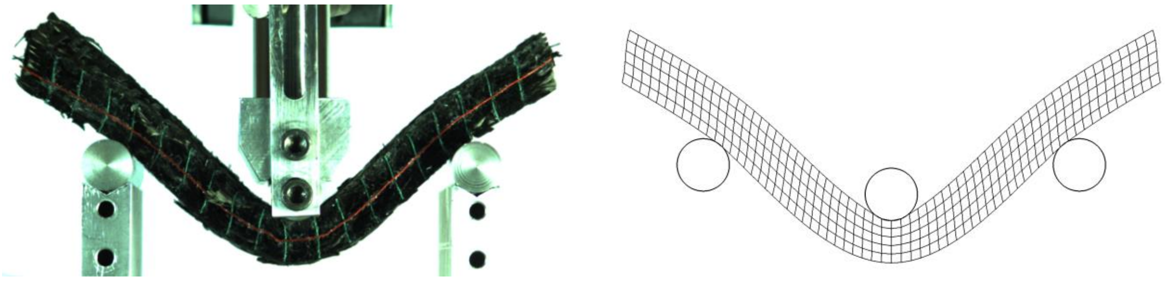
$$\mathbf{M}_l = \begin{cases} \left(D_0 - D_1|\chi|\right)\chi & \text{If } \chi < \dfrac{D_0}{2D_1} \\ \dfrac{D_0}{2}\chi & \text{Otherwise} \end{cases} \quad (8)$$

This non-linear law depends on two coefficients D_0 and D_1. It is a compromise between simplicity and efficiency of the bending simulations. The identification of the bending parameters D_0 and D_1 is accomplished by inverse optimization over the experimental and numerical midplanes of the three point bending test using the Levenberg-Marquardt algorithm.
The result of a simulation using these bending stiffnesses is shown in figure 4 for a 40mm deflection. The final result is consistent with the experiment: the central radius is close to the experimental result, the external parts of the sample are almost aligned with the central part and do not stay horizontal. Above all, no spurious non-physical transverse hourglass modes develop demonstrating the effectiveness and consistency of the proposed addition of local bending stiffness in modeling thick fibrous preforms.



Experiment Simulation

Figure 4: Three point bending – 40mm deflection – Additional bending stiffness

5. Conclusions

Finite element analyses of 3D composite preform deformation based on an anisotropic hyperelastic law induce the development of spurious peaks or bulges in bending-dominated simulations. In order to deal with this issue two solutions have been proposed.
The first one, simple and efficient, relies on the implementation of the F-bar method, an assumed strain formulation which forces the dilatation to be constant at each point of the element. Although not initially designed for this purpose, this technique proves to be effective in avoiding the problematic hourglass modes. The resulting kinematics of the three point bending simulation are closer to the one observed experimentally.
To take into account the multi-scale, intrinsic nature of composite preform behavior, the modeling requires a richer underlying continuum theory. This is accomplished by introducing a dependence on the curvature variation in the medium through a linear bending moment. The overall final constitutive law proves to be more adequate. The use of curvature variations in the thickness prevent the occurrence of bulges induced by transversal hourglass instabilities. In addition, the problems highlighted for thick fibrous media in bending are resolved.

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