DISCRETIZING THE DEFORMATION WITHIN A STRESS BASED FUNCTIONALLY GRADED INTERPHASE IN A MICROMECHANICS COMPOSITE MODEL

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Abstract
A new modeling framework is developed to capture the response of long fibre composites in to the plastic regime. This model separates the deformation in a unit cell into three distinct regions, each undergoing a different deformation and having a different stress state. The material models used for each of the regimes corespond to the response of the natural material. The subdivision of the regions is accomplished through the use of a stress based functionally graded interphase. The model is calibrated to experimental results and shows very good agreement up to the onset of failure for multiaxial loading using a single set of material parameters.

1. Introduction

Composite materials pose a huge potential advantage over conventional metallic materials for use in vehicle structures. They offer a high weight savings potential due to their high specific strength and stiffness. Composite structures also offer additional energy absorption mechanisms [1]. Composites structures have been used in high performance low volume vehicles [2]. Current interest is in development of energy absorbing crash structures for use in automotive applications [3–5]. In order for composite materials to become a viable material for automotive applications, models with better predictive capabilities need to be developed for automotive composites. The authors have developed a new micromechanics framework where a composite is represented by a unit cell containing a single representative fibre, the force transfer is represented through a functionally graded interphase encompassing the representative fibre [6].

Presently no micromechanics model framework allows for the seperation of the individual stress components in the fibre and matrix. One of the purposes of this framework is to allow for the seperation of the stress in the fibre and matrix in order to implement stress based failure criteria for both the fibre and matrix along with the interaction between the two. In Sabiston et al. a method using a functionally graded interphase (FGI) between the representative fibre and bulk matrix is used to combine the stresses

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occurring in each region [6].

In Sabiston et al. it is assumed that the strain field is the same in the fibre and matrix for the unit cell model [6]. This is known to be incorrect as the materials exhibit very different stiffness and they therefore must exhibit different strain fields. The authors have further developed their FGI model to incorporate the difference in strains through partitioning the deformation into distinct regimes.

2. Method

A unit cell model is defined containing a single representative fibre aligned with the x_1 axis, where the coordinate system is defined about the center of a hexahedral element. The faces of the unit cell are at one unit in each direction, allowing the unit cell to be mapped to any deformed hexahedral shape. The stress within the unit cell is defined to change radially as a function of position surrounding the fibre. This change in stress as a function of position is applicable to all possible loading conditions. To accomplish the transition in stress using the two distinct sets of materials properties for the fibre and matrix it is necessary to also change the deformation as a function of position surrounding the fibre. This is accomplished by defining concentric regions surrounding the fibre where the stress and deformation are constant within each region.

For the unit cell, the total displacement field is the sum of the product of the displacement gradient (G_i) and the volume fraction (V_i) over which it acts for each of these regions.

$$G = \sum_{i=1}^{n} G_i V_i \tag{1}$$

This enforces compatibility in the unit cell. Within the domains the stress and strain are constant, therefore a large number of concentric domains are required to accurately capture the correct transition in stress defined by the FGI. If enough concentric domains are used the difference in stress between the subsequent domains is negligible thereby enforcing a pseudo equilibrium. The concentric domains are defined using cylindrical coordinates about the centre of the fibre for the undeformed unit cell. As the displacement field applied to each domain is different the size of these regions is not constant.

In order to accurately model this transition there would also have to be a transition in the material model used and material properties as a function of position surrounding the fibre. This greatly complicates the modelling process and it is difficult to define and validate experimentally. Therefore, it is desirable to only use the known fibre and matrix properties to define the transition in stress within the unit cell. Thus, the deformation within the composite unit cell is divided into three regimes; the bulk matrix deformation, the fibre deformation and the deformation in the matrix portion of the interphase. The deformation of the bulk matrix material is assumed to be the same as the overall unit cell. The deformation in the fibre is determined using Eshelby's method [7] to determine the strain in the inclusion from the far field (bulk matrix) strains. The difference in these strain values is accommodated in the matrix portion of the interphase.

The principle of using this unit cell definition is that by separating the deformation into these three unique regions the average stress in each can be volumetrically combined to obtain the overall unit cell stress. Each of the concentric domains discussed within the interphase zone is characterized to either have the material properties of the fibre or matrix. This devision is completed through integration of the interphase function to find a representative interface radius. This representative interface radius quantifies

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the amount of material acting as fibre material and is defined as

$$r_{im} = \sqrt{\frac{V\Delta\sigma}{\pi(\sigma_f - \sigma_m)} + (kr_f)^2} \tag{2}$$

where, $V\Delta\sigma$ is the volume calculated from the integration of the interphase function, σ_f is the stress in the fibre section, σ_m is the stress in the bulk matrix material, k is a material paring constant with a value between 0 and 1 for the fibre matrix paring and r_f is the representative fibre radius calculated from the fibre volume fraction. The relationship of the fibre volume fraction (V_f) and the representative fibre radius is given in [6] as

$$r_f = \sqrt{\frac{4V_f}{\pi}}. \tag{3}$$

The volume fractions of the three regions are as follows; for the representative fibre

$$V_{rf} = \frac{\pi}{4} r_{im}^2 \tag{4}$$

for the bulk matrix

$$V_{bm} = 1 - \frac{\pi}{4}(lr_f)^2 \tag{5}$$

where l is a material pairing parameter similar to k except the values of l are greater than 1. The final volume fraction for the interphase portion acting as matrix material is determined by subtracting the other two volume fractions from 1 or as

$$V_{im} = \frac{\pi}{4}((lr_f)^2 - r_{im}^2). \tag{6}$$

The interphase functions are used to describe the stress as a function of radial position within the interphase zone. The interphase zone has an inner boundary of kr_f and an outer boundary of lr_f. This definition with proportionality to the fibre radius gives independence on the number of fibres within a unit cell and therefore allows a single representative fibre to account for the response of all the fibres in the region of interest as shown in [6]. The $V\Delta\sigma$ is calculated by finding the volume under the interphase function as

$$V_{\Delta\sigma} = \int_0^{2\pi} \int_{kr_f}^{lr_f} \int_{\sigma_{bm}}^{\sigma(r)} r d\sigma dr d\theta. \tag{7}$$

For examples of interphase functions see Sabiston et al. [6] in terms of elastic constants and [8] in terms of the fibre and bulk matrix stress.

The volume fractions of the three regions are used to calculate the overall unit cell stress as

$$\sigma = V_{bm}\sigma_{bm} + V_{im}\sigma_{im} + V_{rf}\sigma_f \tag{8}$$

where σ_{bm}, σ_{im} and σ_f are the stresses in the bulk matrix, matrix portion of the interphase and the fibre respectively. For the present implementation of this model the stress in the bulk matrix and matrix portion of the interphase are calculated using the glass rubber model for polymers developed by Buckley and Jones [9]. The required input for this model to calculate the stress is the deformation gradient, for the bulk matrix which deforms the same amount as the overal unit cell the deformation gradient (F) is known, the deformation gradient for the matrix portion of the interphase is calculated using the relation shown in Eq. 1 as

$$F_{im} = \frac{(F - I)(1 - V_{bm}) - \varepsilon_f V_{im}}{V_{im}} + I \tag{9}$$

where I is the identity matrix ε_f is the strain in the representative fibre. The stress in the representative fibre is calculated from the representative fibre strain given that the fibre is an isotropic elastic material.

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The strain in the representative fibre is calculated using Eshelby's method based off the initial elastic response of the bulk matrix material. The fibre strain is calculated from the overall unit cell strain as

$$\varepsilon_f = T\varepsilon \tag{10}$$

where T is the transformation tensor. The transformation tensor is calculated using the elastic matrices for the fibre and bulk matrix L_f and L_{bm} respectively and Eshelby's tensor for the fibre shape (S_f) as

$$T = [I + S_f L_{bm}^{-1}(L_f - L_{bm})]^{-1}. \tag{11}$$

3. Results

The model is implemented in the commercial explicit dynamic finite element software LS-DYNA. The model is calibrated to match experimental results for a carbon fibre epoxy composite presented by Hsio and Daniel [10]. In this data set compressive testing was conducted on a unidirectional composite in both the axial, transverse and shear directions.

The model is calibrated in the axial direction to determine the interphase parameters in the initial elastic response. The material properties for the glass rubber constitutive law are set using the transverse compression data. The calibration is validated using the shear direction. A plot of the stress strain response in all three directions is shown in Figure 1. The calibration parameters for the model are given in Table 1, more details on implementation of the models are given in [8, 11].

Table 1. Calibration parameters for the model

Parameter		Value
E_f	GPa	279
G_f	GPa	116.25
ν_f		0.2
G_m	GPa	1.3
K_m	GPa	4.6
ρ_m	$kg\ m^{-3}$	1220
c_m	J kg^{-1}	1422
ΔH_{0_m}	kJ mol^{-1}	102
T_{g_m}	K	383
T_{∞_m}	K	331
C_m	K	2068
V_{s_m}	$m^3\ mol^{-1}$	2.23×10^{-3}
V_{p_m}	$m^3\ mol^{-1}$	2.06×10^{-4}
τ_{0_m}	msec	3.49×10^{12}
$T_{f_{\infty\sigma_m}}$	K	413
$\varepsilon^V_{0_m}$		0.337
N_{c_m}	m^{-3}	4.58×10^{27}
α_m		0.288
k		0.70
l		1.150
m		0.25

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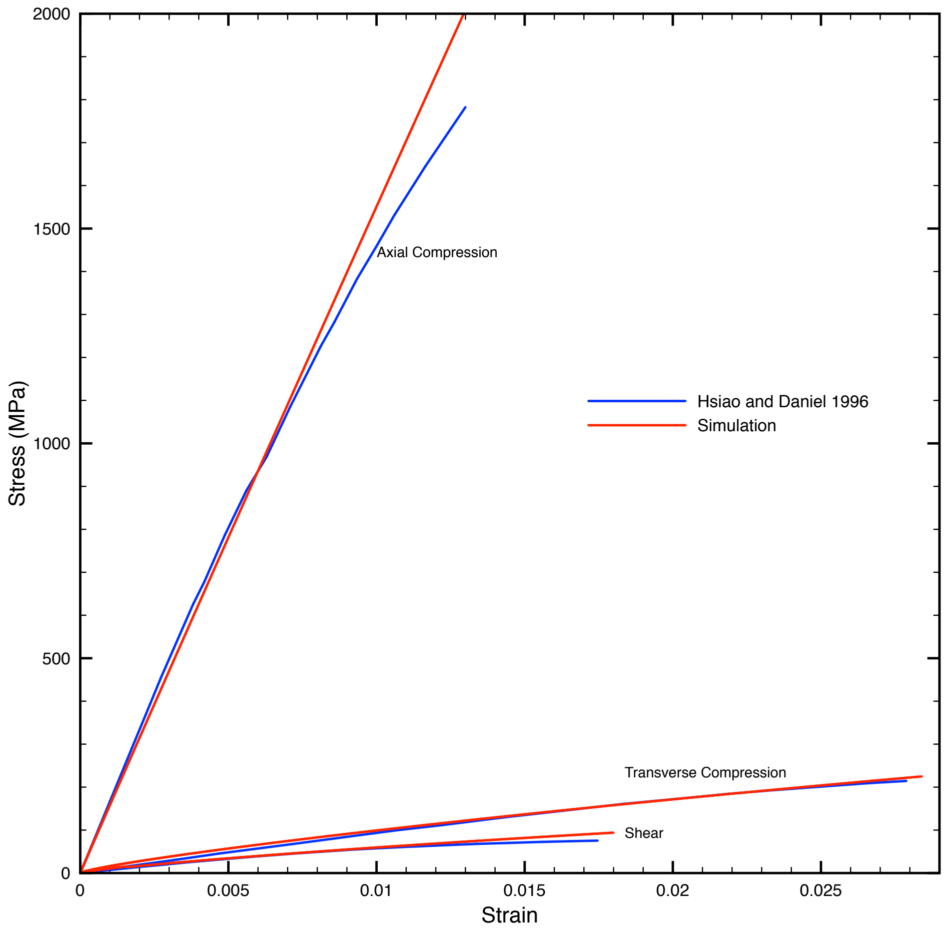

Figure 1. Proposed model calibrated to experimental data from Hsiao and Daniel [10]

The simulation matches the experimental data to a high degree of accuracy for the three loading directions. It is observed that for the axial compression the model predicts a linear response, which differs from the experimental results due to the onset of failure mechanisms not accounted for in the current model, in particular fibre microbuckling is an active failure mode, which is not considered [12]. In the other two directions failure driven from matrix microcracking and fibre matrix debonding lead to the differences between the experiment and simulation at higher strains.

4. Conclusion

A new micromechanics frame work is developed where the deformations within a unit cell are subdivided into three regions to represent the deformation. The stresses in each of these regions is calculated using their respective constitutive law. By using this technique compatibility between strains is maintained and the functionally graded interphase also allows for a quasi preservation of equilibrium.

The model is calibrated to match experimnetal data and shows excellent results for three different load paths. At higher strains the model diverges from the experimental results because failure is not currently implemented. The current implementation offers improvements in being able to capture any loading through the use of a single set of model calibration parameters.

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