A MESO-SCALE MODEL TO STUDY THE COMPRESSIVE STRENGTH OF WOVEN CARBON FIBER REINFORCED PLASTICS

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Keywords: Kink-bands, Compression, Homogenization,

Abstract
Modeling kink-band formation in woven composites using a detailed micro-model is numerically expensive. In order to reduce the computational resources, a method to homogenize fiber-tows is proposed which uses a rules of mixture approach. The method is tested by comparing the stiffness and compressive strength of fiber-tows simulated with the proposed homogenization method and simulations of fiber-tows with the micro-structure modeled in detail. Results obtained for both straight fiber and woven composites indicate that the proposed homogenization technique can be used to predict the stiffness and compressive strength of UD and textile composites.

1. Introduction

Kink band formation is one of the dominant failure mechanisms in woven composites under compressive loading [1]. Detailed numerical studies of the formation of kink-bands in unidirectional composites on the micro-scale, where each fiber is modeled explicitly, have been performed by Kyriakides et al. and Pimenta et al. [2, 3]. Modeling kink-band formation in woven composites on a larger scale, considering multiple stacked layers and taking into account the 3-dimensional nature of the specimen, is challenging as modeling each fiber individually becomes computationally inefficient. The alternative is to homogenize the mechanical properties of the fibers and the epoxy in a tow.
In this paper we will study the effects of this homogenization on the accuracy of the analysis of the stiffness and the strength of unidirectional and woven composites under compression. The paper is organized as follows: in the next section, the homogenization-based constitutive model is presented; in the third and fourth section, the model is applied to a straight fiber geometry and a woven geometry, respectively. The paper ends with some conclusions and an outlook to future applications.

Excerpt from ISBN 978-3-00-053387-7

Table 1. The strength ratios used in Hill's quadratic failure criterium [4]. The ratios are defined with respect to the matrix yield stress σ^y_m and the matrix yield stress in shear τ^y_m.

Yield stress ratio	value	Yield stress ratio	value
$\sigma^y_{11}/\sigma_\mathrm{m}$	327	$\tau^y_{12}/\tau_\mathrm{m}$	1
$\sigma^y_{22}/\sigma_\mathrm{m}$	1	$\tau^y_{13}/\tau_\mathrm{m}$	1
$\sigma^y_{33}/\sigma_\mathrm{m}$	1	$\tau^y_{23}/\tau_\mathrm{m}$	1

2. Methods

As this work aims to analyze the performance of a homogenized tow model, a comparison is made with a detailed micro-model where each individual fiber is modeled. First, the constitutive models used in this study are explained, after which the constitutive relations are applied on a straight fiber geometry.

2.1. Constitutive modeling

Within this study, the mechanical properties of the carbon fibers and the resin are similar to those used by Kyriakides et al. [2]. The carbon fibers are isotropic-elastic with an elastic modulus of $E_\mathrm{f} = 214\,\mathrm{GPa}$ and a Poisson's ratio of $\nu_\mathrm{f} = 0.263$. The matrix is an isotropic-elastic perfectly-plastic medium having an elastic modulus of $E_\mathrm{m} = 6140\,\mathrm{MPa}$, a Poisson's ratio of $\nu_\mathrm{m} = 0.356$ and a yield stress $\sigma^y_\mathrm{m} = 82.1\,\mathrm{MPa}$.
To arrive at a homogenized tow model, a rule-of-mixture approach is used to generate an orthotropic-elastic perfectly-plastic medium. Using the fiber volume fraction V_f, the elastic properties of the tow model can be calculated using:

$$
\begin{aligned}
E_{11} &= V_\mathrm{f} E_\mathrm{f} + (1 - V_\mathrm{f}) E_\mathrm{m}\,, \\
\frac{1}{E_{33}} &= \frac{V_\mathrm{f}}{E_\mathrm{f}} + \frac{1 - V_\mathrm{f}}{E_\mathrm{m}}\,, \\
\frac{1}{G_{13}} &= \frac{V_\mathrm{f}}{G_\mathrm{f}} + \frac{1 - V_\mathrm{f}}{G_\mathrm{m}}\,, \\
\tau^y_{13} &= \tau^y_m = \frac{\sigma^y_\mathrm{m}}{\sqrt{3}}\,, \\
\sigma^y_{33} &= \sigma^y_\mathrm{m}\,, \\
\nu_{13} &= \nu_m\,,
\end{aligned}
\tag{1}
$$

where G is the elastic shear-modulus and τ^y is the yield strength in shear. The subscripts indicate the direction of each quantity such that E_{11} represents the elastic modulus in the 1-direction.
To guarantee that the orthotropic tow will remain elastic in the fiber direction, Hill's quadratic failure criterion [4] is used to determine the onset of plasticity. The yield stress ratios used can be seen in Table 1.

3. Straight fiber model

Before applying the homogenized tow model to a woven geometry, simulations are performed on a straight fiber geometry to investigate if the homogenized tow model is able to predict a similar strength as the micro-model. The geometry and the boundary conditions of the micro-model and the homogenized model are discussed below.

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3.1. Micro-model

The micro-model consists of fibers and matrix layers stacked on top of each other. An overview of the model and the boundary conditions can be seen in Figure 1. The out-of-plane direction of the material is

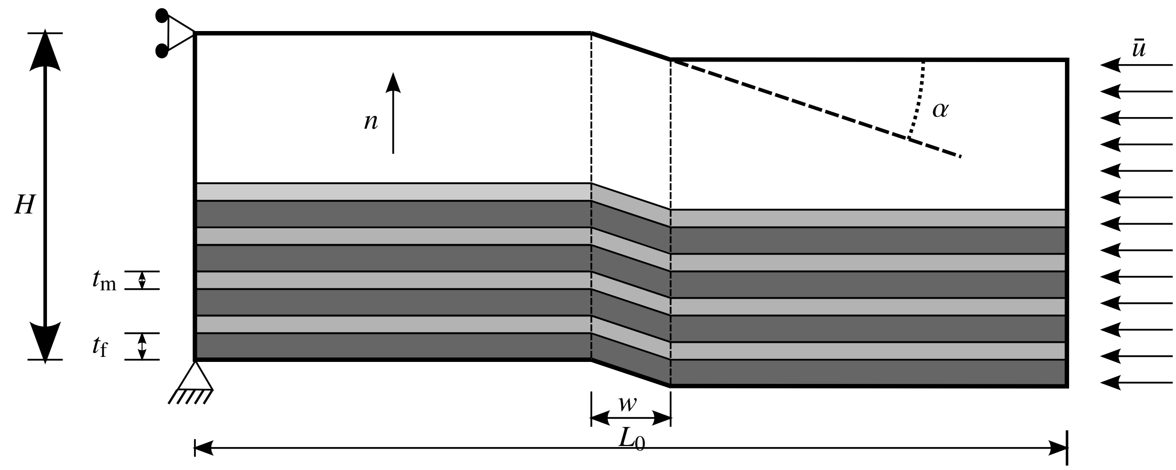

Figure 1. Geometry and boundary conditions of the straight fiber micro model.

modeled using generalized plane strain with unit thickness. The bottom left point of the model is fixed in all directions and the left side of the model is constrained in the fiber direction. The top and bottom edges of the model remain traction free. A displacement $\bar{u}$ is prescribed on the right edge. Throughout the simulations the fiber thickness $t_f = 6\,\mu$m is kept constant. The thickness of the matrix layer between two fibers depends on the fiber-volume-fraction V_f and can be calculated using:

$$t_m = \frac{t_f}{V_f} - V_f\,. \tag{2}$$

The length of the model is $L_0 = 350\,t_f$ and the ratio between the length and the height of the model is kept approximately constant at $\frac{L_0}{H} \approx 2.1$. The number of alternating layers of fibers and matrix needed to approximate this height can be calculated using:

$$n = [\frac{H}{t_f + t_m}] \tag{3}$$

To initiate buckling of the fibers, an imperfection with a width of $w = 33\,t_f$ is added to the middle of the model. Within the imperfection area, the fibers are rotated with a small imperfection angle $\alpha = 5°$.

3.2. Homogenized model

The homogenized model shown in Figure 2 uses a similar geometry as the micro model displayed in Figure 1. The length of the model L_0 is the same as the micro-model, as well as the out of plane unit thickness used for the generalized plane strain formulation. In contrast to the micro-model, the ratio between the length and the height of the model is fixed at $\frac{L_0}{H} = 2.1$. Again, the bottom left node is fixed in all directions and the left edge of the model is constrained in the fiber direction. The prescribed boundary conditions $\bar{u}$ on the right edge of the model are the same as in the micro-model. Although the homogenized model does not have any distinct fibers, a local coordinate system is used to represent a fictitious fiber direction in the orthotropic constitutive formulation. Within the imperfection area w this local coordinate system that is aligned with the fiber orientation is rotated at an angle α, such that the orthotropic material model corresponds to the bent fibers in the micro-model.

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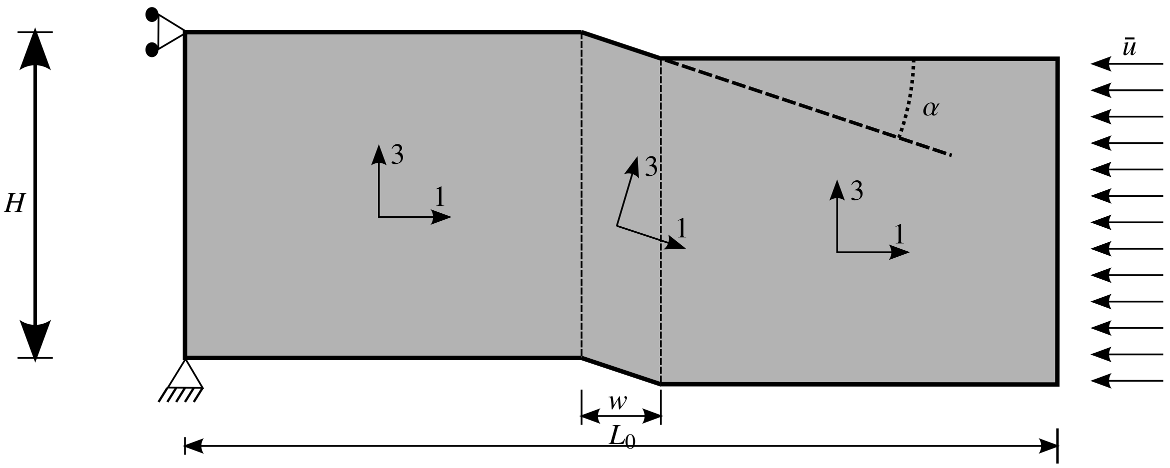

Figure 2. Geometry and boundary conditions of the straight fiber homogenized model.

3.3. Straight fiber results

Within this section, two sets of results are presented. Figure 3 shows the error between the homogenized model and the micro-model for both the compressive strength and the stiffness. The error in strength is defined as:

$$\epsilon_\sigma = \frac{\sigma_{\mathrm{c,hom}} - \sigma_{\mathrm{c,micro}}}{\sigma_{\mathrm{c,micro}}}, \tag{4}$$

where $\sigma_{\mathrm{c,micro}}$ is the compressive strength predicted by the micro model and $\sigma_{\mathrm{c,hom}}$ is the compressive strength predicted using the homogenized model. The error in stiffness is determined by:

$$\epsilon_E = \frac{E_{\mathrm{c,hom}} - E_{\mathrm{c,micro}}}{E_{\mathrm{c,micro}}}, \tag{5}$$

where $E_{\mathrm{c,hom}}$ is the stiffness calculated using the homogenized approach and $E_{\mathrm{c,micro}}$ is the stiffness obtained using the micro model.

Figure 3 shows that the error in the calculated stiffness is negligible. Furthermore, Figure 3 shows

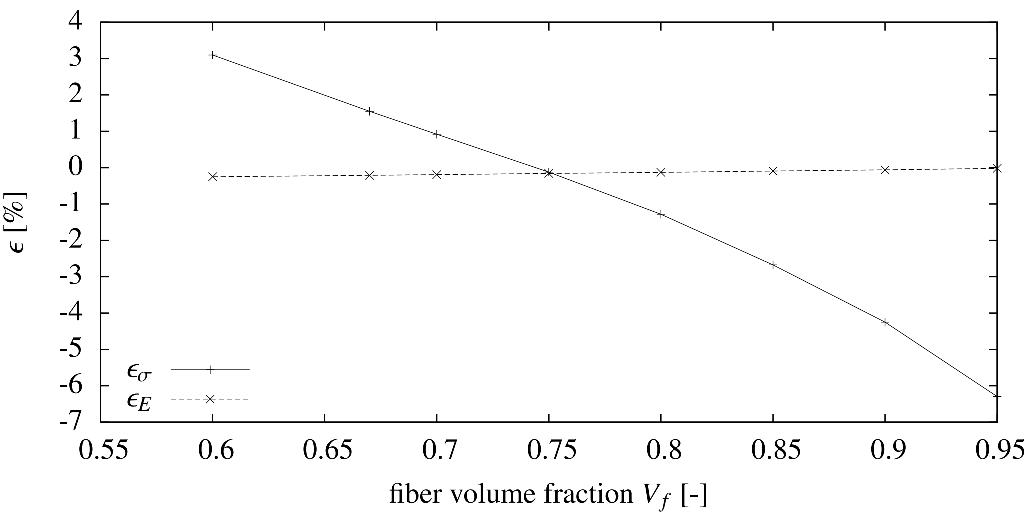

Figure 3. The stress-strain curves generated using the micro model with an imperfection angle of 5°. Various fiber-volume fractions have been simulated.

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that the error in the calculated compressive strength is a function of the fiber-volume fraction. Although the accuracy varies with the fiber-volume fraction, the error is small over the whole range of simulated fiber-volume fractions and near zero for realistic fiber-volume fractions. One side note is that the use of the model is only valid till the point of initial yielding of the matrix. After the initial yielding of the the matrix, the homogenized model misses the relevant micro-geometry, which regularizes the deformation.

4. Woven model

In order to investigate whether the homogenized-model is suited to model the formation of kink-bands in woven laminates, the woven unit cell displayed in Figure 4 is modeled. The light grey areas in Figure 4 indicate resin pocket whereas the striped areas represent weft fiber-tows. The weft fiber-tows have the homogenized fiber-tow properties. The warp fiber-tow is constructed in a similar way as straight-fiber micro-model. A fiber thickness of $t_\mathrm{f} = 6\,\mu\mathrm{m}$ is used and the the matrix layer thickness is determined using (2) in combination with a fiber volume fraction of $V_\mathrm{f} = 0.7$. A complete warp-tow is constructed

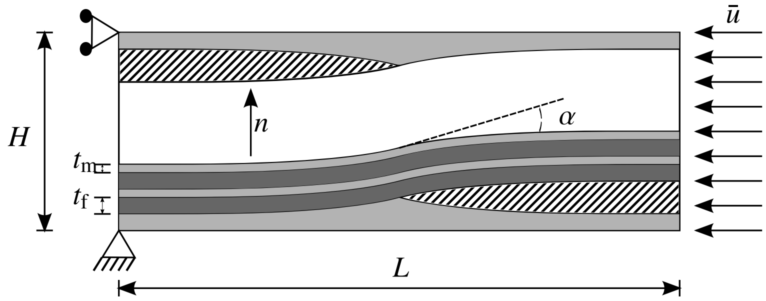

Figure 4. An overview of the geometry and boundary conditions of the woven model geometry.

by stacking a fiber and a resin layer n times. In order to simulate a realistic fabric, the warp undulation angle is set to $\alpha = 10°$ and the unit cell has dimensions $H = 0.48$ mm and $L = 1.7$ mm. Five of these unit cells are stacked on top of each other to model a realistic, 5 layer, composite and to minimize the edge effects. The top edge and the bottom edge of the laminate are traction free. The compressive boundary conditions used on the woven model are the same as those used on the straight-fiber model. A horizontal displacement $\bar{u}$ is acting on the right edge while the left edge is constrained in the compressive direction. One node on the bottom left corner of the model is fixed to prevent rigid body motion.
The homogenized version of the woven model is created by replacing the micro-structure of the warp tow with a homogenized tow in the same way as was done with the homogenized unidirectional model.

4.1. Results

Figure 5 shows the stress-strain curves for both the micro-model and the homogenized model. It can be seen that the homogenized model predicts the same initial stiffness as the micro-model. At the same time predicts a lower peak compressive stress with respect to the micro-model. An explanation of this phenomenon can be found when the mesh-sensitivity of both models is studied. Figure 6 shows the peak compressive strength simulated using both methods as a function of the mesh size. It can be concluded that the homogenized method suffers from localization. The homogenized model lacks the geometry of the fibers that regularize the growth of the kink-band. The figure however also indicates that if the correct mesh size is chosen, the peak stress of a laminate can be approximated well.

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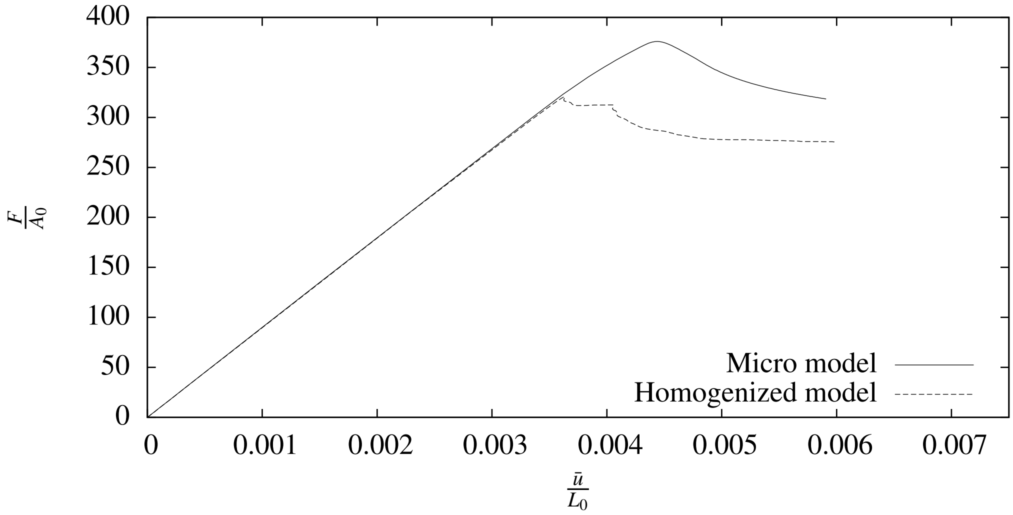

Figure 5. A comparison of the stress-strain curves generated using the micro-model and the homogenized model.

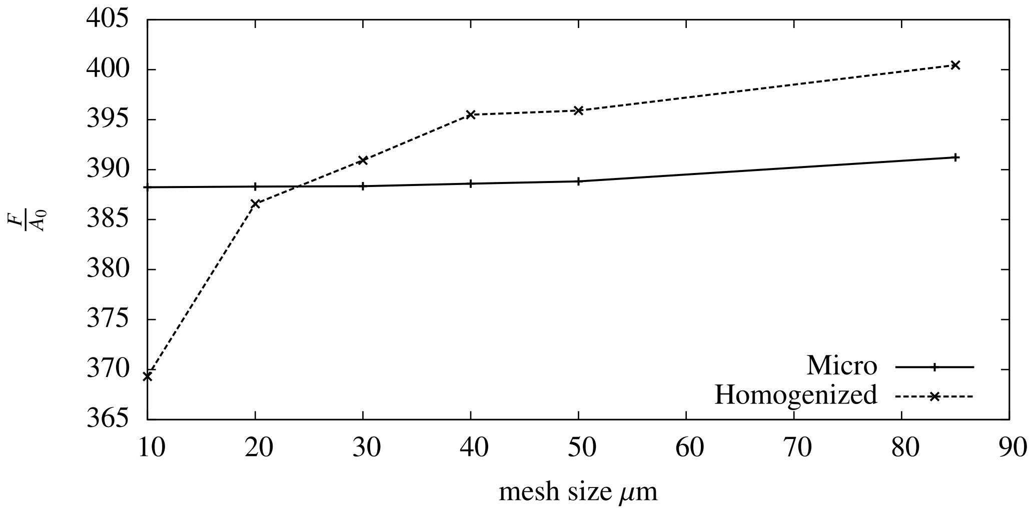

Figure 6. The mesh sensitivity of the micro-model and the homogenized model.

5. Conclusion

This paper sets out to demonstrate that it is possible to use homogenization to reduce the computational cost it takes to simulate a fiber-tow under compression. Using a rules of mixture approach in combination with Hill's quadratic failure criterion enables the simulation of a straight fiber-tow under compression. The results show that the error in the stiffness and compressive strength simulated with the homogenized model are small with respect to the stiffness and compressive strengths simulated with a detailed micro model. Applying the homogenization to a woven geometry show that the homogenized model suffers from localization. The homogenized model is however still valuable if the correct mesh size is used. For each woven geometry and set of material parameters, the mesh size can be calibrated using the micro model. In the future, using this homogenized approach will make it possible to simulate woven composite laminates under compression on a meso-scale.

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Acknowledgments

This research is supported by the Dutch Technology Foundation STW, which is part of the Netherlands Organisation for Scientific Research (NWO), and which is partly funded by the Ministry of Economic Affairs.

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