

# NUMERICAL STUDY OF THE RESPONSE OF DYNAMIC PARAMETERS TO DEFECTS IN COMPOSITE STRUCTURES

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## Abstract

The purpose of this work is to study three different models of delamination in composite plate (the free mode model, the constrained model and the contact model) and applicability of this models to the vibrational method of damage detection based on frequency shifts. The results of numerical simulation have shown that the free mode model leads to abrupt changes in natural frequencies due to non-physical condition of mutual penetration of adjacent volumes in the defect zone. The constrained and the contact models yield qualitative agreement in shifts of natural frequencies with a change of the defect size. All models have shown the necessity of analyzing shifts of high frequencies to detect small size delamination.

## 1. Introduction

The estimation of the mechanical state of a structure during its operation based on detection of internal damages has gained a great importance for such rapidly developing fields of production as mechanical engineering, aerospace industry, etc., whose recent trends are toward an increased use of new types of materials.

The most widely spread type of defects in composite constructions is delamination, which leads to a separation of the object in the defect zone into several parts, which begin to respond to loads independently of each other [1] while the stiffness of each part is significantly lower than the stiffness of the whole object.

The appearance of delamination may be the cause of the deprivation of bearing capacity of the structure, its breakage and decommissioning which indicates the relevance of the problem of damage detection at an early stage of development. The application of visual methods of damage detection is difficult due to internal location of delamination. It is well known that the appearance of defects in a particular place of the structure influences the local stiffness in this zone, which leads to a shift of natural frequencies. This specific feature of structure response is behind the global vibrational methods of damage detection. These methods use different dynamic parameters (eigenfrequencies, mode shapes, curvatures of mode shapes, etc.) to evaluate the state of the investigated structure [2-5].

The methods of numerical simulation allow us to investigate the processes occurring in structures during application of different damage detection methods and receive the answers to many questions without performing expensive experiments. Moreover, it is due to numerical methods that we can track the evolution of different parameters with a change of the defect size.

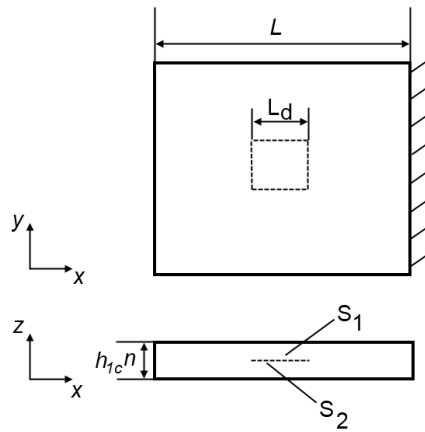
Studies of objects with delamination type defects have received considerable attention in scientific literature. Analytical description of the free mode model of delamination in a beam, which allows the mutual penetration of volumes in the zone of delamination, is given in [6]. This disadvantage is eliminated in the constrained model where areas in the delamination zone have equal vertical displacements [7]. Only few low frequencies are considered in these studies. The presence of multiple delaminations is discussed in [8].

Experimental investigations of vibrations of delaminated structures are performed in [9, 10]. Excitation of vibrations and measurement of the signal must be implemented using actuators and special measurement devices (sensors). The capabilities of using piezoelements for registration of high frequency vibrations are described in [11]. The Authors of this research capture vibrations of the plate up to 40 kHz with the help of a piezoelectric sensor which proves the possibility of using such devices for measuring high frequency vibrations. The method of electromechanical impedance (EMI), which is one of the vibrational methods of damage detection applied to delaminated composite beam, is given in [12]. The need to place the actuator close to the defect and hence the requirement of arrangement of a dense grid of piezoelectric devices or prior knowledge of the location of defect is the main disadvantage of such a kind of damage detection method. On the other hand natural frequencies give integral characteristic of the object of research. In [13-15] vibrations of delaminated structures with different geometries such as beam, cylindrical and conical shells are studied.

A numerical study of the dynamic parameter response to defects of different sizes is carried out in the framework of three models of delamination in a composite structure, the advantages and drawbacks of each model are estimated in the process of simulation. First, we consider a free mode model of delamination, in which the adjacent volumes in the zone of the defect are not coupled with each other and therefore they are assumed to be mutually penetrable. In the second, the so-called constrained model, the coincident nodes in the zone of delamination are coupled by one component of the displacement, while other components of the displacement remain independent. The last one is the model that takes into account the contact forces. The free mode and constrained models allow us to perform a modal analysis for computing the eigenfrequencies of the structure. With contact forces taken into account the problem becomes nonlinear. The algorithm for calculating the spectrum of eigenfrequencies in the nonlinear problem includes the following steps: setting of impact load, transient analysis, measurements of signal at a certain point of the structure with a subsequent conversion of the received response of the structure from the amplitude-time to amplitude-frequency domain with the help of the Fourier Transform

## 2. Numerical modeling

A numerical study was performed on a square plate of size  $L = 0.15 \times 0.15$  m, made of a layered composite material. The thickness of one layer is  $h_{lc} = 0.0003$  m. There are a total of  $n = 15$  layers across the thickness of the plate therefore the total thickness is  $h_{lc} * n = 0.0045$  m (Fig. 1). The square shape delamination of size  $L_d$  m is located between 6 and 7th layers. The center of delamination coincides with the center of the plate. The composite material is modeled as a homogeneous body with the following orthotropic effective mechanical properties (reinforcing direction is not taken into account).  $E_x = 24$  GPa,  $E_y = 18$  GPa,  $E_z = 6$  GPa,  $G_{xy} = 4$  GPa,  $G_{yz} = 3$  GPa,  $G_{xz} = 3$  GPa,  $\nu_{xy} = 0.15$ ,  $\nu_{yz} = 0.18$ ,  $\nu_{xz} = 0.42$ ,  $\rho = 1800$  kg/m<sup>3</sup>. The presence of composite layers is taken into account only by geometrical dimensions. Finite elements with quadratic approximation providing more accurate results in terms of out of plane strains were used in the simulation.



**Figure 1.** Geometrical representation of the plate

The principle of virtual work is used for the mathematical formulation of the problem

$$\int_V (F_i - \rho \frac{\partial^2 u_i}{\partial t^2}) \delta u_i dV + \int_{S_\sigma} p_i \delta u_i dS = \int_V \sigma_{ij} \delta \varepsilon_{ij} dV \quad (1)$$

Where  $u_i$  - components of the displacement vector,  $\varepsilon_{ij} = \frac{1}{2} (\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$  - components of strain tensor,  $\sigma_{ij} = C_{ijkl} \varepsilon_{kl}$  - components of stress tensor,  $C_{ijkl}$  - stiffness tensor,  $\rho$  - material density,  $F_i$  - components of body forces vector,  $p_i$  - components of surface tractions vector.

The boundary condition corresponds to the cantilever fixing of the plate on the border  $x = L$

The free mode model allows the mutual penetration of volumes (Fig. 2), hence surfaces S1 and S2 are free from stresses and boundary conditions for this models have the following form:

$$\sigma_{zz}|_{S_1, S_2} = \sigma_{zx}|_{S_1, S_2} = \sigma_{zy}|_{S_1, S_2} = 0$$

For the constrained model, the coincident nodes associated with the surfaces S<sub>1</sub> and S<sub>2</sub> have equal displacements in the z axis direction while other components of the displacement remain independent therefore the boundary conditions can be written as follows

$$\begin{aligned} \sigma_{zx}|_{S_1, S_2} = \sigma_{zy}|_{S_1, S_2} = 0 \\ U_z|_{S_1} = U_z|_{S_2} \end{aligned}$$

A numerical solution of the dynamic problem of vibration of a plate was found by the finite element method using the commercial package ANSYS. The finite-element formulation in matrix form can be written as follows

$$[M]\{\ddot{u}\} + [K]\{u\} = \{F(t)\} \quad (2)$$

Where  $[M]$  - mass matrix of the system,  $[K]$  - stiffness matrix of the system,  $\{F(t)\}$  - time dependent load function,  $\{u\}$  - the vector of nodal displacements,  $\{\ddot{u}\}$  - the vector of nodal accelerations.

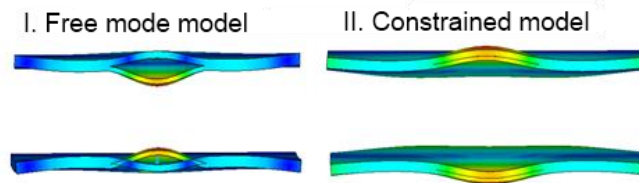
In the absence of external influences, the problem is reduced to the typical problem of finding eigenvalues and eigenvectors

$$u_i(\bar{x}, t) = e^{-i\omega t} \xi_i(\bar{x}) \quad (3)$$

where  $\omega$  - natural frequency,  $\xi_i$  - eigenvector or mode shape. The finite element formulation of the problem has the following form

$$([K] - \omega^2 [M])\{u_0\} = \{0\} \quad (4)$$

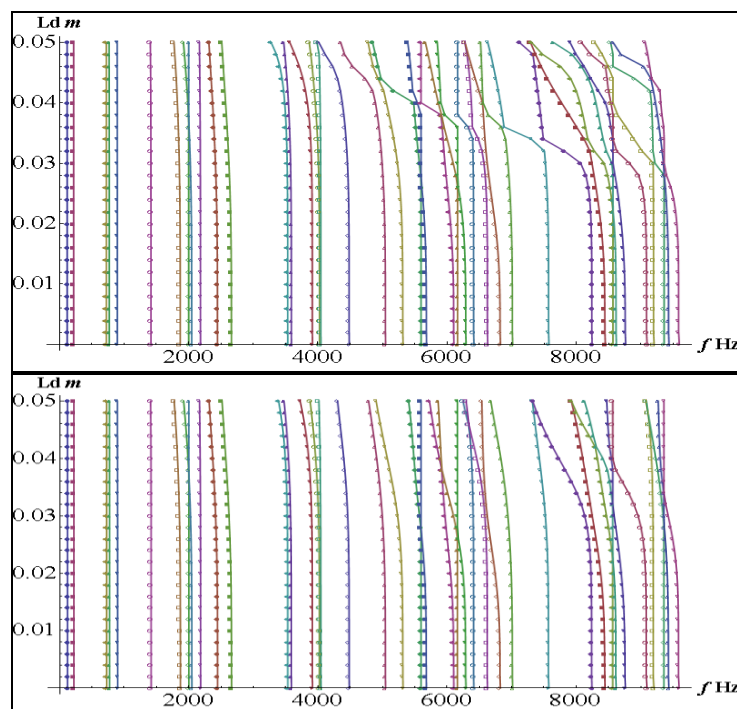
Where  $\{u_0\}$  - the vector of nodal values of natural vibration modes.



**Figure 2.** Models of delamination

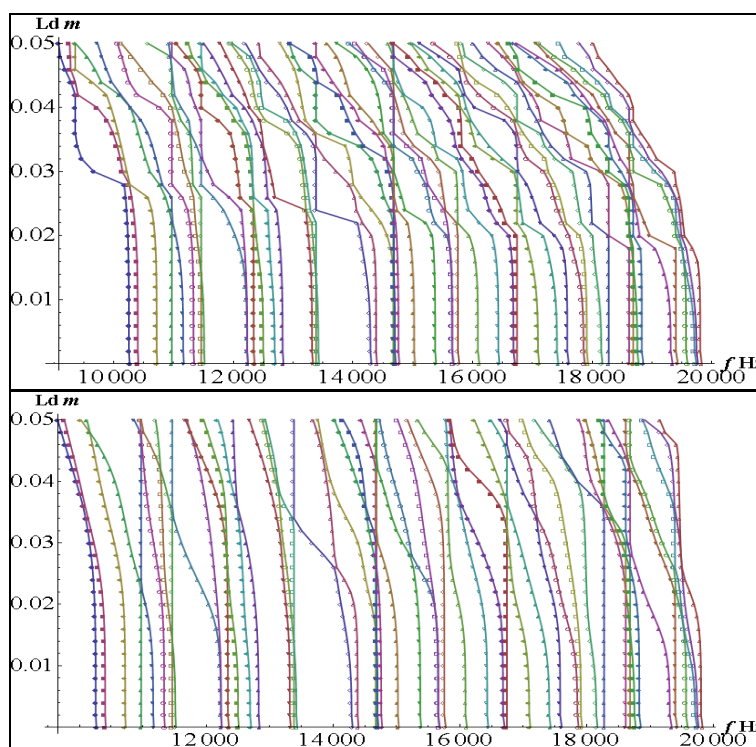
### 3. Results

The frequency range from 0-20 kHz was analyzed. The size of delamination was changed from 0 to 0,05 m with a step of 0.002 m. The change of eigenfrequencies, depending on the size of delamination in the range of 0-20 kHz, is shown in Fig. 3 and 4 (the upper plots show the results for the free mode model of delamination, the lower – for constrained model). The vertical axis represents the defect size, the horizontal axis corresponds to the frequency of oscillations in Hertz. The reaction of the natural frequencies to increase in the size of the defect for the free mode model is not observed up to 4 kHz. At higher frequencies a sudden drop in the natural frequencies for the defect of the large size is observed.



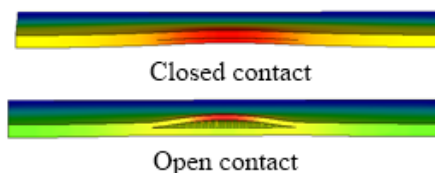
**Figure 3.** Shifts of natural frequencies in the range of 0-10 kHz

In the range of frequencies from 10-20 kHz a similar abrupt pattern of natural frequency shifts is observed (Fig. 4). Restriction applied to the component of displacement in the delamination zone and thereby precluding mutual penetration of the adjacent volumes in this area significantly affects the results. This model is characterized by a more gradual and smaller change in magnitude of natural frequencies with increasing the size of the defect. For both models a clear tendency towards greater sensitivity of high frequency vibrations to the defects of a smaller size is observed, which is explained by the fact that the reaction of frequencies to the appearance of a defect depends not only on its size but also on its location. If a defect is located in the area of small or zero strains of mode shape, the frequency will remain unchanged. The mode shapes of high frequencies have a large number of zones with non-zero strains (bends), providing greater sensitivity of these frequencies to the defect. That's what makes these frequencies more appropriate for detection of small sized defects.



**Figure 4.** Shifts of natural frequencies in the range of 10-20 kHz

The third variant of the model of delamination is the most accurate and complete, as it uses contact elements to model the interaction of adjacent surfaces in the delamination zone. Thus, several options for the contact status are possible (Fig. 5): 1) open contact where the nodes, between which the contact is defined, are divided; 2) closed contact, in this case, the nodes are in contact with each other and their relative behavior is determined by the contact stiffness, by default, equal to the modulus of elasticity of the material multiplied by the size of the element adjacent to the contact surface. In this problem the node to node type of contact (element conta178) is used because mode superposition transient analysis, which supports only this type of nonlinearity, was performed. A significant advantage of this type of calculation is its performance, compared to the full method of transient analysis.



**Figure 5.** State of the contact

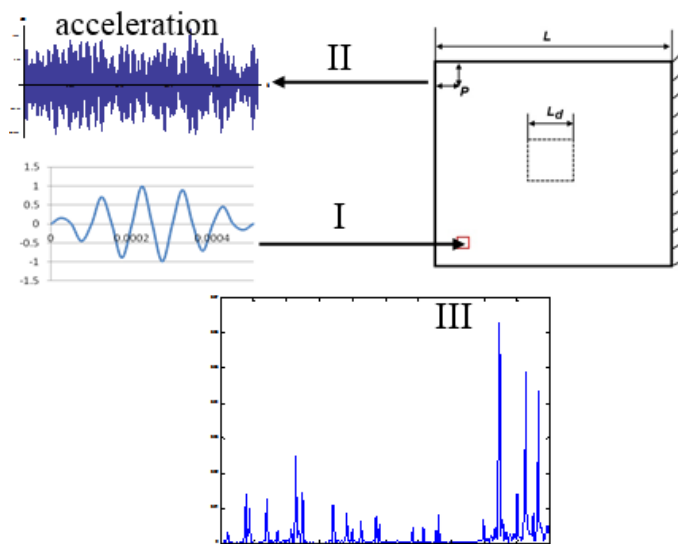
In contrast to the free mode model and constrained model where natural frequencies could be calculated by the modal analysis, the spectrum of frequencies in transient analyses is calculated using the Fourier Transform of a signal recorded at a certain point of the plate. Since accelerometers are usually used as sensors to measure the acceleration in the experiment, in the numerical model the accelerations are also calculated at the point. Increasing the size of the centrally located delamination, the graphs similar to ones in Fig. 3 and 4 are received which reflect the change of the natural frequencies of the spectrum when you change the size of the defect. This type of analysis requires a more profound approach, as a reflection of the resonant frequencies on the spectrum significantly depends on the type of the input signal, the points of impact and measurement. If these points are located on the nodal line of mode shape, this frequency will not be excited and will not affect the resulting spectrum.

The algorithm of calculation of the nonlinear dynamic problem with contact interaction taken into account consists of the following steps (Fig. 6). Impact load is applied to a certain area on the surface of the plate (step I)

$$F(t) = \text{Sin}(2\pi f_m t) \text{Sin}(2\pi f_o t)$$

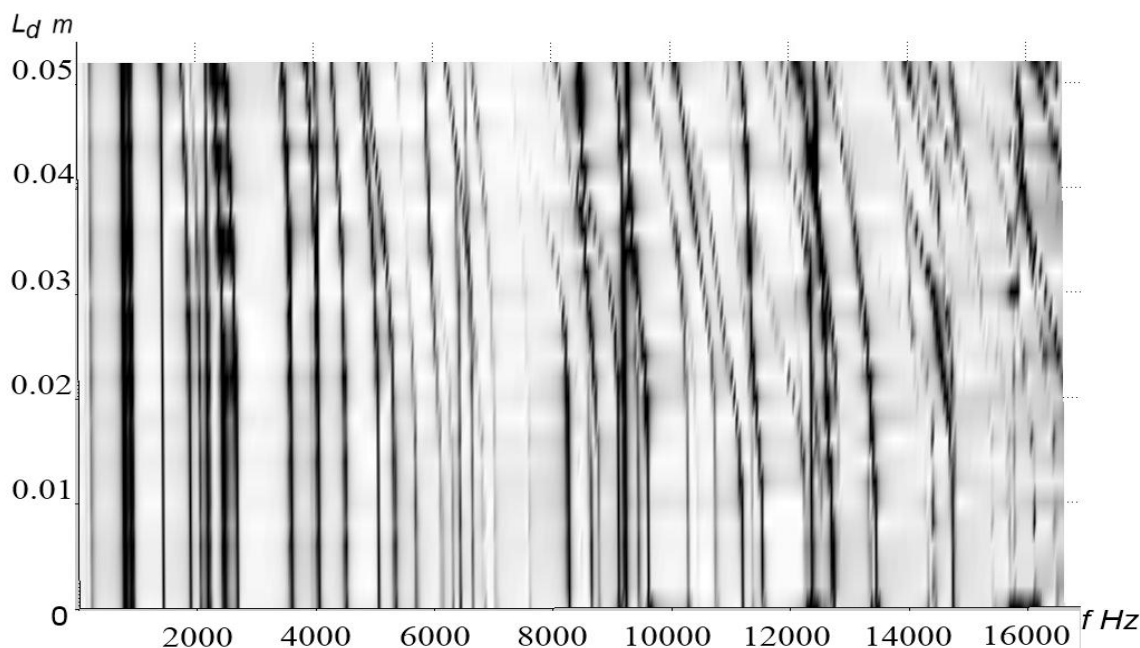
Where  $f_m = \frac{f_o}{2n}$  – modulation frequency,  $f_o$  – central frequency. On the time interval  $T = 0.4$  s.

mode superposition transient analysis is performed. The result of this analysis is the component of acceleration  $a_z$  registered at the node at the specified time period (step II). The received signal includes a set of steady-state oscillations of different frequencies, which is obtained by decomposition of the signal using the Fourier Transform (step III).



**Figure 6.** Algorithm of transient analysis

Resonance peaks on the resulting graph correspond to the eigenfrequencies. Internal dissipation was not considered in the current study, therefore, the analysis of the amplitudes of the resonance peaks was not performed. By implementing this algorithm, and increasing the size of the defect at each step, it is possible to monitor changes in natural frequencies.



**Figure 7.** Shifts of natural frequencies in the contact model of delamination

The general nature of the frequency response to an increase of the damage size is similar to the models described above. We can see the lack of influence of damage on low frequencies and high frequency sensitivity to the defect of a small size. The sharpness of the frequency spectrum picture will significantly depend on relative location of impact and the measurement and type of impact load.

#### 4. Conclusions

The comparison of the three considered models of delamination with respect to their applicability to the solution of the problem of analysis of changes of natural frequencies of a composite plate shows that in spite of the indisputable simplicity of the free mode model, the results show an excessive drop in eigenfrequencies. The constrained and the contact models yield qualitative agreement in shifts of natural frequencies with a change of the defect size. The calculation of the constrained model is much easier and faster. However, the algorithm of calculation of the model with the contact repeats the steps performed during the experiment and more fully reflects special features typical for real structures. The results suggest that for detection of defects at an early stage of their development it is necessary to record the change in the spectrum in the high frequency range (in this case greater than 4 kHz). A numerical model of the studied structure allows to analyse frequency response to the occurrence of the defect and to determine the most sensitive of them to a specific type of defect.

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