A LINEAR THERMO-VISCOELASTIC ORTHOTROPIC CONSTITUTIVE LAW — APPLICATION TO COMPOSITES

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Abstract
A constitutive material law for linear thermo–viscoelasticity in the time domain with orthotropic material symmetry under plane stress assumption is developed and implemented into a commercial FEM package. Full orthotropy is realized for the time dependent relaxation formulation, i.e., both for the elastic as well as for the viscous properties. Thereby, each element of the relaxation tensor is described by its own and independent Prony series expansion. Time dependent thermal expansion relaxation/creep which occurs in composite materials is treated in analogy.
Various tests on isotropic and orthotropic problems are carried out for verification. Homogenized material data as input to the developed material laws are computed from a periodic unit cell approach.

1. Introduction

Viscoelastic effects are widespread in natural as well as in engineering materials. Among them are almost all biological tissues and most polymers, in particular thermoplastic materials. Biological materials are composites "by nature", whereas engineering polymers are often mixed with other constituents to improve their performance. In addition, the field of glass or carbon fiber reinforced plastics has gained great importance in lightweight design and for industrial applications. The latter group of composites is likely to exhibit relaxation/creep type behavior for the thermal expansion, too.
Such composites, natural and man made ones, often have elongated reinforcements with preferred orientation. Consequently, their properties are direction depended and the consideration of anisotropy becomes inevitable. For the mathematical description of a material's behavior a constitutive law is required. Strictly speaking, it must be able to predict the response to any type of loading and any type of loading history. Such constitutive laws at hand do not only contribute to the understanding of the material, they are also necessary to give the material description in course of structural analyses. A widespread and very general approach to the latter is the Finite Element Method (FEM) which is employed in the present work.
A general introduction into viscoelasticity can be found, e.g. in [1–3], which focus predominantly on isotropic behavior and treat the time as well as the frequency domain. Transversely isotropic (and orthotropic) viscoelastic models have been presented, e.g. in [4, 5], based on invariants of the strain representation, which are commonly used for biological soft tissues. Typically, they combine nonlinear orthotropic elasticity with linear isotropic viscosity. Linear viscoelasticity of orthotropic media is pre-

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sented in [6, 7] in the context of geo-materials. Linear and nonlinear orthotropic viscoelasticity adopting not the full set of orthotropic viscous effects is presented in [8, 9] and applied to composites and foam materials, respectively. A linear viscoelastic constitutive law in the time domain for cubic material symmetry is presented in [10] and applied to highly porous structures. Orthotropic material symmetry in the frequency domain is treated in [11]. Relaxation type effects occurring for the thermal expansion behavior are mentioned in [1]. A hygro–thermal expansion relaxation function is presented in [12].

In the present work linear viscoelasticity of orthotropic materials is treated in the time domain. Thereby the elastic as well as the viscous response are considered to behave orthotropically. Temperature dependence is accounted for by a time–temperature shift function. Time dependent, relaxation/creep type thermal expansion is treated. The algorithms are implemented into the commercial FEM program ABAQUS/Standard v6.14 *(Dassault Systemes Simulia Corp., Providence, RI, USA)* as user supplied material law (`UMAT`) and user supplied thermal expansion behavior (`UEXPAN`), respectively. The treatment is done under the assumption of plane stress states, but the approach is general and can be extended to tri-axial problems and general anisotropy straightforwardly.

2. Linear Viscoelastic Orthotropic Plane Stress Constitutive Model

The hereditary integral in tensorial form reads,

$$\sigma_i(t) = \int_0^t R_{ij}(t-s)\dot{\varepsilon}_j(s)ds \qquad \text{with} \qquad \sigma_i = (\sigma_{11}\ \sigma_{22}\ \sigma_{12})^T \quad , \quad \varepsilon_i = (\varepsilon_{11}\ \varepsilon_{22}\ \gamma_{12})^T \quad , \tag{1}$$

adopting plane stress and Voigt notation. The time dependent material tensor for orthotropic materials,

$$R_{ij}(t) = \begin{pmatrix} R_{11}(t) & R_{12}(t) & 0 \\ R_{21}(t) & R_{22}(t) & 0 \\ 0 & 0 & R_{33}(t) \end{pmatrix} \quad , \quad R_{ij}(t) = R_{ji}(t) \quad , \tag{2}$$

is composed by individual relaxation functions,

$$R_{ij}(t) = R_{ij\,0}[1 - \sum_k r_{ij\,k}(1 - \exp(-t/\tau^{r_{ij\,k}}))] \quad \text{(no sum on } ij) \quad , \tag{3}$$

which are given by Prony series expansions with $R_{ij\,0}$ being the instantaneous values, i.e. the elasticity tensor elements giving the short term behavior. The sum over k Prony terms contain the relative relaxation elements, $r_{ij\,k}$, and the corresponding characteristic times, $\tau^{r_{ij\,k}}$.
Note, that in the present formulation the relaxation tensor, eqn. (2), possesses off-diagonal terms which can exhibit their own independent relaxation behavior. This requires extended treatment in contrast to the isotropic (or uni-axial) case which can be formulated by uncoupled scalar equations. Moreover, these off-diagonal terms implicitly contain some Poisson relaxation which is not necessarily monotonic, see e.g. [13].

2.1. Decoupled Shear Behavior

Inspection of eqn. (2) shows that for orthotropic material symmetry treated in the principal material coordinate system, the shear behavior is decoupled from the other tensor components. Thus, the corresponding scalar equation can be handled by the standard means.
For the case of the decoupled shear behavior, eqn. (1) can be interpreted as scalar equation with $i = j = 3$. The same applies to the relaxation function as a simplification of eqn. (3) for the index 33. Then the R_{33} and r_{33} variables pertain to shear modulus quantities. Applying these simplifications, eqns. (1) and (3) can be combined to give the shear stress shear angle relation,

$$\sigma_3(t) = R_{33\,0}[\varepsilon_3 - \sum_k \frac{r_{33\,k}}{\tau^{r_{33\,k}}} \int_0^t \exp(-s/\tau^{r_{33\,k}})\varepsilon_3(t-s)ds] \quad , \tag{4}$$

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where the expressions in the summation are the contributions by the Prony terms k, and the "relative creep shear angle" can be introduced as,

$$e^3_{3\,k} = \frac{1}{\tau^{r_{33\,k}}} \int_0^t \exp(-s/\tau^{r_{33\,k}}) \varepsilon_3(t-s) ds \quad , \tag{5}$$

where the superscript indicates the cause (i.e. the applied strain) and the subscript denotes the stress component which is affected. By introducing the "total creep shear angle",

$$\epsilon^3_3 = \sum_k r_{33\,k}\, e^3_{3\,k} \quad , \tag{6}$$

as the sum over all Prony terms, the shear stress shear angle relation reads, finally,

$$\sigma_3 = R_{33\,0}[\varepsilon_3 - \epsilon^3_3] \quad . \tag{7}$$

The relative creep shear angle contributions of each Prony term, $e^3_{3\,k}$, are interpreted as internal state variables concerning the shear response. Additional state variables pertaining to normal strains will be introduced later.

2.2. Coupled Normal Behavior

The normal stresses can be expressed by combining eqn. (1) and (3), here exemplified for one normal stress component, as,

$$\begin{array}{ll} \sigma_1(t) & = R_{11\,0}[\varepsilon_1 - \sum_k \frac{r_{11\,k}}{\tau^{r_{11\,k}}} \int_0^t \exp(-s/\tau^{r_{11\,k}}) \varepsilon_1(t-s) ds] \\ & + R_{12\,0}[\varepsilon_2 - \sum_k \frac{r_{12\,k}}{\tau^{r_{12\,k}}} \int_0^t \exp(-s/\tau^{r_{12\,k}}) \varepsilon_2(t-s) ds] \end{array} \quad , \tag{8}$$

which reveals the coupling. The first term is the contribution by the normal strain ε_1 and the decay expressed by the diagonal element $R_{11}(t)$ from eqn. (3). So far it is formally equivalent to the decoupled shear behavior discussed above. However, the second term states the coupling effect caused by the normal strain ε_2 and the corresponding decay is expressed by the off-diagonal element $R_{12}(t)$. The latter, in the present formulation, can have its own and independent relaxation behavior. The relative creep contributions in the Prony terms can be given as,

$$e^1_{1\,k} = \frac{1}{\tau^{r_{11\,k}}} \int_0^t \exp(-s/\tau^{r_{11\,k}}) \varepsilon_1(t-s) ds \qquad \text{and} \qquad e^2_{1\,k} = \frac{1}{\tau^{r_{12\,k}}} \int_0^t \exp(-s/\tau^{r_{12\,k}}) \varepsilon_2(t-s) ds \quad , \tag{9}$$

and summarized to the "total creep contributions" as,

$$\epsilon^1_1 = \sum_k r_{11\,k}\, e^1_{1\,k} \qquad \text{and} \qquad \epsilon^2_1 = \sum_k r_{12\,k}\, e^2_{1\,k} \quad . \tag{10}$$

Finally, the stress component can be given as,

$$\sigma_1 = R_{11\,0}[\varepsilon_1 - \epsilon^1_1] + R_{12\,0}[\varepsilon_2 - \epsilon^2_1] \quad . \tag{11}$$

The stress component σ_2 can be derived likewise, involving ϵ^1_2 and ϵ^2_2 type expressions.
Note, that the expressions ϵ^j_i and $e^j_{i\,k}$ cannot be interpreted as creep strains, since they pertain to some 3×3 matrix (in Voigt notation) which, moreover, does not need to be symmetric. Consequently, they are rather property type quantities carrying information on how much of the relaxation potential has been consumed until time t. Nevertheless, the $e^j_{i\,k}$ for every Prony term k represent the internal state variables. The temperature dependence of viscoelastic material data is treated by introducing a reduced time, $t_{\text{red}} = t/A(T)$ with the shift function, $A(T)$, which depends on the temperature, T.

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2.3. Thermal Expansion Relaxation/Creep Model

Materials may show thermal expansion which exhibit time dependence. For composites such an effect is obvious since the effective thermal expansion is, among others, a function of the elastic properties of the constituents. Once the latter are time dependent, the thermal expansion of the composite is it, too.
In linear thermo–(visco)elasticity assuming small strains, additive decomposition can be applied, which gives the total strain,

$$\varepsilon^{\text{tot}} = \varepsilon^{\text{ve}} + \varepsilon^{\text{th}} \quad , \tag{12}$$

as the sum of the viscoelastic $^{\text{ve}}$ and the thermal $^{\text{th}}$ contribution.
Following [1], the time dependent thermal expansion relaxation is given as,

$$\varepsilon_i^{th}(t) = \int_0^t \alpha_i(t-s)\dot{\vartheta}(s)ds \quad , \tag{13}$$

with $\alpha_i(t)$ being the time dependent coefficients of thermal expansion, ϑ is the temperature change from some (stress free) starting temperature, and temperature independent behavior is assumed. For the time independent case, the usual thermal expansion results from integration of eqn. (13) as, $\varepsilon^{th} = \alpha\vartheta$. Note that eqn. (13) exactly resembles the hereditary integral in eqn. (1) for the time dependent stresses under strain loading. Thus, the further treatment goes in perfect analogy to linear viscoelasticity as treated in Section 2.1.
The time dependent thermal expansion tensor (in Voigt notation) for orthotropic materials under plane stress reads,

$$\alpha_i(t) = [\ \alpha_{11}(t)\ \alpha_{22}(t)\ \ 0\ ]^T \quad . \tag{14}$$

The components are given by the individual relaxation functions,

$$\alpha_i(t) = \alpha_{i\,0}[1 - \sum_k a_{i\,k}(1 - \exp(-t/\tau^{a_{i\,k}}))] \quad \text{(no sum on } i\text{)} \quad , \tag{15}$$

with the instantaneous thermal expansion coefficient $\alpha_{i\,0}$ and the relative relaxation values $a_{i\,k}$ as well as characteristic times $\tau^{a_{i\,k}}$ for the Prony series expansion with k terms. For a typical composite, the relative relaxation values, $a_{i\,k}$, are negative. Since the thermal expansion involves second rank material tensors and the scalar valued temperature there is no coupling among thermal expansion coefficients. For the present case of plane stress orthotropy two independent relaxations are present. Applying eqns. (4) to (7) analogously, one obtains the thermal expansion as,

$$\varepsilon_i^{th} = \alpha_{i\,0}[\vartheta - \sum_k a_{i\,k}\ \theta_{i\,k}] \quad . \tag{16}$$

The "relative creep temperatures", $\theta_{i\,k}$, have no direct physical meaning, however, they carry the information on the amount of relaxation and, consequently, are used as internal state variables. Temperature dependent thermal expansion relaxation is not treated here.
The viscoelasticity and the thermal expansion relaxation have to be solve simultaneously, fulfilling eqn. (12). Within the framework of an implicit, incremental FEM approach the thermal expansion increment is subtracted from the total strain increment, eqn. (12). The remaining strain enters the viscoelastic constitutive material law from which the incremental stress update is computed.

3. Application

The implementation of the viscoelastic model is carried out with the `UMAT` option, the thermal expansion with the `UEXPAN` option, for ABAQUS/Standard v6.14 *(Dassault Systemes Simulia Corp., Providence, RI, USA)*. The implementation follows the ABAQUS Manual. For the handling of the coupling terms,

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Table 1. Isotropic linear thermo–viscoelastic material data as input to the ABAQUS material law; instantaneous elastic moduli, bulk and shear relaxation by one Prony term each, and coefficient of thermal expansion.

$E_0 = 2500\text{MPa} \quad \nu_0 = 0.25 \quad (G_0 = 1000\text{MPa})$
$k = 0 \quad (\tau^k = 1.0\text{s}) \quad g = 0.5 \quad \tau^g = 1.0\text{s}$
$\alpha_0 = 1 \times 10^{-4}/°\text{C}$

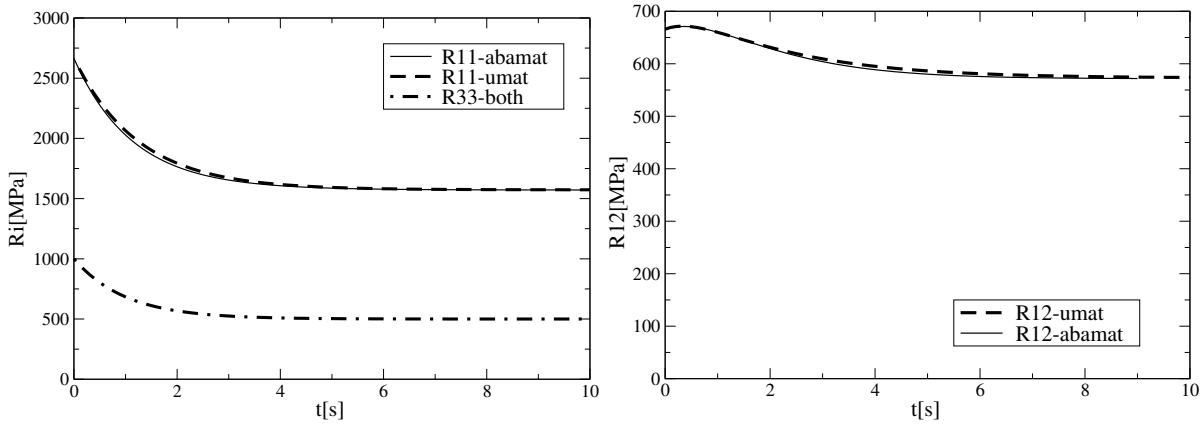

Figure 1. Single element predictions of the relaxation functions $R_{ij}(t)$ as response to Heaviside step strains by the ABAQUS on board material law ("abamat") and the developed constitutive material law ("umat") calibrated to the former one.

appropriate extension and modifications are introduced. Stress update and the consistent material Jacobian, as well as an automatic time stepping scheme are handled.
All simulations employ four noded plane stress elements, using reduced integration for single element tests and full integration for the structural simulations. The element thickness is 1 mm.

3.1. Isotropic Material — Single Element Tests

The ABAQUS on-board material is used to obtain reference solutions, the (generic) material data is given in Table 1, which shows shear relaxation only, plotted in Fig. 1 (left) as "R33". Single element simulations are performed to compute the material parameters pertaining to the developed material model, and which are the input to the `UMAT`. For this purpose the response to uni-axial strain loading is sought for. A Heaviside step function is applied at $t = 0$ and kept constant afterwards as $\varepsilon_{11}(t > 0) = 1$. The other in-plane strain components are $\varepsilon_{22} = 0$ and $\gamma_{12} = 0$ at any time. The relaxation response, i.e. the time dependent elements in the relaxation matrix, are shown in Fig. 1 by thin solid lines. Note that the response which contains the Poisson effect, R_{12}, is non-monotonous. The relaxation behavior is now fitted by applying Prony series representations. These resulting material parameters for the `UMAT` are listed in Table 2, the corresponding relaxation response is shown in Fig. 1.

3.2. Isotropic Material — Structural Simulations

A quadratic patch of unit length is discretized with 20×20 plane stress elements. The patch contains two parallel rectangular "voids" with 0.2×0.6 mm, see Fig. 2. The boundary conditions are "unit cell" like (to be used later for homogenization). The structure can be interpreted as porous material with orthotropic properties. Its isotropic matrix is modeled by the ABAQUS on-board material, Table 1. The orthotropic

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Table 2. Isotropic linear viscoelastic material data as input to the UMAT; instantaneous elasticity matrix elements, relaxation matrix elements by two Prony terms each.

$R_{11\,0} = 2666$MPa	$R_{22\,0} = 2666$MPa	$R_{12\,0} = R_{21\,0} = 666$MPa	$R_{33\,0} = 1000$MPa
$r_{11} = 0.41$	$r_{22} = 0.41$	$r_{12} = r_{21} = 0.25$	$r_{33} = 0.25$
$\tau^{r_{11}} = 1.25$s	$\tau^{r_{22}} = 1.25$s	$\tau^{r_{12}} = \tau^{r_{21}} = 2.0$s	$\tau^{r_{33}} = 1.0$s
$r_{11} = 0$	$r_{22} = 0$	$r_{12} = r_{21} = -0.11$	$r_{33} = 0.25$
$\tau^{r_{11}} = 1.25$s	$\tau^{r_{22}} = 1.25$s	$\tau^{r_{12}} = \tau^{r_{21}} = 0.6$s	$\tau^{r_{33}} = 1.0$s

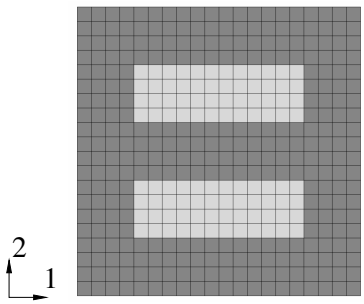

Figure 2. Structure with voids, also representing a unit cell of an orthotropic material.

relaxation response of the structure is presented in Fig. 3 (denoted by "unit cell"). The off-diagonal terms are equal, i.e. $R_{12}(t) = R_{21}(t)$. The fitted material parameters are listed in Table 3. These material parameters together with the developed UMAT are now employed to run single element tests. Comparison to the original input is shown in Fig. 3. Excellent agreement can be seen for all elements, even for the off-diagonal element $R_{12}(t)$.

3.3. Thermo-Viscoelastic Orthotropic Composite

In this section an example is presented to study the thermal expansion relaxation. To this end the geometrical model of the structure from the previous examples is used, but the regions of the voids are now filled with quasi-rigid material. The isotropic coefficient of thermal expansion is set to $1 \times 10^{-6}/°\mathrm{C}$. This way, some model composite is obtained. For the matrix material the data from Table 1 is used. As in the previous examples, the unit cell is employed to perform material characterization, i.e. to compute the effective linear viscoelastic properties of the composite. The calibrated homogenized material data is listed in Table 4, graphical presentation is omitted.
In addition, the effective thermal expansion behavior of the composite can be predicted by the unit cell simulations. A Heaviside temperature step, $\vartheta(t > 0) = 1$, from an undeformed, stress free configuration is applied and the thermal expansion response is evaluated. Since the constituents' stresses and strains show relaxation and creep, the effective thermal expansion shows "relaxation/creep", too. The corresponding unit cell predictions are presented in Fig. 4 (thin lines). To obtain input data to the UEXPAN

Table 3. Orthotropic linear viscoelastic material data as input to the UMAT; instantaneous elasticity matrix elements and relaxation matrix elements by one Prony term each.

$R_{11\,0} = 1693$MPa	$R_{22\,0} = 1124$MPa	$R_{12\,0} = R_{21\,0} = 227$MPa	$R_{33\,0} = 242$MPa
$r_{11} = 0.440$	$r_{22} = 0.439$	$r_{12} = r_{21} = 0.234$	$r_{33} = 0.453$
$\tau^{r_{11}} = 1.09$s	$\tau^{r_{22}} = 1.09$s	$\tau^{r_{12}} = \tau^{r_{21}} = 1.98$s	$\tau^{r_{33}} = 1.08$s

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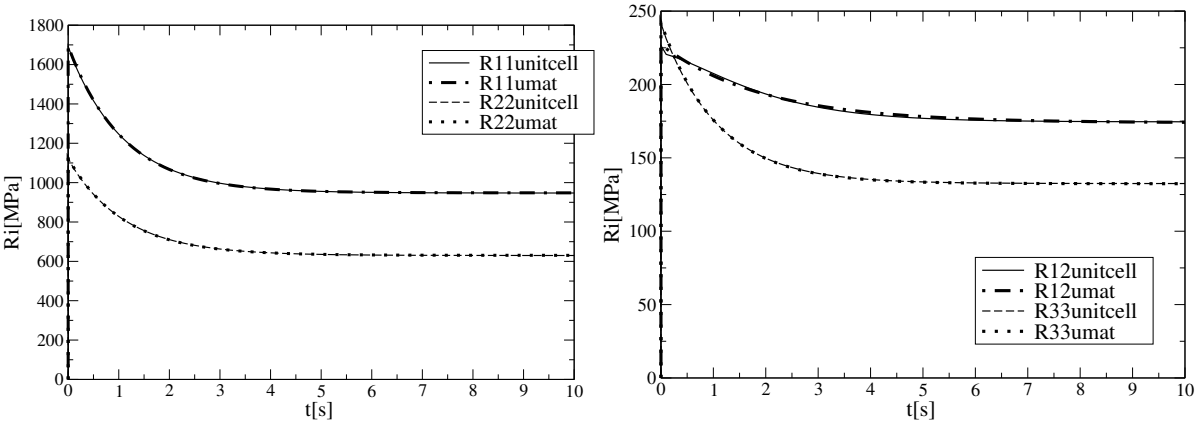

Figure 3. Orthotropic relaxation functions $R_{ij}(t)$ as response to Heaviside step strains; Homogenization of a voided structure ("unit cell"), and single element predictions by the developed constitutive material law ("umat") calibrated to the homogenized behavior.

Table 4. Orthotropic linear thermo–viscoelastic material data of a rigid inclusion reinforced model composite as input to the `UMAT` and the `UEXPAN`; instantaneous elasticity matrix elements, relaxation matrix elements, instantaneous coefficients of thermal expansion, and thermal expansion relaxation data.

$R_{11\,0} = 5301\text{MPa}$	$R_{22\,0} = 3857\text{MPa}$	$R_{12\,0} = R_{21\,0} = 817\text{MPa}$	$R_{33\,0} = 714\text{MPa}$
$r_{11} = 0.419$	$r_{22} = 0.413$	$r_{12} = r_{21} = 0.203$	$r_{33} = 0.434$
$\tau^{r_{11}} = 1.20\text{s}$	$\tau^{r_{22}} = 1.20\text{s}$	$\tau^{r_{12}} = \tau^{r_{21}} = 2.00\text{s}$	$\tau^{r_{33}} = 1.00\text{s}$
$r_{11} = 0.000$	$r_{22} = 0.000$	$r_{12} = r_{21} = -0.034$	$r_{33} = 0.000$
$\tau^{r_{11}} = 1.20\text{s}$	$\tau^{r_{22}} = 1.20\text{s}$	$\tau^{r_{12}} = \tau^{r_{21}} = 0.35\text{s}$	$\tau^{r_{33}} = 1.00\text{s}$
$\alpha_{1\,0} = 5.16 \times 10^{-5}/°\text{C}$		$\alpha_{2\,0} = 7.58 \times 10^{-5}/°\text{C}$	
$a_1 = -0.019$		$a_2 = -0.045$	
$\tau^{a_1} = 1.00\text{s}$		$\tau^{a_2} = 1.30\text{s}$	

subroutine, the thermal expansion response is fitted and the material data is listed in Table 4. The fitted behavior is presented in Fig. 4 (thick dashed lines).

4. Summary

A constitutive material law for linear thermo–viscoelasticity in the time domain with orthotropic material symmetry under plane stress assumption is developed and implemented into FEM. For the linear viscoelastic material behavior the orthotropic material symmetry is not only considered for the elastic part. Also for the relaxation response, full orthotropy is realized with the appropriate number of independent material parameters. The formulation is based on a time dependent elasticity tensor for which each element possesses its own relaxation function. This way, the mutual coupling of the normal components, i.e. a Poisson type effect, is accounted for. The relaxation function for each tensor element is prescribed by its individual Prony series expansions.
The thermal expansion relaxation is treated in analogy to the mechanical, i.e. viscoelastic, model. The material behavior is described by instantaneous coefficients of thermal expansion and their individual relaxation functions. The time dependent thermal strain is modeled as function of the applied temperature change. Various tests on isotropic and orthotropic problems are carried out for verification successfully.

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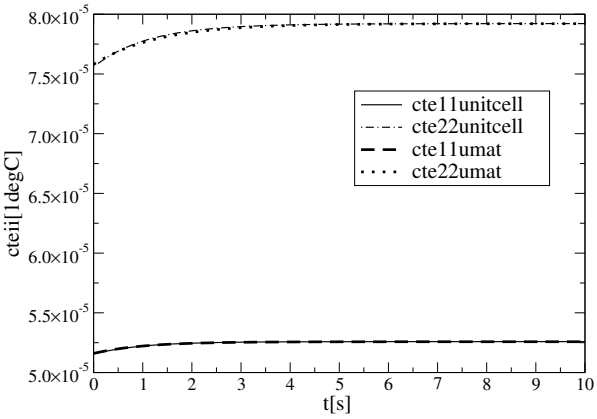

Figure 4. Orthotropic thermal expansion relaxation/creep functions $\alpha_i(t)$ as response to unity temperature Heaviside step; homogenization of a rigid inclusion reinforced model composite ("unit cell"), and single element predictions by the developed thermal expansion law ("uexpan") calibrated to the homogenized behavior.

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