Benefits, applying Tsai's Ideas 'Trace', 'Double-Double' and 'Omni Failure Envelope' to UD-multiply composed Laminates?

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Abstract:

Objective: 'Simpler' linear-elastic Pre-design of composite structures with walls composed of uni-directional (UD) plies of endless fibers, thereby achieving a practical determination of the Reserve Factor.

An important and lasting task of structural mechanics is an as simple as possible design of laminates. To this task, S. Tsai has introduced an important innovation, which, however, does not receive the attention it deserves everywhere. This is partly due to the fact that the designations are not yet neither consistent nor enough standard-adapted, and also because it is only after a long processing time that it is possible to make oneself sufficiently understandable to the potential user. It is important to find out where the open questions on the new topic arise and then to incorporate the answers into further processing.

Invariants and their linear combinations are helpful quantities in mechanics because they are independent of the CoS used in analysis. In this sense Tsai developed his 'Trace'-idea to more systematically estimate the stiffness quantities of novel laminates lacking of sufficient data in pre-design. Objective is to enable new approaches by the application of normalized stiffness matrices which allow for a composite design being independent of actual laminate thickness and CFRP material. A 'Trace'-based sizing approach is possible.

Lay-up symmetry is usually required for the laminate in order to maximally avoid warping, spring-in and have minimum problems in the case of adding repair layers. For optimum strength performance minimum layer (ply) thickness is desired to reduce Micro-fracture mechanics-induced micro-cracking. In the above context the classical 'Quad-laminate' family [0/ ± 45/90] offers not the practical optimum. Here, Tsai-Melo's idea of the 'Double-Double (DD) laminate' comes in. This can be realized with today's UD prepreg materials or with the newly available C-ply material DD represents a sub-laminate of two angle-plies or two Doubles, respectively, where 2 angle-plies of different fiber angles form a four-ply sub-laminate $\{\varphi / -\varphi / \psi / -\psi\}$. DD is automatically balanced, needs no ten percent rule, no stacking sequence, and homogenization due to the number of repetitions makes mid-plane symmetry unnecessary.

Computing 'all' possible combinations of ply-orientation (loading representing) angles and ply-types a so-called failure stress-based 'Omni-(*principal FPF strain*) failure envelope' is obtained with an intact Non-FPF area within. First Ply Failure FPF (*includes Fiber Failure FF and Inter-Fiber-Failure IFF*) envelopes are obtained for a distinct composite material, which covers all its potential laminate stacks. The chosen strength failure criterion significantly determines the shape of the envelope. Dimensioning is performed by showing that the design loading-caused principal strains are lying within the Non-FPF area, which means that the material Reserve Factor is $f_{RF} > 1$. A more conservative procedure, termed 'Unit-circle'-approach, uses the radius of the internal circle of the '*Omni failure envelope*'. Recently, for the FPF-envelope a formula could be derived by Cuntze. It enables to by-pass the effortful ply-by-ply analysis of multiple-ply laminates.

The ideas of Stephen Tsai have been followed in order to get a deeper mechanical feeling for laminates when designing them to First-Ply Failure (*FPF*), an approximately linear–elastic level. This would enable to reduce the effort for Design Dimensioning regarding optimization and finally also for Design Verification considering analysis and testing.

1 Introduction

1.1 Motivation

Some history from R. Cuntze: As early as 2014 Steve Tsai sent him some pre-information and in 2015 the book *Tsai S W and Melo J D*: "*Composite Materials Design and Testing - unlocking mystery with invariants on the Trace idea"* [*Tsa15*]. He thought at that time: This is an excellent idea to improve laminate optimization in cases where the UD-material remains the same in the laminate stack. Then, in order to make it edible in our notation, I transferred it - still in 2014 - into the designations described in the guideline VDI 2014 (*and in the Mil Handbook 17*) which he issued in 2006 and which are also used in the German Aerospace Handbook HSB (*Fundamentals and Methods for Aeronautical Design and Analyses*). Unfortunately, the HSB-responsible working group IASB did not pick up the 'Trace' idea, but then K. Rother at the Hoch-Schule Munich did, using Tsai's terminology. Later, the second author E. Kappel joined the Double-Double working group and created Chapter 3, Unique Manufacturing Opportunity, in [*Tsa22*].

1.2 Terminology

"A general system of signs and symbols is of high importance for a logically consistent universal language for scientific use !" ***Gottfried Wilhelm Leibniz*** (about 1800)

Desired as models are 'homogeneous' solids, however, reality is much more complicated. Practically, all materials are composites. One distinguishes two structural composite types: Material Composites and Composite Materials. A structural material usually is the model on the envisaged scale of a homogenized complex solid that became 'smeared' to usually obtain an engineering-like macro-model. *Fig.1-1* presents composite products used in mechanical and civil engineering.

Modeling the variety of laminates is a challenge. In this context, essential for the interpretation of the failures faced after testing, is the knowledge about the lay-up (stack) of the envisaged laminate, because crimped fabrics and non-crimped NCF-materials behave differently. It is further extremely necessary to provide the material-modeling design engineer and his colleague in production (*for the Ply Book*) with a clear, distinguishing description of UD-lay-ups being Non Crimp Fabrics NCFs (*stitched multi-UD-layer*) or Fabric layers (*crimped*). Due to unclear descriptions unfortunately one can often not use valuable test results of fiber-reinforced materials. One could distinguish the various types by a clear optical designation, a square bracket [..] and a wavy bracket {..}, in order to enable a realistic material modelling in the case of ply-by-ply analyses, that optically helps to distinguish NCF **{**stitched UD-stack**}** from those woven fabrics where one practically cannot mechanically separate the single woven layers within one fabric layer as in the case of *plain weave* binding, $\begin{bmatrix} 0 \\ 90 \end{bmatrix}$, which is symmetric in itself. Applied this means:

* Single UD-layers-*deposited* stack $[0/90]_S = [0/90/90/0]$-lay-up, prepreg

* Semi-finished product, *stitched* NCF: $\{0/90\} + \{90/0\}$ symmetrically stacked, dry;

deliverable 'building blocks' are $\{0/45/\text{-}45/90\}$, novel C-ply^{TM} $\{\varphi/\text{-}\psi/\text{-}\varphi/\psi\}$,

as DD building block and sub-laminate i.e. $\{75/-75/-15/15\}_r$ with r = repetitions.

The production of the balanced angle-ply (BAP) double-double semi-finished products requires machines that can produce non-crimp fabrics (NCF), as it is the case with Karl Mayer GmbH. The later investigated specific 'ply' C-PLY™ is produced at the company Chomarat, France.

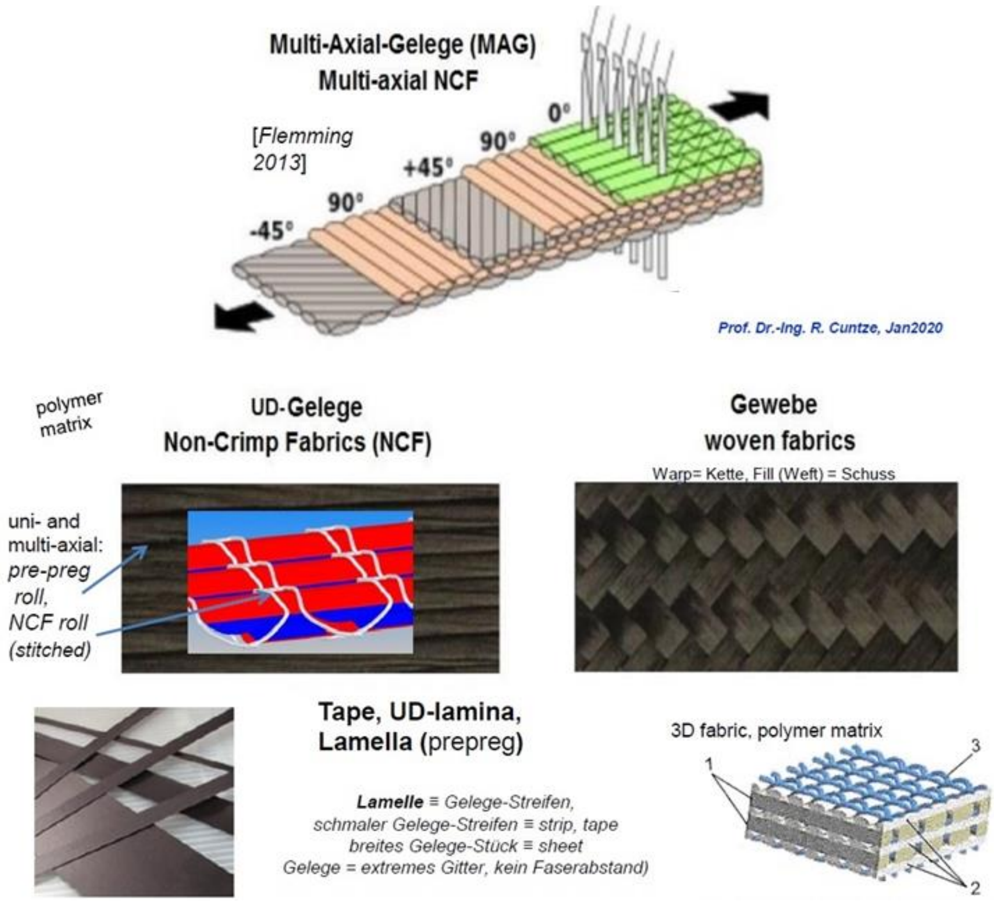


Fig. 1-1: Some composites with designations

Some specific terms for a better common understanding need to be added here:

1.3 Tsai Notations, an Application-Bottleneck for some Structural Engineers in industry

Despite of the fact that the following designations are later used they are put here.

Please, mind at first the differences in [Tsa22, Sha20, VDI 2014]. Figure and text below show the opposite designation of VDI 2014 and Tsai in UD notation:

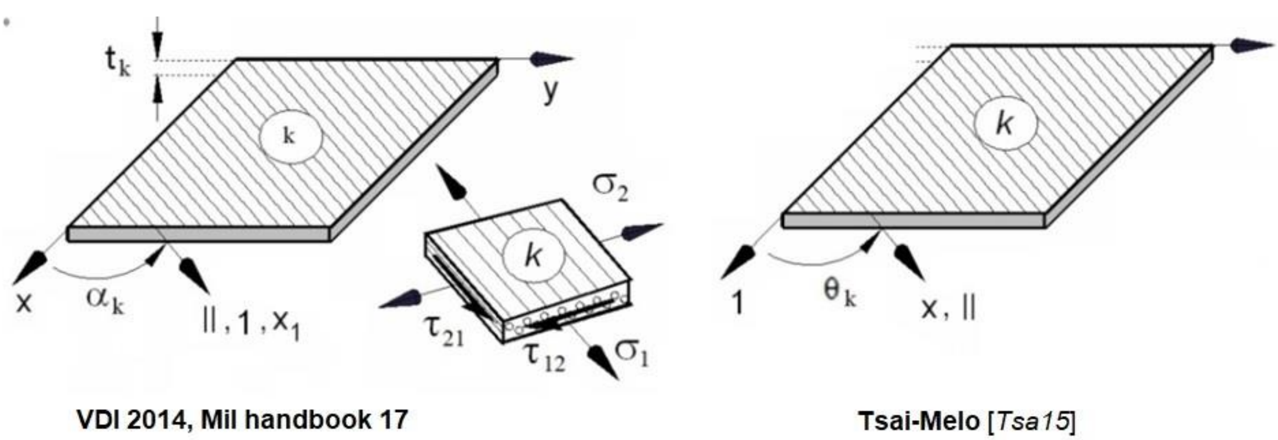


Fig.1-1: Opposite designation of coordinates

In this context it is to bring the following forward still here, because it important to draw attention to avoid misuse:

(1) Differently applied suffix 1 for the coordinates and a different positive angle direction

(2) Trace invariant: In [Sha20] it reads $Tr = trace([Q^{Tr}]) = Q_{11} + Q_{22} + 2 \cdot Q_{66}$ *and in [Tsa22]* $trace([Q^{Tr}]) = Q_{xx} + Q_{yy} + 2 \cdot Q_{SS}$. *This is confusing for the user, because in the younger publication [Tsa22] x and y were used according to* $x = 1 = ||$, $y = \perp$. *Sha20-contributors also contributed to [Tsa22]. Above formulation still uses tensor notation, indicated by the factor 2, and not matrix notation as it is normal practice and applied in the VDI 2014*

(3) Contributions of a lamina (ply) to the laminate stiffness is performed by firstly rotating [Q] into [Q'], which means from the material CoS into the laminate CoS and then summed up by the CLT. The laminate CoS is a structural CoS and in mechanics the axes are required to be indexed by ***x****, y*

(4) For the components in the sub-matrices of the inverse, [K]$^{-1}$*, the denotations A*, B*, D* are often internationally used, since decades! This leads to a conflict in the latter case considering Tsai's star * to mark the thickness*-normalized sub-matrices of *[K]. Cuntze recommends the roof* $\left[A\right]^* \rightarrow \left[\hat{A}\right]$ which is still applied for the thickness-normalized nominal stress $\left[\hat{\sigma}\right]$ of the composite plate

(5) The definition of the larger Poisson's ratio ν changed within the last 4 decades twice. Apply Maxwell-Betti $\nu_{12} \cdot E_1 = \nu_{21} \cdot E_2$ *to get to know what is meant. Also the notation of the UD natural axes changed. This would not have been a problem if one outlines the symbols* $||, \perp$ as well

(6) A UD-layer may consist of several UD-plies, a C-ply-layer contains several differently oriented UD-plies becoming a building-block of the laminate

(7) In Tsa15, page 53, the prime is used as the well-known classical rotation index (as applied in the VDI 2014), and on page 14 to indicate compressive or negative !?

(8) Bar over $\bar{R}$ *is used in some literature as an average value, representing the statistic mean*

(9) [Tsa15]: Index 0 belongs to a general reference plane. This <u>*might*</u> *be the mid-plane, possibly*

(10) UD-invariants I_1*,* I_2 *are fixed for decades in UD mechanics as* $I_1 = \sigma_1,\ I_2 = \sigma_2 + \sigma_3$.

(11) On top: the choice of J_2*= trace[S] confuses with the Mises invariant for isotropic materials.*

In order to bypass above conflicts, the author uses in the following text the notations of the VDI 2014 guideline. These had been carefully checked by the co-workers of the guideline working group in the eighties by regarding international publications.
Moving from tensor to contracted engineering notation under the presumption 'Symmetry of stress, strain and stiffness elasticity matrix' for the transfer to the stiffness matrix no correction factor is necessary but for the compliance matrix, due to:

Shear strain $\varepsilon_{12} = 0.5 \cdot (\partial u / \partial y + \partial v / \partial x)$ *tensor* $\rightarrow$ $\varepsilon_6 \equiv \gamma_{12} = 2 \cdot \varepsilon_{12}$ *engineering* .

This is important, because the derivation of the 'Trace' idea requires tensor formulations.

1.4 'Quad'-Laminate and Double-Double (DD) Laminate Lay-ups

Beside so-called 'Quad-laminates' (*standard laminates with 0°, 90°, 45°, -45° fiber orientations)* Tsai investigated a novel semi-finished product, termed C^{TR}-Ply, and created the promising 'Double-Double (DD) laminate (see [*Kap22*] and [*Cun23a*]). In the latter document the not simply to perform transfer of Tsai's notation on stresses and strengths has been executed compatible to the German Standard VDI 2014.

Tsai's Idea was: Laminate parameter plots can efficiently former carpet plots, because now all laminates can be portrayed on one plot offering faster design decisions.

Whereas the 'Quad'-laminate family is well known the novel 'DD'-laminate family has to be presented. Double-Double means a sub-laminate of two angle-plies or two Doubles, respectively: Two angle-plies of different fiber angles form a four-ply sub-laminate. It is a multi-ply semi-fished product identified by the brackets {..} to discriminate it from [..] for the UD-layer pre-preg stacks.
DD is automatically balanced, needs no ten percent rule, no stacking sequence Homogenization makes mid-plane symmetry unnecessary. In stress analysis the repeated double angle-ply sub-laminate and the full laminate could be modelled ply-wise as $\{\varphi / -\varphi / \psi / -\psi\}$ in each sub-laminate stack. A stack $\pm\varphi,\ \pm\psi$ corresponds to the ω-angle in net-theory $\pm\omega_1, \pm\omega_2$, where $\alpha_1 = \omega_1,\ \alpha_2 = \text{-}\omega_1,\ \alpha_3 = \omega_2\ ,\ \alpha_4 = \text{-}\omega_2$).

1.5 Tensor Relations for using 'Trace'

Invariants are later used to estimate a 'normalized Master-ply' stiffness matrix [Q] for the estimation of laminate stiffness quantities, helpful in the case of a novel UD material lacking some lateral elasticity properties and by the co-author E. Kappel for the invariants'-based estimation of laminate-CTE values.
Therefore, before coming to details it seems to be helpful to shortly present the manifold use of invariants. Most often in engineering invariants of rank two tensors are applied. So-called principal invariants of a second rank tensor $\boldsymbol{T} \equiv [T]$ are the coefficients of the characteristic polynomial

$$\text{“ } p(\lambda) = det(\boldsymbol{T} - \lambda\cdot\boldsymbol{I}) = -\lambda^3 + I_1\cdot\lambda^2 - I_2\cdot\lambda + I_3 \text{ “} \quad \text{with } [\boldsymbol{T}] = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix},\ [\boldsymbol{I}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and $\boldsymbol{I}$ being the identity tensor and λ the polynomial's eigenvalues as solutions of p(λ) = 0. The derived Principal Invariants are (T_{ii} considers Einstein's sum convention)

$$I_1 = trace(T) = T_{ii} = T_{11} + T_{22} + T_{33}\ ,$$
$$I_2 = T_{11}\cdot T_{22} + T_{11}\cdot T_{33} + T_{33}\cdot T_{22} - T_{12}\cdot T_{21} - T_{13}\cdot T_{31} - T_{23}\cdot T_{32}\ ,$$
$$I_3 = T_{11}\cdot(T_{22}\cdot T_{33} - T_{23}\cdot T_{32}) + T_{12}\cdot(T_{23}\cdot T_{31} - T_{21}\cdot T_{33}) + T_{13}\cdot(T_{21}\cdot T_{32} - T_{22}\cdot T_{31})$$

Main invariants are functions of the principal ones which means coefficients of the characteristic polynomial "$\boldsymbol{T} - trace(\boldsymbol{T}/3)$".

In the isotropic material case Main Invariants used read: I_1 , $J_2 = I_1^2/3 - I_2$, $J_3 = 2\cdot I_1^3/27 - I_2\cdot I_1/3 + I_3$, mixed invariants, such as sums or differences are used for distinct applications.

For transversely-isotropic UD material invariants of the stress tensor σ_{ik}, of the strain tensor ε_{ik} and of the material elasticity stiffness tensor Q_{ik} or its inverse the compliance tensor S_{ik} are employed. Invariants of the stress tensor are used to establish *stress*-based strength failure criteria and invariants of the strain tensor for *strain*-based strength failure criteria.

It can be concluded: Using invariants in the stiffness domain helps to get information for a more reliable estimation of pre-design properties for a novel material in pre-design and thus saving time and reducing test effort. The following UD invariants will be used by Cuntze for his SFCs:

$$I_1 = I_1^{\sigma} = \sigma_1,\ I_2 = \sigma_2 + \sigma_3, \quad I_3 = \tau_{31}{}^2 + \tau_{21}{}^2, \quad I_4 = (\sigma_2 - \sigma_3)^2 + 4 \cdot \tau_{23}{}^2,$$

$$I_5 = (\sigma_2 - \sigma_3) \cdot (\tau_{31}{}^2 - \tau_{21}{}^2) - 4 \cdot \tau_{23} \cdot \tau_{31} \cdot \tau_{21} \quad \text{(from A. Boehler, applied in the FMC)}.$$

A special focus here will be the rank-two tensor Q_{ik} being the *2D-reduced rank-four elasticity stiffness tensor* C_{ikjl} .

Design Verification demands for reliable reserve factors *RF* and these - beside a reliable structural analysis - demand for reliable SFCs. Such a SFC is the mathematical formulation $F = 1$ of a failure curve or of a failure surface (body). Generally required are a yield condition and fracture strength conditions. The *yield* SFC usually describes just one mode, i.e. for isotropic materials the classical 'Mises' describes shear yielding SY. *Fracture* SFCs usually must describe two independent fracture modes, shear fracture SF and normal fracture NF in the simple isotropic case. For the here focused transversely-isotropic UD material a so-called material-inherent 'generic' number 5 for fracture seems to be given [*Cun23a, Cun22*]. This means for UD altogether 3 Inter Fiber failure (IFF) and 2 Fiber Failure (FF) modes and further 5 strengths, too. Considering the design with brittle UD material this means a set of Strength (*fracture*) Failure Criteria (SFC) has to be provided.

Principally, in order to avoid either to be too conservative or too un-conservative, a separation is required of the always needed 'analysis of the average structural behaviour' in Design Dimensioning (*using average properties and average stress-strain curves*) in order to obtain the best possible information (= 50% expectation value) from the mandatory single Design Verification analysis of the final design. There statistically minimum values for strength and minimum, mean or maximum values for the task-demanded other properties are applied as Design Values.

To achieve Structural Integrity by a successful Design Verification it is to demonstrate that 'No relevant Limit State is met'. The paper at hand is based on well-modelling test data by the SFCs applied. In these SFC formulations each strength quantity is an average strength consequently indicated by a bar over $\bar{R}$. The letter R is applied in a general formulation and for the strength Design Allowables. Design verification with respect to Static Strength is performed here on material level by a material reserve factor f_{RF} using stresses in the critical location of undisturbed areas such as stress uniform material areas.

For performing an accurate designing it is to note:

* The present stress-based design verifications - i.e. in Aerospace - requires stress criteria and as input A- or B-strength Design Allowables R.
* A strain-based design verification as precondition for certification, would firstly need permission of the FAA including authority-accepted strain criteria coupled to *Strain* Design Allowables (*also statistically reduced*), which are not available as official values in material data sheets and this is the objection here. A special Strain-based Design makes just sense if the material has some ductility and if the part is just a few cycles submitted to an extreme loading beyond the 'plastic' limit of the material such as a pipe under earthquake loading. On top this would require a Damage Tolerance Proof
* UD internal principal strains and stresses: These have no physical meaning but are practical quantities to represent the stress state of the laminate's plies. If linear-elasticity can be assumed up to the FPF-level then $\varepsilon \sim \sigma \equiv \text{loading}$ and the Proportional loading Concept can be applied.

The required relationships are listed in the following subchapter (*t is laminate thickness*).

Table 1-1: Transfer of the UD elasticity properties.
After VDI 2014, the Stiffness quantities in matrix spelling and intentionally also in symbolic spelling, k is running ply number of the stack

Relationships of the k^th Lamina strains and stresses

$x_1 \equiv 1 \equiv \parallel$, $2 \equiv \perp$, *the prime' indicates the rotated lamina*. Mind Maxwell-Betti : $\nu_{21} \cdot E_2 = \nu_{12} \cdot E_1$.

The stress and strain relations for the UD lamina in lamina CoS and rotated CoS read

$$\{\sigma\} = (\sigma_1, \sigma_2, \tau_{12})^T \ , \quad \{\sigma'\} = (\sigma_x, \sigma_y, \tau_{xy})^T = [T_\sigma] \cdot \{\sigma\}$$

$$\{\sigma'\} = [T_\sigma] \cdot \{\sigma\} = [T_\sigma]\,[Q]\,\{\varepsilon\} = [T_\sigma] \cdot [Q] \cdot [T_\varepsilon]^{-1}\,\{\varepsilon'\} = [T_\sigma]\,[Q]\,[T_\sigma]^T\,\{\varepsilon'\} \ = [Q']\,\{\varepsilon'\}.$$

* In the lamina (ply) CoS: [Q] is denoted 'reduced 3D stiffness matrix' [C].

$$\{\varepsilon\}_k = [S]_k \cdot \{\sigma\}_k , \qquad \{\sigma\}_k = [Q]_k \cdot \{\varepsilon\}_k \qquad \text{with} \quad [Q]_k = [S]_k^{-1} , \ [T_\varepsilon]^{-1} = [T_\sigma]^T$$

$$\{\varepsilon'\}_k = [S']_k \cdot \{\sigma'\}_k , \ \{\sigma'\}_k = [Q']_k \cdot \{\varepsilon'\}_k \quad \text{with} \quad [Q']_k = [T_\sigma]_k \cdot [Q]_k \cdot [T_\sigma]_k^T$$

$$[Q] = \begin{bmatrix} \frac{E_1}{1-\nu_{21}\cdot\nu_{12}} & \frac{\nu_{21}\cdot E_2}{1-\nu_{21}\cdot\nu_{12}} & 0 \\ \frac{\nu_{12}\cdot E_1}{1-\nu_{21}\cdot\nu_{12}} & \frac{E_2}{1-\nu_{21}\cdot\nu_{12}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & 1\cdot Q_{66} \end{bmatrix} \text{ and } [Q]^{-1} = [S] = \begin{bmatrix} \frac{1}{E_\parallel} & \frac{-\nu_{\parallel\perp}}{E_\perp} & 0 \\ \frac{-\nu_{\perp\parallel}}{E_\parallel} & \frac{1}{E_\perp} & 0 \\ 0 & 0 & \frac{1}{G_{\parallel\perp}} \end{bmatrix} .$$

$$\{\varepsilon\}_k = \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}_k = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix}_k \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{21} \end{Bmatrix}_k = [S]_k \cdot \{\sigma\}_k , \ \{\sigma\}_k = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{21} \end{Bmatrix}_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}_k$$

with τ_{21} as failure driving shear stress and not τ_{12}.

* In the 'rotated' laminate CoS, applying the transformation matrices $[T_\sigma]^T = [T_\varepsilon]^{-1} , \ [T_\sigma]_k^{-1} = [T_\varepsilon]$

$$[T_\sigma] = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix}, \ [T_\sigma]^T = \begin{bmatrix} c^2 & s^2 & sc \\ s^2 & c^2 & -sc \\ -2sc & 2sc & c^2 - s^2 \end{bmatrix}, \ [T_\sigma]^{-1} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix}, [T_\varepsilon] = \begin{bmatrix} c^2 & s^2 & -sc \\ s^2 & c^2 & sc \\ 2sc & -2sc & c^2 - s^2 \end{bmatrix}$$

and using the strain condition $\{\varepsilon'\}_k = \{\varepsilon'\}$ of the k^th lamina embedded in the laminate stack,

the 'rotated' lamina stresses $\{\sigma'\}_k$ can be derived

$$\{\sigma'\}_k = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} Q'_{11} & Q'_{12} & Q'_{16} \\ \bullet & Q'_{22} & Q'_{26} \\ (symm) & \bullet & Q'_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k = [Q']_k \cdot \{\varepsilon'\}_k \quad \text{with} \quad [Q'] = [T_\sigma] \cdot [Q] \cdot [T_\sigma]^T$$

and from them $\{\sigma\}_k = [T_\sigma]_k^{-1} \cdot \{\sigma'\}_k$ as input for the SFC insertion in order to compute *Eff*.

Engineering and tensor stress-strain formulations (*mandatory for invariant determination*) read:

1.6 Design Verification by demonstrating a Reserve Factor RF > 1

The Reserve Factor *RF* in mechanical engineering is a load-defined factor, defined as ratio of a 'resistance value' and an 'action value'. In this context some notes:

(1) Resistance value means here Predicted or Measured 'failure load / (design factor of safety x Design Limit Load)'.

(2) If linear analysis is permitted RF will correspond to the material reserve factor, derived from 'f_{RF} = strength / design stress'. A value higher than one would allow an increase of loading.

(3) For brittle behaving materials, the decisive static limit state is the Design Ultimate Load case, suffix $_{ult}$. The Design's strength is demonstrated if (a) no relevant strength failure, respectively limit state of any failure mode, is met and (b) all dimensioning load cases are respected by the formulas below, reaching values > 1 = 100%.

(4) The Final Failure Load in the non-linear case is reached when Eff becomes 100% in the critical stress 'point'.

(5) Assumption in usual deterministic procedure is most often: 'Worst case scenario' with respect to loading, temperature and moisture).

If linear analysis is a sufficient solution (presumption): $\sigma \sim$ load $\rightarrow RF \equiv f_{\mathrm{RF}} = \dfrac{1}{Eff}$

material reserve factor $\quad f_{\mathrm{RF},\,ult} = \dfrac{\text{Strength Design Allowable } R}{\text{Stress at } j_{ult} \cdot \text{Design Limit Load}} > 1,$

Non-linear analysis required: σ not proportional to load

reserve factor (load-defined) $\quad RF_{ult} = \dfrac{\text{Predicted Failure Load at computing } Eff = 100\%}{j_{ult} \cdot \text{Design Limit Load}} > 1.$

A very simple example for a Design Verification of an applied stress state in a critical UD lamina location of a distinct laminate wall design shall depict the *RF*-calculation as most essential task in design which streamlines every procedure when generating a design tool in the following chapters:

Asssumption: Linear analysis permitted, design FoS $j_{\mathrm{ult}} = 1.25$

* Design loading (action): $\{\sigma\}_{\mathrm{design}} = \{\sigma\} \cdot j_{\mathrm{ult}}$

* 2D-stress state: $\{\sigma\}_{\mathrm{design}} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T \cdot j_{\mathrm{ult}} = (0,\ -76,\ 0,\ 0,\ 0,\ 52)^T$ MPa

* Residual stresses: 0 $\left(\textit{effect vanishes with increasing micro-cracking}\right)$

* Strengths (resistance) : $\{\bar{R}\} = (\bar{R}_{||}^t, \bar{R}_{||}^c, \bar{R}_{\perp}^t, \bar{R}_{\perp}^c, \bar{R}_{\perp||})^T$

$= (1378,\ 950,\ 40,\ 125,\ 97)^T$ MPa average from mesurement

statistically reduced $\{R\} = (R_{||}^t, R_{||}^c, R_{\perp}^t, R_{\perp}^c, R_{\perp||})^T = (1050,\ 725,\ 32,\ 112,\ 79)^T$ MPa

* Friction value(s) : $\mu_{\perp||} = 0.3,\ (\mu_{\perp\perp} = 0.35)$, Mode interaction exponent: $m = 2.7$

$$\{Eff^{\,mode}\} = \left(Eff^{||\sigma},\ Eff^{||\tau},\ Eff^{\perp\sigma},\ Eff^{\perp\tau},\ Eff^{\perp||}\right)^T = \left(0.88,\ 0,\ 0,\ 0.21,\ 0.20\right)^T$$

$$Eff^{\,m} = (Eff^{||\sigma})^m + (Eff^{||\tau})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp||})^m = 100\% \ .$$

The results above deliver the following material reserve factor $f_{\mathrm{RF}} = 1/Eff$

* $Eff^{\perp\sigma} = \dfrac{\sigma_2 + |\sigma_2|}{2 \cdot \bar{R}_{\perp}^t} = 0, \quad Eff^{\perp\tau} = \dfrac{-\sigma_2 + |\sigma_2|}{2 \cdot \bar{R}_{\perp}^c} = 0.60, \quad Eff^{\perp||} = \dfrac{|\tau_{21}|}{\bar{R}_{\perp||} - \mu_{\perp||} \cdot \sigma_2} = 0.55$

$$Eff = [(Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp||})^m]^{1/m} = 0.80.$$

$\Rightarrow \quad f_{\mathrm{RF}} = 1/Eff = 1.25 \rightarrow$ RF $= f_{\mathrm{RF}}$ (if linearity permitted) $\rightarrow$ MoS $= RF - 1 = 0.25 > 0$!

The certification–relevant load-defined Reserve Factor *RF* corresponds in the given linear case to the material reserve factor f_{RF}, the value of which is 1.25 > 1 → *Laminate wall design is verified!*

> Steve Tsai's hope for future laminate design: *"Materials and laminates are equivalent and the same entity with different views. They are interchangeable through their single parameters all locked in through their transformation and interpolation properties in a compact, elegant, continuous field, totally different from a collection of so-called discrete 'Quad' laminates. Lack of data can no longer derail innovations".*

The introduction of tapes with two variable fiber orientations opens the possibility of formulating via angle-dependent functions for the stiffness Each of the two angles in a laminate must be optimized in order to "earn its place", which is done via a "best-of" search [*Rot22*]. This is a new architecture of sub-laminates.

The use of thin-plies further reduces the thickness of such simple basic sub-laminates. The advantage is demonstrated later. The stiffness of 'Quad- laminates' can at least be well approximated by appropriate DD-alternatives.

Scalar invariants do not change if the CoS is changed. The following figure presents the various stresses faced with laminas and laminates and a stress transformation for the normal stress into an inclined structural CoS x(y), exemplarily.

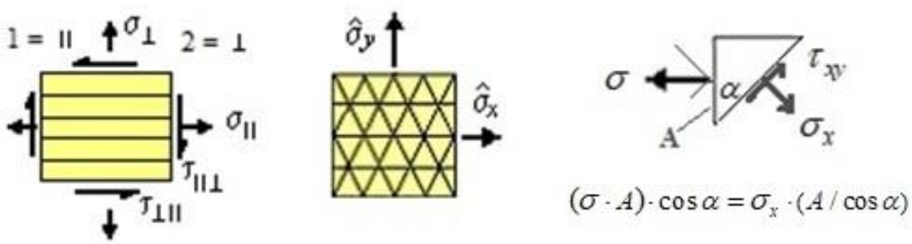


Fig.1-2: Some stress denotations

From equilibrium at the inclined section plane of a tensioned UD lamina test specimen, balancing the forces in normal and tangential direction, follow the equations below compiled in the standard matrix shape and adapted to UD material, with $[T_\sigma]$ as defined in VDI 2014.

$$\{\sigma'\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} \sigma_1 \cdot c^2 + \sigma_2 \cdot s^2 - 2\tau_{12} \cdot sc \\ \sigma_1 \cdot s^2 + \sigma_2 \cdot c^2 + 2\tau_{12} \cdot sc \\ \sigma_1 \cdot sc - \sigma_2 \cdot sc + \tau_{12} \cdot (c^2 - s^2) \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix} \cdot \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = [T_\sigma] \cdot \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}.$$

The later necessary tensor formulations of the stress-strain relations are derived as follows

$$\{\sigma'\} = [Q'] \cdot \{\varepsilon'\} = [T_\sigma] \cdot [Q] \cdot [T_\sigma]^T \cdot \{\varepsilon'\} = [T_\sigma] \cdot \left[Q^{Tr}\right] \cdot [R]^{-1} \cdot \{\varepsilon'\} \quad \text{with the}$$

$$\text{Reuter Matrix } [R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \ [R]^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}; \ \varepsilon_{12} = \frac{\gamma_{12}}{2}, \ \sigma_{12} \equiv \tau_{12}.$$

The factor on the shear stress arises from the classical definition of shear strain, which is twice the tensor shear strain. Being editor of the VDI 2014, sheet 3, the author had to use the more generally applied VDI notation. This caused some problems because literature does not show full consistency. For the sake of survey some more relevant formulas shall be provided:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}Q_{11}' & Q_{12}' & Q_{16}'\\ Q_{12}' & Q_{22}' & Q_{26}'\\ Q_{16}' & Q_{26}' & Q_{66}'\end{bmatrix}\cdot\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{bmatrix}Q_{11}^{Tr}{}' & Q_{12}^{Tr}{}' & 2Q_{16}^{Tr}{}'\\ Q_{12}^{Tr}{}' & Q_{22}^{Tr}{}' & 2Q_{26}^{Tr}{}'\\ Q_{16}^{Tr}{}' & Q_{26}^{Tr}{}' & 2Q_{66}^{Tr}{}'\end{bmatrix}\cdot\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{Bmatrix}=\left[Q^{Tr}{}'\right]\cdot\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{Bmatrix}=\left[Q^{Tr}{}'\right]\cdot[R]^{-1}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}$$

$$\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix}=[R]\cdot\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_{12}\end{Bmatrix}=[R]\cdot[T_\varepsilon]^{-1}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{Bmatrix}=[R]\cdot[T_\sigma]^T\cdot\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{Bmatrix}=[R]\cdot[T_\sigma]^T\cdot[R]^{-1}\cdot\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}.$$

The index $^{\mathrm{Tr}}$ in $\left[Q^{Tr}{}'\right]$ was introduced in order to distinguish the 'Trace'-associated one from [Q].

Pleasant Memory:

ICCM conference, 1986, where we had to co-chair a session.

"***Ralf, please chair, I will switch the light on and off***".

And decades later, he switched a mechanical light on with

'Trace', 'Double-Double' and the 'Omni failure envelope'.

2 'Trace', Tsai's 'generic' Invariant-Idea

2.1 Idea behind 'Trace'-invariant with Proof for being invariant

For the author, the UD invariant 'Trace' $= Q_{11} + Q_{22} + 2 \cdot Q_{66}$, numerically *Tr*, is successfully to use as a 'generic laminate quantity', presuming the same UD material is used building a monolithic laminate. This reminds him of his FMC, where he found that isotropic materials have a 'generic' number of 2 *(elastic constants, strengths, strength failure modes, decisive invariants, fracture-angle stable fracture toughness properties*) and the transversely-isotropic UD material a generic number of 5. Such knowledge effortful guides engineering work, reduces the test amount and makes engineering life simpler and more practical like Steve's 'Trace idea in laminate design dimensioning or sizing, respectively. The desired novel stiffness quantities are assumed to be fractions of the invariant 'Trace'. *Table 2-1* shows the derivation that 'Trace' = trace$[Q^{\mathrm{Tr}}]$, is an invariant.

Invariants and their linear combinations are helpful quantities in mechanics because they are independent of the CoS used in analysis. Unifying stiffness analyses by an invariant–based theory makes design analysis and testing simpler. It is further welcomed to hopefully more systematically estimate the stiffness quantities of novel laminates lacking of sufficient data in pre-design.

LL:

1. *'Trace' = Tr =* $trace([Q^{Tr}]) = (Q_{11}^{Tr}{}' + Q_{22}^{Tr}{}' + 2Q_{66}^{Tr}{}') = (Q_{11} + Q_{22} + 2 \cdot Q_{66}) \neq Q_{11}' + Q_{22}' + 1 \cdot Q_{66}'$ *is UD lamina (ply) material-related and not dependent on the rotation angle α of fiber direction*
2. *In general, a trace sums up the elements on the main diagonal of a square matrix such as [Q].*
3. *If the matrix is a tensor, then trace becomes an invariant, termed 'Trace' by Tsai.*
4. *Lay-up (stacking sequence) and thickness are geometric quantities.*
5. *Trace could be seen as an independent stiffness property. Hence, it seems that one can advantageously use 'Trace' as a factor Tr of the stiffness quantities in the sub-matrices A, B, D of the laminate stiffness matrix[K].*

To the honor of Steve, in [Sha20] the authors termed,
- as analogous quantity to the Young's modulus E in the isotropic case –
the transversely-isotropic UD invariant Trace
'Tsai modulus' .

2.2 Thickness- and 'Trace'-normalized Laminate Relations

Based on test data, Tsai and Melo investigated, that CFRP laminates have common 'generic' stiffness properties after normalization with above Trace invariant of the 2D-stiffness elasticity matrix [Q]. They found that Trace offers a basic measure to capture the behavior of the UD ply material independently of the CoS and of any lay-up. Replacing [Q] by the invariant-dedicated $[Q^{\mathrm{Tr}}]$ leads to benefits in laminate design, especially the optimization of the stack.

For stiff fibers, like carbon-fibers, Q_{11} dominates 'Trace', whereas the matrix-dominated stiffness terms contribute just a little. This is an essential benefit that can be exploited when designing laminates. 'Trace' can be used to show that a wide range of materials have almost identical entries if one looks 'Trace'-normalized at the elasticity matrix coefficients.

Table 2-1 presents a derivation that 'Trace' is an invariant.

Table 2-1: Derivation to prove 'Trace' is an invariant

Engineering formulations, VDI2014: Rotation from UD-lamina CoS $\|, \perp$ to Laminate CoS x, y

$$\{\sigma'\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T_\sigma]\cdot\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix},\ \{\sigma\} = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = [Q]\cdot\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & 1\cdot Q_{66} \end{bmatrix}\cdot\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}, \{\varepsilon'\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [T_\varepsilon]\cdot\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}.$$

To apply the Trace idea one uses a tensorial formulation σ, ε and also within engineering quantities σ, τ, γ.

engineering: $\{\sigma'\} = [T_\sigma]\cdot\{\sigma\},\ [Q'] = [T_\sigma]\cdot[Q]\cdot[T_\sigma]^T,\ \{\varepsilon\} = [T_\varepsilon]^{-1}\cdot\{\varepsilon'\},\ [T_\varepsilon]^{-1} = [T_\sigma]^T,\ [Q^{Tr}] = [Q]\cdot[R]$

tensorial: $\{\sigma'\}_{\text{tens}} = [T_\sigma]\cdot[Q^{Tr}]\cdot[T_\sigma]^T\cdot\{\varepsilon'\}_{\text{tens}} = [T_\sigma]\cdot[Q]\cdot[R]\cdot[T_\sigma]^T\cdot[T_\varepsilon]^{-1}\cdot\{\varepsilon'\}_{\text{tens}}$,

with the abbreviation $[Q^{Tr}\,'] = ([T_\sigma]\cdot[Q]\cdot[R])\cdot[T_\sigma]^T$, $\varepsilon_{xy} = \gamma_{xy}/2$

At first $([T_\sigma]\cdot[Q]\cdot[R])$ is derived as follows

$$\bullet \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \text{ matrix multiplication } \bullet \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2-s^2 \end{bmatrix} = \begin{bmatrix} c^2Q_{11}+s^2Q_{12} & c^2Q_{12}+s^2Q_{22} & -2sc\cdot Q_{66} \\ c^2Q_{12}+s^2Q_{11} & s^2Q_{12}+c^2Q_{22} & 2sc\cdot Q_{66} \\ sc(Q_{11}-Q_{21}) & sc(Q_{12}-Q_{22}) & (c^2-s^2)\cdot Q_{66} \end{bmatrix} = \begin{bmatrix} c^2Q_{11}+s^2Q_{12} & c^2Q_{12}+s^2Q_{22} & -2sc\cdot Q_{66}\cdot 2 \\ c^2Q_{12}+s^2Q_{11} & s^2Q_{12}+c^2Q_{22} & 2sc\cdot Q_{66}\cdot 2 \\ sc(Q_{11}-Q_{21}) & sc(Q_{12}-Q_{22}) & (c^2-s^2)\cdot Q_{66}\cdot 2 \end{bmatrix}$$

and then $[Q^{Tr}\,'] = ([T_\sigma]\cdot[Q]\cdot[R])\ \cdot\ [T_\sigma]^T$ as the final multiplication

$$\bullet \begin{bmatrix} c^2 & s^2 & sc \\ s^2 & c^2 & -sc \\ -2sc & 2sc & c^2-s^2 \end{bmatrix}$$

$$\begin{bmatrix} c^2Q_{11}+s^2Q_{12} & c^2Q_{12}+s^2Q_{22} & -2sc\cdot Q_{66}\cdot 2 \\ c^2Q_{12}+s^2Q_{11} & s^2Q_{12}+c^2Q_{22} & 2sc\cdot Q_{66}\cdot 2 \\ sc(Q_{11}-Q_{21}) & sc(Q_{12}-Q_{22}) & (c^2-s^2)\cdot Q_{66}\cdot 2 \end{bmatrix} = \begin{bmatrix} c^2\left[c^2Q_{11}+s^2Q_{12}\right]+s^2\left[c^2Q_{12}+s^2Q_{22}\right]-2sc\cdot[-2scQ_{66}]\cdot 2 & . & . \\ . & . & . \\ . & . & . \end{bmatrix}$$

resulting in the matrix

$$[Q^{Tr}\,'] = \begin{bmatrix} c^2\left[c^2Q_{11}+s^2Q_{12}\right]+s^2\left[c^2Q_{12}+s^2Q_{22}\right]-2sc\cdot[-2scQ_{66}]\cdot 2 & . & . \\ . & Q_{22}^{Tr}\,' & . \\ . & . & Q_{66}^{Tr}\,' \end{bmatrix}.$$

$Q_{11}^{Tr}\,' = c^2[c^2Q_{11}+s^2Q_{12}]+s^2[c^2Q_{12}+s^2Q_{22}]-2sc[-2sc\cdot Q_{66}\cdot 2]$, $Q_{22}^{Tr}\,' = s^2[c^2Q_{12}+s^2Q_{11}]+c^2[s^2Q_{12}+c^2Q_{22}]+$ $+\,2sc[2sc\cdot Q_{66}\cdot 2]$, $Q_{66}^{Tr}\,' = sc[sc(Q_{11}-Q_{21})]-sc[sc(Q_{12}-Q_{22})]+(c^2-s^2)^2\cdot[Q_{66}\cdot 2]$ which delivers

$$\text{'Trace'} = trace([Q^{Tr}\,']) = \Sigma\ c^2[c^2Q_{11}+s^2Q_{12}]+s^2[c^2Q_{12}+s^2Q_{22}]+8s^2c^2[Q_{66}]+$$

$$+\ s^2[c^2Q_{12}+s^2Q_{11}]+c^2[s^2Q_{12}+c^2Q_{22}]+8s^2c^2[Q_{66}]+\ 2\cdot(s^2c^2[Q_{11}-2Q_{21}+Q_{22}]+(c^2-s^2)^2[2Q_{66}])$$

$$= (c^4+s^4+2s^2c^2)[Q_{11}]+(c^4+s^4+2s^2c^2)[Q_{22}]+s^2c^2(+0\cdot Q_{12})+(8s^2c^2+2(c^2-s^2)^2)[Q_{66}]$$

$$= \Sigma(Q_{11}+Q_{22}+2Q_{66}) = \Sigma(Q_{11}^{Tr}\,'+Q_{22}^{Tr}\,'+2Q_{66}^{Tr}\,'),\ \text{tensorial q.e.d.}$$

Addition theorems: $C = c^2-s^2 = \cos 2\alpha,\ c^2 = C+s^2 = C+(1-c^2),\ 4s^2c^2 = (1+C)\cdot(1-C) = 1-C^2$

$8s^2c^2+2(c^2-s^2)^2 = 2(1-C^2)+2C^2 = 2,\ c^4+s^4+2s^2c^2 = (c^2-s^2)^2+4s^2c^2 = C^2+(1-C^2) = 1.$

For a later application some combinations of the elasticity coefficients are provided, [*Tsa22*, chapter 2]:

$$U_1 = 3/8 \cdot (Q_{11} + Q_{22}) + 1/4 \cdot (Q_{12} + 2Q_{66}),\ U_2 = 1/2 \cdot (Q_{11} - Q_{22}),\ U_3 = 1/8 \cdot (Q_{11} + Q_{22}) - 1/4 \cdot (Q_{12} + 2Q_{66})$$

.

Thickness-Normalization:

Under mechanical loading the general equilibrium conditions of the in-plane loaded plate read (^{0} *means reference plane which might be the mid-plane if of advantage*)

$$\{n\} = [K] \cdot \{\varepsilon\} = \begin{Bmatrix} \mathrm{n} \\ \mathrm{m}^{\circ} \end{Bmatrix} = \begin{bmatrix} \mathrm{A} & \mathrm{B} \\ \mathrm{B}^{\mathrm{T}} & \mathrm{D} \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} \text{ with } \{\mathrm{n}^0\} = \mathrm{t} \cdot \{\hat{\sigma}\},\ [Q']_{\mathrm{k}} = [T_\sigma]_{\mathrm{k}} \cdot [Q]_{\mathrm{k}} \cdot [T_\sigma]_{\mathrm{k}}^T$$

$$[A] = 1 \cdot \sum_{n=1}^{n} [\mathrm{Q}'_k] \cdot (\mathrm{z}_k - \mathrm{z}_{k-1}),\ [B] = \frac{1}{2} \cdot \sum_{n=1}^{n} [\mathrm{Q}'_k] \cdot (\mathrm{z}_k{}^2 - \mathrm{z}_{k-1}{}^2),\ [D] = \frac{1}{3} \cdot \sum_{n=1}^{n} [\mathrm{Q}'_k] \cdot (\mathrm{z}_k{}^3 - \mathrm{z}_{k-1}{}^3)\ .$$

Thickness-Normalization of the [*K*]-sub-matrices shall be now applied in order to achieve the same units, in GPa or MPa, in all sub-matrices and also to numerically achieve homogenized laminates. The resulting sub-matrices are marked by a roof sign analogous to the roof sign used to indicate homogenized (smeared) laminate stresses $\hat{\sigma}$ or strains $\hat{\varepsilon}$ used in the VDI 2014.
In *Table2-3* the sub-matrices after the thickness-normalization are depicted for the classical (here just [*A*] and for the normalized case the definitions of Tsai. Eventually, the external loading representing principal strains are derived.

Table 2-2: Thickness-normalization of laminate sub-matrices, [*Q*]*-based. t = laminate thickness*

*From the force loadings the external laminate principal strains are determined

$$[A] = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \text{ in } \frac{N}{mm} \quad \text{with} \quad [A] = \sum_{k=1}^{n} [Q']_k \cdot \mathrm{t}_k = \frac{[\hat{A}]}{t} \cdot \begin{Bmatrix} \hat{\sigma}_x \\ \hat{\sigma}_y \\ \hat{\tau}_{xy} \end{Bmatrix}$$

$$\text{Definitions: } [\hat{A}] = [A] / \mathrm{t},\ [\hat{B}] = [B] \cdot 2 / \mathrm{t}^2,\ [\hat{D}] = [D] \cdot 12 / \mathrm{t}^3, \text{ all in } \frac{N}{mm^2} = \text{MPa}\ .$$

$$\{\varepsilon'\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [A]^{-1} \cdot \begin{Bmatrix} n_x \\ n_y \\ n_{xy} \end{Bmatrix} \rightarrow \begin{Bmatrix} \varepsilon_I \\ \varepsilon_{II} \\ 0 \end{Bmatrix} = \{\varepsilon_{pr}\} = [T_\varepsilon] \cdot \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [T_\varepsilon] \cdot \{\varepsilon'\}$$

Secondly the prosperous use of ‘Trace’ shall be induced. This procedure is termed ‘Trace’-normalization.

‘Trace’-Normalization:

For the UD-material IM7-977 (*data from Tsai-Melo*) in *Table 2-3*, co-author Kappel provided an example of ‘Trace’-normalization. The example is a ‘Quad laminate’ from aerospace industry with the classical 4 fiber directions.

LL: Above relations require that any performed transformation from engineering to tensor quantities must be considered in order to use standard CLT-programs.

Table 2-3: Trace-normalized lamina $[Q^{Tr}]$-based, IM7-977

Lay-up $[0_2/45/-45/90]_{4s}$, $t_k = 0.125$ mm, 40 layers or UD-laminas, t =5 mm

$\{\bar{R}\} = (3250,\ 1600,\ 62,\ 98,\ 75)^T$ MPa, $\{E\} = (191000,\ 191000,\ 9940,\ 9940,\ 7790)^T$ MPa, $\nu_{21} = 0.35$.

Relations:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix} = [Q]\cdot\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{21} & Q_{22} & 0\\ 0 & 0 & 1\cdot Q_{66}\end{bmatrix}\cdot\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix} \Rightarrow \begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{Bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{21} & Q_{22} & 0\\ 0 & 0 & 2\cdot Q_{66}\end{bmatrix}\cdot\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{12}\end{Bmatrix} = [Q^{Tr}]\cdot\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{12}\end{Bmatrix}$$

$$\left[Q^{Tr}\right] = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{21} & Q_{22} & 0\\ 0 & 0 & 2\cdot Q_{66}\end{bmatrix} = \begin{bmatrix}192.2 & 3.501 & 0\\ 3.502 & 10.008 & 0\\ 0 & 0 & 2\cdot 7.790\end{bmatrix}\text{ GPa}$$

$$\rightarrow \text{'Trace'} = Tr = 192.2 + 10.008 + 2\cdot 7.790 = 217.81\text{ GPa}$$

$$\text{or}\qquad [Q]\cdot[R] \;=\; \left[Q^{Tr}\right] = Tr\cdot\begin{bmatrix}0.883 & 0.016 & 0\\ 0.016 & 0.046 & 0\\ 0 & 0 & 2\cdot 0.036\end{bmatrix}\text{ GPa.}$$

$$\rightarrow \text{'Trace'} = 217.8 = Tr\cdot(0.883 + 0.046 + 2\cdot 0.036)\text{ GPa} = Tr\cdot 1\text{ GPa}.$$

The classical elements of the A-B-D-submatrices *(A in N/mm, B in N, D in N·mm)* read

$$[A] = \begin{bmatrix}514651 & 99540 & 0\\ 99540 & 332429 & 0\\ 0 & 0 & 120983\end{bmatrix},\ [B] = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\ [D] = \begin{bmatrix}1227866 & 192634 & 0\\ 192634 & 566364 & 0\\ 0 & 0 & 237308\end{bmatrix}$$

and after Trace-normalization with Thickness-Normalization *(in MPa)*.

$$\left[\hat{A}^{Tr}\right] = Tr\cdot\begin{bmatrix}0.473 & 0.091 & 0\\ 0.091 & 0.305 & 0\\ 0 & 0 & 0.111\end{bmatrix},\ \left[\hat{B}^{Tr}\right] = Tr\cdot\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\ \left[\hat{D}^{Tr}\right] = Tr\cdot\begin{bmatrix}0.541 & 0.085 & 0.006\\ 0.085 & 0.250 & 0.006\\ 0.006 & .0006 & 0.105\end{bmatrix}.$$

where the last expressions assume identical thickness for all *n* plies → laminate thickness $t = n\cdot t_k$.
The check of the traces proofs that the sum of the diagonal terms

$$\text{trace}\,[\hat{A}^{Tr}]\,/\text{Tr} = 0.473 + 0.305 + 2\cdot 0.111 = 1 = \text{trace}\,[\hat{D}^{Tr}]\qquad\text{if 'Trace'-normalized.}$$

For completing information: Tsai and Melo gave in [*Tsa15, pages 62 and 65*] rotation relations when rotating from the UD-material Coordinate System (CoS) to a so-called rotated CoS, rotated by an angle α (c = cosα, s = sinα):

$$\begin{aligned}
Q_{11}' &= c^2(c^2 Q_{11} + s^2 Q_{12}) + s^2(c^2 Q_{12} + s^2 Q_{22}) + 4\cdot c^2 s^2 Q_{66}\\
Q_{22}' &= s^2(s^2 Q_{11} + c^2 Q_{12}) + c^2(s^2 Q_{12} + c^2 Q_{22}) + 4\cdot c^2 s^2 Q_{66}\\
Q_{66}' &= c^2 s^2(Q_{11} + Q_{22}) - 2\cdot c^2 s^2 Q_{12} + (c^4 - 2c^2 s^2 + s^4) Q_{66}\\
Q_{12}' &= c^2 s^2(Q_{11} + Q_{22}) + (c^4 + s^4) Q_{12} - 4\cdot c^2 s^2 Q_{66}\quad .
\end{aligned}$$

From the stiffness elasticity invariants using the UD tensor matrix $[Q^{tr}]$ follow the formulations:

$$\begin{aligned}
Tr &= \text{'}Trace\text{'} = trace([Q^{tr}]) = Q_{11} + Q_{22} + 2\cdot Q_{66}\ ,\ \text{see }[Tsa15,\ p.\ 65]\\
I_2^{\,Q} &= 0.5\cdot\left((trace[Q^{tr}])^2 - trace[Q^{tr}]^2\right) = 2\cdot Q_{66}\cdot(Q_{11} + Q_{22}) + Q_{11}\cdot Q_{22}\\
I_3^{\,Q} &= \text{determinant}[Q^{tr}] = 2\cdot Q_{66}\cdot(Q_{11}\cdot Q_{22} - Q_{12}^{\,2})\quad .
\end{aligned}$$

2.3 Kappel's Invariants Extension

E. Kappel extended the invariant idea and observed that $I^Q = Q_{11} + Q_{22} + Q_{66} + Q_{12}$ is also an invariant. This means, that one can fully stick to a linear sum of all engineering stiffness quantities this invariant takes the full Q-set into account in contrast to Trace. However mind, this is still not a complete set of invariants. Hence, E. Kappel tried to generalize the determination of UD elasticity stiffness invariants by applying an optimization procedure. As approach he set up a four-parameter objective function

$$\left| p_1 \cdot (Q_{11}' - Q_{11}) + p_i \cdot (Q_{22}' - Q_{22}) + p_i \cdot (Q_{12}' - Q_{12}) + p_i \cdot (Q_{66}' - Q_{66}) \right|$$

with the constraints $p_i > 0$. The parametric solution reads:

$$b \cdot Q_{11}' + (b-a) \cdot Q_{12}' + b \cdot Q_{22}' + (b+a) \cdot Q_{66}' = b \cdot Q_{11} + (b-a) \cdot Q_{12} + b \cdot Q_{22} + (b+a) \cdot Q_{66}$$

with the parameter set (a,b).

Some special cases of above parametric solution equation capture all invariants below.

Table 2-4: Invariants derived by Kappel, see [Kap23]

$8 \cdot U_1$	$= 3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}$	.........Case b =3, a = 1
$8 \cdot U_4$	$= 2Q_{11} + 2Q_{22} + 6Q_{12} - 4Q_{66}$	.........Case b =1, a = -5
$\text{'Trace'} = trace[Q^{tr}] = \text{Tr} =$	$Q_{11} + Q_{22} + 2Q_{66}$	.......................Case b =1, a = 1
I_{12}	$= Q_{11} + Q_{22} + 2Q_{12}$	.... .Case b =1, a = -1
$2 \cdot I^Q = \text{'Trace'} + I_{12}$	$= 2 \cdot (Q_{11} + Q_{22} + Q_{12} + Q_{66})$	.... .Case b =1, a = 0 .

Taking all data sets into account the main finding, analogous to Tsai-Melo is, that the fiber-dominated Coefficient of Variation CoV is pretty small and counts just 2.25 %. The higher matrix-dominated CoVs do not contribute to Tr or I^Q that much, demonstrated by the following examples:

$$[Q]_{\text{ply}} = Tr \cdot ([Q^{Tr}]_{\text{ply}}) = 167.7 \cdot \begin{bmatrix} 0.888 & 0.017 & 0 \\ 0.017 & 0.052 & 0 \\ 0 & 0 & 0.028 \end{bmatrix} \text{GPa} \rightarrow (0.888 + 0.052 + 2 \cdot 0.028) = 1.$$

$$[Q]_{\text{ply}} = I^Q \cdot ([Q^{IQ}]_{\text{ply}}) = 166.0 \cdot \begin{bmatrix} 0.899 & 0.017 & 0 \\ 0.017 & 0.053 & 0 \\ 0 & 0 & 0.029 \end{bmatrix} \text{GPa} \rightarrow (0.899 + 0.053 + 0.029 + 0.017) = 1 .$$

The advantage of using Kappel's invariant I^Q is that the Coefficient of Variation (CoV) of the 'Trace'-normalized stiffness coefficients is further reduced which is significant in design.
Kappel took all stiffness matrix elements into account and also proved the invariance of I^Q. A significant further reduction of the CoV of the dominating fiber-dominated stiffness $E_{\parallel}^t$ from 2.25% to 1.51%, such improving the Trace-based results (see [*Kap23*]).

Table 2-5 presents material data sets for very different UD CFRP (epoxy) materials. The columns represent the elasticity properties, the 'Trace' (Tr = *scalar* value) and the 'Trace'-normalized elements Q_{ij} / Tr of the UD laminas. Invariant elastic properties give comprehensive information about the in-plane stiffness potential of a laminate consisting of a distinct composite material. All the elastic stiffness quantities are now fractions of the invariant I^Q.

Table 2-5: Prepreg specific elasticity composite (RP) moduli, normalized stiffness coefficients.
$Tr = Q_{11} + Q_{22} + 2 \cdot Q_{66}$, $I^Q = Q_{11} + Q_{22} + Q_{66} + Q_{12}$, *Tsai-Melo [Tsa15, black entries] and Kappel (red entries).*

Material	E_1	E_2	ν_{12}	G_{12}	Tr	Q_{11}/Tr	Q_{22}/Tr	Q_{12}/Tr	Q_{66}/Tr
IM7/977-3	191.0	9.9	0.35	7.79	217.8	0.883	0.046	0.016	0.036
T800/Cytec	162.0	9.0	0.40	5.00	182.5	0.895	0.050	0.020	0.027
T700 C-Ply 55	121.0	8.0	0.30	4.70	139.2	0.875	0.058	0.017	0.034
T700 C-Ply 64	141.0	9.3	0.30	5.80	162.8	0.871	0.057	0.017	0.036
AS4/3501	138.0	9.0	0.30	7.10	162.0	0.857	0.056	0.017	0.044
IM6/epoxy	203.0	11.2	0.32	7.10	229.6	0.889	0.049	0.016	0.031
AS4/F937	148.0	9.7	0.30	4.55	167.7	0.888	0.058	0.017	0.027
T300/N5208	181.0	10.3	0.28	7.17	206.5	0.880	0.050	0.014	0.035
IM7/8552	171.0	9.1	0.32	5.29	191.6	0.897	0.048	0.015	0.028
IM7/MTM45	175.0	8.2	0.33	5.50	195.1	0.901	0.042	0.014	0.028
IMA/M21E	154.0	8.5	0.32	4.20	171.8	0.901	0.050	0.016	0.024
AS4/8552	141.0	9.8	0.27	5.20	161.9	0.875	0.061	0.016	0.032
700GC/M21	110.0	8.0	0.31	8.20	135.2	0.819	0.060	0.018	0.061
T700/M21	135.0	8.5	0.33	4.20	152.9	0.889	0.056	0.018	0.027
T700/M21 m	117.0	7.8	0.33	3.50	132.7	0.888	0.059	0.020	0.026
T800H/3900-2	129.1	7.5	0.33	3.52	144.5	0.899	0.052	0.017	0.024
IM7/977-2	159.0	9.2	0.25	4.37	177.6	0.899	0.052	0.013	0.025
CoV	16.95	10.62	10.3	26.2	16.1	2.25	9.83	11.1	27.3
Average Tr	151.5	8.99	0.314	5.48	172.4	0.883	0.053	0.017	0.032
Median	148.0	9.0	0.32	5.2	167.7	0.888	0.052	0.017	0.028
CoV	16.95	10.62	10.3	26.2	16.1	1.51	10.09	11.07	28.55
Average I^Q	151.5	8.99	0.314	5.48	169.79	0.897	0.054	0.017	0.033
Median	148.0	9.0	0.32	5.2	166.04	0.899	0.053	0.017	0.029

'Trace' values are not only given above for the 'bar-over marked' statistical mean (average) $\overline{Tr}$ but for the statistical median, too. It is not yet known which way delivers the better estimation. The authors follow here the median value.

2.4 *Application of 'Trace' to Estimate the Stiffness Matrix [Q^{Tr}] of a Novel UD Lamina*

Some Lessons Learned which help to perform an advantageous application:

1. In the elastic domain the Q_{ij} are theoretically identical in the tensile and the compressive domain.
2. Q_{11} is the main driving entity.
3. For the bulk of standard CFRP materials a 'common Master Ply' exists, possessing a low CoV such as to see above in *Table 2-5*.

Above information encourages establishing a procedure for a novel CFRP: Measure just Q_{11} and put it together with the missing Master Ply $Q^{Tr}{}_{ij}$-values according to

$$Tr^{novel} = Q_{11}^{novel}(E_1) + \bar{T}r \cdot (Q_{22}^{\bar{T}r(master)} + 2 \cdot Q_{66}^{\bar{T}r(master)}) \quad \text{with} \quad Q_{11}^{novel} = E_1^{novel} / \left(1 - \nu_{21} \cdot \nu_{12}\right).$$

For pre-design with a new UD material one can work with the computed value Tr (or I^Q) from the measured [Q] of the new UD material and the derived Master Ply elasticity coefficients applying

Table 2-6 shows that the CoVs of the A-coefficients are very small. Using 'Trace' they are found to be between 0.76% and 6.52%. It can be seen that the master-ply data is very close to the determined median values. Choosing $I^Q = (Q_{11} + Q_{22} + Q_{66} + Q_{12})$, the results are a little better because the CoV for the dominating fiber-stiffness Q_{11} still becomes a little smaller than with Tr.

Table 2-6: $[0_2/45/\text{-}45/90]_{4S}$ *–CFRP laminates*

Material	$\tilde{A}_{11}$	$\tilde{A}_{22}$	$\tilde{A}_{66}$	$\tilde{A}_{12}$	I^Q
IM7/977-3	0.47	0.31	0.11	0.09	217.8
T800/Cytec	0.48	0.31	0.11	0.10	182.5
T700 C-Ply 55	0.47	0.31	0.11	0.09	139.2
T700 C-Ply 64	0.47	0.31	0.11	0.09	162.8
AS4/3501	0.47	0.31	0.11	0.09	162.0
T300/N5208	0.47	0.31	0.11	0.09	206.5
IM6/epoxy	0.47	0.31	0.11	0.09	229.6
AS4/F937	0.48	0.31	0.11	0.10	167.7
IM7/8552	0.48	0.31	0.11	0.10	191.6
IM7/MTM45	0.48	0.31	0.11	0.09	195.1
IMA/M21E	0.48	0.31	0.11	0.10	171.8
AS4/8552	0.47	0.31	0.11	0.09	161.9
T700GC/M21	0.46	0.30	0.12	0.08	135.2
T700/M21	0.48	0.31	0.11	0.10	152.9
T700/M21-measured	0.48	0.31	0.11	0.10	132.7
T800H/3900-2	0.48	0.31	0.11	0.10	144.5
IM7/977-2	0.48	0.31	0.11	0.10	177.6
Master ply	0.47	0.31	0.11	0.10	168.4
CoV	1.30	0.76	2.13	6.52	16.11

LL regarding the scatter of the numerically obtained 'Trace'-based stiffness quantities:

- If the average behavior of a laminate shall be modelled and a prediction for a new laminate is to provide - as best basis - average properties are to use in CLT analysis, which alone guarantees the optimally achievable estimation, namely 50% reliability.
- Considering the production-based scatter, Automated Fabrication (AF) of the semi-finished CF-plies will keep scatter lower
- One can further conclude that laminates usually have smaller CoVs. This is due to the favorable compensation of the effect of the flaws across the laminate thickness.
- The average value may become slightly lower, but the CoV-influence has a 2.3 times higher effect in the calculation of the design value.
- Normalization leads to insensitivity among many laminas which justifies the creation of a 'Master-ply', helpful when predesigning with novel UD laminas of the same fiber family. Hence, certification may permit lesser tests, at least of the smooth coupon test specimen campaigns.
- The Master ply idea fully corresponds to statistics, where the best prediction is achieved with maximum information about the parent distribution, preferably the CoV.

2.5 *Application to estimate Laminate CTEs with Kappel's Invariant* I^Q

Before, the impact of 'Trace' was on the reduction of warping. Now, reduction of Process-induced Distortion (PiD) shall be the objective. This means, to check whether the Master-ply concept may help to obtain sufficiently good CTE estimations and further help to reduce the test effort necessary for pre-design (see [*E. Kappel: On invariant combinations of Q_{ij} coefficients and a novel invariant I^Q), Composites Part C: Open Access 10 (2023) 100335], 4 pages*).

When laminate stiffness is focused, the chosen stiffness-normalizing invariant has a strong effect, however, the CTE determination process is variant-independent. This will be shown in the following paragraph. Hence, the table indicates that the exact ply data is not essential for adequate CTE- predictions, as the scatter of the different materials in terms of the thickness-normalized coefficients is very small. Focusing Process-induced Distortion PiD, from general equilibrium of the plate the membrane loading relation is of interest. From this equation the laminate CTEs can be determined for zero mechanical loading under the presumption that the temperature change is constant over the laminate thickness

For deriving the laminate-CTEs the thermal relations are of interest:

$$\{\varepsilon'\} = (\varepsilon_x, \varepsilon_y, \gamma_{xy})^T = [T_\varepsilon] \cdot \{\varepsilon\} = [T_\varepsilon] \cdot ([S] \cdot \{\sigma\} + \{\varepsilon_T\}), \quad [T_\sigma]^{-1} = [T_\varepsilon]^T, [T_\varepsilon]^{-1} = [T_\sigma]^T$$

$$\{\sigma'\} = (\sigma_x, \sigma_y, \tau_{xy})^T = [T_\sigma] \cdot \{\sigma\} = [T_\sigma] \cdot [Q] \cdot (\{\varepsilon\} - \{\varepsilon_T\}) = [Q'] \cdot (\{\varepsilon'\} - \{\varepsilon_T'\}) \quad \text{with}$$

$$\{\varepsilon_T'\} = \{\alpha_T'\} \cdot \Delta T = [T_\varepsilon] \cdot \{\varepsilon_T\} = [T_\varepsilon] \cdot \{\alpha_T\} \cdot \Delta T; \quad \{\alpha_T'\} = (\alpha_{Tx}, \alpha_{Ty}, \alpha_{Txy})^T, \quad \{\alpha_T\}_k = (\alpha_{T\parallel}, \alpha_{T\perp}, 0)_k^T .$$

<u>*Table 2-7*</u> presents all further relationships to determine the CTEs.

The shear portion α_{Txy} only becomes zero if symmetrical stacking with a balanced angle ply of the angle ω is given. The rotated CTEs are just dependent on the material choice and not dependent on the invariant I_Q .

Laminate CTEs are found independent of *Tr or* $I^{Q,}$ which explains the observation, that laminates with a certain stacking, made from different pre-pregs show usually very similar laminate CTEs, even though ply engineering constants (properties) differ strongly.

<u>*LL*</u>*:*

* *The Master-ply concept can be extended to the application for CTE determination but needs further investigation*
* *Above equations are basis for determining high-quality CTE-estimates of laminates without knowing the exact UD-ply properties*
* *Laminate CTEs are found independent of Tr or I^Q, which explains the observation, that laminates with a certain stacking, made from different prepregs show usually very similar laminate CTEs, even though ply engineering constants (properties) differ strongly*
* *An approximate homogenization of the laminate is the first task in order to avoid i.e. PID etc. This is performed by the right sub-laminate stacking.*

Table 2-7: All further relationships for the determination of the CTEs

$$\{n^0\} = [A]\cdot\{\varepsilon^0\} - \{n_T{}^0\} \;\Rightarrow\; \{n^0\} = [A]\cdot\{\varepsilon^0\} - \{n_T{}^0\} = 0 \;\rightarrow\; \{\varepsilon^0\} = [A]^{-1}\cdot\{n_T{}^0\}$$

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \begin{bmatrix} Q'_{11} & Q'_{12} & Q'_{16}\\ & Q'_{22} & Q'_{26}\\ (symm) & & Q'_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix},\; [T_\sigma] = \begin{bmatrix} c^2 & s^2 & -2sc\\ s^2 & c^2 & 2sc\\ sc & -sc & c^2-s^2\end{bmatrix},\; [T_\varepsilon] = \begin{bmatrix} c^2 & s^2 & -sc\\ s^2 & c^2 & sc\\ 2sc & -2sc & c^2-s^2\end{bmatrix}$$

The lamina-rotated expressions are $\{\varepsilon^0\} = \{\alpha_T{}'\}^{\text{laminate}}\cdot\Delta T = [T_\varepsilon]\cdot\{\alpha_T\}\cdot\Delta T$

For the laminate: Same ply material is used!

$$\{\varepsilon^0\} = \{\alpha_T{}'\}^{\text{laminate}}\cdot\Delta T = [T_\varepsilon]\cdot\{\alpha_T\}\cdot\Delta T \text{ and also } \{\varepsilon^0\} = [A]^{-1}\cdot\{n_T{}^0\}\ .$$

$$\{n_T{}^0\} = \sum [Q']_k\ [T_\varepsilon]_k\ \cdot\{\alpha_T\}\cdot\Delta T\cdot t_k,\quad [Q']_k = [T_\sigma]_k\cdot[Q]_k\cdot[T_\sigma]_k^T, [T_\sigma]^T = [T_\varepsilon]^{-1}$$

$$\Rightarrow\ \{\alpha_T{}'\}^{\text{laminate}}\cdot\Delta T = [A]^{-1}\cdot\{n_T{}^0\} = [A]^{-1}\cdot\sum_{k=1}^{n}[Q']_k\cdot\ [T_\varepsilon]_k\ \cdot\{\alpha_T\}\cdot\Delta T\cdot t_k$$

$$\{\alpha_T{}'\}^{\text{laminate}}\cdot\Delta T = [A]^{-1}\cdot\sum_{k=1}^{n}[T_\sigma]_k\cdot[Q]_k\cdot[T_\sigma]_k^T\cdot\ [T_\varepsilon]_k\ \cdot\{\alpha_T\}\cdot\Delta T\cdot t_k.$$

$$\{\alpha_T{}'\}^{\text{laminate}} = [A]^{-1}\cdot\sum_{k=1}^{n}[T_\sigma]_k\cdot[Q]_k\cdot 1\ \cdot\{\alpha_T\}\cdot t_k$$

$$(\alpha_{Tx},\alpha_{Ty},\alpha_{Txy})^T = \left(\sum_{n=1}^{n}[Q'_k]\cdot t_k\right)^{-1}\cdot\{\alpha_T\}\cdot t_k = \left(\sum_{n=1}^{n}[T_\sigma]_k\cdot[Q]_k\cdot[T_\sigma]_k^T\cdot t_k\right)^{-1}\cdot\{\alpha_T\}\cdot t_k.$$

Introduction of all elements involves I_Q

$$(\alpha_{Tx},\alpha_{Ty},2\alpha_{Txy})^T = \left(\sum_{n=1}^{n}[T_\sigma]_k\cdot I_{1+2}{}^Q\cdot[Q_Q^{Master}]\cdot[T_\sigma]_k^T\cdot t_k\right)^{-1}\cdot\sum_{n=1}^{n}[T_\sigma]_k\cdot I_{1+2}{}^Q\cdot[Q_Q^{Master}]\cdot\{\alpha_T\}\cdot t_k$$

$$(\alpha_{Tx},\alpha_{Ty},2\alpha_{Txy})^T = \left(\sum_{n=1}^{n}[T_\sigma]_k\cdot\ I_{1+2}{}^Q\cdot[Q_Q^{Master}]\cdot[T_\sigma]_k^T\right)^{-1}\cdot\sum_{n=1}^{n}[T_\sigma]_k\cdot I_{1+2}{}^Q\cdot[Q_Q^{Master}]\cdot\{\alpha_T\}$$

$$(\alpha_{Tx},\alpha_{Ty},2\alpha_{Txy})^T = \left(\sum_{n=1}^{n}[T_\sigma]_k\cdot\ [Q_Q^{Master}]\cdot[T_\sigma]_k^T\right)^{-1}\cdot\sum_{n=1}^{n}[T_\sigma]_k\cdot[Q_Q^{Master}]\cdot\{\alpha_T\}\quad .$$

3 'Double-Double' Sub-laminate Family Idea

Citation of Steve Tsai:

"***Off-axis ply angles other than 45° may open up great opportunities for design. Instead of the standard discrete plies angles, we propose the use of ply angles with angles much shallower than the 45°*** ".

In 2022 a customization and industrialization of Trace followed enhanced by the DD working group, generating an easily applicable tool and giving the community "*Double-Double - A New Perspective in the Manufacture and Design of Composites*", [*Tsa22*], ISBN 978-0-98192-43-2-9 e-book, incorporating the basic contents.

$$(0°, 45°,-45°, 90°) \text{ 'Quad' family in aerospace } \rightarrow (\pm\varphi°, \pm\psi°) \text{ proposed novel family}$$

Double-Double (DD) Laminates Idea = coupling and mass reduction concept
Double-Double means two angle-plies or means two doubles.
Two angle-plies of different fiber angles form a four-ply sub-laminate
as building block of a monolithic laminate wall

A full DD-laminate consists of sub-laminate building blocks, which are repeated as indicated by the repetition parameter r . In order to approximately achieve UD-strength high performance quality (*no crossings which cause micro-crack damage*), angle-ply layers are applied in production. A balanced angle-ply set of two angle-plies builds up a sub-laminate $[\pm\varphi / \pm\psi]$ as a building block, counting r = 1. This means that two angle-ply layers are laid upon another and then stitched for a good handling. There are different NonCrimpFabric (NCF) possibilities for the NCF-C-ply™ $\{\varphi/\text{-}\psi/\text{-}\varphi/\psi\}$:

$$\{\varphi/\text{-}\psi/\text{-}\varphi/\psi\} = (\varphi/\text{-}\varphi) + (\psi/\text{-}\psi) \quad \text{balanced angle ply semi-finished product}$$
$$\{\varphi/\text{-}\psi/\text{-}\varphi/\psi\} = (\varphi/\text{-}\psi) + (\text{-}\varphi/\psi) \quad \text{unbalanced angle ply semi-finished product .}$$

The angle values are determined by solving an optimization task. The C-PLY™ represents such a deliverable balanced angle-ply set. This specific 'ply' is developed using the most advanced technology and tow spreading process [see *Composites World*]. The C-PLY, *produced at Chomarat, France,* is comprised of unidirectional or multidirectional stitched plies from 50 g/m^2 to 600 g/m^2 per ply. The used carbon tows possess 12, 24, 48 k filaments and are provided in the domains HS (*High Strength*), IM (*Intermediate Modulus*) or HM (*High Modulus*). There are only two DD stacking sequences to be selected and not, classically, stacks from hundreds of variations.

The concept provides a novel stacking method of a new family of laminates for optimal sizing. Trace idea and Double-Double concept intend to fulfill the traditional design requirements by a novel procedure 'approximated decoupling by achieving $[B] \cong 0$' and finally optimally targeting the relatively "simple" isotropic [K]-laminate stiffness matrix. The quality of the procedure depends on the repeats r.

3.1 Reduction of Coupling in [K] by use of semi-finished DD-Stacks

The laminate stiffness matrix [K] is composed of the three sub-matrices [A], [B], [D]. *Fig.3.1* allocates the effect of each single element in the sub-matrices on the deformation behavior of the laminated wall. The elements determine whether a laminate experiences undesired twisting and warping for in-service loadings forces and moments, temperature and moisture, fabrication with curing.

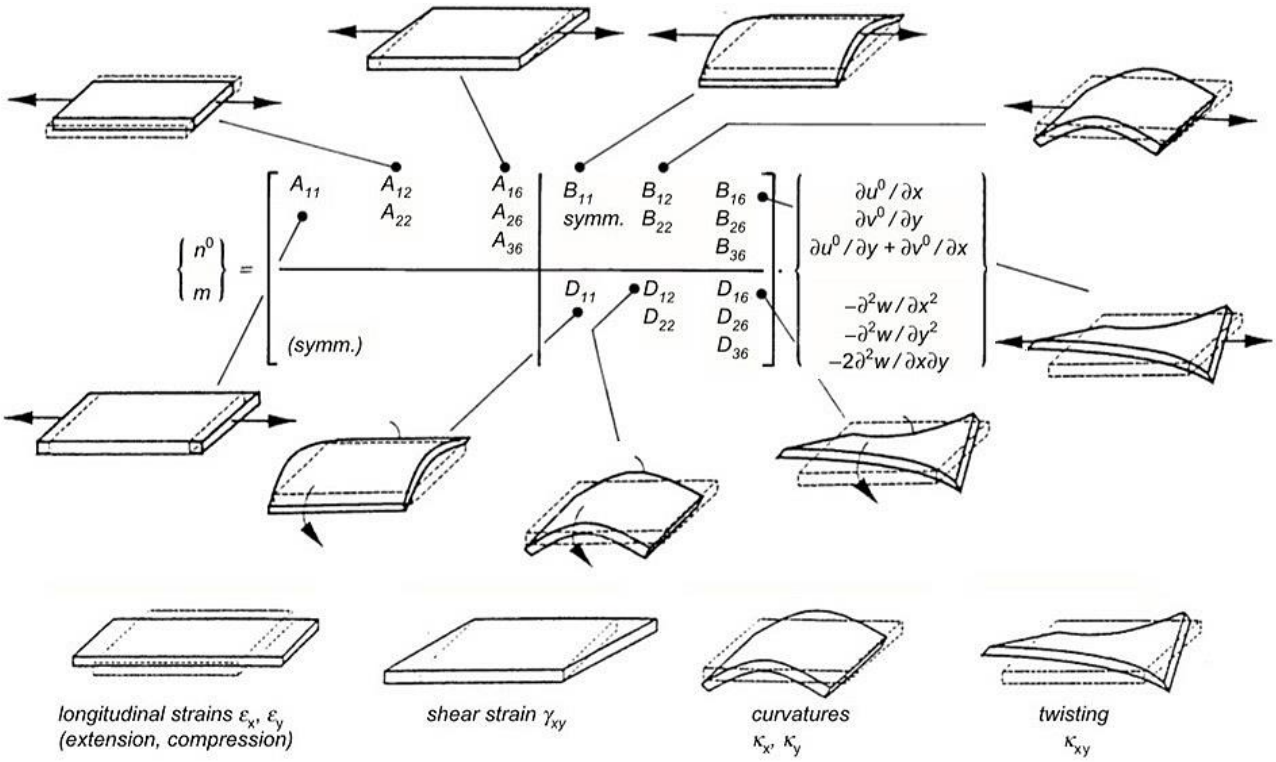


Fig.3-1: Effects of the stiffness components in [K]

Reduction of coupling encourages the optimization of [K] reducing the coupling responsible B_{ij} elements. The sub-matrix [A] is not dependent on the stack in contrast to [D]. A symmetric stack makes [B] zero (blue dots in *Fig.3-2*), however, a design requirement symmetrization is a mass bottleneck for classical design and production, because it cannot be realized economically. Appropriate stacking of a non-symmetric laminate to reduce the size of the [B] matrix is possible by using many thin layers compared to fewer thick layers. Thereby coupling will be reduced and mass saving can be obtained. *Fig.3-2* presents the filling of the sub-matrices regarding a UD-ply material – composed laminate (left) and an isotropic material (right), where no coupling is faced.

$$\text{UD-laminate: [K]} = \left\{\begin{array}{c|c} A & B \\ \hline B & D \end{array}\right\} = \begin{bmatrix} \bullet & \bullet & & \bullet & \bullet & \bullet \\ \bullet & \bullet & & \bullet & \bullet & \bullet \\ & & \bullet & \bullet & \bullet & \bullet \\ & & & \bullet & \bullet & \bullet \\ & sym & & \bullet & \bullet & \bullet \\ & & & \bullet & \bullet & \bullet \end{bmatrix} \Rightarrow \text{ isotropic: } \begin{bmatrix} \bullet & \bullet & & & & \\ \bullet & \bullet & & & & \\ & & \bullet & & & \\ & & & \bullet & \bullet & 0 \\ & sym & & \bullet & \bullet & 0 \\ & & & 0 & 0 & \bullet \end{bmatrix}$$

Fig.3-2: Occupancy (filling) of laminate stiffness matrix [K]
in the transversely-isotropic UD case and in the isotropic case, or if being optimally homogenized

Modern tapes composed of NonCrimpFabrics (*stitched multi-UD-layers, still representing sub-laminates*), allow now the use of thin plies, whereby coupling effects and costs can be reduced. The conflict, between the designer (*he desires many thin layers in order to reduce the fracture-mechanics-based low lateral micro-cracking level in transversal ply direction*) and the production engineer who prefers fewer 'thick' layers for production cost reasons, is not a big issue anymore.

Traditional UD-prepreg ply composed laminate families, for decades used in aerospace, have the layer angles (0°, $\pm 45°$, 90°), which means 4 fixed fiber directions α. They are designed according to rules, built up from experience. Basic rules are to:

(1) get D_{16} and D_{26} approximately zero in order to avoid coupling between bending and twisting,
(2) achieve symmetric laminates to obtain $[B] = 0$, and
(3) have balanced stacks which decouple extension and shear, $A_{16} = A_{26} = 0$. Thereby coupling will be reduced and mass saving can be obtained.

The ABD-matrix of the laminate determines whether a laminate experiences undesired twisting and warping for in-service loads and also due to manufacturing. Homogenization goal of Double-Double DD: A minimally-filled ABD matrix, enabled by repeated 4-ply building blocks, where full mid-plane symmetry is no longer needed. The DD-tool to realize this is a novel lay-up strategy that incorporates a 4-ply sub-laminate and uses building blocks stacked upon each other. 4 plies = 1 physical layer ≡ 1 numerical lamina = sub-laminate = building block of the full laminate Its definition is $\{\varphi / -\varphi / \psi / -\psi\}$ or $\{\varphi / -\psi / -\varphi / \psi\}$, which means a double balanced angle ply, with the angles counted relative to the length orientation. In the laminate stack the given angles are referred to the difference of the orientation angle and the laminate CoS x,y. The angular difference of successive layers should be as small as possible in order to keep the shear stresses in the interface low. A feature of the balanced angle ply, like C-Ply™ , compared to the bunch of usually applied 'Quads' is that it fits directly to the production of '$[B] = 0$ DD-laminates'.

Table 3-1 impressively informs about the homogenization process due to reduced ply thickness and increasing repeats. C-Ply-application reduces $[B]$ and thereby offers advantages especially for repair. Classical Quad-stacks are compared to DD-stacks, representing a repeat number $r = 1$ and $r = 8$. The material input is the CFRP IM7-977. Computation of $[A]$ is driven here from the pre-condition 'Equal membrane stiffness in all examples'.

To reduce above bottleneck problem, the target is the 'homogenization of the laminates'. This means to generate a laminate-stiffness matrix ABD that is approximately filled like the isotropic one but nevertheless provides different stiffness and strength capacities for plates and shells in the different directions and laminate stiffness and strength resistance will not become quasi-isotropic but remain oriented. Warping and twisting of a laminate can be suppressed by a sufficient number of repeats and the ABD-laminate stiffness matrix will approximately look like the simple isotropic one. Hence, the homogenization goal is a minimally-filled ABD matrix, enabled by repeated 4-ply building blocks, where full mid-plane symmetry is no longer needed.

The novel C-PLY™ (TM is Trade Mark, see *Fig.3-3*), a dry, multi-axial, gap-free, semi-finished NCF (stitched by a chain polyester, which harms the stack a little) with an EP-powder binder fully cured later in the final resin-system infusion process.

Table 3-1: Examples classical aerospace 'Quad-laminate' versus two 'BB-stacks'

IM7/977: $\{E\} = (191000,\ 191000,\ 9940,\ 9940,\ 7790)^T$ MPa, $\nu_{21} = 0.35$. Elasticity stiffness matrix

$$\left[Q^{Tr}\right] = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & 2\cdot Q_{66} \end{bmatrix} = \begin{bmatrix} 192.2 & 3.501 & 0 \\ 3.502 & 10.008 & 0 \\ 0 & 0 & 2\cdot 7.790 \end{bmatrix} \text{ GPa in 'Trace format':}$$

Lay-up $[0/\ 45/\text{-}45/90]_{4s}$, $t_k = 0.125$ mm, 32 layers or UD-laminas, t = 4 mm

The elements of the classical A-B-D-submatrices read (A in N/mm, B in N, D in N·mm)

$$[A] = \begin{bmatrix} 322425 & 96089 & 0 \\ 96089 & 322425 & 0 \\ 0 & 0 & 113193 \end{bmatrix},\ [B] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ [D] = \begin{bmatrix} 499515 & 126770 & 11389 \\ 126770 & 362849 & 11389 \\ 11389 & 11389 & 149643 \end{bmatrix}$$

Invariant, after Trace-normalization with Thickness-normalization (in MPa)

$$\left[\hat{A}^{Tr}\right] = Tr\cdot\begin{bmatrix} 0.370 & 0.110 & 0 \\ 0.110 & 0.370 & 0 \\ 0 & 0 & 0.130 \end{bmatrix},\ \left[\hat{B}^{Tr}\right] = Tr\cdot\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \left[\hat{D}^{Tr}\right] = Tr\cdot\begin{bmatrix} 0.430 & 0.109 & 0.010 \\ 0.109 & 0.312 & 0.010 \\ 0.010 & 0.010 & 0.129 \end{bmatrix}.$$

Check of traces proved, that the 'Trace sum' of the diagonal terms, meaning factor 2 for the third, becomes 1.

$$1 = 0.430 + 0.312 + 2\cdot 0.129 \quad \text{or} \quad \text{trace}\,[\hat{A}^{Tr}] = \text{trace}\,[\hat{D}^{Tr}] = 217810 \text{ MPa.}$$

Lay-up $\{22.5/\ \text{-}22.5/67.5/\text{-}67.5\}_r$, $r = 1$, $t_k = 1.0$ mm, 4 layers → t = 4 mm

The elements of the A-B-D-submatrices read (A in N/mm, B in N, D in N·mm)

$$[A] = \begin{bmatrix} 322425 & 96089 & 0 \\ 96089 & 322425 & 0 \\ 0 & 0 & 113193 \end{bmatrix},\ [B] = \begin{bmatrix} -257700 & 0 & -64425 \\ 0 & 257700 & -64425 \\ -64425 & -64425 & 0 \end{bmatrix},\ [D] = \begin{bmatrix} 429900 & 128051 & 82033 \\ 128051 & 429900 & -82033 \\ 82033 & -82033 & 150924 \end{bmatrix}$$

After Trace-normalization with Thickness-normalization (in MPa = N/mm²)

$$\left[\hat{A}^{Tr}\right] = Tr\cdot\begin{bmatrix} 0.370 & 0.110 & 0 \\ 0.110 & 0.370 & 0 \\ 0 & 0 & 0.130 \end{bmatrix},\ \left[\hat{B}^{Tr}\right] = Tr\cdot\begin{bmatrix} -0.148 & 0 & -0.037 \\ 0 & 0.148 & -0.037 \\ -0.037 & -0.037 & 0 \end{bmatrix},\ \left[\hat{D}^{Tr}\right] = Tr\cdot\begin{bmatrix} 0.370 & 0.110 & 0.071 \\ 0.110 & 0.370 & -0.071 \\ 0.071 & -0.071 & 0.130 \end{bmatrix}$$

The check of the 'Trace sum' of the normalized terms delivers 1.

Lay-up $\{22.5/\ \text{-}22.5/67.5/\text{-}67.5\}_r$, $r = 8$, $t_k = 0.125$ mm, 32 layers or UD-laminas → t = 4 mm

The elements of the A-B-D-submatrices read (A in N/mm, B in N, D in N·mm)

$$[A] = \begin{bmatrix} 322425 & 96089 & 0 \\ 96089 & 322425 & 0 \\ 0 & 0 & 113193 \end{bmatrix},\ [B] = \begin{bmatrix} -32213 & 0 & -8053 \\ 0 & 32213 & -8053 \\ -8053 & -8053 & 0 \end{bmatrix},\ [D] = \begin{bmatrix} 429900 & 128051 & 1282 \\ 128051 & 429900 & -1282 \\ 1282 & -1282 & 150924 \end{bmatrix}$$

After Trace-normalization with Thickness-normalization (in MPa)

$$\left[\hat{A}^{Tr}\right] = Tr\cdot\begin{bmatrix} 0.370 & 0.110 & 0 \\ 0.110 & 0.370 & 0 \\ 0 & 0 & 0.130 \end{bmatrix},\ \left[\hat{B}^{Tr}\right] = Tr\cdot\begin{bmatrix} -0.018 & -0.005 & -0.005 \\ -0.005 & 0.018 & -0.005 \\ -0.005 & -0.005 & 0.000 \end{bmatrix},\left[\hat{D}^{Tr}\right] = Tr\cdot\begin{bmatrix} 0.370 & 0.110 & 0.001 \\ 0.110 & 0.370 & -0.001 \\ 0.001 & -0.001 & 0.130 \end{bmatrix}$$

The check of the 'Trace sum' of the normalized terms delivers 1.

.

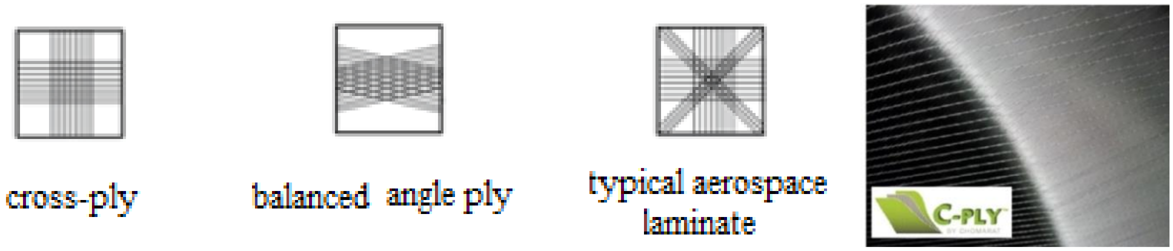


Fig. 3-3: (left) Lay-up types; ((right) C-PLY™ example (from DD-book).

Fig.3-4 presents a plate composed according to the DD concept and thereby replacing a *C-PLY™* ‘building block’ by a similarly built four UD-layer ‘building block’. The material is UD M21E/IMA (*from Hexcel, A350 aircraft material*) medium-grade prepreg of 0.184 mm, $t_{building\ block}$ = 0.736 mm. In the figure fine polishing of the surface was performed to identify the individual plies of the building block-stack. The inclined cut through the cross-section of the laminate, see upper picture, shows the individual plies in one building block which is here a UD-ply-building block. The optical folding is the result of a flatter cut. From the first displayed building block, indicated r = 3, the thickness is unfortunately not visible on the image. Indexing

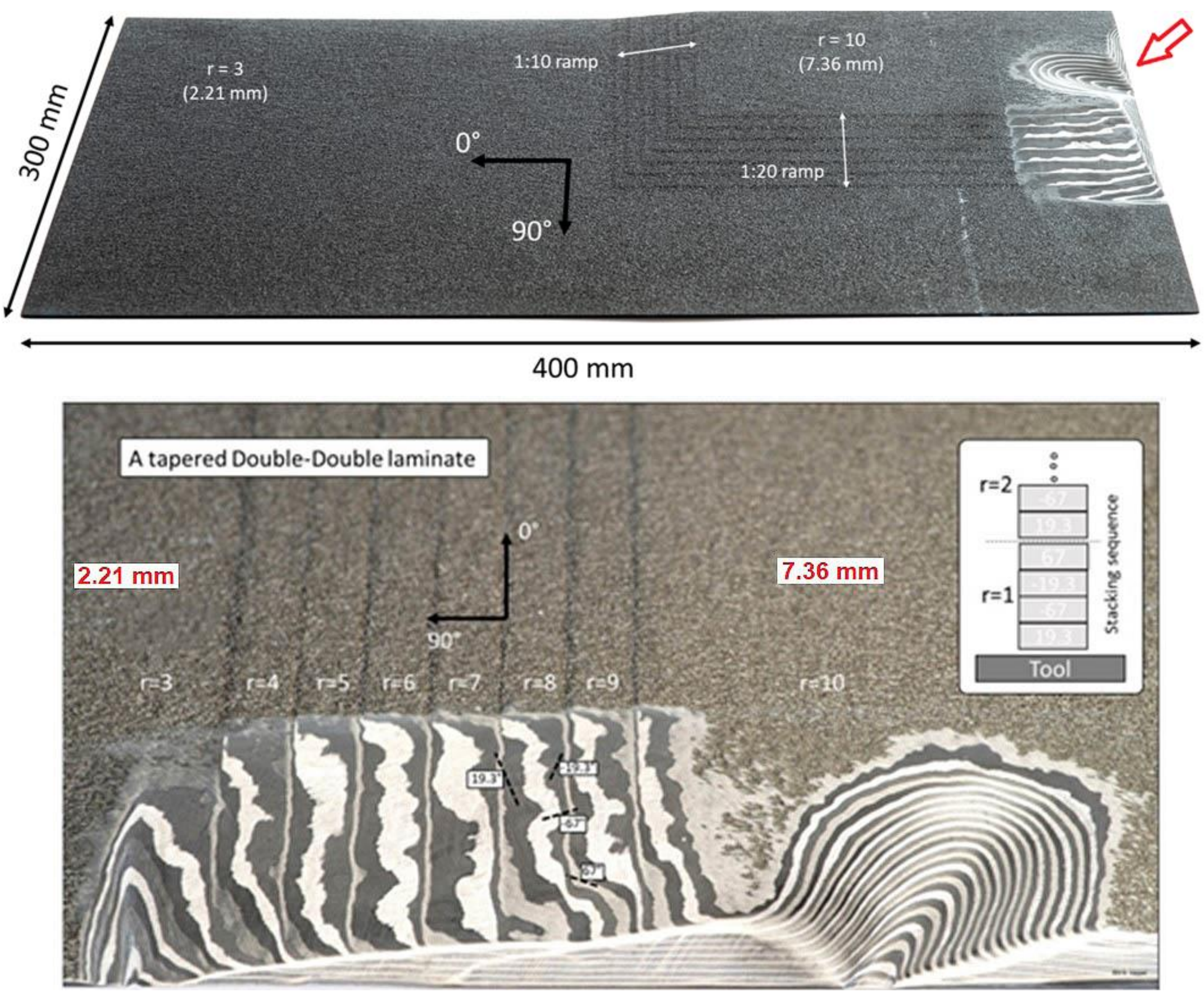


Fig.3-4, Tapered ‘Double-Double laminate’: (up) Test specimen, lay-up: {19.3/ -67/ -19.3/ 67}$_{3\text{-}10}$; (down) inclined cross-section cut (r ≡ classical running index k). [Kappel, DLR] runs via $1 < k < n$ with k the running layer and n the total number of C-PLY™ -layer repetitions. In classical UD-CLT it reads $1 < k < n$ with n the total number of UD layers.

Table 3-2: 2D Numerical examples: Classical aerospace 'Quad-stack-laminate' versus 'DD-stack', t = 4 mm

Input: $\{\bar{E}\} = (191000,\ 191000,\ 9940,\ 9940,\ 7790)^T$ MPa, $\nu_{21} = 0.35$, MPa = N/mm^2

Elasticity stiffness matrix $[Q] = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} = \begin{bmatrix} 192200 & 3501 & 0 \\ 3502 & 10008 & 0 \\ 0 & 0 & 7790 \end{bmatrix}$ MPa.

The elements of the classical A-B-D-submatrices read (A in N/mm, B in N, D in N·mm).

Lay-up [0/ 45/-45/90], prepreg, t_k = 1 mm, 4 layers or UD-laminas, t = 4 mm

The elements of the classical A-B-D-submatrices read (A in N/mm, B in N, D in N·mm)

$$[A] = \begin{bmatrix} 322425 & 96089 & 0 \\ 96089 & 322425 & 0 \\ 0 & 0 & 113193 \end{bmatrix},\ [B] = \begin{bmatrix} -273333 & 0 & -45555 \\ 0 & 273333 & -45555 \\ -45555 & -45555 & 0 \end{bmatrix},\ [D] = \begin{bmatrix} 511034 & 46018 & 0 \\ 46018 & 511034 & 0 \\ 0 & 0 & 68891 \end{bmatrix}$$

Lay-up $[0/\ 45/-45/90]_2$, t_k = 0.5 mm, 8 layers or UD-laminas, t = 4 mm

$$[A] = \begin{bmatrix} 322425 & 96089 & 0 \\ 96089 & 322425 & 0 \\ 0 & 0 & 113193 \end{bmatrix},\ [B] = \begin{bmatrix} -136666 & 0 & -22778 \\ 0 & 136666 & -22778 \\ -22778 & -22778 & 0 \end{bmatrix},\ [D] = \begin{bmatrix} 450409 & 107543 & 0 \\ 107543 & 450409 & 0 \\ 0 & 0 & 130416 \end{bmatrix}$$

Lay-up $[0/\ 45/-45/90]_S$, t_k = 0.5 mm, 8 layers or UD-laminas, t = 4 mm, symmetric stack

$$[A] = \begin{bmatrix} 322425 & 96089 & 0 \\ 96089 & 322425 & 0 \\ 0 & 0 & 113193 \end{bmatrix},\ [B] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ [D] = \begin{bmatrix} 723741 & 107543 & 45555 \\ 107543 & 177076 & 45555 \\ 45555 & 45555 & 130416 \end{bmatrix}$$

Lay-up $[0/\ 45/-45/90]_{4S}$, t_k = 0.125 mm, 32 layers or UD-laminas, t = 4 mm, symmetric stack

$$[A] = \begin{bmatrix} 322425 & 96089 & 0 \\ 96089 & 322425 & 0 \\ 0 & 0 & 113193 \end{bmatrix},\ [B] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ [D] = \begin{bmatrix} 499515 & 126770 & 11389 \\ 126770 & 362849 & 11389 \\ 11389 & 11389 & 149643 \end{bmatrix}$$

Lay-up $\{22.5/\ -22.5/67.5/-67.5\}_r$, NCF, r = 1, t_k = 1.0 mm, 4 layers $\rightarrow$ t = 4 mm

$$[A] = \begin{bmatrix} 322425 & 96089 & 0 \\ 96089 & 322425 & 0 \\ 0 & 0 & 113193 \end{bmatrix},\ [B] = \begin{bmatrix} -128850 & 0 & -32213 \\ 0 & 128850 & -32213 \\ -32213 & -32213 & 0 \end{bmatrix},\ [D] = \begin{bmatrix} 429900 & 128051 & 20508 \\ 128051 & 429900 & -20508 \\ 20508 & -20508 & 150924 \end{bmatrix}$$

Lay-up $\{22.5/\ -22.5/67.5/-67.5\}_r$, r = 2, t_k = 0.5 mm, 8 layers $\rightarrow$ t = 4 mm

$$[A] = \begin{bmatrix} 322425 & 96089 & 0 \\ 96089 & 322425 & 0 \\ 0 & 0 & 113193 \end{bmatrix},\ [B] = \begin{bmatrix} -64425 & 0 & -41017 \\ 0 & 64425 & 41017 \\ -41017 & 41017 & 0 \end{bmatrix},\ [D] = \begin{bmatrix} 429900 & 128051 & 32213 \\ 128051 & 429900 & 32213 \\ 32213 & 32213 & 150924 \end{bmatrix}$$

Lay-up $\{22.5/\ -22.5/67.5/-67.5\}_r$, r = 8, t_k = 0.125 mm, 32 layers or UD-laminas $\rightarrow$ t = 4 mm

$$[A] = \begin{bmatrix} 322425 & 96089 & 0 \\ 96089 & 322425 & 0 \\ 0 & 0 & 113193 \end{bmatrix},\ [B] = \begin{bmatrix} -32213 & 0 & -8053 \\ 0 & 32213 & -8053 \\ -8053 & -8053 & 0 \end{bmatrix},\ [D] = \begin{bmatrix} 429900 & 128051 & 1282 \\ 128051 & 429900 & -1282 \\ 1282 & -1282 & 150924 \end{bmatrix}$$

3.2 Traditional 'Quad-Laminates' versus 'DD-Laminates'

Traditional UD-ply composed laminate families, for decades used in aerospace, have the layer angles (0°,45°,-45°,90°), which means 4 fixed fiber directions α. They are designed according to rules, built up from experience. Basic rules are to get D_{16} and D_{26} approximately zero in order to avoid coupling between bending and twisting and to achieve symmetric laminates obtaining $[B] = 0$. Further, balanced stacks decouple extension and shear $A_{16} = A_{26} = 0$.

The concept provides a novel stacking method of a new family of laminates for optimal sizing. Trace idea and Double-Double concept intend to fulfill the traditional design requirements by a novel procedure. The aim is an approximated decoupling by achieving $[B] \cong 0$ and finally optimally targeting the relatively "simple" isotropic K-matrix. The quality of the procedure depends on the repeats *r*. The coefficients $\boldsymbol{B_{ij}}$ depend on 1/r, whereas the $\boldsymbol{D_{16}}$ and $\boldsymbol{D_{26}}$ (*bold black-dotted in above Fig.3-1)* depend on $1/r^2$ and thus the decaying effect by r^2 is stronger! For more details, especially on the applied Thickness-/'Trace'-normalized sub-matrices, see [*Kap22*]. Therein, it is found for an Omega stringer profile that a B-matrix-minimal lay-up is $\{\varphi / -\psi / -\varphi / \psi\}$.

In *Table 3-2* a classical laminate stack was compared to two DD-stacks, representing a repeat number *r* = 1 and *r* = 8. The material input is the CFRP above, namely IM7-977. Computation of [A] is driven here from the pre-condition 'Equal membrane stiffness in all three examples'.

3.3 Optimum stack determination

Final challenge is the DD-application in optimization considering Minimum Mass, several Design Load Cases and production Side Constraints.

Of course, to obtain an optimal stack in the sizing phase of the design usually requires the consideration of numerous permutations. This number of permutations can be reduced by applying in the optimization the Trace-normalized stiffness quantities. After optimization, several sub-laminate stacks may be optimal and one has to decide which one should be taken. The quadratic distribution of the inter-laminar shear stress across the thickness of an isotropic cross-section under shear delivers some measure for the achieved homogenization of the laminate stack.

In general there are two different tasks:

1. DD-substitution of a conventional quad-stack laminate and
2. Fully free DD-optimization, performed analogously to the sizing of sheets. According to the fact that each DD-sublaminate is balanced the normal strains in the plane are decoupled from the shear strains. This simplifies extremely the optimization procedure.

1. DD-substitution case

Two objective functions based on $\left[\widehat{A}\right], \left[\widehat{D}\right]$ for i = 1, 2, 6 it is to search

$$\rightarrow \quad \min\left(\left|\widehat{A}_{ii} - \widehat{A}_{ii,ref}\right|\right) \quad \text{and} \quad \min\left(\left|\widehat{D}_{ii} - \widehat{D}_{ii,ref}\right|\right) .$$

If the structural task is an in-plane problem then one can apply $\min\left(\left|\widehat{A}_{ii} - \widehat{A}_{ii,ref}\right|\right)$ and for a bending problem $\min\left(\left|\widehat{D}_{ii} - \widehat{D}_{ii,ref}\right|\right)$.

. One big advantage of DD sub-laminates is that their stiffness matrix elements can be described analytically which is different to traditional carpet plot quasi-isotropic ‘Quad’ sub-laminates but opens a novel idea for laminate design including optimization. Multiple FEA’s can be avoided by employing stiffness transformations within the continuous field of DD sub-laminates *instead of multiple FE analyses within the discrete design alternatives for the traditional ’Quad’sub-laminates.*

The replacement of an existing laminate, such as the ‘Quad’ one, can be performed via the transformation functions

$$\hat{V}_1 = (\hat{A}_{11} - \hat{A}_{22})/(2U_2),\ \hat{V}_2 = (\hat{A}_{11} + \hat{A}_{22} - 2U_1)/(2U_3),\ \hat{V}_3 = (\hat{A}_{61} + \hat{A}_{62})/U_2,\ \hat{V}_4 = (\hat{A}_{61} - \hat{A}_{62})/(2U_3)$$

inserting

$$U_1 = 3/8 \cdot (Q_{11} + Q_{22}) + 1/4 \cdot (Q_{12} + 2Q_{66}),\ U_2 = 1/2 \cdot (Q_{11} - Q_{22}),\ U_3 = 1/8 \cdot (Q_{11} + Q_{22}) - 1/4 \cdot (Q_{12} + 2Q_{66})$$

regarding $cos\, 2\psi = \hat{V}_1 + [-\hat{V}_1^2 + 0.5 \cdot \hat{V}_2 + 0.5],\ \ cos\, 2\varphi = 2\hat{V}_1 - cos\, 2\psi$

with the layer fiber volume-bounds: 0°, 90° < 50 %.

Chapter 13 in [*Tsa22*] presents Case Studies using MicMac for a ‘Quad’-‘DD’-replacement (→1.) and Lamsearch to find the best DD (→2.). Here in *chapter A4*.

2. Free DD-optimization case

Fully free DD-optimization case with, due to the design task, differently combined side constraints. For the design requirements Loading and Stiffness are to provide:

* Side constraint “Stability”: Buckling condition,
* Side constraint “Stress limit (= strength)”: Strength failure condition (criterion)
* Side constraint “Strain limit”: Strain failure condition
* Side constraint “Deformation limit”: Deformation limit.

Thereby, it is to discriminate a structural limit state from a material-linked design limit state.

A procedure, obtaining optimum fiber-oriented DD-sub-laminate stacks was published in [*Rot22*] and modified by Cuntze, *Fig.3-5*.

A laminate search algorithm for the best fiber angles is to provide. Here, the program Lamsearch is free available, which significantly minimizes the numerical effort involved in optimization. See *Annex A-4*.

A ‘Trace’-based direct sizing approach firstly selects a basic sub-laminate (*building block of the laminate* with an initial thickness. Then, the required stiffness is to realize and for each Design Load Case a linear elastic finite element analysis (FEA) is performed considering all significant failure modes, not only strength wherefore in each FE element a material reserve factor is determined. Finally, in order to fully meet the all design requirements regarding stiffness and loading the initial thickness can be linearly scaled, in case of in-plane loading. Also another material may be used after a new material screening..

When structural designing mind again, please:

* Whereas the modelling is performed with average properties and average stress-strain curves, in the verification of the final laminate design - task-required - upper or lower or average properties are to insert in the analysis.

* The present <u>stress-based</u> design verification in Aerospace requires stress criteria and A- or B-strength design allowables. A <u>strain-based</u> design verification as precondition for certification, would need permission of the FAA and the EASA (European Union Aviation Safety Agency) including strain criteria coupled to agency-permitted strain design allowables.

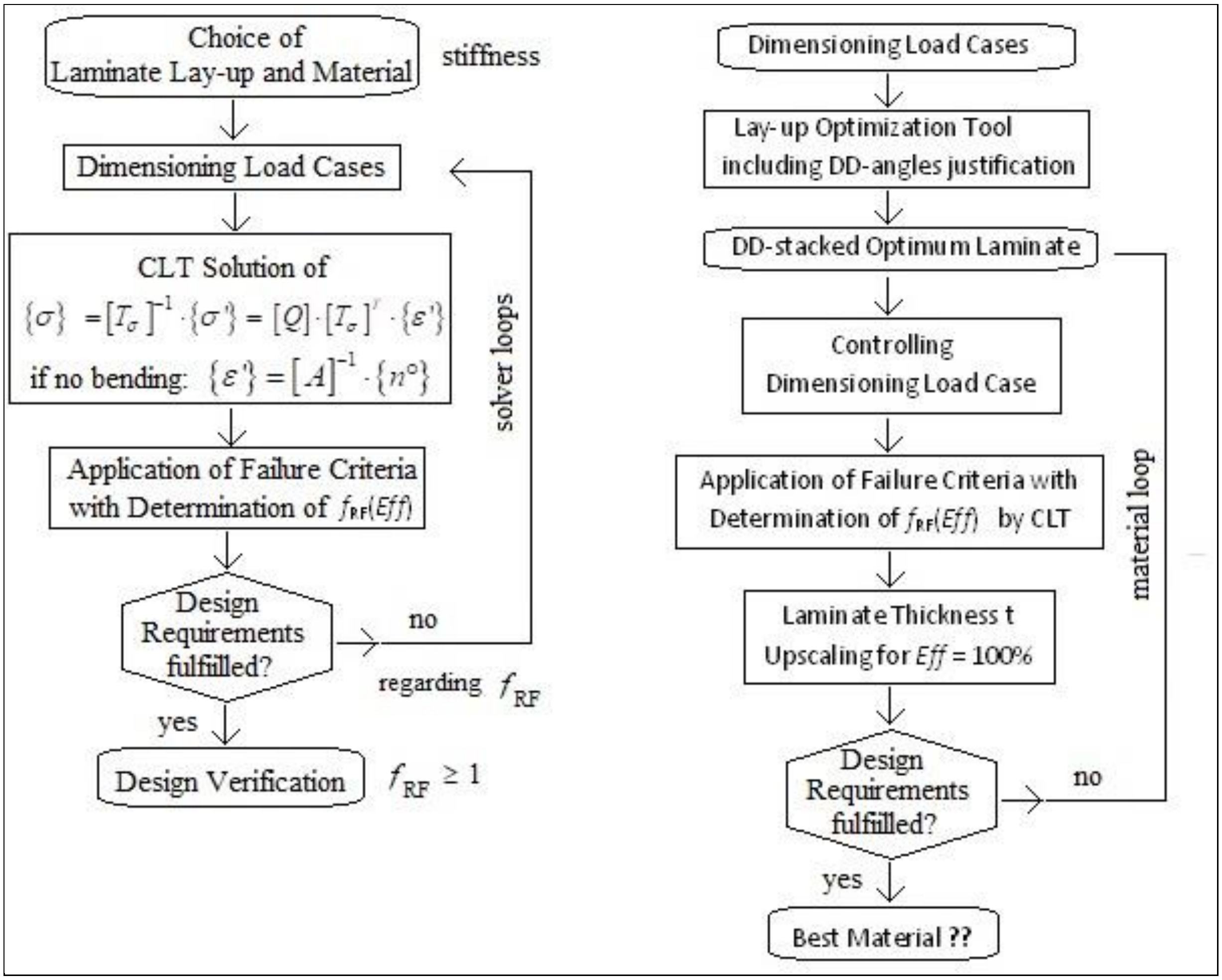


Fig.3-5: (left) Traditional design flow, such as 'Quad' (rigid), (right) DD Lamsearch-based design flow, (material open structure, see also[Rot22]

3.4 Optimum Patching

Repair requires local thickening of the existing laminate. For design reasons, the stiffness matrix must not be changed.

The DD-procedure with building-blocks (4-plies at once) is of interest for the upcoming production methods Automated Fiber Placement AFP, Automated Tape Laying ATL and AFPP (Automated Fiber Patch Placement). These AF methods permit to reduce the stress concentration problems at ply-drops, resin pockets and other flaw locations. *Fig.3-6* [*CUN22*]

The figure below presents a procedure when using Automated Fiber Patch Placement. Why not moving here from the varying quad-stack family (0°, 45°, 90°) to DD-stacks?

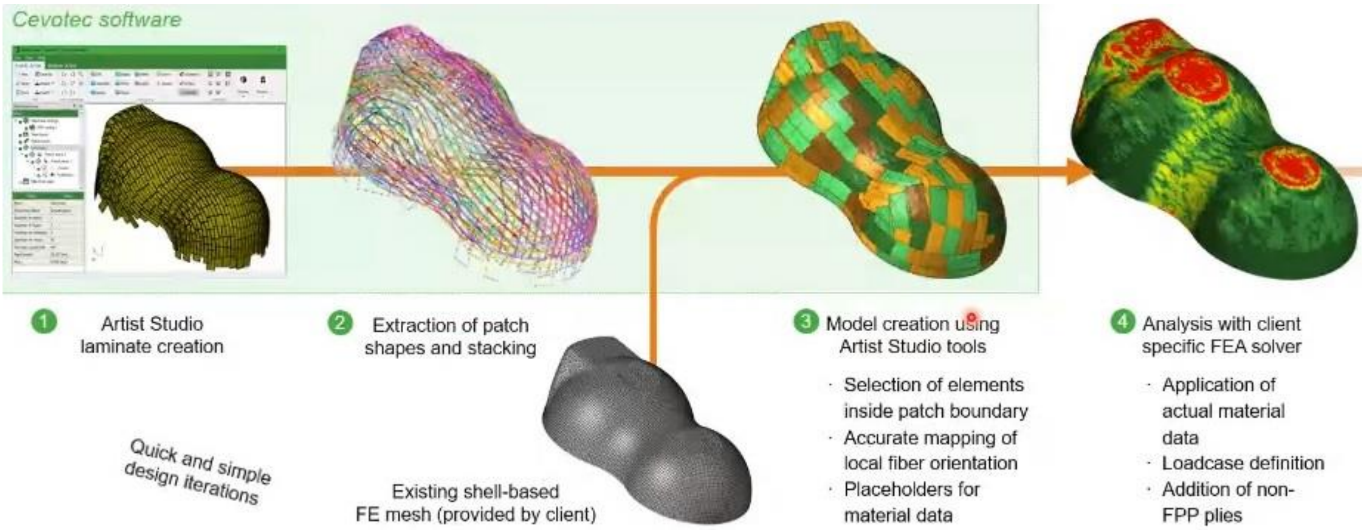


Fig.3-6: Automated Fiber Patch Placement [courtesy Cevotec Software]

LL:

- *Trace-based Master-ply stiffness and strength makes optimization possible and practical.*
- *The influence of the repeat factor r with 1/r and 1/r² is clearly shown by the decreasing off-diagonal elements of the two laminates with r =1 and r = 8, see Table 3-5.* $[\hat{A}^{Tr}]$ *and* $[\hat{D}^{Tr}]$ *become narrower for increasing repeats r which is a desired homogenization effect.*

Precautionary Material Data Table:

Composite properties V_f = 0.60	$E_{\parallel}$ GPa	$E_{\perp}$ GPa	$G_{\parallel\perp}$ GPa	$\nu_{\perp\parallel}$	$\bar{R}^t_{\parallel}$ MPa	$\bar{R}^c_{\parallel}$ MPa	$\bar{R}^t_{\perp}$ MPa	$\bar{R}^c_{\perp}$ MPa	$\bar{R}_{\perp\parallel}$ MPa	$\mu_{\perp\parallel}$
1 Toray T300/Ep, 7	135	5.6	1.3	0.32	1850	1470	40	125	95	0.2
2 T800/ Cytec, 7	162	9.0	5.0	0.32	2700	1570	63	145	98	0.2
3 IM7/ 977-3, 7	191	9.9	7.8	0.35	3250	1600	62	98	75	0.2
4 T700/M21GC, 7	126	8.3	4.1	0.3	2230	1537	71	202	78	0.2
5 Toray M60J, 5µm	365	6	4	0.3	2010	785	32	165	>39	0.2
6 E-glass/MY750,	46	16.2	5.8	0.28	1280	800	40	145	73	0.2
GFRP, Vetrotex P	49	15	6.5	0.27						
CFRP Pitch K13C6k/Ep	900	9	5	0.3						
BsFRP Basalt/Ep	55	12	4.6	0.3						

4 ‘Omni (*principal FPF strain-linked*) failure envelope’

4.1 Derivation of the ‘Omni failure envelope’ using the Tsai Procedure

<u>Background of Tsai’s Idea with its Envelope Derivation Procedure:</u>

In contrast to stresses, strains are linearly distributed over the thickness at least of thin laminates. This behavior could be a design advantage when laying out laminates. In this context, Tsai’s idea was to derive on basis of a generally loaded single ply a strain-formulated Non-FRP area and using this area to check whether the principal strains of a critical lamina (ply) location of the designed laminate lies within this area. Such an application works for all lay-ups. The procedure uses for the derivation of the ‘Omni failure envelope’ average strength properties $\bar{R}$. for the single lamina the following steps are to go for each principal loading ratio, applying <u>*Fig.4-1*</u>:

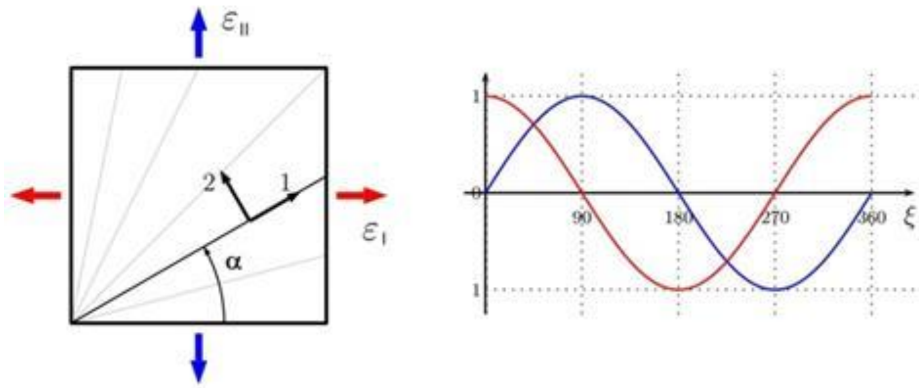


Fig.4-1: Procedure performed for each ply-orientation 0° ≤ α ≤ 90° and principal strain loading ratio angle ξ

A superposition of the envelopes of all conceivable layer orientations, see <u>*Fig.4-2*</u>, finally results in a conservative firm principal strain envelope. This firm envelope is termed Tsai’s ‘Omni failure envelope’ (*omni means all*). These principal FPF strains are force loading-representatives. They are derived by using a FPF strength criterion, see <u>*Table 4-1*</u>.

Table 4-1: Derivation of the Non-FPF area as area inside of the FPF- ‘Omni failure envelope’

- Take the external lamina (ply) principal strains (*laminate, k=1, single ply*) ε_I, ε_{II} as varying representatives of the force loading and as coordinates of the envisaged graph ‘Non-FPF area’ inside Tsai’s so-called ‘Omni failure envelope’
- Determine values of *Eff* $^{\text{modes}}$ for each ply, oriented under the loading angle α, and of the principal strain ratio angle ξ, regarding *Fig. 4-1*
- Determine FPF failure strains $\varepsilon_{I,\text{FPF}}, \varepsilon_{II,\text{FPF}}$ from applying a strength criterion SFC

  $Eff_{\text{FPF}} = [(Eff^{\parallel\sigma})^m + (Eff^{\parallel\tau})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\parallel})^m + (Eff^{\perp\tau})^m]^{m^{-1}} = 1$ Cuntze or Tsai-Wu

  $$\frac{\sigma_1^2}{\bar{R}_\parallel^t \cdot \bar{R}_\parallel^c} + \sigma_1 \cdot \left(\frac{1}{\bar{R}_\parallel^t} - \frac{1}{\bar{R}_\parallel^c}\right) + \frac{2F_{12}}{\sqrt{\bar{R}_\parallel^t \cdot \bar{R}_\parallel^c \cdot \bar{R}_\perp^t \cdot \bar{R}_\perp^c}} \cdot \sigma_1 \cdot \sigma_2 + \frac{\sigma_2^2}{\bar{R}_\perp^t \cdot \bar{R}_\perp^c} + \sigma_2 \cdot \left(\frac{1}{\bar{R}_\perp^t} - \frac{1}{\bar{R}_\perp^c}\right) = 1$$
- For all the i (ξ,α)-combinations from $Eff_{\text{FPF},i}$ compute the factor $f_{RF,\,i} = 1/Eff_{\text{FPF},i}$
- Store data and determine strain FPF-envelope points and map the full envelope.

<u>LL</u>: The ‘Omni failure envelope’ is a strength criterion-based failure curve that is displayed by graphs using principal strain coordinates, which proportionally represent the failure stress loading due to the linear elasticity model.

Exemplarily, for 3 UD plies out of an arbitrary stack *Fig.4-2* presents the associate 3 FPF principal strain envelopes according to the associated principal FPF-stresses. This means that the failure strains are elastically derived from the failure stresses. In the figure some principal stress state points $(\sigma_{\mathrm{I}}, \sigma_{\mathrm{II}})$ are attached onto the principal strain state points curve $\varepsilon_{\mathrm{II}}(\varepsilon_{\mathrm{I}})$.
In the isotropic case the magnitude of the stress normal to the principal plane (at zero shear stress) is termed principal stress and the associated strain is called principal strain. In the cases of anisotropy this does not work anymore.

For design verification the strength Design Allowables *R* are to apply.

The internal area of the 3 plies (**0°, 45°, 90°**) in *Fig.4-2* can be termed Non-FPF failure area or intact FPF-free area and is limited by a failure envelope. This area becomes a general one, if all *i* combinations are treated and the failure envelope becomes the 'Omni-failure envelope', which will be the focus now. *Fig.4-3* presents the intact FPF-free area for the two strength failure criteria of Tsai-Wu and Cuntze. It displays different 'butterflies' a name, how the Cuntze termed the bunch of ***i*** FPF-curves, derived by applying above two strength failure criteria SFCs.

There are some significant differences, where the reasons of which are still to investigate. The figure visualizes the (ξ,α)-combinations to be executed, *i* = 361 strain states were evaluated and the corresponding point on the envelope.

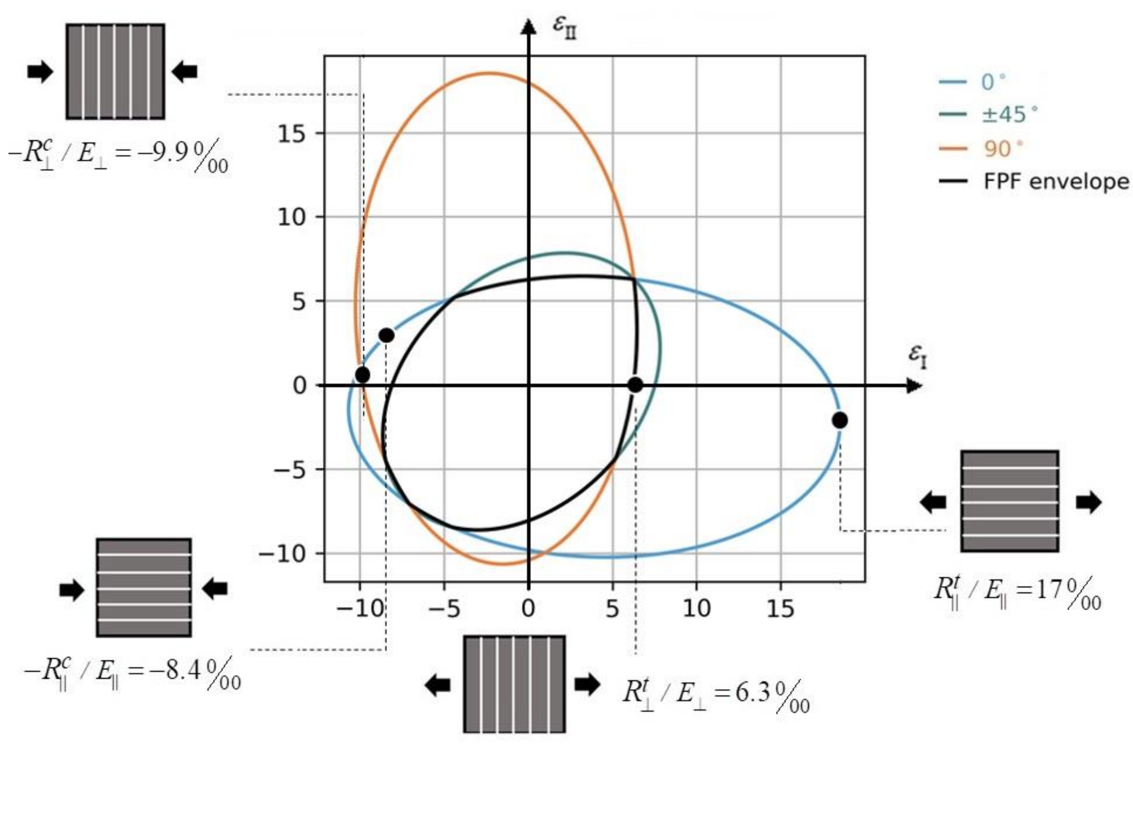


Fig.4-2, FPF, Tsai-Wu: FPF-envelopes Eff = 100% of single UD-laminas (3 ply angles) under 4 different stress states potentially leading to FPF in terms of FPF failure stresses-linked equivalent principal strains. ε in ‰. IM7/ 977-3

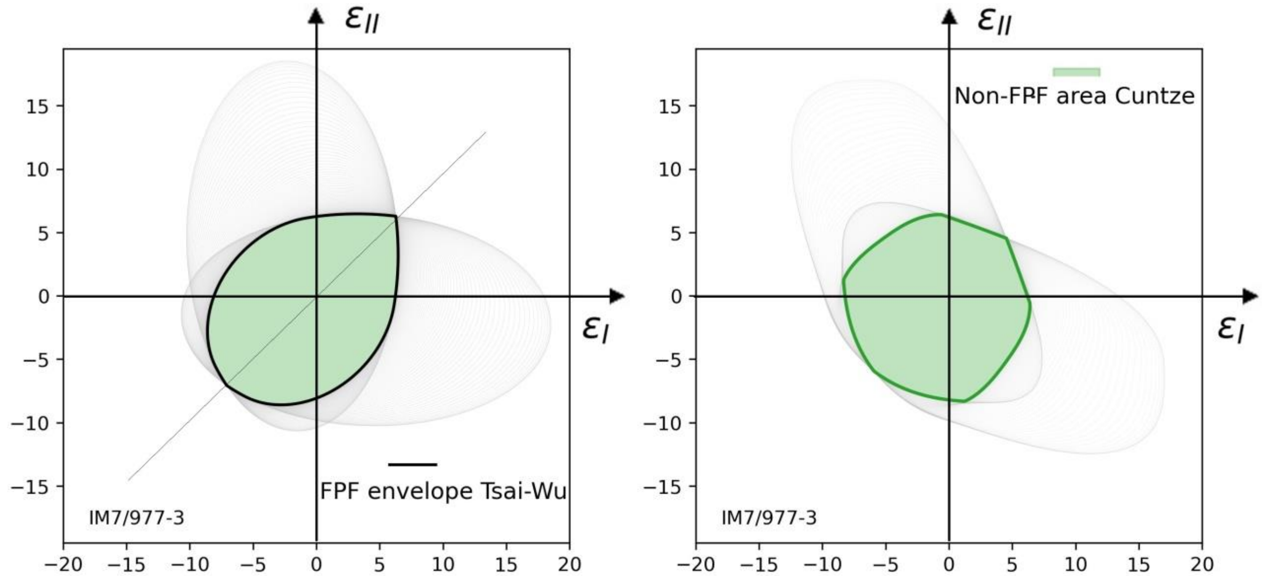


Fig.4-3: Bundle of all FPF envelopes = 'butterflies': All ply FPF-envelopes enclosing a non-FPF failure area; 0°< α < 90° (91 ply angles). Principal strain in ‰, suffix FPF is skipped. CFRP IM7/977-3. In the pictures: (left) Tsai-Wu with $\mu_{\perp\|} = 0,\ F_{12} = -0.5$ *and (right) Cuntze with* $\mu_{\perp\|} = 0.2,\ m = 2.7$.

Fig.4-4 depicts the Non-PDF areas for two 'higher performance' CFRP materials. The associate Tsai-Wu envelope has been implemented and shows a significant effect of the SFC used. The different properties determine the shape of the obtained symmetrical 'butterfly'.

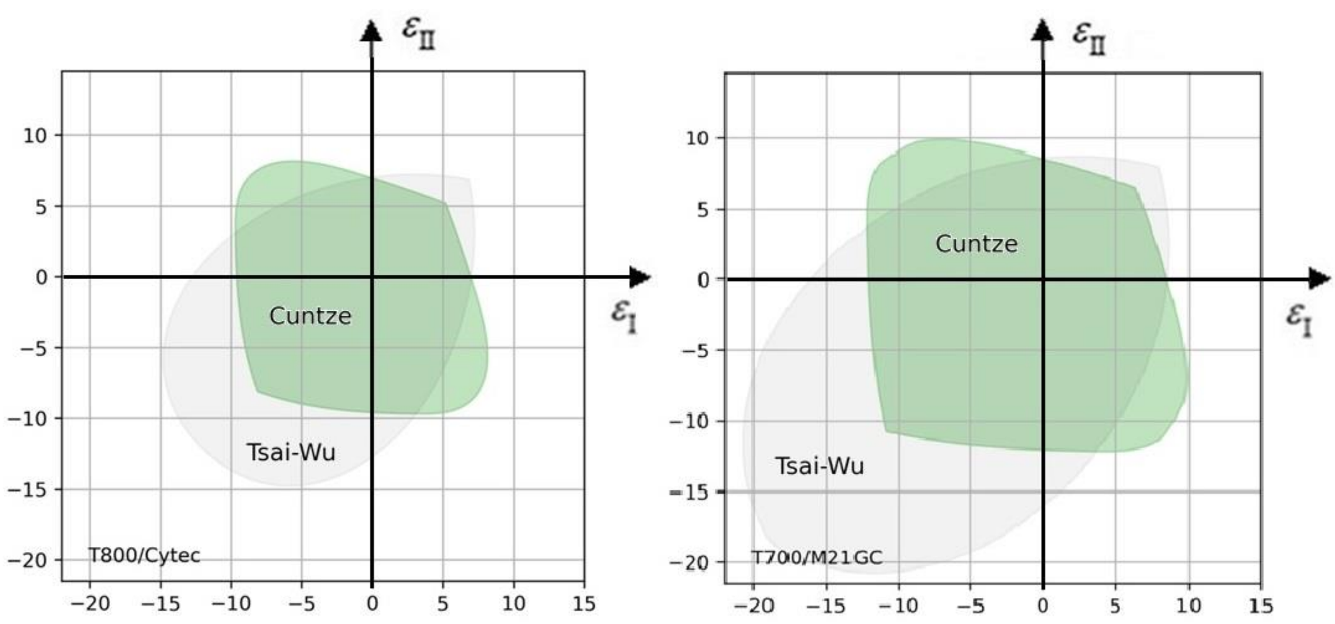


Fig.4-4: 'Non-FPF area' of two UD materials, Tsai-Wu (grey) versus Cuntze (green): (left) T800/Cytec, (right) T700/M21GC, ε in ‰

Finally *Fig.4-5 (left)* comprises the Non-FPF areas of five materials and *Fig.4-5 (right)* intentionally provides for comparison reasons the area of a very stiff CFRP. Drawing the right conclusions here is a task that still needs to be done later. The difference of the shapes of a standard modulus CFRP with GFRP seems to come from the fact that the GFRP is less anisotropic. It can be further concluded that the difference Tsai-Wu to Cuntze becomes smaller with decreasing anisotropy as it is the case with GFRP.

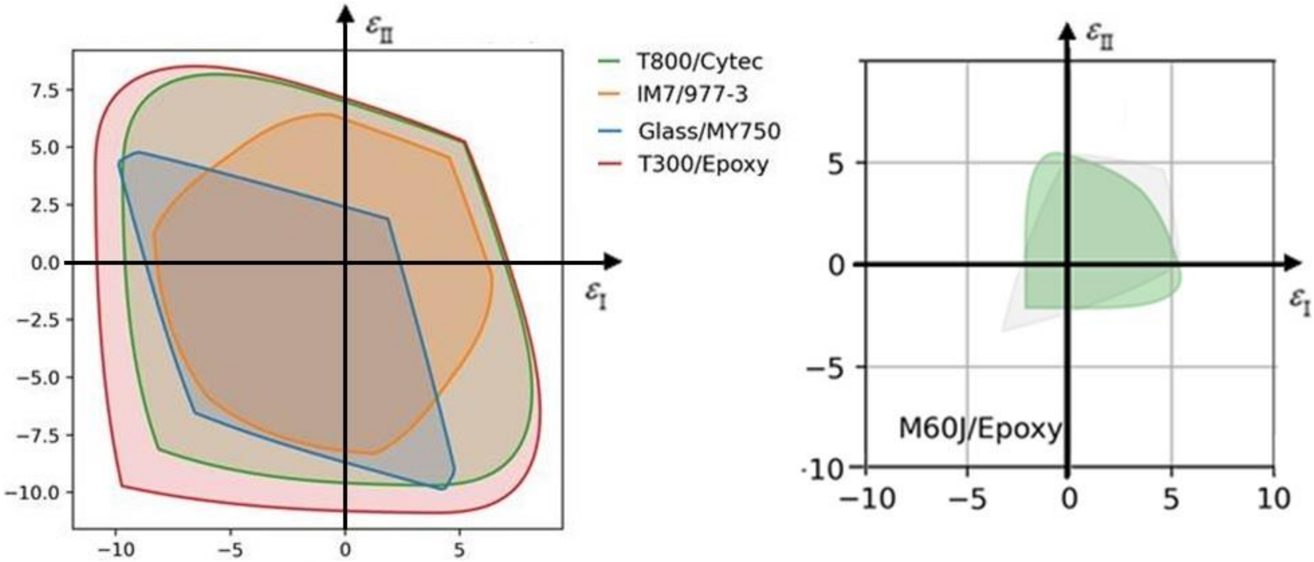


Fig.4-5: 'Non-FPF areas: (left Cuntze) Compilation T300+ IM7 +T800 + glass, (right Tsai-Wu (grey) with Cuntze (green)) very stiff PAN-UHM CFRP (Toray M60J/Ep); ε in ‰

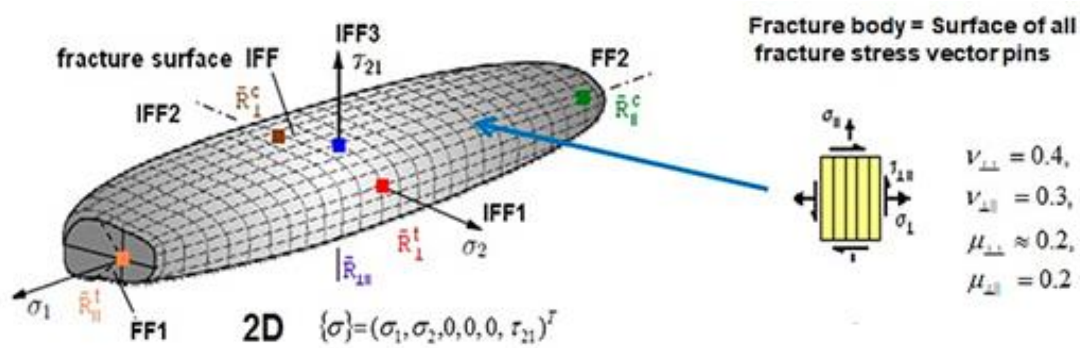
4.2 Derivation of the 'Omni FPF envelope' using Cuntze's direct Procedure

Meanwhile developed a formula for the failure envelope containing a procedure of derivation the Non-FPF domain and of the material reserve factor f_{RF}

<u>Background of the Procedure</u>

- The well validated 2D UD failure body $(\sigma_2, \sigma_1, \tau_{21})$ is the physical basis of the non-FPF area $\varepsilon_{II}(\varepsilon_I)$ inside of the 'Omni FPF envelope'
- Cuntze's hope: There is a distinct 'master' plane τ_{21} = constant of the failure body that determines the minimum non-FPF area $\varepsilon_{II}(\varepsilon_I)$ advantageously applicable in linear elastic pre-design.

<u>*Fig.4-6*</u> *up* below depicts 4 relevant (*left after full checking*) horizontal length cross-sections of the FPF body $(\sigma_2, \sigma_1, \tau_{21})$, below.



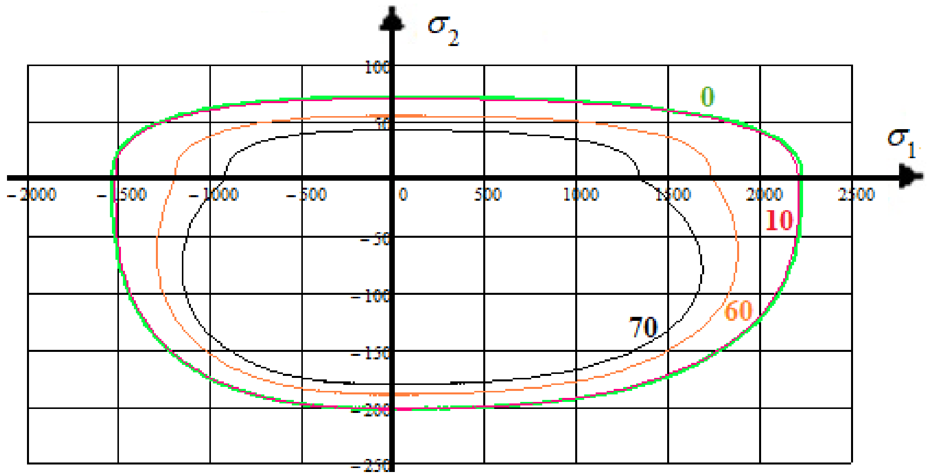


Fig.4-6: (up) FPF body. (below) FPF-envelopes $\sigma_2(\sigma_1, \tau_{21} = const)$ *) for 4 planes* τ_{21}*= const. T700*

LL: The investigation of various cross–sections τ_{21}*=constant proved that* τ_{21}*=0 delivers the smallest non-FPF area, thus making a simpler pre-design of arbitrary laminates possible*

Determination of the 'Omni FPF principal strain envelope'

Fig.4-7 (left) presents the resulting Omni principal strain FPF curves $\varepsilon_{II}(\varepsilon_I)$ with a not unambiguously solution for each parameter level τ_{21} = const. $\varepsilon_{II}(\varepsilon_I)$

In the right graph in *Fig.4-7* the second solution-linked additional outer curve parts are to exclude. Eventually *Fig.4-7* (right) shows the 'cleaned-up' envelope, representing *Eff* = 100%, of the non-FPF area. The cleaned-up graph above is identical to the non-FPF area obtained by the Tsai procedure. Domains of the envelope could be dedicated to the locally faced failure mode FF or IFF.

Table 4-2 presents both the procedures the 'butterfly one and the direct one.

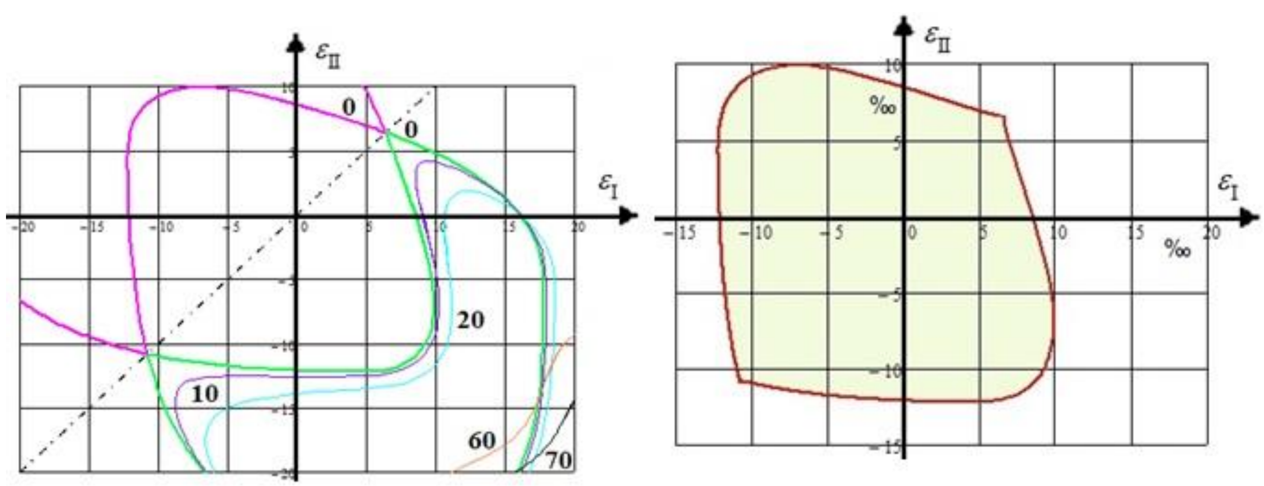


Fig.4.-7: Mirror-inverted envelope of the Non-FPF area (Cuntze procedure) IM7/977-3

Of highest interest is the reserve factor. Does the 'Principal strain procedure' deliver smaller values than the classical 'Ply-by-ply procedure' and thus remaining on the Safe Side when applying?

Below follows the application of Cuntze's direct procedure to determine the 'Omni failure curve'. In *Table 4-3* a more detailed description of the basic equations of Cuntze's direct procedure' is given. The solution is: Solving the quadratic equation $a \cdot Eff^2 + b \cdot Eff + c = 0$.

Table 4-3: Direct procedure to determine the 'Omni failure curve' of Cuntze and of Tsai-Wu

Ply strains: $\varepsilon_1 = s_{11}\cdot\sigma_1 + s_{12}\cdot\sigma_2$, $\varepsilon_1 = s_{12}\cdot\sigma_1 + s_{22}\cdot\sigma_2$, $(\gamma_{12} = 0)$ with $s_{11} = \frac{1}{E_1}$, $s_{12} = \frac{-\nu_{21}}{E_1}$, $s_{22} = \frac{1}{E_2}$

Principal strains: $\varepsilon_I = 0.5\cdot(\varepsilon_1+\varepsilon_2) + \sqrt{(\varepsilon_1-\varepsilon_2)^2/4}$, $\varepsilon_{II} = 0.5\cdot(\varepsilon_1+\varepsilon_2) - \sqrt{(\varepsilon_1-\varepsilon_2)^2/4}$

**Cuntze :* $\{\sigma\} = (\sigma_1, \sigma_2, \tau_{12})^{\mathrm{T}}$, $\{\bar{R}\} = (\bar{R}_{||}^t, \bar{R}_{||}^c, \bar{R}_{\perp}^t, \bar{R}_{\perp}^c, \bar{R}_{\perp||})^T$ with friction $\mu_{\perp||} = 0.2$, $m = 2.7$

$$\left(\frac{(\sigma_1 + |\sigma_1|)\cdot E_{||}}{2\cdot R_{||}^t}\right)^m + \left(\frac{(-\sigma_1 + |\sigma_1|)\cdot E_{||}}{2\cdot R_{||}^c}\right)^m + \left(\frac{\sigma_2 - |\sigma_2|}{2\cdot\bar{R}_{\perp}^t}\right)^m + \left(\frac{-\sigma_2 + |\sigma_2|}{2\cdot\bar{R}_{\perp}^c}\right)^m = 1 = 100\%$$

Execution of one implicit solution by applying Mathcad 15 to obtain $\rightarrow \sigma_2(\sigma_{1,\mathrm{FPF}}) \Rightarrow \varepsilon_{II}(\varepsilon_{I,\mathrm{FPF}})$.

**Tsai-Wu :* $\{\sigma\} = (\sigma_1, \sigma_2, \tau_{12} = 0)^{\mathrm{T}}$, $\{\bar{R}\} = (\bar{R}_{||}^t, \bar{R}_{||}^c, \bar{R}_{\perp}^t, \bar{R}_{\perp}^c, \bar{R}_{\perp||})^T$, $F_{12} = -0.5$, $R\cdot Eff = \sigma$

$$\frac{\sigma_1^{\,2}}{\bar{R}_{||}^t\cdot\bar{R}_{||}^c} + \sigma_1\cdot\left(\frac{1}{\bar{R}_{||}^t} - \frac{1}{\bar{R}_{||}^c}\right) + \frac{2F_{12}}{\sqrt{\bar{R}_{||}^t\cdot\bar{R}_{||}^c\cdot\bar{R}_{\perp}^t\cdot\bar{R}_{\perp}^c}}\cdot\sigma_1\cdot\sigma_2 + \frac{\sigma_2^{\,2}}{\bar{R}_{\perp}^t\cdot\bar{R}_{\perp}^c} + \sigma_2\cdot\left(\frac{1}{\bar{R}_{\perp}^t} - \frac{1}{\bar{R}_{\perp}^c}\right) = 1$$

Solution by solving the quadratic equation $a/Eff^{\,2} + b/Eff = 1 \rightarrow \sigma_2(\sigma_{1,\mathrm{FPF}}) \Rightarrow \varepsilon_{II}(\varepsilon_{I,\mathrm{FPF}})$.

with $a = \frac{\sigma_1^{\,2}}{\bar{R}_{||}^t\cdot\bar{R}_{||}^c} + \frac{2\cdot F_{12}\cdot\sigma_1\cdot\sigma_2}{\sqrt{\bar{R}_{||}^t\cdot\bar{R}_{||}^c\cdot\bar{R}_{\perp}^t\cdot\bar{R}_{\perp}^c}} + \frac{\sigma_2^{\,2}}{\bar{R}_{\perp}^t\cdot\bar{R}_{\perp}^c}$, $b = \sigma_1\cdot\left(\frac{1}{\bar{R}_{||}^t} - \frac{1}{\bar{R}_{||}^c}\right) + \sigma_2\cdot\left(\frac{1}{\bar{R}_{\perp}^t} - \frac{1}{\bar{R}_{\perp}^c}\right)$.

Table 4-2: Full presentation of all the procedures used, T700

Relationships: 2D linear elasticity applied: Objective is the material reserve factor

Muli-axial Stresses : $f_{RF} = \dfrac{\text{Strength Design Allowable } R}{\text{Stress at } j \cdot \text{Design Limit Load}} > 1$ (uni-axial) and in

Principal strains : $f_{RF} = \sqrt{(\varepsilon_{I,FPF}{}^2 + \varepsilon_{II,FPF}{}^2)} / \sqrt{(\varepsilon_I{}^2 + \varepsilon_{II}{}^2)} = \varepsilon_{I,FPF} / \varepsilon_I$ ($\Leftarrow$ beam $f_\varepsilon = \dfrac{\varepsilon_{II}}{\varepsilon_I}$) .

Ply strains: $\varepsilon_1 = s_{11} \cdot \sigma_1 + s_{21} \cdot \sigma_2$, $\varepsilon_2 = s_{21} \cdot \sigma_1 + s_{22} \cdot \sigma_2$, $\gamma_{12} = s_{66} \cdot \tau_{21}$; $\{\sigma\} = (\sigma_1, \sigma_2, \tau_{12})^T$

with $s_{11} = 1/E_1$, $s_{21} = -\nu_{21}/E_1$, $s_{22} = 1/E_2$, $s_{66} = 1/G_{12}$.

$$[T_\sigma]_k = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix}_k, \quad [T_\varepsilon]_k = \begin{bmatrix} c^2 & s^2 & -sc \\ s^2 & c^2 & sc \\ 2sc & -2sc & c^2 - s^2 \end{bmatrix}_k, \quad [Q]_k \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_k$$

Principal laminate strains activated by the loaded laminate given in the structural CoS(x,y), c = cosγ, s = sinγ

$$\varepsilon_I = 0.5 \cdot [(\varepsilon_1 + \varepsilon_2) + \sqrt{(\varepsilon_1 - \varepsilon_2)^2 + \gamma_{12}{}^2}], \quad \varepsilon_{II} = 0.5 \cdot [(\varepsilon_1 + \varepsilon_2) - \sqrt{(\varepsilon_1 - \varepsilon_2)^2 + \gamma_{12}{}^2}]$$

$$\{\varepsilon'\}_k = [T_\varepsilon]_k^{-1} \cdot \{\varepsilon_{pr}\}, \quad \{\sigma'\}_k = [Q']_k \cdot \{\varepsilon'\}_k \quad \text{with } [Q'] = [T_\sigma] \cdot [Q] \cdot [T_\sigma]^T, \quad \{\sigma\}_k = [T_\sigma]_k^{-1} \cdot \{\sigma'\}_k$$

If linear analysis delivers a sufficiently good solution, then $\sigma \sim$ load and $RF \equiv f_{RF} = 1/Eff$

Stress-strain relation reads $\sigma_1 = (s_{21} \cdot \varepsilon_{II} - s_{22} \cdot \varepsilon_I)/(s_{21}{}^2 - s_{11} \cdot s_{22})$, $\sigma_2 = \varepsilon_{II} - s_{21} \cdot \sigma_1)/s_{22}$.

Here Model Mapping $\{\bar{R}\}$, $\mu_{\perp\parallel}$ $\Rightarrow$ Design Verification $\{R\} = (R_\parallel^t, R_\parallel^c, R_\perp^t, R_\perp^c, R_{\perp\parallel})^T$, $\mu_{\perp\parallel}$

Input, T700, $\{\bar{R}\} = (\bar{R}_\parallel^t, \bar{R}_\parallel^c, \bar{R}_\perp^t, \bar{R}_\perp^c, \bar{R}_{\perp\parallel})^T \rightarrow (2230, 1537, 71, 202, 78)^T$ MPa,

$\{\bar{R}\}$, friction $\mu_{\perp\parallel} = 0.2$, $m = 2.7$, example loading $\varepsilon_I = 0.007$, $\varepsilon_{II} = 0.005 = f_\varepsilon \cdot \varepsilon_I$, $f_\varepsilon = \varepsilon_{II} / \varepsilon_I$.

Stress-principal strain relations read like above .

*Stress Procedure Cuntze: Lamina task, solved by ply-by-ply failure analysis

$Eff^m = [(Eff^{\parallel\sigma})^m + (Eff^{\parallel\tau})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m]$ with the mode portions inserted, 2D,

$$Eff = [(\frac{(\sigma_1 + |\sigma_1|)}{2 \cdot \bar{R}_\parallel^t})^m + (\frac{(-\sigma_1 + |\sigma_1|)}{2 \cdot \bar{R}_\parallel^c})^m + (\frac{\sigma_2 + |\sigma_2|}{2 \cdot \bar{R}_\perp^t})^m + (\frac{-\sigma_2 + |\sigma_2|}{2 \cdot \bar{R}_\perp^c})^m + (\frac{|\tau_{21}|}{\bar{R}_{\perp\parallel} + 0.5 \cdot \mu_{\perp\parallel} \cdot (-\sigma_2 + |\sigma_2|)})^m]^{1/m}$$

Example 0°-ply, $\varepsilon_I = \varepsilon_x = \varepsilon_\parallel \rightarrow \sigma_1 = 900$ MPa , $\sigma_2 = 59$ MPa $\rightarrow Eff = 0.70 \Rightarrow f_{RF} = 1/Eff = 1.4$.

*Principal Strain Procedure Cuntze: classical laminate task, solved by a laminate failure analysis; $\tau_{12} = 0$

Due to $\varepsilon_1 = \varepsilon_I$, $\varepsilon_2 = \varepsilon_{II}$ for the 2 failure determining stresses follows

$\sigma_1 = (s_{21} \cdot \varepsilon_2 - s_{22} \cdot \varepsilon_1) / (s_{21}{}^2 - s_{11} \cdot s_{22})$ and $\sigma_2 = (\varepsilon_2 - s_{21} \cdot \sigma_1) / s_{22}$ which is to insert into

the FPF-criterion-based 'Omni principal strain failure envelope' formula

$$(\frac{(\sigma_1 + |\sigma_1|)}{2\bar{R}_\parallel^t})^m + (\frac{(-\sigma_1 + |\sigma_1|)}{2\bar{R}_\parallel^c})^m + (\frac{\sigma_2 + |\sigma_2|}{2\bar{R}_\perp^t})^m + (\frac{-\sigma_2 + |\sigma_2|}{2\bar{R}_\perp^c})^m + (0)^m = 1 = 100\% \ .$$

The equations above still take into account – in anticipation – that shear and shear strain need not to consider.

*Original Principal Strain Procedure Tsai-Wu : $\{R\} = (\bar{X}^t, \bar{X}^c, \bar{Y}^t, \tilde{Y}^c, \bar{S}_{12})^T$, $F_{12} = -0.5$; $\tau_{12} \neq 0$

$$\frac{\sigma_1{}^2}{\bar{R}_\parallel^t \cdot \bar{R}_\parallel^c} + (\frac{\sigma_1}{\bar{R}_\parallel^t} - \frac{\sigma_1}{\bar{R}_\parallel^c}) + \frac{2F_{12}\sigma_1 \cdot \sigma_2}{\sqrt{\bar{R}_\parallel^t \cdot \bar{R}_\parallel^c \cdot \bar{R}_\perp^t \cdot \bar{R}_\perp^c}} + \frac{\sigma_2{}^2}{\bar{R}_\perp^t \cdot \bar{R}_\perp^c} + (\frac{\sigma_2}{\bar{R}_\perp^t} - \frac{\sigma_2}{\bar{R}_\perp^c}) + \frac{\tau_{12}{}^2}{\bar{R}_{\perp\parallel}{}^2} = 1$$

4.3 Pre-design Example by using the 'Omni Non-FPF area'

Laminate Design Verification is traditionally performed by a 'ply-by-ply' analysis, assessing the obtained ply (lamina) stresses $\{\sigma\}$ in the critical location of the critical plies (see [*Kap24*]). Now, a simpler more global assessment is possible by using in-plane principal strains of the laminate, strains which represent the loading. Such principal strains are a standard output of modern FE software.

Execution of the Design Check under the Presumption: Linear Analysis, proportional stressing is permitted, $\sigma \sim \varepsilon$, see *Table 4-4*.

Table.4-4: Procedure of checking a probably critical design stress state

A Non-FPF area within an 'Omni failure envelope' is given for the chosen laminate material ➢ FEA delivers the maximum state of the 3 strains of the laminate stack ➢ Transformation into the 2 principal strains as coordinates of the Non-FPF area ➢ Check, whether the strain point $(\varepsilon_I, \varepsilon_{II})$ lies within the Non-FPF area, ➢ Eventually. Determination of the material reserve factor f_{RF} = vector length ratio of *failure strain/design strain.*

Tsai's so-called 'Omni principal strain strain envelope' surrounds a Non-FPF or a Non-LPF area, respectively. FPF is required if the design requirement asks to fulfill a First-Ply-Failure in the critical locations of the plies of the laminate.

The LPF, if to apply, is required to fulfill a Last-Ply-Failure limit. However, this usually involves a non-linear analysis up to the ultimate failure load of the structural part, in order to cope with the previously still given reserve factor definition:

$$\text{stress-defined} \quad f_{RF} = \frac{\text{Strength Design Allowable } R}{\text{Stress at } j \cdot \text{Design Limit Load}} > 1 \quad \Rightarrow$$

$$\text{load-defined} \quad RF_{\text{ult}} = \frac{\text{non-linearly determined ultimate failure load at } Eff = 100\%}{j_{\text{ult}} \cdot \text{Design Limit Load}}.$$

In *Fig.4-8* for three single plies the FPF *failure strain envelopes* are displayed. Four 'loading' points are added to visualize some uni-axial failure stress-based principal strain points $(\varepsilon_I, \varepsilon_{II})$ on the FPF-envelopes. The right part of the figure presents the area which is free of FPF (intact) regarding the 3 plies, termed 'Omni failure envelope' by Tsai. In addition, for a chosen load level in order to outline the different reserves a strain-based material Reserve Factor f_{RF} are marked. The Reserve Factors are given by the vector length ratio = *failure point value* divided by the *load point value*. According to the assumed linearity load or stress versus strain the load-defined *RF* is determined linearly.

Table 4-5 collects the obtained values for the classical 'Ply-by-ply procedure' regarding very different ply orientations and for the 'Principal strain procedure' with just one computation. The numbers were obtained by a Mathcad 15-calculation.

Due to the still envisaged comparison of procedures, namely the classical 'Ply-by-ply' and the 'Principal FPF-principal strain' procedure, the Design Factor of Safety (FoS) is focused. For simplicity reasons the FoS j is kept 1 and as strength values the average values are used, see *Table 4-5*. As SFCs those of Cuntze and Tsai-Wu were foreseen to apply.

For the real Design Verification as FoS j may to be applied 1.20 and as strengths the Strength Design Allowables. This shrinks the strength failure body (3D) for the 'Ply-by-ply' procedure and the non-FPF principal strain area (2D) for the other one.

Table 4-5: Proof, that the application of the FPF envelope is on the safe side.(T700)
Eff corresponds to the so-called 'Tsai strength ratio' R, which is not the strength ratio R^c/R^t

α °	ε_I ‰	ε_{II} ‰	σ_1 MPa	σ_2 MPa	τ_{21} MPa	Eff_σ	f_{RF}	$\varepsilon_{I,FPF}$ ‰	$\varepsilon_{II,FPF}$ ‰	$f_{RF,\varepsilon}$	σ_1 MPa	σ_2 MPa
Cuntze, FailureModeConcept-based 'Modal' UD criteria set												
0	6	5	773	57	0	0.60	1.66	7.2	6.0	**1.20**	932	68
30			680	61	-4	0.71	1.41					
45			649	63	0	0.75	**1.33**					
60			680	61	+4	0.71	1.41					
90			773	57	0	0.60	1.66					
0	5	6	773	57	0	0.60	1.66	5.6	6.7	**1.11**	722	70
30			680	61	-4	0.71	1.41					
45			680	63	0	0.75	**1.33**					
90			773	57	0	0.60	1.66					
0	6	-5	748	-27	0	0.06	17.7	17.4	-14.5	**2.9**	2167	-77
30			-277	21	-39	0.20	**4.90**					
45			-2.77	38	0	0.19	5.29					
90			-277	21	+39	0.20	4.90					
Tsai-Wu, 'Global' UD criterion												
0	6	5	773	57	0	0.56	1.78	9.0	7.5	**1.50**	1163	85
30			680	61	-4	0.63	1.58					
45			649	63	0	0.65	**1.54**					
90			773	57	0	0.63	1.58					

Fig.4-8 presents the procedure used within Mathcad 15 and applying Cuntze's SFC.

Vorgabe

$$epIFPF := 0.002 \qquad \sigma1 := 100 \qquad \sigma2 := 10$$

$$epIFPF = s11\cdot\sigma1 + s21\cdot\sigma2 \qquad f\varepsilon\cdot epIFPF = s21\cdot\sigma1 + s22\cdot\sigma2$$

$$\left(\frac{\sigma1+|\sigma1|}{2R1t}\right)^{mint} + \left(\frac{\sigma2+|\sigma2|}{2R2t}\right)^{mint} + \left(\frac{-\sigma1+|\sigma1|}{2R1c}\right)^{mint} + \left(\frac{-\sigma2+|\sigma2|}{2R2c}\right)^{mint} = 1$$

$$M := Suchen(epIFPF, \sigma1, \sigma2) \qquad M = \begin{pmatrix} 0.00723 \\ 931.69522 \\ 68.4298 \end{pmatrix}$$

$$M_0 = 7.231\times10^{-3} \qquad M_1 = 932 \qquad M_2 = 68$$

$$\varepsilon IFPF := M_0 \qquad \sigma1 := M_1 \qquad \sigma2 := M_2$$

$$\boxed{\varepsilon IFPF = 0.0072} \qquad \boxed{\sigma1 = 932} \qquad \boxed{\sigma2 = 68}$$

$$\boxed{\varepsilon I = 0.006} \qquad \boxed{\varepsilon II = 0.005} \qquad \boxed{f\varepsilon = 0.833}$$

$$\varepsilon IIFPF := f\varepsilon\cdot\varepsilon IFPF \qquad \varepsilon IIFPF = 0.00603 \qquad fRF := \frac{\varepsilon IFPF}{\varepsilon I} \qquad \boxed{fRF = 1.205}$$

Fig.4-8: Determination of the material reserve factor employing Cuntze's SFCs, T700

LL: *The 'Principal strain procedure' is on the 'Safe Side', due to 1.2 < 1.4! The novel Direct determination of the 'Omni-envelope' works well.*

Fig.4-9 may give (again) an explanation for the differences of the material reserve factor values from the SFCs of Tsai-Wu and of Cuntze.

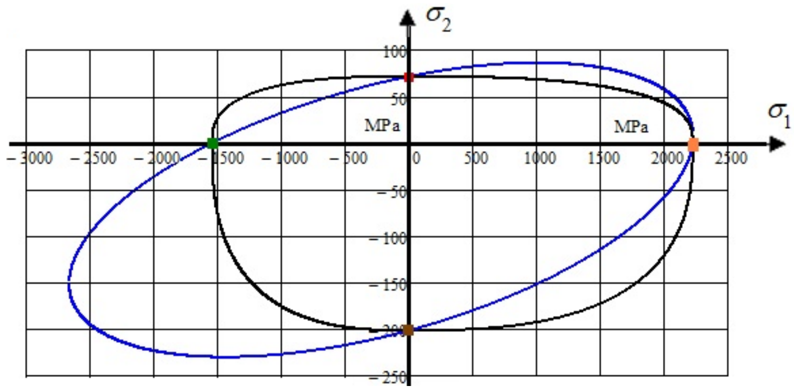


Fig. 4-9: Cross-section $\sigma_2(\sigma_1)$, Cuntze and Tsai-Wu, T700

4.4 Pre-Design Tool based on the internal Circle of the 'Omni FPF envelope'

Fig.4-10 informs how for 3 differently oriented UD plies pre-design verification could be obtained with a material reserve factor $f_{RF} > 1$. The left part figure uses the full Non-FPF area and the right part figure the so-called 'Unit Circle' as Pre-Design Tool. Accepting to be more conservative the idea arose to use the internal circle with the radius r_{FPF} as design tool.

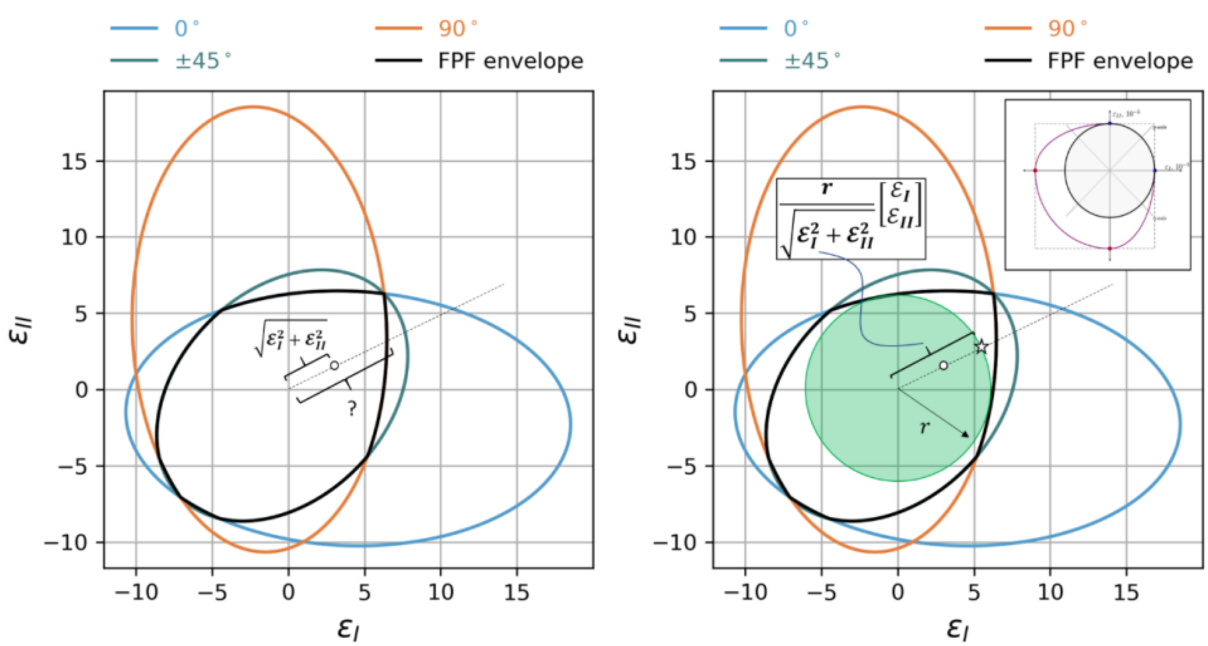


Fig.4-10: FPF-strain envelopes of 0°, 90°, ±45° plies with (left) a chosen lamina design load point • and an associate FPF –envelope point. ε in ‰, IM7/ 977-3, (right) Display of the Tsai-Melo circle radius r. The bold black line is envelope surrounding the Non-FPF area

Tsai and Melo proposed the 'Unit-Circle Criterion' (UCC) as a conservative approximation of the complex envelope shape. Nettles proposed a circle (marked by the subscript $_{NC}$) as a simplification of the UCC (see [*Kap22b*]). Its radius is defined by the tensile-anchor point of the envelope $r_{NC} = |(\varepsilon_1, 0)|$ Introducing the NC simplifies the strain-state assessment. *Fig.4-10* shows the circle in green colour. The comparison of the NC radius and the current strain-state magnitude allows for a direct determination of the material reserve factor f_{RF} .

Mind. please: This unit-circle pre-design tool itself is not a failure criterion, as sometimes cited.

5 Benefits, Conclusions, Findings

5.1 *'Trace' with Master-Ply*

Tsai and Melo showed in [*Tsa15*] that the diagonal trace of a specified stiffness matrix of a uni-directional ply trace$\left[Q^{Tr}\right]$ is invariant to coordinate transformation, see *Table 5-1*. The name of this matrix elements' summation was termed 'Trace' = *Tr* (*numerically*) by Tsai.

This initiated the idea to formulate a so called 'master ply', defined by four specific elements $Q^{Tr}\,/\,Tr$. For a list of several common CFRP materials the 'Trace'-normalized longitudinal stiffness element has a very small average CoV of 1.5% while the $\overline{Tr}$ (averaged *Tr*) lesser participating transverse and shear components appeared to show larger CoVs of up to 16.4% [*Kap23*].

The same holds true for the *Tr* of thickness-normalized in-plane laminate stiffness quantities and flexural laminate stiffness quantities. The application of the normalized stiffness matrices allows for composite design independent of actual laminate thickness and CFRP material. This shortly will be demonstrated on the next pages.

Naturally, the following body text including the tables and especially Annex A-4 will use some text and tables of the Double-Double book but is enriched by the authors' comments.

Table 5-1: Benefits of using 'Trace' with a Master-Ply

Focus: Properties of the physical ply or the computational element lamina

Ply (lamina) material properties: elasticity matrix

$$\text{IM7/977:}\ \{\overline{E}\} = (E_1, E_2, G_{21}, \nu_{21}, \nu_{23})^T = (191000,\ 9940, 7790, 0.35, -)^T\ \text{MPa}$$

$$[Q] = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & 1\cdot Q_{66} \end{bmatrix} = \begin{bmatrix} 192.2 & 3.501 & 0 \\ 3.502 & 10.008 & 0 \\ 0 & 0 & 7.790 \end{bmatrix}\ \text{GPa}$$

$$Trace':\ Tr = Q_{11} + Q_{22} + 2\cdot Q_{66} = \text{diagonal of } [Q^{Tr}]$$

$$\rightarrow \left[Q^{Tr}\right] = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & 2\cdot Q_{66} \end{bmatrix} \quad \text{and} \quad \text{trace}\,[Q^{Tr}] = 217810\ \text{MPa}.$$

'Master Ply' values: incorporate minimum scattering *Tr*-normalized elasticity values $Q_{ij}^{TR}\,/\,Tr$

- ➢ All stiffness quantities are fractions of 'Trace' $= Tr = trace([Q^{Tr}]) = Q_{11} + Q_{22} + 2\cdot Q_{66}$
- ➢ This enables a stiffness unification, welcomed for pre-design when using novel UD materials

For pre-design with a new material one can work with the computed value *Tr* of just the measured novel Q_{11} of the new UD material and the derived Master Ply elasticity coefficients. Putting it together with the known Master Ply Q^{Tr}_{ij}-values according to (*Tables 2-5, 5-1)*

$$Tr^{novel} = Q_{11}^{novel}(E_1) + \overline{Tr}\cdot(Q_{22}^{\overline{Tr}(master)} + 2\cdot Q_{66}^{\overline{Tr}(master)}) \quad \text{with} \quad Q_{11}^{novel} = E_1^{novel}/\left(1-\nu_{21}\cdot\nu_{12}\right)$$

Laminate properties of thin plies'-composed DD-sub-laminates:

Thickness-normalized [*K*] sub-matrices: $\left[\hat{A}\right] = \left[A\right]/\,\text{t},\ \left[\hat{B}\right] = \left[B\right]\cdot 2\,/\,\text{t}^2,\ \left[\hat{D}\right] = \left[D\right]\cdot 12\,/\,\text{t}^3.$

Lay-up $\{22.5/\text{-}22.5/67.5/\text{-}67.5\}_r$, r = 1, t_k = 1.0 mm, 4 layers → t = 4 mm

Elements of the A-B-D-submatrices with A in N/mm, B in N, D in N·mm) after normalization all in N/mm^2 = MPa

$$\left[\hat{A}^{Tr}\right] = Tr \cdot \begin{bmatrix} 0.370 & 0.110 & 0 \\ 0.110 & 0.370 & 0 \\ 0 & 0 & 0.130 \end{bmatrix}, \left[\hat{B}^{Tr}\right] = Tr \cdot \begin{bmatrix} -0.148 & 0 & -0.037 \\ 0 & 0.148 & -0.037 \\ -0.037 & -0.037 & 0 \end{bmatrix}, \left[\hat{D}^{Tr}\right] = Tr \cdot \begin{bmatrix} 0.370 & 0.110 & 0.071 \\ 0.110 & 0.370 & -0.071 \\ 0.071 & -0.071 & 0.130 \end{bmatrix}$$

'Trace sum' : $\text{trace}\,[Q^{Tr}] = \text{trace}\,[\hat{A}^{Tr}] = \text{trace}\,[\hat{D}^{Tr}] = 0.370 + 0.370 + 2 \cdot 0.130 = 1$

Lay-up $\{22.5/\text{-}22.5/67.5/\text{-}67.5\}_r$, r = 8, t_k = 0.125 mm, 32 layers or UD-laminas → t = 4 mm

$$\left[\hat{A}^{Tr}\right] = Tr \cdot \begin{bmatrix} 0.370 & 0.110 & 0 \\ 0.110 & 0.370 & 0 \\ 0 & 0 & 0.130 \end{bmatrix}, \left[\hat{B}^{Tr}\right] = Tr \cdot \begin{bmatrix} -0.018 & -0.005 & -0.005 \\ -0.005 & 0.018 & -0.005 \\ -0.005 & -0.005 & 0.000 \end{bmatrix}, \left[\hat{D}^{Tr}\right] = Tr \cdot \begin{bmatrix} 0.370 & 0.110 & 0.001 \\ 0.110 & 0.370 & -0.001 \\ 0.001 & -0.001 & 0.130 \end{bmatrix}.$$

- Mid-plane requirement is obsolete due to the possible through-the-thickness homogenization
- Delamination-decisive inter-laminar stresses are the same for all homogenized DD-composed laminates (*V. Tan, DD book*)
- Application of the normalized stiffness sub-matrices allows for a composite design independent of actual laminate thickness
- 'Trace'-normalized membrane stiffness sub-matrix $\left[\hat{A}\right]$ and bending stiffness matrix $\left[\hat{B}\right]$ show the similarity: $\text{trace}\,[\hat{A}^{Tr}] = \text{trace}\,[\hat{D}^{Tr}] = \text{trace}\left[Q^{Tr}\right] = Tr$ (see *Table 2-6)*
- A check of these traces proofs that the sum of the diagonal terms is 1, if 'Trace'-normalized
- A general DD-laminate can be neither termed matrix- nor fiber-dominated. All UD-modes are faced.

Tsai: *"Each laminate is identified by only two ply-angles, and, more importantly, can be characterized by a single mechanical property, Tsai's Modulus 'Trace', just like classic isotropic materials"*

5.2 'Double-Double' Laminates

Table 5-2 depicts the benefits od using 'DD' instead of 'Quad' laminates.

For fiber composite structural parts with endless fibers, this novel tool offers advantages for design and will simplify fabrication and repair. Homogenization is of significant advantage in design. Using homogenized asymmetric stacking sequences makes the 'Trace'-based sizing approach possible and simplifies manufacturing rules. If thermo-mechanically induced warping (spring-in) appears it became a smaller challenge when using asymmetric stacking sequences. The use of DD sub-laminates with the possibility of asymmetrical stacking sequences but homogenization of laminates further simplifies the design and manufacturing process of such structures. Spring-in is used for distortion resulting from the manufacturing process, mainly due to chemical shrinkage of the matrix during curing and to a certain extent due to the comparatively high thermal expansion in the direction of the laminate thickness. For DD it can be said that distortion problems due to "coupling" are reduced with increasing r

Traditional laminate design rules usually lead to mass penalties in design and more complexity in production:

1. Laminates must have mid-plane symmetry in order to avoid warpage

2. Laminates must be balanced = orthotropic, possessing material symmetry is easier to model
3. Laminates must have 10 percent in each of quad angles to guard against secondary loadings for fiber-dominated 'well-designed' laminates to prevent matrix failure
4. Inter-ply angle should be 45 degree or less to minimize interlaminar stresses.

Table 5-2: Benefits of using 'Double-Double instead of 'Quad-Laminates'

Focus: Properties of the to be designed laminate

'DD' sub-laminate family: 2 angle-plies of fiber angles $\{\varphi / -\varphi / \psi / -\psi\}$ as building block

'Quad' sub-laminate family: 4 fiber angles (0°, 45°,-45°, 90°) Prepreg as laminate building block.

Design: General

- A 'Trace-based' direct sizing approach causes a reduced complexity of the development process
  The coefficients B_{ij} scale proportionally with the factor 1/r, whereas the D_{16} and D_{26} are scaling proportionally with the factor $1/r^2$ and thus the decaying effect by r^2 is stronger. The influence of the repeat factor r with 1/r and 1/r² is clearly shown by the decreasing off-diagonal elements of the two laminates with r =1 and r = 8 (Fig. 5-1). For more details, especially on the applied Thickness-/'Trace'-normalized sub-matrices, see [*Kap22*], wherein is found for Omega stringer profile that a B-matrix-minimal lay-up is $\{\varphi / -\psi / -\varphi / \psi\}$
- DD is a homogenization tool reaching an acceptable homogenization after only a few repetitions *r* of the DD building block and thus, being intrinsically symmetric, eliminating the mid-plane symmetry design constraint
- Homogenization, performed by a DD sub-laminate offers also a quality measure in laminate design
- One can only work with relatively thick full 'Quad' sub-laminates, which is a mass-bottleneck of 'Quad' design
- A basic DD sub-laminate with an initial thickness has to be selected. Linear-elastic finite element analysis is to perform for each design load case (*Fig.5-2*)
- Homogenization improves the resistance to unforeseen loading variation, transverse impact with delamination
- Finally, the initial thickness can be linearly scaled, in case of in-plane loading to fulfil the design requirements
- The final design may be scaled to any other material or to meet higher stiffness or strength requirements
- Lay-up with step transitions (*Fig.5-1*) shows higher performance due to lesser built-in disturbances
- The thickness of the semi-finished thin-ply sub-laminate determines the laminate thickness *t*.

Production, Lay-up optimization

- The use of thin DD sub-laminates reduces the overall laminate thickness, which is often an obstacle to the use of 'Quad'-CFRP sub-laminates because the resulting total wall thicknesses offer too few advantages compared to metals
- Tapering: Tapering of laminates can be locally executed in lesser stressed areas. This is the more essential for relatively thick 'Quad' sub-laminates as laminate building blocks. In other words: Each structural element could have its minimally necessary thickness realized by the number *r* of repeats
- Stacked double-double sub-laminates can be deposited completely independently of symmetry requirements, e.g. in the mould from the inside to the outside or from the outside to the inside. In this way, a step-less, smooth part surface is possible
- The use of Double-Double simplifies and is particularly predestined for Automated Fiber Placement (AFP) and Automatic Tape Placement (ATP)

- The use of thin DD sub-laminates reduces the overall laminate thickness, which is often an obstacle to the use of 'Quad'-CFRP sub-laminates because the resulting total wall thicknesses offer too few advantages compared to other metals
- Single ply-drops can be located at the surfaces of the mold reducing discontinuities such as voids, neat resin pockets and wrinkles
- Ply-drop step size reduced together with material property change in adjacent locations.

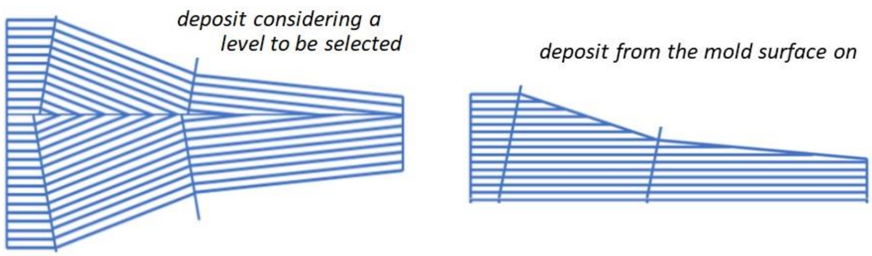


Fig.5-1: Laying strategies when using homogenized laminates, shown in cross-section through the laminates Neutral plane, from [*Roh22*]

<u>*Fig..5-2*</u> , from [*Tsa22*], presents an excellent view of the stacking variations faced with 'Quad' and 'DD' laminates. In-plane stiffness of a 'Quad' can be matched by 'DD' exactly in most cases. Also strength is matched well.

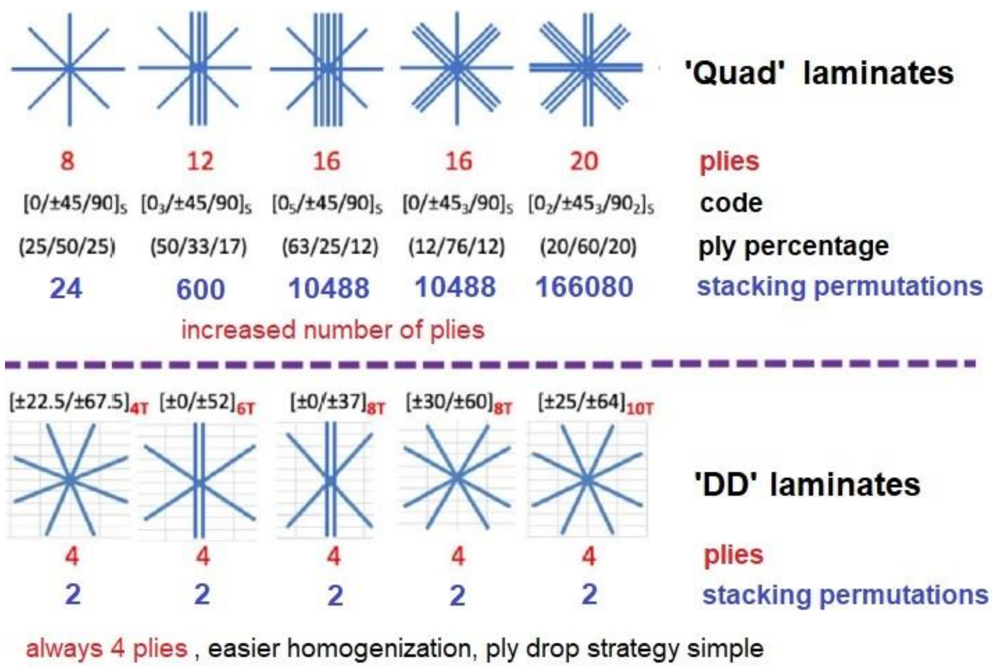


Fig.5-2: Comparison 'Quad' with 'DD'

5.3 *'**Omni** (principal FPF strain) **failure envelope**'*

Finally the focus are the benefits of using the failure stress-based 'Omni-(strain)failure envelopes' FPF envelopes obtained for a distinct composite material. The envelope globally covers all its potential laminate stacks.

Table 5-3: Benefits of using the 'Omni failure envelope' for determination of a reserve factor

Focus: Design Verification of the final design laminate at its strength-critical lamina locations

I Original Procedure. Tsai-Melo: $\tau_{12} \neq 0 \rightarrow$ One ε-envelope = failure curve (index *i*) of the 'butterfly'

- Take the external principal lamina (ply) strains (*laminate, k=1, single ply*) ε_I, ε_{II} as varying representatives of the force loading and as coordinates of the envisaged graph 'Non-FPF (*intact*) area' inside Tsai's so-called 'Omni *principal FRP strain* failure envelope'
- Determine values for each ply, oriented under the angle α, and of the strain ratio angle ξ which is linked to the beam angle f_ε, regarding *Fig. 4-11*
- Determine FPF failure strains $\varepsilon_{I,\mathrm{FPF}}, \varepsilon_{II,\mathrm{FPF}}$ from applying a Strength Criterion (Tsai-Wu or ?)
- Determine material reserve factor for Design Verification

*For all the *i* (ξ,α)-combinations compute the strains for insertion into $Eff_{\mathrm{FPF},i}$

$$\text{Tsai-Wu}: \frac{\sigma_1^2}{\bar{R}_{||}^t \cdot \bar{R}_{||}^c} + (\frac{\sigma_1}{\bar{R}_{||}^t} - \frac{\sigma_1}{\bar{R}_{||}^c}) + \frac{2F_{12}\sigma_1 \cdot \sigma_2}{\sqrt{\bar{R}_{||}^t \cdot \bar{R}_{||}^c \cdot \bar{R}_{\perp}^t \cdot \bar{R}_{\perp}^c}} + \frac{\sigma_2^2}{\bar{R}_{\perp}^t \cdot \bar{R}_{\perp}^c} + (\frac{\sigma_2}{\bar{R}_{\perp}^t} - \frac{\sigma_2}{\bar{R}_{\perp}^c}) + \frac{\tau_{12}^2}{\bar{R}_{\perp||}^2} = 1$$

*Store data and determine the strain FPF-envelope points, map envelope, determine circle radius

- Determine material reserve factor for Design Verification

$$* \quad f_{\mathrm{RF}} = r_{NC} \,/\, \sqrt{(\varepsilon_I^2 + \varepsilon_{II}^2)} \quad \text{with} \quad r_{NC}(\varepsilon_{I,\,FPF}\,,\ \varepsilon_{II} = 0)$$

II Novel direct Procedure Cuntze: $\tau_{12} = 0\,! \ \Rightarrow$ finally $\varepsilon_{II}(\varepsilon_{I,\,FPF})$

$$\text{Tsai-Wu}: \frac{\sigma_1^2}{\bar{R}_{||}^t \cdot \bar{R}_{||}^c} + (\frac{\sigma_1}{\bar{R}_{||}^t} - \frac{\sigma_1}{\bar{R}_{||}^c}) + \frac{2F_{12} \cdot \sigma_1 \cdot \sigma_2}{\sqrt{\bar{R}_{||}^t \cdot \bar{R}_{||}^c \cdot \bar{R}_{\perp}^t \cdot \bar{R}_{\perp}^c}} + \frac{\sigma_2^2}{\bar{R}_{\perp}^t \cdot \bar{R}_{\perp}^c} + (\frac{\sigma_2}{\bar{R}_{\perp}^t} - \frac{\sigma_2}{\bar{R}_{\perp}^c}) + (0) = 1$$

$$\text{Cuntze:} \qquad Eff_{\mathrm{FPF}} = [(Eff^{||\sigma})^m + (Eff^{||\tau})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp||})^m + (0)^m\,]^{m^{-1}} = 1$$

With both the criteria $\rightarrow \varepsilon(\sigma_{FPF})$ is to derive $\Rightarrow$ with vanishing strain beam f_ε finally

$$* \quad f_{\mathrm{RF}} = \sqrt{(\varepsilon_{I,\mathrm{FPF}}^2 + \varepsilon_{II,\mathrm{FPF}}^2)} \,/\, \sqrt{(\varepsilon_I^2 + \varepsilon_{II}^2)} = \varepsilon_{I,\mathrm{FPF}} / \varepsilon_I\,, \quad f_\varepsilon = \frac{\varepsilon_{II,\mathrm{FPF}}}{\varepsilon_{I,\mathrm{FPF}}} = \frac{\varepsilon_{II}}{\varepsilon_I} \ \text{vanishes}$$

the following steps are to go for each principal loading ratio (*force or strain*). Before all steps - as guiding parameter input - the determination of the relationship of the forces-representative principal strains is to perform, due to *Table 5-3*.

The 'old' Carpet Plots shall be replaced by novel 'Omni strain envelope' plots

► At minimum, a valuable linear-elastic Predesign Tool for the full laminate is provided !

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Annex

A1 UD Failure modes, 2D Failure Criteria (Tsai-Wu with Cuntze) and some Test Data

Fracture Failure modes

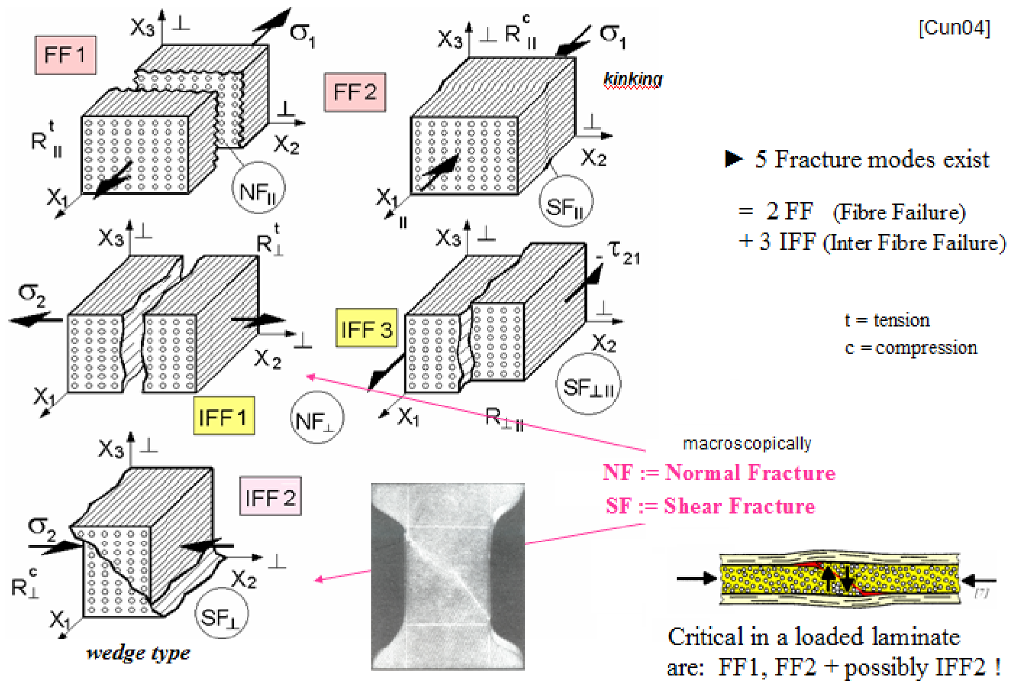


Fig. A-1: UD fracture failure modes of UD material, NF = Normal fracture, SF = Shear fracture

Stress-based 2D Strength Failure Criteria of Tsai-Wu and Cuntze

For obtaining the envisaged Design Sheets just the 2D versions of the two stress-strength-based SFCs are of interest. Strain-based SFCs (procedure: strain ε < failure strain e) are generally not permitted regarding present authority regulations which require strength design allowables (procedure: stress σ < failure stress = strength R).

2D Tsai-Wu : After the insertion of the parameters F_{ij} the reduced ‘global SFC reads

$$\{\sigma\} = (\sigma_1,\sigma_2,\tau_{12})^{\mathrm{T}}, \quad \{\bar{R}\} = (\bar{R}_{||}^t, \bar{R}_{||}^c, \bar{R}_{\perp}^t, \bar{R}_{\perp}^c, \bar{R}_{\perp||})^T,$$

$$\frac{\sigma_1^{\,2}/Eff^2}{\bar{R}_{||}^t\cdot\bar{R}_{||}^c} + \frac{\sigma_1}{Eff}\cdot\left(\frac{1}{\bar{R}_{||}^t}-\frac{1}{\bar{R}_{||}^c}\right) + \frac{2F_{12}}{\sqrt{\bar{R}_{||}^t\cdot\bar{R}_{||}^c\cdot\bar{R}_{\perp}^t\cdot\bar{R}_{\perp}^c}}\cdot\frac{\sigma_1\cdot\sigma_2}{Eff^2} + \frac{\sigma_2^{\,2}/Eff^2}{\bar{R}_{\perp}^t\cdot\bar{R}_{\perp}^c} + \frac{\sigma_2}{Eff}\cdot\left(\frac{1}{\bar{R}_{\perp}^t}-\frac{1}{\bar{R}_{\perp}^c}\right) + \frac{\tau_{12}^{\,2}/Eff^2}{\bar{R}_{\perp||}^{\,2}} = 1$$

2D Cuntze:

$$\{\sigma\} = (\sigma_1,\sigma_2,\tau_{12})^{\mathrm{T}}, \quad \{\bar{R}\} = (\bar{R}_{||}^t, \bar{R}_{||}^c, \bar{R}_{\perp}^t, \bar{R}_{\perp}^c, \bar{R}_{\perp||})^T, \ \mu_{\perp||}.$$

$$Eff = [(Eff^{||\sigma})^m + (Eff^{||\tau})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp||})^m + (Eff^{\perp\tau})^m]^{m^{-1}} = 1$$

$$Eff^{||\sigma} = \frac{(\sigma_1 + |\sigma_1|)}{2\cdot R_{||}^t}, \quad Eff^{||\tau} = \frac{(-\sigma_1 + |\sigma_1|)}{2\cdot R_{||}^c}, \quad Eff^{\perp\sigma} = \frac{\sigma_2 + |\sigma_2|}{2\cdot \bar{R}_{\perp}^t}, \quad Eff^{\perp\tau} = \frac{-\sigma_2 + |\sigma_2|}{2\cdot \bar{R}_{\perp}^c}, \quad Eff^{\perp||} = \frac{|\tau_{21}|}{\bar{R}_{\perp||} + 0.5\cdot\mu_{\perp||}\cdot(-\sigma_2 + |\sigma_2|)}$$

For the parameter F_{12}, in order to bypass an open failure surface, the value -0.5 is applied. Here, the *Eff* corresponds to the so-called 'Tsai strength ratio' R.

Note: When automatically inserting the FEA stress output $\{\sigma\}$ into the *Eff*-equations some *Effs* may become negative which mechanically means zero *Eff*. In order to make an automatic use of the FMC-based fracture SFCs also in a 3D state of stresses possible and to avoid complicate queries in the computer program absolute values are used in order to avoid a sign query. Due to successful comparison with the 3D-reduced SCF (*suffix 3 dropped*) the 3D-reduced shear failure $Eff^{\perp\parallel}$ could be further simplified to the above Mohr-Coulomb formulation. Negative Effs are physical nonsense and are to make zero. The interaction exponent is taken m = 2.6. For the friction value the same value is inserted for all materials with $\mu_{\perp\parallel} = 0.2$.

A reminder for the numeric procedure:

Determination of material Stressing Effort $Eff \neq 1$: $Eff = [\Sigma\,(Eff^{\text{modes}})^m]^{m^{-1}}$

Determination of failure curve, surface of failure body $Eff = 1$: $1 = \Sigma\,(Eff^{\text{modes}})^m$.

<u>Test data validation of Cuntze's FMC-based SFCs and Main Cross-sections of FPF failure Body</u>

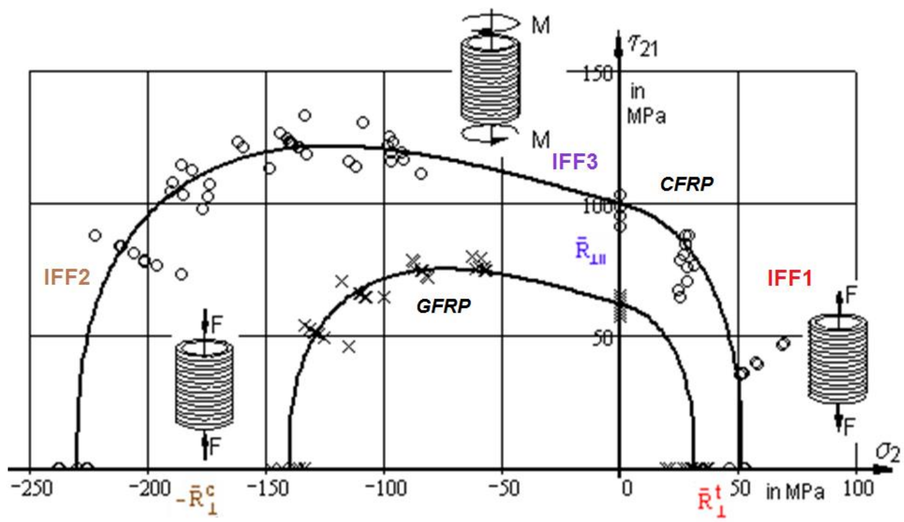


Fig.A-3, IFF test results: 2 GFRP, 1 CFRP test series (test data from MAN Technologie research project on Puck's IFF criterion, [Cun97], m =2.7. E-glass / LY556, HT976, DY070; CFRP: T300 / LY556, HT976. The main cross-section of the UD fracture failure body is mapped by the Cuntze SFCs

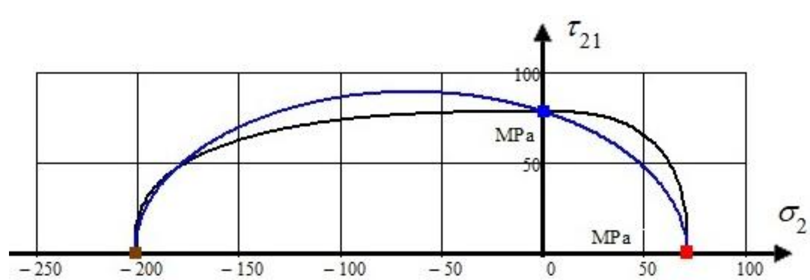


Fig. A-4: Cross-section $\tau_{21}(\sigma_2)$ *. Cuntze, Tsai-Wu. T700*

$$\{\bar{R}\} = (\bar{R}_\parallel^t, \bar{R}_\parallel^c, \bar{R}_\perp^t, \bar{R}_\perp^c, \bar{R}_{\perp\parallel})^{\mathrm{T}} = (\mathrm{X}, X', Y, Y', S;\ \mathrm{S}_{23})^{\mathrm{T}} = (,,,,)^{\mathrm{T}}$$

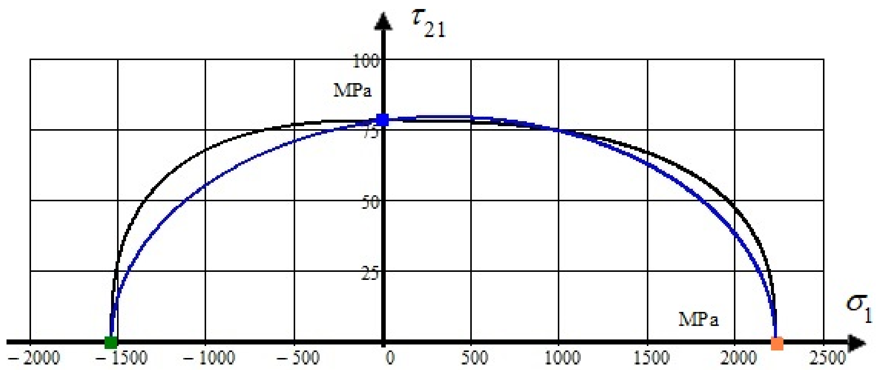
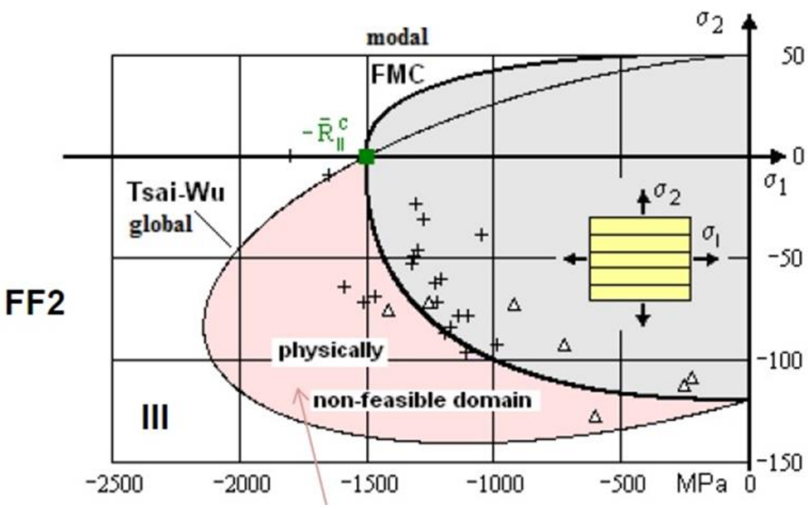


Fig. A-5: Cross-section $\tau_{21}(\sigma_1)$ *, Cuntz and Tsai-Wu. T700*



[Data, quadrant III: courtesy, Knops, IKV Aachen, 2003]

Fig.A-6, WWFE-II: Mapping of $\sigma_2(\sigma_1)$ *test data (test results: M. Knops, IKV Aachen, [Kno3, Kno07]*

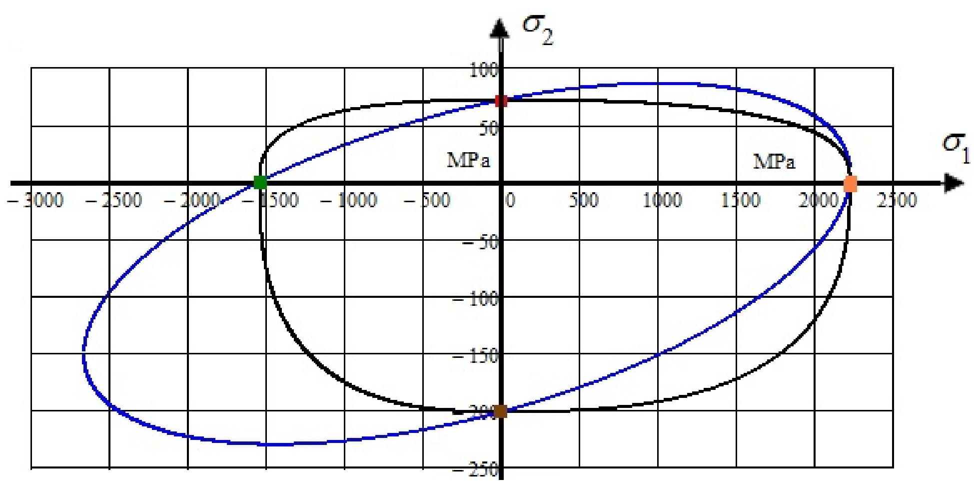


Fig. A-7: Cross-section $\sigma_2(\sigma_1)$ *, Cuntze and Tsai-Wu. T700*

For the following computations Cuntze adapted an old WWFE –I program created by his former co-worker Andreas Freund. For the two envisaged SFCs Tsai-Wu and Cuntze 'Omni failure envelopes' are to compute.

A2 Solution procedures for the SFCs of Tsai-Wu and Cuntze

Table A-1: MathCad 12 procedure for Cuntze to determine the associate FPF envelope

Cuntze

$$\text{Lamina} := $$

$i \leftarrow 0$

for $x \in \text{start}, \text{start} + \text{inkr} \,..\, \text{schluss}$

$\quad 1 \leftarrow 0$

$\quad \text{end} \leftarrow 0$

$\quad \text{ende} \leftarrow \text{stop} + 1$

$\quad$ while $\text{end} < \text{ende}$

$\qquad 1 \leftarrow \text{end}$

$\qquad \sigma_1 \leftarrow 1 \cdot \text{loadstep} \cdot \cos(x \cdot \text{Grad}) \cdot \text{SIG1}$

$\qquad \sigma_2 \leftarrow 1 \cdot \text{loadstep} \cdot \sin(x \cdot \text{Grad}) \cdot \text{SIG2}$

$\qquad \tau_{21} \leftarrow 0$

$\qquad \tau_{31} \leftarrow 0$

$\qquad \sigma_3 \leftarrow 0$

$\qquad \text{Eff}_{1t} \leftarrow \left(|\sigma_1| + \sigma_1\right) \cdot \dfrac{0.5}{R_{1t}}$

$\qquad \text{Eff}_{1c} \leftarrow \left(-\sigma_1 + |\sigma_1|\right) \cdot \dfrac{0.5}{R_{1c}}$

$\qquad \text{Eff}_{2t} \leftarrow \left(|\sigma_2| + \sigma_2\right) \cdot \dfrac{0.5}{R_{2t}}$

$\qquad \text{Eff}_{2c} \leftarrow \left(-\sigma_2 + |\sigma_2|\right) \cdot \dfrac{0.5}{R_{2c}}$

$\qquad \text{Eff}_{21} \leftarrow \dfrac{|\tau_{21}|}{\left[R_{21} + 0.5 \cdot \mu_{21} \cdot \left(-\sigma_2 + |\sigma_2|\right)\right]}$

$\qquad \varepsilon_1 \leftarrow S_{11} \cdot \sigma_1 + S_{21} \cdot \sigma_2$

$\qquad \varepsilon_2 \leftarrow S_{21} \cdot \sigma_1 + S_{22} \cdot \sigma_2$

$\qquad \varepsilon_I \leftarrow 0.5 \cdot (\varepsilon_1 + \varepsilon_2) + \sqrt{0.25 \cdot (\varepsilon_1 - \varepsilon_2)^2 + (0)^2}$

$\qquad \varepsilon_{II} \leftarrow 0.5 \cdot (\varepsilon_1 + \varepsilon_2) - \sqrt{0.25 \cdot (\varepsilon_1 - \varepsilon_2)^2 + (0)^2}$

$\qquad \text{Eff}_{res} \leftarrow \left| \left[(\text{Eff}_{1t})^{m°} + (\text{Eff}_{1c})^{m°} + (\text{Eff}_{2t})^{m°} + (\text{Eff}_{2c})^{m°} + (\text{Eff}_{21})^{m°} \right]^{\frac{1}{m°}} \right|$

$\qquad \text{end} \leftarrow \text{ende} \;\text{ if }\; \text{Eff}_{res} \geq 1$

$\qquad \text{end} \leftarrow \text{end} + 1$

$\quad \text{Kurve}^{\langle i \rangle} \leftarrow [[[(x \;\; \sigma_1 \;\; \sigma_2 \;\; \tau_{21} \;\; \sigma_3 \;\; \tau_{31} \;\; \text{Eff}_{1t} \;\; \text{Eff}_{1c} \;\; \text{Eff}_{2t} \;\; \text{Eff}_{2c} \;\; \text{Eff}_{21} \;\; \varepsilon_I \;\; \varepsilon_{II} \;\; \text{Eff}_{res})]]]^T$

$\quad i \leftarrow i + 1$

$[[[[((\text{Kurve}))]]]]^T$

Materialdaten

$R_{1t} \equiv 2230 \quad R_{2t} \equiv 71 \quad R_{1c} \equiv 1537 \quad R_{2c} \equiv 202 \quad R_{21} \equiv 78 \quad R_{1c} \equiv 1537 \quad R_{2c} \equiv 202 \quad F12 := -0.5$

$E1 := 126000 \quad E2 := 8300 \quad G21 := 4100 \quad \nu 21 := 0.30 \quad \mu_{21} \equiv 0.2 \quad m° \equiv 2.7$

$S11 := \dfrac{1}{E1} \quad S21 := \dfrac{-\nu 21}{E1} \quad S22 := \dfrac{1}{E2} \quad S66 := \dfrac{1}{G21} \quad S_{11} \equiv 0.00000794 \quad S_{21} \equiv -0.00000238 \quad S_{22} \equiv 0.0001205$

$S_{66} \equiv 0.0002439$

Auslesen der Result-Matrix für Diagramm

$k := 0 \,..\, \dfrac{\text{schluss} - \text{start}}{\text{inkr}} \quad i1 := 0 \,..\, 78 \quad i2 := 0 \,..\, 57 \quad i3 := 0 \,..\, 70$

$SIG1_k := \text{Lamina}_{k,1} \quad SIG2_k := \text{Lamina}_{k,2} \quad TAU21_k := \text{Lamina}_{k,3} \quad EPS1_k := \text{Lamina}_{k,11} \quad EPS2_k := \text{Lamina}_{k,12}$

Schalter für Wahl der Inputloads

SIG1 ≡ 1 SIG2 ≡ 1 TAU21 ≡ 0

Wahl der Diskretisierung der Kurve

start ≡ 0 schluss ≡ 360 inkr ≡ 0.3 (Winkel des Lastvektors Grad)

Loadstepping

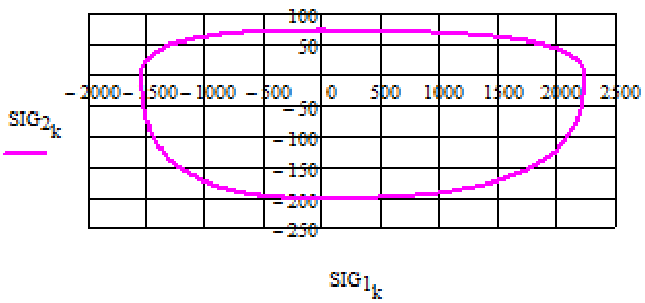
loadstep ≡ 1 in MPa

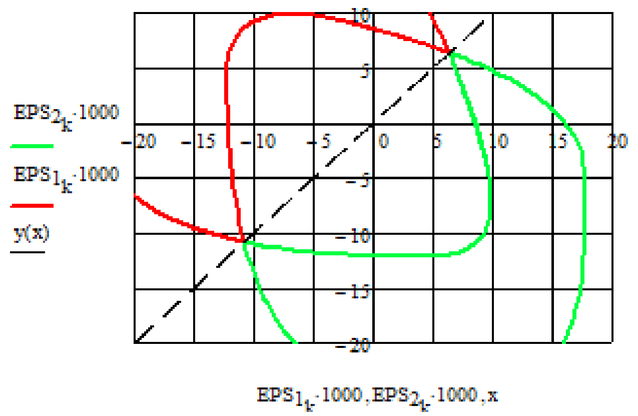
Runtime-Limit

stop ≡ 2230

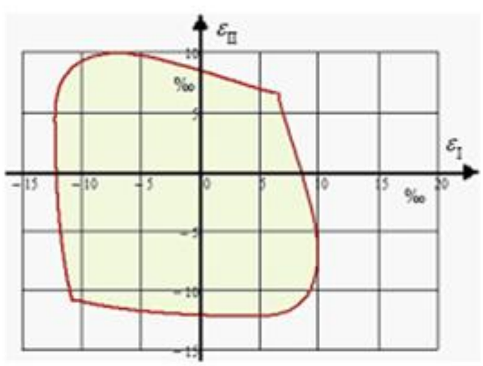
Computation results y(x) := x

Mastercurve, tau21 = 0

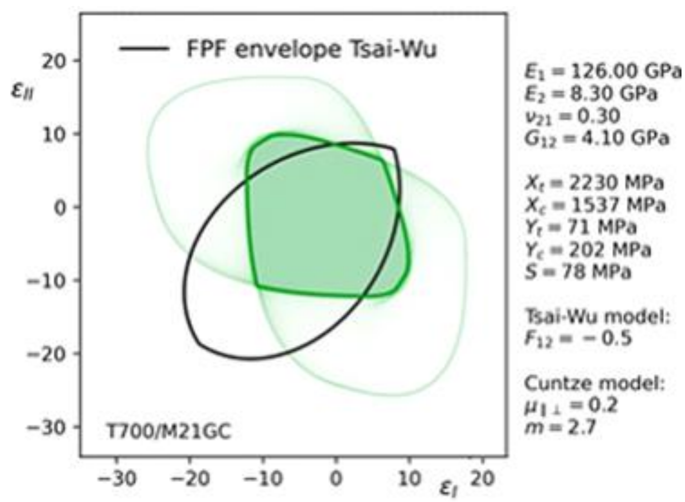


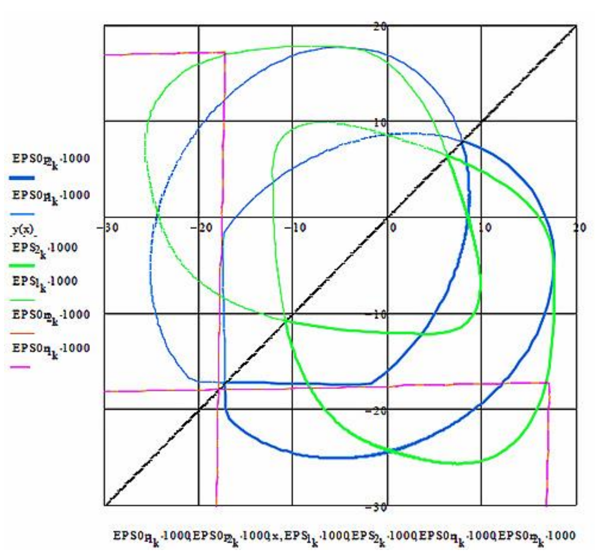


The following two figures depict the task to be solved, to achieve an efficient computation of the envelope curve. In the case of Cuntze the multi-solution could be solved by Mathcad and the full curve obtained.



CFRP of IM7/977-3

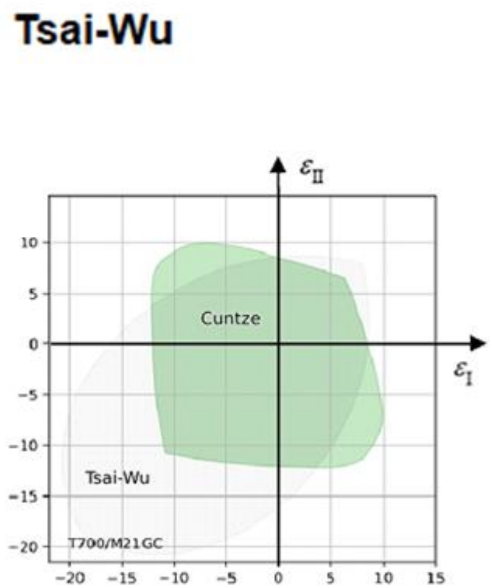




Difficulties arose for MathCad to present all Tsai-Wu FPF envelope domains due to the multiple roots faced (see figure above), which does not fit in not conservative domain after *Fig.A-7*.

Table A-2: MathCad 12 procedure for Tsai-Wu to determine the associate FPF envelope

```
Lamina := | i ← 0
          | for x ∈ start, start + inkr .. schluss
          |   | l ← 0
          |   | end ← 0
          |   | ende ← stop + 1
          |   | while end < ende
          |   |   | l ← end
          |   |   | σ1 ← l·loadstep·cos(x·Grad)·SIG1
          |   |   | σ2 ← l·loadstep·sin(x·Grad)·SIG2
          |   |   | τ21 ← 0
          |   |   | τ31 ← 0
          |   |   | σ3 ← 0
          |   |   | a ← σ1^2/(R1t·R1c) + 2F12/√[R1t·R1c·(R2t·R2c)]·σ1·σ2 + σ2^2/(R2t·R2c)
          |   |   | b ← σ1·(1/R1t − 1/R1c) + σ2·(1/R2t − 1/R2c)
          |   |   | c ← −1
          |   |   | d ← 1
          |   |   | e ← 1
          |   |   | ε1 ← S11·σ1 + S21·σ2
          |   |   | ε2 ← S21·σ1 + S22·σ2
          |   |   | εI ← 0.5·(ε1 + ε2) + √(0.25·(ε1 − ε2)^2 + (0)^2)
          |   |   | εII ← 0.5·(ε1 + ε2) − √(0.25·(ε1 − ε2)^2 + (0)^2)
          |   |   | Eff_res ← a / (0.5·(−b + √(b^2 − 4·a·c)))
          |   |   | end ← ende if Eff_res ≥ 1
          |   |   | end ← end + 1
          |   | Kurve^⟨i⟩ ← [[[(x σ1 σ2 τ21 σ3 τ31 a b c d e εI εII Eff_res)]]]^T
          |   | i ← i + 1
          | [[[[((Kurve))]]]]^T
```



$$ep = \frac{0.5}{a}\cdot\left(-b + \sqrt{b^2 - 4\cdot a\cdot c}\right)$$

$$c := -1$$

$$en = \frac{0.5}{a}\cdot\left(-b - \sqrt{b^2 - 4\cdot a\cdot c}\right)$$

$$e^2\cdot(a) + e\cdot(b) + c = 0$$

$$Ef := \frac{1}{e}$$

$EPS0p1_k := Lamina_{k,11}$ $EPS0p2_k := Lamina_{k,12}$

$SIG601_k := Lamina_{k,1}$ $SIG602_k := Lamina_{k,2}$ $TAU21_k := Lamina_{k,3}$

(Cuntze tried to take another way of solution in order to get all solution branches, however a software crash demolished the respective folder with the Mathcad programs).

A3 Collection of various Property-Tables

Table A-3a: Test data sets provided by Tsai-Melo (mind indexing),
[Composite Double-Double and Grid/Skin Structures], Ch.1

Table 1.2 Trace normalized engineering constants and [Q] and master ply

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
2	CFRP	Ex	Ey	Es	nu	Qxx	Qyy	Qxy	Qss	Trace	Ex*	Ey*	Es*	Qxx*	Qyy*	Qxy*	Qss*	Trace*
3	IM6/epoxy	203	11.20	8.40	0.32	204	11.3	3.6	8.4	232.2	0.874	0.048	0.036	0.879	0.049	0.016	0.036	1.000
4	IM7/977-3	191	9.94	7.79	0.35	192	10.0	3.5	7.8	217.8	0.877	0.046	0.036	0.883	0.046	0.016	0.036	1.000
5	T300/5208	181	10.30	7.17	0.28	182	10.3	2.9	7.2	206.5	0.877	0.050	0.035	0.880	0.050	0.014	0.035	1.000
6	IM7/MTM45	175	8.20	5.50	0.33	176	8.2	2.7	5.5	195.1	0.897	0.042	0.028	0.901	0.042	0.014	0.028	1.000
7	T800/Cytec	162	9.00	5.00	0.40	163	9.1	3.6	5.0	182.5	0.888	0.049	0.027	0.895	0.050	0.020	0.027	1.000
8	IM7/8552	159	8.96	5.50	0.32	160	9.0	2.9	5.5	179.9	0.884	0.050	0.031	0.889	0.050	0.016	0.031	1.000
9	T800S/3900	151	8.20	4.00	0.33	152	8.2	2.7	4.0	168.1	0.898	0.049	0.024	0.903	0.049	0.016	0.024	1.000
10	T300/F934	148	9.65	4.55	0.30	149	9.7	2.9	4.6	167.7	0.883	0.058	0.027	0.888	0.058	0.017	0.027	1.000
11	T700 C-Ply 64	141	9.30	5.80	0.30	142	9.4	2.8	5.8	162.8	0.866	0.057	0.036	0.871	0.057	0.017	0.036	1.000
12	AS4/H3501	138	8.96	7.10	0.30	139	9.0	2.7	7.1	162.0	0.852	0.055	0.044	0.857	0.056	0.017	0.044	1.000
13	T650/epoxy	139	9.40	5.50	0.32	140	9.5	3.0	5.5	160.4	0.866	0.059	0.034	0.872	0.059	0.019	0.034	1.000
14	T4708/MR60H	142	7.72	3.80	0.34	143	7.8	2.6	3.8	158.3	0.897	0.049	0.024	0.903	0.049	0.017	0.024	1.000
15	T700/2510	126	8.40	4.20	0.31	127	8.5	2.6	4.2	143.7	0.877	0.058	0.029	0.883	0.059	0.018	0.029	1.000
16	AS4/MTM45	128	7.93	3.65	0.30	129	8.0	2.4	3.7	144.0	0.889	0.055	0.025	0.894	0.055	0.017	0.025	1.000
17	T700 C-Ply 55	121	8.00	4.70	0.30	122	8.0	2.4	4.7	139.2	0.869	0.057	0.034	0.875	0.058	0.017	0.034	1.000
18	New CFRP																	
19	New CFRP																	
20										Average	0.880	0.052	0.031	0.885	0.052	0.017	0.031	1.000
21										cv %	1.5%	10.0%	17.9%	1.5%	10.1%	9.6%	17.9%	0.0%
22													MASTER PLY: CARBON/EPOXY					

Table A-3b: Test data sets provided in the book "Composite Laminates – Theory and practice of analysis, design and automated layup "from Stephen W. Tsai, José Daniel D. Melo, Sangwook Sihn, Albertino Arteiro, Robert Rainsberger, Verlag Stanford Aeronautics & Astronautics, 2017, ISBN 0986084530, 356 pages

Table 2.3. Strength of various composite materials in SI.

Type	CFRP	BFRP	CFRP	GFRP	KFRP	CFRTP	CFRP	CFRP	CCRP	CCRP
Fiber/cloth	T300	B(4)	AS	E-glass	Kev 49	AS 4	IM6	T300	T300	T300
Matrix	5208	N5505	H3501	epoxy	epoxy	PEEK	epoxy	F 934	F 934	F 934
Engineering constants, GPa or dimensionless								4-mil	13-mil	7-mil
E_x, GPa	181.0	204.0	138.0	38.6	76.0	134.0	203.0	148.0	74.00	66.00
E_y, GPa	10.30	18.50	8.96	8.27	5.50	8.90	11.20	9.65	74.00	66.00
ν_x	0.28	0.23	0.30	0.26	0.34	0.28	0.32	0.30	0.05	0.04
Es, GPa	7.17	5.59	7.10	4.14	2.30	5.10	8.40	4.55	4.55	4.10
v_f	0.70	0.50	0.66	0.45	0.60	0.66	0.66	0.60	0.60	0.60
Sp Gravity	1.60	2.00	1.60	1.80	1.46	1.60	1.60	1.50	1.50	1.50
ho, mm	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.100	0.325	0.175
Max stress (MPa)										
X	1500	1260	1447	1062	1400	2130	3500	1314	499	375
X'	1500	2500	1447	610	235	1100	1540	1220	352	279
Y	40	61	52	31	12	80	56	43	458	368
Y'	246	202	206	118	53	200	150	168	352	278
S	68	67	93	72	34	160	98	48	46	46
Max strain, ε^* x 10^{-3}										
x	8.29	6.18	10.49	27.51	18.42	15.90	17.24	8.88	6.74	5.68
x'	8.29	12.25	10.49	15.80	3.09	8.21	7.59	8.24	4.76	4.23
y	3.88	3.30	5.77	3.75	2.18	8.99	5.00	4.46	6.19	5.58
y'	23.88	10.92	22.99	14.27	9.64	22.47	13.39	17.41	4.76	4.21
s	9.48	11.99	13.10	17.39	14.78	31.37	11.67	10.55	10.11	11.22

Table A-4: Test data sets provided in the ***WWFE***

Fibre type	IM7	T300	A-S	S2-glass	E-Glass
Matrix	**8551-7**	**PR-319**	**Epoxy1**	**Epoxy2**	**MY750**
Fibre volume fraction V_f (%)	60	60	60	60	60
Longitudinal modulus E_1 (GPa)	*165**	*129*	*140**	*52*	*45.6*
Transverse modulus E_2 (GPa)	*8.4*	*5.6+*	*10*	*19*	*16.2*
Through-thickness modulus E_3 (GPa)	8.4	5.6+	10	19	16.2
In-plane shear modulus G_{12} (GPa)	*5.6**	*1.33+*	*6**	*6.7**	*5.83**
Transverse shear modulus G_{13} (GPa)	5.6*	1.33+	6*	6.7*	5.83*
Through-thickness shear modulus G_{23} (GPa)	2.8	1.86	3.35	6.7	5.7
Major Poisson's ratio υ_{12}	*0.34*	*0.32*	*0.3*	*0.3*	*0.28*
Major transverse Poisson's ratio υ_{13}	0.34	0.32	0.3	0.3	0.28
Through-thickness Poisson's ratio υ_{23}	*0.5*	*0.5*	*0.49*	*0.42*	*0.4*
Longitudinal tensile strength X_T (MPa)	*2560*	*1378*	*1990*	*1700*	*1280*
Longitudinal compressive strength X_C (MPa)	*1590*	*950*	*1500*	*1150*	*800*
Transverse tensile strength Y_T (MPa)	*73*	*40*	*38*	*63*	*40*
Transverse compressive strength Y_C (MPa)	*185***	*125***	*150***	*180***	*145***
Through-thickness tensile strength Z_T (MPa)	63	40	38	50	40
Through-thickness compressive strength Z_C (MPa)	185**	125**	150**	180**	145**
In-plane shear strength S_{12} (MPa)	*90***	*97***	*70***	*72***	*73***
Transverse shear strength S_{13} (MPa)	90**	97**	70**	72**	73**
Through-thickness shear strength S_{23} (MPa)	57	45	50	40	50
Longitudinal tensile failure strain ε_{1T} (%)	*1.55*	*1.07*	*1.42*	*3.27*	*2.81*
Longitudinal compressive failure strain ε_{1C} (%)	*1.1*	*0.74*	*1.2*	*2.21*	*1.75*
Transverse tensile failure strain ε_{2T} (%)	*0.87*	*0.43*	*0.38*	*0.33*	*0.246*
Transverse compressive failure strain ε_{2C} (%)	*3.2*	*2.8*	*1.6*	*1.5*	*1.2*
Transverse tensile failure strain ε_{3T} (%)	0.76	0.43	0.38	0.263	0.25
Through-thickness compressive failure strain ε_{3C} (%)	3.2	2.8	1.6	1.5	1.2
In-plane shear failure strain γ_{12u} (%)	*5*	*8.6*	*3.5*	*4*	*4*
Transverse shear failure strain γ_{13u} (%)	5	8.6	3.5	4	4
Through-thickness shear failure strain γ_{23u} (%)	2.1	1.5	1.5	0.59	0.88
Longitudinal thermal coefficient α_1 (10^{-6}/°C)	-1	-1	-1	8.6	8.6
Transverse thermal coefficient α_2 (10^{-6}/°C)	18	26	26	26.4	26.4
Through-thickness thermal coefficient α_3 (10^{-6}/°C)	18	26	26	26.4	26.4
Energy release rates G_{Ic}, G_{IIc} (J/m^2 = 1N/m)	200				240, 1500
mixed (fracture mechanics) mode to be assumed					
Stress free temperature (°C)	177	120	120	120	120
Test Case	TC10,11,12	TC2,3,4	TC7	TC6	TC1,5,8,9

* Initial modulus. ** Nonlinear behaviour and stress strain curves and data points are provided

+ These values are considered to be low, compared with typical data for the same material published somewhere else or quoted by the manufacturers. We have not attempted to change them in order to facilitate a comparison with test data in Part B.

Tables A1 and A2 from T. Dickhut (2013) *Beitrag zur Auslegung und Gestaltung von Antriebswellen aus Faser-Kunststoff-Verbunden*. Diss. D17; TU Darmstadt, Shaker Aachen

Tabelle A.1: Kohlefaser–Epoxidharz–Verbund, TohoTenax STS40 24K/LY556, Faservolumenanteil $\varphi = 60\,\%$

$E_{\parallel}$	144080	N/mm²	$R_{\parallel}^{+}$	1920	N/mm²
$E_{\perp}$	8916	N/mm²	$R_{\parallel}^{-}$	1000	N/mm²
$E_{\perp s,fe=1(+)}$	5000	N/mm²	$R_{\perp}^{+}$	40	N/mm²
$E_{\perp s,fe=1(-)}$	3587	N/mm²	$R_{\perp}^{-}$	150	N/mm²
$G_{\parallel\perp}$	5557	N/mm²	$R_{\perp\parallel}$	60	N/mm²
$G_{\parallel\perp s,fe=1}$	2236	N/mm²	$p_{\parallel\perp}^{+}$	0,25	
$\nu_{\perp\parallel}$	0,26		$p_{\parallel\perp}^{-}$	0,3	
$f_{E,thr(\sigma 2+)}$	0,05		$\eta_r E_{\perp}$	0,03	
$f_{E,thr(\sigma 2-)}$	0,05		$\eta_r G_{\parallel\perp}$	0,6	
$f_{E,thr(\tau 21)}$	0,1		m	0,5	
$n_{(\sigma 2+)}$	2,18		s	0,5	
$n_{(\sigma 2-)}$	2,18		$\alpha_{\parallel}$	-1e-6	
$n_{(\tau 21)}$	1,59		$\alpha_{\perp}$	2,6e-5	
$Ci_{(\tau 21+)}$	0,74				
$Ci_{(\tau 21-)}$	0,74				
$Ci_{(\sigma 2)}$	0,53				

Tabelle A.2: Glasfaser–Epoxidharz–Verbund, Vetrotex P192 1200 Tex/LY556, Faservolumenanteil $\varphi = 60\,\%$

$E_{\parallel}$	49280	N/mm²	$R_{\parallel}^{+}$	1100	N/mm²
$E_{\perp}$	15650	N/mm²	$R_{\parallel}^{-}$	1000	N/mm²
$E_{\perp s,fe=1(+)}$	12941	N/mm²	$R_{\perp}^{+}$	55,75	N/mm²
$E_{\perp s,fe=1(-)}$	6574	N/mm²	$R_{\perp}^{-}$	150	N/mm²
$G_{\parallel\perp}$	6543	N/mm²	$R_{\perp\parallel}$	67,54	N/mm²
$G_{\parallel\perp s,fe=1}$	1748	N/mm²	$p_{\parallel\perp}^{+}$	0,3	
$\nu_{\perp\parallel}$	0,27		$p_{\parallel\perp}^{-}$	0,35	
$f_{E,thr(\sigma 2+)}$	0,43		$\eta_r E_{\perp}$	0,03	
$f_{E,thr(\sigma 2-)}$	0,127		$\eta_r G_{\parallel\perp}$	0,25	
$f_{E,thr(\tau 21)}$	0,27		m	0,5	
$n_{(\sigma 2+)}$	2,36		s	0,5	
$n_{(\sigma 2-)}$	2,36		$\alpha_{\parallel}$	-8,6e-6	
$n_{(\tau 21)}$	2,08		$\alpha_{\perp}$	2,64e-5	
$Ci_{(\tau 21+)}$	0,49				
$Ci_{(\tau 21-)}$	0,49				
$Ci_{(\sigma 2)}$	0,5				

Basalt fibres	
Mechanical Property	*Value*
Density ρ	2.75 g/cm³
YOUNG's modulus E_1	89 GPa
YOUNG's modulus E_2	89 GPa
Shear modulus G_{12}	21.7 GPa
Shear modulus G_{23}	21.7 GPa
POISSON's ratio ν_{12}	0.26
POISSON's ratio ν_{23}	0.26

Epoxy resin	
Mechanical Property	*Value*
Density ρ	1.15 g/cm³
YOUNG's modulus E	2.65 GPa
Shear modulus G	0.98 GPa
POISSON's ratio ν	0.35

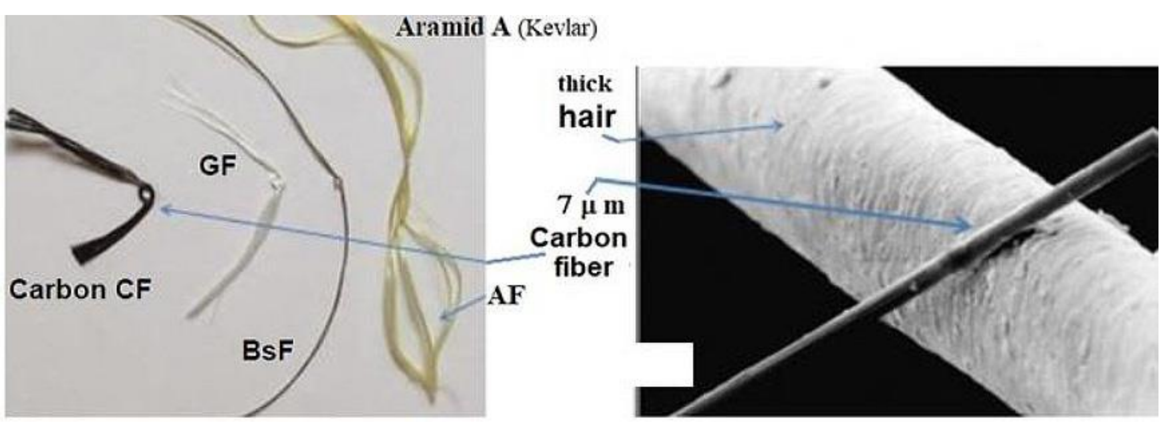
Table 1: Values of the structural mechanical properties: Left: Basalt fibres [18]. Right: Epoxy resin [17].

LL, *application of micro-mechanical properties: Warning!*
These properties can be only used if the associate micro-mechanical formulas are given. These formulas were not provided in the WWFE (to use in a WWFE Test Case) *and led to a discrepancy of the factor two, when using:* Non-creeping anisotropic fiber [*Schuermann*, *Puck*]. Modelling material macro properties on basis of micro-mechanical constituent properties [*VDI 2014*, p.29] with V_f $(=\varphi)$ is fiber volume fraction. The superscripts ^{f} and ^{m} stand for fiber and matrix.

$$E_{\parallel} = E_{\parallel f}\cdot V_f + E_m\cdot(1-V_f) \cong E_f\cdot V_f\,, \quad \text{within regime } 0.3 \le V_f \le 0.65\,;$$
$$\nu_{\perp\parallel} = \nu_{\perp\parallel f}\cdot V_f + \nu_m\cdot(1-V_f)\,, \quad G_m = E_m/(2+2\cdot\nu_m).$$
$$E_{\perp} = \frac{E_m}{1-\nu_m^{\,2}}\cdot\frac{1+0.85\cdot V_f^{\,2}}{(1-V_f)^{1.25}+V_f\cdot E_m/(E_{\perp f}\cdot(1-\nu_m^{\,2}))}\;;\quad G_{\perp\parallel} = \frac{G_m\cdot(1+0.4\cdot V_f^{\,0.5})}{(1-V_f)^{1.5}+V_f\cdot G_m/G_{\perp\parallel f}}\,.$$

Remark considering Carbonfibers:

From "Thoughts of a 'Carbon Fiber-living' Structural Engineer about Application-generated hazardous CarbonFiber-WHO-size Fragments" in the draft [*Cun23d, in German*] *
During machining and operation, mechanical processing or thermal stress (oxidation) of the brittle CFRP components can produce CF fragments that meet the so-called WHO criterion: Filament-fracture particles with a diameter Ø < 3 µm, a length L > 5 µm and a ratio L/ Ø > 3/1



Glass GF (AR glass= alcali-resistant in concrete) **Basalt BsF** (alcali-resistant in concrete by ZrO_2)

Fig.: Different strengthening fibers and comparison CF with human hair. ASTM D3217/D3217M-20 Standard Test Methods for Breaking Tenacity of Manufactured Textile Fibers in Loop or Knot Configurations

The level of graphitization, assumed by the author, determines the modulus of elasticity of the carbon fiber (CF) and this correlates with a risk of fragmentation according to previous findings of the German research program ‘CarboBreak’. This clearly is fulfilled by the investigated meso-phase Pitch fibers (*i.e. used for space applications; (E_f > 550 GPa)*. These CF lead to WHO 'fiber'-shaped particles with a respirable Particulate Matter of aerodynamic diameter < 2.5 µm, termed PM 2.5.
The UltraHighModulus CF (E_f > *380 GPa*) are less gaphitized and have a lower Young’s modulus than the Pitch fibers. For the application of this type of CF a hazard may be possible, which is hopefully going to be cleared in the near future by a German research program.
For the CFs HT, IM and HM (*200 < 330 GPa*) no such a hazard is reported.

A4 Examples for a ’Quad‘ replacement and a Free DD Design Optimization *(from [Tsai22])*

<u>Use of ’DD’, replacement of an Existing ‘Quad’ laminate</u>

As partly still mentioned, it is of advantage to switch from a ‘Quad’ laminate to a ‘DD’ laminate substitute due to simplification of the stacking and to homogenization enabling an easier ply-drop and tapering in manufacturing.
In order to understand the benefits of the DD idea as a numerical example the replacement of an existing ’Quad’ reference laminate by a ‘DD’ laminate will be presented. This example is copied from [*DD Chapter 13*] and designation-adapted. Optimization objective is equivalent stiffness (**in-plane [$\hat{A}$], *** flexural bending [$\hat{D}$]). Side constraint for the decision, what is the optimum angle-ply sub-laminate, is the minimum FPF failure stress of the DD substitute laminate. The numerical example considers a usual residual stress value from cooling down of -100K and a moisture pick-up of 0.5% (*as sometimes applied in the DD book*). However the two effects somewhat cancel out for each other.

	* $\hat{A}$	$\hat{D}$	$\hat{B}$	** $\hat{A}$	*** $\hat{D}$
basis	GPa	GPa	GPa	GPa	GPa
11	94,2	104,6	0	94,2	104,6
22	72,7	70,3	0	72,7	70,3
21	27,9	23,9	0	27,9	23,9
66	32,7	28,7	0	32,7	28,7
61	0	-1,8	0	0	0
FPF	404.8 MPa			404.4	468 MPa

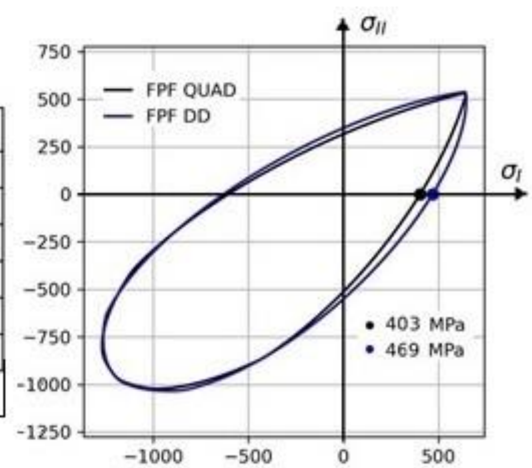


*Fig.A-5, IM6/Ep: (left, Chapter 13) Thickness-normalized stiffness sub-matrices of the laminate and achieved strength capacity. (right) **Laminate FPF failure stress due to in-plane stiffness equivalence substitute (E. Kappel* *mit -100^C, 0.5% mositure berechnet oder ohne = ‘neat ‘ laminate?)*

*$[0_3/45/90_2/-45_3/0/-45/45_2/90/45/-45/0/45]_{2S}$, **$\{20/-20/62/-62\}_{18T}$; ***$\{14/-14/61/-61\}_{18T}$

For **in-plane [$\hat{A}$]: The best ’Quad’-DD-substitute is $[20/-20/62/-62]_{18T}$ and for ***flexural bending [$\hat{D}$] it is $[14/-14/61/-61]_{18T}$, where the substitute delivers a higher value (computation *E. Kappel*). The right figure involves the multi-axial failure stress envelope obtained at FPF level.

The term R in the associated figure in [*Tsa22*] is not to distinguish with the technical strength, to be internationally termed *R*, because the resistance *R* is a standard-fixed technical value.

Use of the Free Software Lamsearch: from DD book, Chapter 13 and Think Composites

Lamsearch is an Excel-based open software search engine developed to find the optimum 'Quad' and 'Double-Double' laminates under multiple loads from 7 to maximally 49 independent load cases, depending on the version. It is a tool to find the best DD laminate for a given set of $\hat{A}_{ij}$. It presents a straightforward application of classical laminated plate theory and UD failure criteria (*other SFCs may be implemented*). Side constraint for the decision of the optimum angle-ply sub-laminate is a minimum FPF failure stress of the DD substitute laminate (not a maximum strength as was also recorded in the DD).

	[±Ψ]	2	3	4	5	6	7	8	9	10	11	12	13	
[±Φ]	0,0	7,5	15,0	22,5	30,0	37,5	45,0	52,5	60,0	67,5	75,0	82,5	90,0	max
0,0	56	57	60	67	84	117	169	243	332	404	411	368	338	411
7,5	57	58	60	68	84	116	168	243	333	388	394	359	336	394
15,0	60	60	64	69	84	114	167	244	336	395	410	392	378	410
22,5	67	68	69	76	87	114	165	240	337	421	446	439	431	446
30,0	84	84	84	87	99	118	160	233	334	447	**495**	491	485	495
37,5	117	116	114	114	118	136	166	231	332	422	430	436	439	439
45,0	169	168	167	165	160	166	197	244	283	288	298	305	308	308
52,5	243	243	244	240	233	231	244	240	207	202	207	212	214	244
60,0	332	333	336	337	334	332	283	207	176	157	155	156	157	337
67,5	404	388	395	421	447	422	288	202	157	140	130	127	127	447
75,0	411	394	410	446	**495**	430	298	207	155	130	121	115	114	495
82,5	368	359	392	439	491	436	305	212	156	127	115	111	109	491
90,0	338	336	378	431	485	420	289	201	150	125	114	109	108	485
max	411	394	410	446	495	436	305	244	337	447	495	491	485	495

Fig.A-6: Relationship of the DD angles' choice and the associate obtained minimum FPF failure stress

In [*Tsa22*] is cited: *"The optimal DD laminate is {30/-30/75/-75}. It appears in the Table above in bold face and rounded in red is given the "maximum strength" of 495 MPa* (?? *No, it seems to be the minimum FPF failure stress of the DD substitute laminate*) *of the laminate. A very practical feature of 'Lamsearch' is the search for alternative solution. If the optimal laminate is not suitable, one can look for other solutions which do not degrade the strength. The colourful zone is where the solution does not differ too much from the optimum. The clear zone goes away from the optimum. In this case going from {30/-30/75/-75} to {30/-30/90/90} solution degrades only by 2% from the optimum".*

*A5 Specific Terms, Glossar**

A general system of signs and symbols is of high importance for a logically consistent universal language for scientific use ! Gottfried Wilhelm Leibniz (about 1800)

From experience with applications intentionally put here in order to guide a right execution.

A general system of signs and symbols is of high importance for a logically consistent universal language for scientific use ! Gottfried Wilhelm Leibniz (about 1800)

Design Dimensioning: static and cyclic sizing

A-Basis (strength) Design Allowable (or "A"-Value): statistically-based material property, above which at least with a probability P = 99% of the population of values is expected to fall, with a confidence level of C = 95% . For failure-redundant laminates often the higher "B"-value is permitted, where P = 90%, C= 95%

Allowable Stress: notion that belonged to a 'retired' Safety Concept. (Shall not be used anymore since 1926, when applying modern safety concepts. The term is confusing and causes significant errors). See [HSB 02000-01]*

Angle-ply: balanced laminate, consisting of plies at arbitrary angles of plus and minus, where α is the angle of the fibers with the principal laminate axis

Average strength: $\sigma_{\text{failure}} \Rightarrow \bar{R}$, example UD material: $\{\bar{R}\} = (\bar{R}_{\parallel}^{t}, \bar{R}_{\parallel}^{c}, \bar{R}_{\perp}^{t}, \bar{R}_{\perp}^{c}, \bar{R}_{\perp\parallel})^{T}$

Balanced laminate: composite laminate in which all laminae at angles other than 0° and 90° occur only in ± pairs, and not necessarily adjacent. (balanced laminates may be not symmetric)

Brittle material: material, when subjected to especially tensile stress, will break without significant plastic deformation. A multi-axial laminate is not too brittle due to the some ductility giving stack

Building block (of laminate): Sub-laminates 'Quad' and 'DD'

Comparison Test Prediction: average values $\bar{R}$ and average stress-strain curves have to be applied in order to obtain the maximum expectation value 50%

Composite material: combination of constituent materials, different in composition, where the constituents retain their identities in the composite

(Strength) Design Allowable: see before

Design Dimensioning: static and cyclic sizing

Design Load: maximum amount of a loading (force, temperature, moisture, stiffness etc.) a (load-carrying) system is to be designed to

Delamination: separation of material layers within a laminate or also in a textile reinforced concrete (may be local or may cover a large area of the laminate)

Design Principle: design of a structure is the result of the design principle chosen. Such design principles are 'fail safe', 'safe life', 'damage tolerant'

Design Strain: classically in aerospace $3\,{}^{0}\!/_{00} = 0.3\,{}^{0}\!/_{0}$ valid for $\parallel, \perp$ multiple fiber direction stack

Design Value: value of a property used in design input which is assumed to respect its uncertainty. Value of a design variable which is used in a design verification

Design Verification (from Latin, veritas facere): fulfillment of a design requirement data set (for a deformation, a frequency, design load, etc)

(dimensioning) Design Load Cases: relevant load cases, to be extracted from the numerous load cases given by single loads, load combinations, stiffness requirements etc.

'Double-Double (DD) laminates': Two angle-plies of different fiber angles form a four-ply sub-laminate or building-block, respectively (for instance C-ply™ from Chomarat)

Elasticity quantities (usually average values): isotropic $\{\bar{E}\} = (E, \nu)^{T}$, UD $\{\bar{E}\} = (E_1, E_2, G_{21}, \nu_{21}, \nu_{23})^{T}$

Engineering stress: ratio of acting load and initial (non-deformed area

Equivalent stress: (1) Equivalent to a multi-axial stress state combining the effects of those stresses that are active in a distinct failure mode. (2) The uni-axial scalar σ_{eq}-value can be compared to the mode-'reigning' associated uni-axial 'basic' strength *R*

Failure: state of inability of an item to perform a required function in its limit state

(strength) Failure Condition: Condition on which a failure becomes effective, meaning $F = 1$ for one limit state. Mathematical formulation of the failure surface that takes the form $F = 1 = 100\ \%$. *Most often meant is a strength failure condition SFC. Aim of a SFC is to assess multi-axial states of stresses*

Fail-Safe: design philosophy in which products are designed in such a way that failure, prior to the required operational life, is not catastrophic

(strength) Failure Criterion (SFC): Distinctive feature defined as a condition for one of the 3 states, taking the form F > 1, F = 1, F < 1. A SFC capture one failure mode just once. Multi-fold acting failure modes , for instance $\sigma_2 = \sigma_3$, must be considered additionally, because the danger to fail multiplies

Failure function *F* : mathematical formulation of the failure event, $F = 1$and surface of the failure body

Failure Index: Originally just value of the failure function used with polymer composites which fits to *Eff* only in cases where the considered stress terms are linear (mathematically homogeneous) in the SFC). (Nowadays it corresponds to the material stressing effort *Eff corresponds to Tsai's Strength Ratio R*

Failure Modes (UD material): observable effect of the mechanism through which the failure occurs.

Failure Mode Concept (FMC)**:** invariant, failure mode-based general concept to generate strength failure conditions for single failure modes. It is a 'modal' formulation in contrast to 'global' concepts where all failure modes are mathematically linked and a concept for materials that can be homogenized (smeared). The applicability of a SFC ends if homogenization as pre-requisite of modeling is violated

Failure type: basically addressed are Normal Fracture NF, Shear Fracture SF under compression and Crushing Fracture CrF under compression. With UD material these are 2 Fiber Failures (FF) and 3 InterFiberFailures (IFF)

Fiber: term used to refer to filamentary materials.

Filament: thinly spun single fiber, extruder material feeding in '3D printing'

First-Ply-Failure (FPF): First Failure in a lamina of the laminate capturing FF and IFF

friction values, UD: transversely-isotropic UD lamina 'strength-impacting' property (with $0.05 < \mu_{\parallel\perp} < 0.3$ and $0.05 < \mu_{\perp\perp} < 0$)

Homogenization of a material conglomerate: descriptive term for a material of uniform composition throughout. Here: Achievement of a quasi-isotropic [*K*]-matrix

Interaction: process of a combined action of stresses, or loadings, or failure modes

Interaction exponent *m*: (Weibull modulus) entity, which captures the common effect of modes

Interface: boundary or surface between the individual, physically distinguishable constituents of a composite. (Note: Surface between filament and matrix and also used for the surface (2D) that separates two parts or two laminate layers)

Invariant: Combination of stresses or strains. Its value does not change when altering the coordinate system. The stresses in the invariants may be powered (exponents may 2, 3 or 4) or not powered. Invariants are advantageous when formulating the usually desired scalar failure conditions. Such material-associated invariants are given for isotropic, transversely-isotropic and orthotropic materials.

'Generic' number: Witnessed material symmetry knowledge seems to tell: There might exist a 'generic' (term was chosen by the author) material inherent number for material families,, namely 2 for isotropic and 5 for transversely-isotropic materials

Lamina: analytical designation of the single UD ply as computational element of the laminate, used as laminate subset or building block for modelling. It might capture several equal physical layers (plies)

lamina properties (isolated): properties obtained from traditional 'isolated' test specimens for UD lamina material. (Notes: (1) Values, which are used in analysis despite of the fact that they cannot consider the effect of embedding in the laminate which may improve the property values. (2) Embedded lamina properties are obtained from special sub-laminate test specimens, where the test ply is embedded between other plies which break at higher load levels than the test ply

Laminate Factor:

Last-Ply-Failure (LPF): Failure state where finally the last ply fractures, usually by Fiber Fracture (due to the fact that the matrix with interface influence onset of final failure LPF usually requires a non-linear analysis, which can be used to save a design. Just setting matrix elasticity properties zero means application of 'Net theory' which is a simple approach. (Application of Net Theory in the fiber tension domain $\sigma_{||}^{t}$ and of stability theory in the compression domain $\sigma_{||}^{c}$ because it is not a strength problem anymore)

Layer, ply: deposit from winding, tape-laying process etc.

Lay-up, stack: process of fabrication involving the assembly of successive layers of fiber-reinforced material, dry or prepreg

Limit state: state in which a structure or a material comes to a distinct limit such as FF, IFF

Loading: loads (including normal and shear forces, moments, torques), pressures, temperature and moisture applied to the structural system

Macro-mechanics: here is an approach in which the layers are considered homogeneous, size range of mm

Margin of Safety *MoS*: $MoS = RF - 1 > 0$

Master-Ply: incorporates the minimum scattering *Tr*-normalized stiffness values Q_{ij}^{TR} / Tr

Material: usually the model of a homogenized more complex solid material. (Note: On the considered scale (level) the homogenized model of the envisaged complex solid is modelled as a smeared solid. On engineering level a macro-model is preferred and normally used)

Material Properties: 'Agreed' values to achieve a common and comparable design basis. Must be provided with average value and coefficient of variation cov

Material Stressing Effort *Eff* ($\neq$ material utilization): artificial term, generated in the UD World Wide Failure Exercises in order to get an English term for the excellent, meaningful German term Werkstoff-Anstrengung. Tsai's so-called Strength Ratio R *(an otherwise still fixed letter R was chosen)* corresponds to *Eff*

Maxwell-Betti theorem: reciprocal work theorem (Gegenseitigkeit der Verschiebungsarbeiten).

(Note: Reads for the example UD material $\nu_{\perp||} \cdot E_{\perp} = \nu_{||\perp} \cdot E_{||}$ *and is applicable for the degraded elasticity matrix, too. Thereby, it is showing symmetry to the diagonal of the elasticity matrix [C or Q])*

Meso-scale: artificially chosen intermediate scale for so-called multi-scale analyses

Micro-mechanics: here, an approach in the filament size range of µm

Model: Theoretical conception of a real process

Non-Crimp Fabrics (NCF): type of non-woven fabric that consists of plies of UD-material laid up at any required angles and held together by a bonding agent or cross stitching (z-threads).

Omni failure envelope: Tsai's envelope of an intact Non-failure area concerning FPF and LPF

'Omni Non FPF domain':

Orthotropic: having three mutually perpendicular planes of elastic symmetry

PAN-CF: precursor PolyAcrylNitril-based CF (basic CF type)

Ply: physical fiber-reinforced material part

Ply-by-ply analysis: term used in laminate analysis if each ply (lamina) is analysed

Poisson's ratio ν: ratio of transverse strain and longitudinal strain of a uni-axially tensioned test specimen

Principal strains σ_I, σ_{II} : remaining components of the strain tensor after the original basis is transformed in such a way that the shear strain vanishes (makes no sense for material internal application, just for external application in order to achieve 2 coordinates for a plain visualization

Process-induced distortion:

Progressive Failure, ply, lamina: behaviour after onset of degradation of a ply of the loaded laminate.

Properties: 'Agreed' values to achieve a common and comparable design basis. Must be provided with average value and coefficient of variation CoV

Proportional Loading: loading situation, when all the external loads are applied simultaneously and when these increase remains in proportion to one another throughout the loading history .

'Quad laminates': (0°, 45°,-45°, 90°) sub-laminate family as laminate building block in aerospace etc

Quasi-isotropic laminate: laminate approximating isotropy by orientation of plies in several directions like [0/60/-60]

Redundant structure, where all of the unknowns cannot be found from equilibrium considerations alone.

Reserve Factor *RF*: load-defined value $RF_{\text{ult}} = \textit{final failure load} \ / \ \textit{design load DL}$, $RF > 1$

Repeat factor r: number of repetitions of a double angle DD–ply in a stack

(material) Reserve factor f_{RF}: $f_{\text{RF}} = \textit{strength design allowable R} \, / \, \textit{stress at design load DL}$

Robust design: design that performs optimally under the variable operating conditions during lifetime or optimally captures the scatter of the design parameters

Roving, tow, strand: number of yarns or ends collected in a parallel bundle with approximately no twist. (The cross-section of a roving is an oval, round cross-sections are caused by protection twist of about 10 rotations. The roving must be through-impregnated not only surface-coated in order to equally load each single filament. It is marked in thousands (k) of filaments. Instead of roving the term tow is often used in construction industry)

Safety concepts: deterministic, semi-probabilistic or even probabilistic concepts (formats) to capture uncertainties in order to implement structural reliability into the design

Safety Factor concept or factor of safety concept: deterministic concept using one single factor by which the level of the given loading is increased. The applied so-called Factors of Safety *FoS* are design load-increasing factors, see [*Cun12*].

Semi-finished product SFP: intermediate product which is further processed to become a final product.

Statistical distribution: arrangement of values of a variable showing their frequency of occurrence. (Note: A function describing the probability that a given value will occur is called the probability density function PDF, and the function describing the cumulative probability that a given value or any value smaller than it will occur is called the distribution function or cumulative distribution function, abbreviated CDF. Applicable for strength are the Weibull distribution and the logarithmic normal distribution. For loads, extreme value distributions are used)

Strength: Maximum uni-axial technical stress or failure stress, which is termed Resistance *R* (one mode). Strength values in general and strength design allowables are not marked by a 'bar over' but by the neat *R. For UD materials* $\{R\} = (R_{\parallel}^{t}, R_{\parallel}^{c}, R_{\perp}^{t}, R_{\perp}^{c}, R_{\perp\parallel})^{T}$

(compressive) Strength value: H. Schürmann and H. Bansemir showed that $R_{\parallel}^{c} \rightarrow R_{\parallel}^{t}$ if the tests are accurate (effortful) , enabling the fibers to lie straight. In the real world however there are undulation to face and stitching harms on top. This is valid for manual fabrication (manufacture) and to a less extent even for automatic fabrication. Conclusion: Compression is not a strength problem anymore but a micro-mechanic instability problem and highly impacts FF-linked failure envelopes (LPF)

Stress component: Term, that exactly should read stress tensor component or very simple just stress (*only a shear stress, like later the transversal shear stress* $\tau_{\perp\perp}$ *, can be composed of a tensile shear stress component jointly acting with a compressive shear stress component. The stress component with the larger failure danger due to the respective mode SFC will basically determine the fracture plane angle*)

Strength Design Allowables: statistically reduced average values such as A- and B-values or 5% fractiles in civil engineering

Strength Ratio: ratio of compressive strength to tensile strength R^{c}/R^{t}

Stress ratio R: ratio of minimum stress (mathematically, less positive) to maximum stress, R(σ)= minσ / maxσ, under cyclic (dynamic) fatigue loading

Subscripts: For the shear stresses τ, in accordance with international usage, the first subscript indicates the direction of the plane normal with respect to the plane upon which the shear stress is acting. The 2nd subscript indicates the direction of the shear force from the stress under consideration

Superscripts: Stress σ or stress τ , indicating the failure causing stress of normal fracture NF or shear SF

Tailored Fiber Placement TFP: textile manufacturing technique based on the principle of sewing for a continuous placement of fibrous material for composite components.

Tape: usually narrow UD-prepreg strip, however further multiaxial NCF tapes (fabricated in widths up to 1200 mm wide for carbon (UD tapes, NCF tapes, CF/PA6 tapes etc.)

'Trace' (Tr): specific trace of the [Q^{Tr}]-matrix is $Tr = Q_{11}^{\ Tr} + Q_{22}^{\ Tr} + 2 \cdot Q_{66}^{\ Tr}$, normalizing stiffness quantities

trace [Q] $= Q_{11} + Q_{22} + 1 \cdot Q_{66}$ with [Q] the 2D stiffness elasticity matrix

Transversely-isotropic material (UD, uni-directional): material model assumption, where the plane 2-3 is quasi-isotropic and due to that UD is termed transversely-isotropic

UD-lamina: lamina (ply) with a unidirectional reinforcement, being the building block of a laminate.

Undulation: waviness of yarns, tows

Ultimate Tensile Strength (UTS): A- or B-Strength Design Allowable

Validation of a model (from validus = strong): 'qualification' of a created model by well mapping physical test results with the derived model (*here material failure model*)

Verification (from Latin, veritas facere): Proof, that the product fulfils the product requirements data, defined in the performance requirements specification

Yarn: group, bundle or assembly of twisted or practically un-twisted filaments suitable in fabrication

S, T: Symmetric, Trials of angle-plies.

Notes on designations: As a consequence to isotropic materials (European standardization) the letter R (≡ f: in construction) has to be used for strength. US notations for UD material with letters X (*direction* 1, ||) and Y (*direction* 2, ⊥) confuse with the structural axes' descriptions X and Y. R_{m} := 'resistance maximale' (French) = tensile fracture strength (superscript ^{t} is usually skipped because in mechanical engineering design runs in the tensile domain, which is opposite to civil engineering, where fiber reinforcement is coming up viewing carbon concrete), R is a strength. Composites are most often brittle and only slightly porous! In the following Table, on basis of investigations of the VDI-2014 Working Group and on investigations for the formerly planned novel ESA Materials Handbook, Cuntze proposed internationally not confusing terms for strengths and physical properties. These self-explaining symbolic designations read.

Property type	**UD quantities**	**'generic' number**
fracture strength properties + friction properties	$\{R\} = (R_{\parallel}^{t}, R_{\parallel}^{c}, R_{\perp}^{t}, R_{\perp}^{c}, R_{\perp\parallel})^{\mathrm{T}}$, $\mu_{\perp\parallel}$, $\mu_{\perp\perp}$	5 2
elasticity properties	$\{E\} = (\mathrm{E}_{\parallel}, E_{\perp}, G_{\parallel\perp}, \nu_{\perp\parallel}, \nu_{\perp\perp})$	5
hygrothermal properties	$\{\alpha\} = \mathrm{CTE}\ (\alpha_{\parallel}^{T}, \alpha_{\perp}^{T})$; $\mathrm{CME}\ (\alpha_{\parallel}^{M}, \alpha_{\perp}^{M})$	2 ; 2

Short Curriculum Vitae: ***Dr.-Ing. Erik Kappel,*** erik.kappel@dlr.de

- Diploma degree in Mechanical Engineering from the Leibniz University of Hannover in 2009. Currently compiling his Habilitation at the Technical University of Braunschweig.
- Visiting researcher at the Materials-Engineering Department of the University of British Columbia (UBC) in 2013.
- Doctoral degree in Mechanical Engineering from Otto v. Guericke University Magdeburg in 2013
- Research Associate at German Aerospace Center's (DLR) Institute of Composite Structures and Adaptive Systems in Braunschweig.
- Research Interest:
- Composite design, analysis and manufacturing with a strong focus on process-induced distortions (PID) of CFRP components in aerospace applications, efficient numerical PID prediction and compensation-measure definition.

Curriculum Vitae: *Prof. Dr.-Ing. habil. Ralf Cuntze VDI,* Ralf_Cuntze@t-online.de

1939 born Sept 8 in Erfurt. Survived bombing and a machine gun fire from a U.S. tank, March 31, 1945

1964: Dipl.-Ing. Civil Engineering CE (construction, ***TU Hannover***). 1968: Dr.-Ing. in Structural Dynamics (CE). 1978: Dr.-Ing. habil. Venia Legendi in Mechanics of Lightweight Structures (TU-M)

1980-1983: Lecturer at Universität der Bundeswehr München: on 'Fracture Mechanics' in the construction faculty and 1990-2002 on 'Composite Lightweight Design' in aerospace faculty

1987: Full professorship 'Lightweight Construction', *not started in favor of industry*

1998: Honorary professorship at Universität der Bundeswehr München

1968-1970: FEA-programming (***DLR***-Essen/Mühlheim)

1970-2004: MAN-Technologie (München and Augsburg). Headed the Main Department 'Structural and Thermal Analysis'. 50 years of life with fibers CF, AF, GF, BF, BsF.

*Theoretical fields of work: structural dynamics, finite element analysis, rotor dynamics, structural reliability, partial/deterministic safety concepts, material modeling and model validation, fatigue, fracture mechanics, design development 'philosophy' & design verification

*Mechanical Engineering applications at ***MAN****:* ARIANE 1-5 launcher family (design of different parts of the launcher stages, inclusively Booster) Cryogenic Tanks, High Pressure Vessels, Heat Exchanger in Solar Towers (GAST Almeria) and Solar Field, Wind Energy Rotors (GROWIAN Ø103 m, WKA 60, AEROMAN. Probably the first world-wide wind energy conferences organized in 1979, 1980 with Dr. Windheim), Space Antennas, Automated Transfer Vehicle (Jules Verne, supplying the space station ISS), Crew Rescue Vehicle (CMC application) for ISS, Carbon and Steel Gas-Ultra-Centrifuges for Uranium enrichment. Filament Winding theory. Material Databank etc.

*Civil Engineering applications: Supermarket statics, armoring plans, pile foundation, 5th German climbing garden (1980 designed, concreted and natural stone-bricked)

1971-2010*:* Co-author of ***ESA/ESTEC***-Structural Materials Handbook, Co-author and first convener of the ESA-Buckling Handbook and co-author in Working Groups WGs for ESA-Standards 'Structural Analysis', 'High Pressure Vessels' (metals and composites) and 'Safety Factors'

1972–2015, ***IASB***: Luftfahrt-Technisches Handbuch HSB 'Fundamentals and Methods for Aeronautical Design and Analyses'. Author and Co-author of numerous HSB sheets and about 2006-2008 co-transfer with co-translation of the HSB aerospace structural handbook into its present English version.

1980-2011: **Surveyor/Advisor** for German BMFT (MATFO, MATEC), BMBF (LuFo), DFG

1980-2006: **VDI Guideline 2014**, co-author of Parts 1 and 2, Beuth Verlag '*Development of Fiber-reinforced Plastic Components';* Part 3 *'Analysis*', editor/convener/co-author

1986 and 1889: One week lecture, each, on composite design in Pretoria, SA

2019:***GLOSSAR.** "Fachbegriffe für Kompositbauteile - *technical terms for composite parts*". Springer2019. Edited at the suggestion of carbon concrete colleagues to help to better understand each other

2000-2013: World-Wide-Failure-Exercises WWFE on UD materials' strength: WWFE-I (*2D stress states)* non-funded winner against institutes of the world, WWFE-II (*3D states*) top-ranked

2009-2021 linked to ***Carbon Composites e.V***. at Augsburg, later ***Composites United CU e.V.*** and to TUDALIT Dresden. Since 2011 working on the light weight material Fiber-reinforced (polymer) Carbon Concrete. **Founded and headed the working groups**: (1) 2009: 'Engineering' linked to the WG Non-Destructive Testing and the WG Connection Technologies, mechanical engineering. (2) 2010: 'Composite Fatigue'. In 2010 the author held an event that was excellently attended by international speakers. (3) 2011: 'Design Dimensioning (*Auslegung*, *Bemessung*) and Design Verification (*Nachweis*)' mainly for carbon concrete. This working group was the foundation stone for the later specialist network ***CU Construction***, aiming at *"Fiber-based lightweight construction*". (4) 2017: 'Automated fabrication in construction including serial production' *(3D-Print)*. (5) 2020, 2021: Forum 'Carbon concrete for practice' at 'Ulm Concrete Days'.

2022:* *Life-Work Cuntze - a compilation from the author's papers, presentations, published and non-published design sheets and project works in industry* (850 Pages)

2024:* Curriculum Vitae Ralf Cuntze, comprising Career, Scientific Findings & Personal Pictures