Leibniz Universität Hannover
Teaching assistant at construction Mechanics Chair

20. Münchner Leichtbauseminar, 2023, 40 min + 10 min

Static 3D-Strength Failure Criteria for the Structural Material Families *Isotropic*, *Transversely–isotropic* UD-Lamina and *Orthotropic* Fabrics

on basis of Cuntze's Failure-Mode-Concept (FMC)

1 Introduction to Strength Failure Criteria (*SFC*)
2 Motivation for the SFC-Generation
3 'Global' SFCs versus 'Modal' SFCs
4 Basics of Cuntze's Failure-Mode-Concept (*FMC*), tool for SFC derivation
5 Application Isotropic: Foam, Concretes, Plexiglass
6 Application Transversely-isotropic UD: FRP Lamina (= focus)
7 Application Orthotropic Fabric: Ceramic
Some Conclusions with Findings

Just delivery of background + SFC-application.
No SFC formula details with discussions ▶

Results of a time-consuming never funded "hobby". ***Since 1970 in the FRP composite business.***

Prof. Dr.-Ing. habil. Ralf Cuntze VDI, formerly MAN-Technologie AG, linked to Carbon Composite e.V. (CCeV) Augsburg, heading the WGs "Engineering" (Mechanical Engineering, since 2009, '" Dimensioning and design verification of composite parts" in Civil Engineering' since 2011, and in Composites United Bau "Automated Manufacturing in Civil Engineering", since 2017.

For me, the presentation shall give an overarching understanding. I will only go a little more detailed into the UD SFC-formulas.

Note on designations and used terms:

Since the author is looking at all 3 material families at the same time,

(*Which author has done this before*?)

he used a self-explanatory, symbolic indexing,

as he sensibly defined it as editor of VDI 2014, Sheet 3 'Analysis‘ 2006,

on the basis of already well-known old designations

together with his working group colleagues, such as A. Puck.

This will make understanding over the material & discipline fences possible**!**

Good 'Design Dimensioning' (Auslegung) **+ 'Design Verification'** (Nachweis)
that a distinct Strength Limit has not yet been reached
requires the application of Validated Strength Failure Criteria (SFC).

*This captures for **ductile** behavior*

Yield SFCs for

Non-linear Analyses and for **Yield Limit Design Verification**

representing a test data-validated failure envelope, described by the

Failure Function ***F*****,** such as with the SFC Mises: $F^{\text{Mises}} = \sqrt{3J_2} \,/\, R_{0.2} = 1 = 100\ \%$

*and for **brittle** behavior*

SFCs for Fracture Limit Design Verification ***F = 1 = 100%***

(Failure Function *F* mathematically describes the Surface of the Fracture Body.

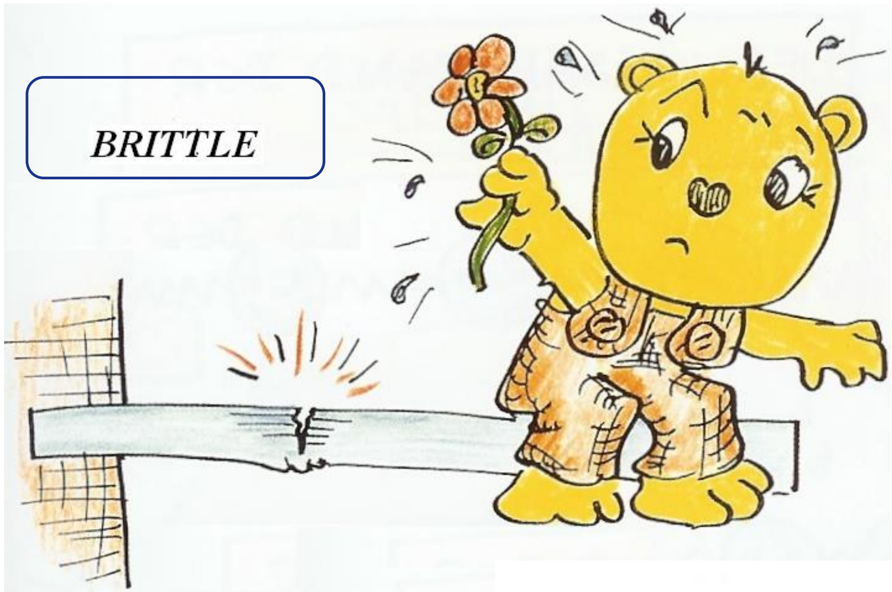
F consists of one or more functional parts.
The surface is the smoothed shape of the multi-axial *failure stress vector* ends)

► Strength Failure Criteria capture yield and fracture !

The Visualization of Novel Failure Bodies
will be one essential subject of the presentation !

How may one principally discriminate *Material Behaviour* ?

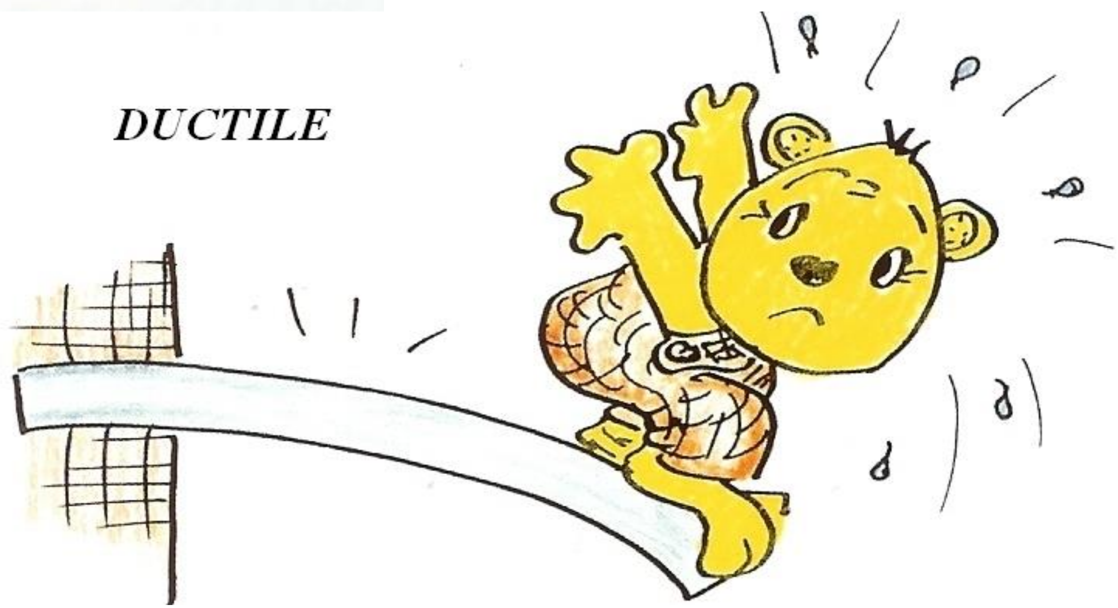
Focus here



One feels good until sudden fracture occurs

Courtesy: Prof. C. Mattheck

DUCTILE



Ductile Fracture = type of a failure mode in a material or structure generally preceded by a large amount of plastic deformation

„What is a basic Structural Design Verification Task in industry ?“

The Achievement of a Reserve Factor *RF >1* against a Limit State in order to achieve Certification for the Production of the Structural Part

---

For each designed structural part it is to compute

for each distinct ‘Load Case‘ with its various Failure Modes

Reserve Factor (load-defined) **:** ***RF = Failure Load / applied Design Load***

Material Reserve factor : ***f_{RF} = Strength / Applied Stress***

if linear analysis: ***f_{RF} = RF = 1 / Eff***

Material Stressing Effort *: ***Eff = σ / R = 100% if RF = 1***

(Werkstoff-Anstrengung, a very expressive German Term)

*** equivalent in English an artificial technical term being created in 2003 together with QinetiQ during the World-Wide-Failure-Exercise I .**

$$\text{Relationship of } F \text{ with } Eff = \sigma / R:$$

$$\text{SFC Mises}: F^{\text{Mises}}(\text{uniaxial}) = \sqrt{3J_2} / R = \sqrt{3 \cdot 2\sigma^2 / 6} / R = \frac{\sigma}{R} \quad \Rightarrow F = Eff.$$

Motivation 1 for the investigation: Advantageous Use of the Material Stressing Effort

≡ *'Modal'* material stressing effort *

$$Eff^{mode} = \sigma_{eq}^{mode} / \overline{R}^{mode}$$

equivalent mode stress

mode associated average strength

$$Eff^{fracture\ mode} = \sigma_{eq}^{fracture\ mode} / R_m$$

$$Eff^{Mises} = \sigma_{eq}^{Mises} / R_{po.2}$$

v. Mises, modal SFC

<u>Advantages</u>: In the case of 'Modal' SFCs

- *Eff* and σ_{eq} are always clearly to define
- *Eff* is linearly <u>and</u> non-linearly applicable !

see →

***Eff* is necessary to interact the mode failure portions !**

Motivation 2: Achieving Equivalent Stresses σ_{eq} !

→ *This has 2 aspects for the author:*

(1) σ_{eq} **captures the common action *Eff* (Werkstoffanstrengung)**

of a multi-axial stress state, active in a distinct failure mode

is equal to the multi-axial stress state ***as in***

**** Mises σ_{eq} : ductile, Mode 'Shear stress Yielding',***

**** Maximum σ_{eq} : brittle, Mode 'Normal Fracture' etc.***

(2) The value of σ_{eq} is

comparable to a strength value R

belonging to the activated failure mode.

What have the ancestors already found for enabling a physical derivation of Strength Failure Criteria ?

Motivation 3: Knowledge from Beltrami and Mohr-Coulomb for SFCs

„Isn't a SFC-derivation basically just the application of *Beltrami ?*
(strain energy W in a solid cubic element of a material will consist of two portions, namely
isotropic W_{volume}, I_1^2 + W_{shape}, J_2)

and of *Mohr/Coulomb ?* "
(as third portion, friction is to consider under compression-caused shear stressing)

Hencky-
Mises-
Huber

Richard von Mises
1883-1953
Mathematician

Eugenio Beltrami
1835-1900
Mathematician

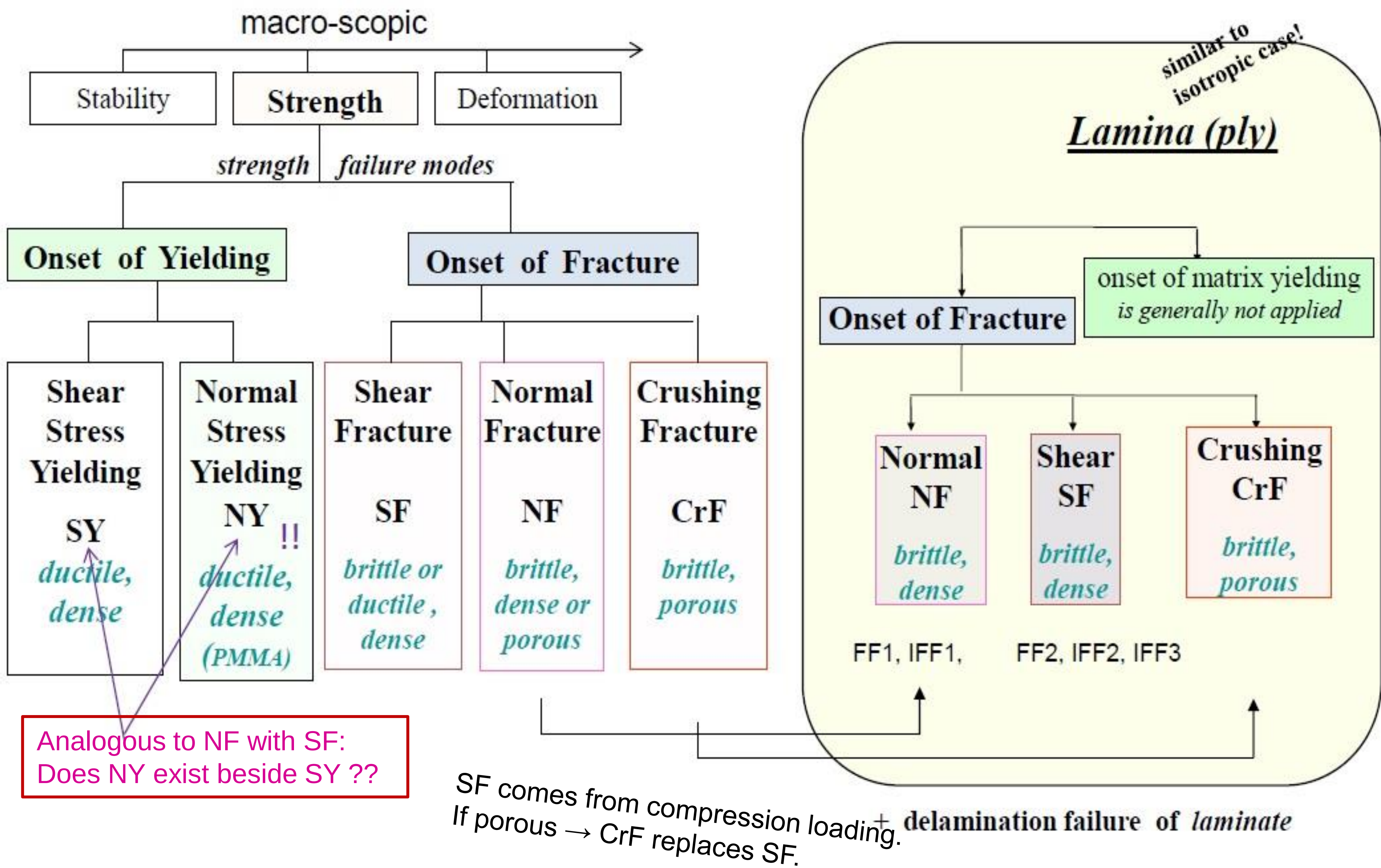
Otto Mohr
1835-1918
Civil Engineer

Charles de Coulomb
1736-1806
Physician

'Onset of Yielding'

'Onset of Cracking'

Motivation 4: Checking by test results, whether Cuntze's system of Failure Modes (assumed 1990) is sensible ?



„*Which SFC Types are used?*“ So-called ‘Modal‘ and ‘Global‘ (pauschal) SFCs

Cuntze’s ‘Play on Words’

All modes are married in the Global formulation.
Any change hits all mode domains NF and SF of the fracture body surface

Drucker-Prager, Ottosen, Willam-Warnke, Tsai-Wu,
Altenbach/Bolchun/ Kulupaev, Yu , etc.

1 Global SFC :	$F\left(\{\sigma\},\{R\}\right)=1$	global formulation, usually
Set of Modal SFCs :	$F\left(\{\sigma\},\{R^{\text{mode}}\}\right)=1$	model formulation in the FMC

Mises,Puck,Cuntze

All modes are separately formulated.
Any change hits only the relevant domain of the fracture body surface

$F\left(\{\sigma\},\{R^{\text{mode}};\ \mu^{\text{mode}}\}\right)=1$ more precise formulation **Novel**

by direct introduction of the friction value
considering Mohr-Coulomb for brittle materials under compression

$$UD:\quad \{\sigma\}=(\sigma_1,\sigma_2,\sigma_3,\tau_{23},\tau_{31},\tau_{21})^T,\ \ \{\overline{R}\}=(\overline{R}_{\|}^t,\overline{R}_{\|}^c,\overline{R}_{\perp}^t,\overline{R}_{\perp}^c,\overline{R}_{\perp\|};\ \mu_{\perp\|},\mu_{\perp\perp})^T$$

$$Isotrop:\{\sigma\}=(\sigma_x,\sigma_y,\sigma_z,\tau_{yz},\tau_{zx},\tau_{xy})^T=(\sigma_I,\sigma_{II},\sigma_{III})^T,\ \ \{\overline{R}\}=(\overline{R}^t,\overline{R}^c;\ \mu)^T$$

Needs an interaction of Failure Modes:
This is performed by a probabilistic approach (series failure system) in the transition zones between neighboring modes NF and SF

FMC-based creation of SFCs : How can the Driving Ideas below realized?

performed by the author analogously to **:**

- **failure mode-wise** *(shear yielding failure, etc.)*	**Mises, Hashin, Puck etc.**
- **stress invariant-based** *(J_2 etc.)* using *physical content of the distinct Invariant*	**Mises, Tsai, Hashin, Christensen, etc.**
- **use of material symmetry** demands	**Christensen**
- **obtaining equivalent stresses** (treated)	**Mises for shear yielding, Rankine for fracture**

Details of the first 3 points ▶

► Failure mode-wise based **Features of the FMC** (1995)

It could be found:

- **Each failure mode represents 1 independent failure mechanism**

  and thereby 1 piece of the complete *failure surface*

- **Each failure mechanism is governed by 1 basic strength** (**is observed!**)

- **Each failure *mode* can be represented by 1 strength failure *criterion*** ***(SFC).***

  Therefore, equivalent stresses can be computed for each mode !!

► Stress Invariants-based (example isotropic)

Invariants (see Mises) are linked to a physical mechanism of the deforming solid !

Following Beltrami, Mises and Mohr-Coulomb *for isotropic materials*

- volume change : $\mathbf{I_1}^2$ … ***(dilatational energy)*** **relevant if porous**
- shape change : $\mathbf{J_2}$ (**Mises**) … ***(distortional energy)*** **relevant if material element shape changes**
- friction : $\mathbf{I_1}$ … ***(friction energy)*** **relevant if brittle**

Mohr-Coulomb

Analogous for transversely-isotropic UD materials !! [CUN §1]

Isotropic invariants:

$$I_1 = (\sigma_I + \sigma_{II} + \sigma_{III})^T = f(\sigma) \ ,$$

$$6J_2 = (\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2 = f(\tau)$$

$$27J_3 = (2\sigma_I - \sigma_{II} - \sigma_{III}) \cdot (2\sigma_{II} - \sigma_I - \sigma_{III}) \cdot (2\sigma_{III} - \sigma_I - \sigma_{II})$$

► Use of material symmetry demands ('generic number' as novel idea)

There seems to exist (after intensive investigations of the author)
a 'generic' (term was chosen by the author)
material inherent number for the 3 Material Families:

Isotropic Material: 2

- 2 *elastic 'constants', 2 strengths, 2 strength failure modes* (NF,SF; NY,SY) and just 2 *fracture toughnesses* $K_{Icrit}^{NF} \equiv K_{\mathbf{Ic}rit}$ and K_{IIcrit}^{SF} (defined here as modes, where the crack plane <u>does not turn</u> , some proof in *[CUN §4.2]). Beside* $K_{\mathbf{Ic}rit}$ *the terms* K_{IIcrit} *and* $K_{IIIcrit}$ *are 'just ' model parameters of the classical tension-linked formula)*.

Transversely-Isotropic Material: 5

- 5 *elastic 'constants', 5 strengths, 5 strength failure modes* (NFs with SFs) , *5 fracture mechanics modes*

Orthotropic Material: 9

The Full Proof of the existence of a 'generic ' number will significantly simplify the *Structural Mechanics Building* !

Choice of a Modal Concept: → Requires Interaction of the single Modal SFCs

Multi-axial stress states usually activate more than one failure mode.

This **Interaction in the 'mode transition zones'** of

adjacent Failure Modes *is captured by a* ***series failure system*** **model**

= 'Accumulation' of interacting *failure danger portions* Eff^{mode}

$$Eff = \sqrt[m]{(Eff^{\mathrm{mode\ 1}})^m + (Eff^{\mathrm{mode\ 2}})^m + \ldots} = 1 = 100\,\%, \text{ if failure}$$

with a mode-interaction exponent *2.5 < m < 3* **,** *from mapping experience*

It is assumed engineering-like: *m* takes the same value for all mode transition zones captured by the interaction formula above

<u>In the context of above a Note on the difference of</u> ***Eff*** **and |*F*|** :
Applying an interaction equation to consider all micro-damage causing portions of all activated modes makes to move from the absolute value of the Failure Function |*F*| to *Eff !*

* For a mathematically homogeneous Failure Function F using $Eff = \sigma / R$ it reads

$$F^{\mathrm{Mises}}(\mathrm{uniaxial}) = \sqrt{3J_2}\,/\,R = \sqrt{3 \cdot 2\sigma^2 / 6}\,/\,R = \frac{\sigma}{R} = 1 \quad \text{or} \quad Eff = 1 \quad \Rightarrow F \equiv Eff = \frac{\sigma}{R}.$$

* For a mathematically non-homogeneous F such as

$$F = c_1 \cdot \frac{\sigma^2}{R^2} + c_2 \cdot \frac{\sigma}{R} \quad \text{or} \quad F = c_1 \cdot Eff^2 + c_2 \cdot Eff \qquad \Rightarrow \mathrm{F} \neq Eff.$$

|*F*| *was formerly often termed Failure Index*

Pre-requisites, required for Generating FMC-based Strength Failure Criteria (SFC)

An SFC $\boldsymbol{F = 1}$ is the mathematical formulation of the described failure surface !

Pre-requisites for the establishment of the **F**ailure function *F* are:

- simply formulated, numerically robust,
- **physically-based,** and therefore, need only few information for pre-dimensioning
- shall allow for a simple determination of the design driving *RF* or *Eff*
- all **model parameters should be measurable**.

Prerequisites, especially required for UD Material Modelling and Validation

- The UD-lamina is homogenized to a macroscopically homogeneous solid or the lamina is treated as a ‘smeared‘ material

- The UD-lamina is transversely-isotropic:
  On planes transverse to the fiber direction it behaves quasi-isotropically

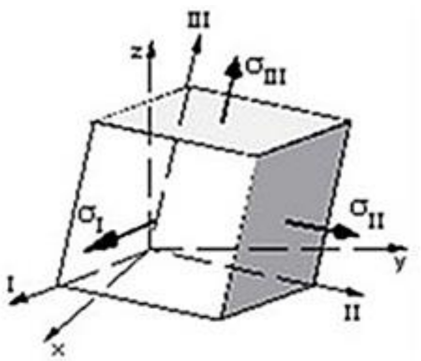
- For validation of the model a uniform stress state about the critical stress ‘point‘ location is mandatory.

Which are the derived SFCs?
At first the formula set + test data mapping for the isotropic material family? ⇨

SFCs for Dense + Porous Isotropic Materials *(SFCs for use)*

Dense

$$F^{SF} = c_{1\Theta}^{SF} \cdot \frac{3J_2 \cdot \Theta^{SF}}{\bar{R}^{c2}} + c_{2\Theta}^{SF} \cdot \frac{I_1}{\bar{R}^c} = 1$$



isotropic

$$\{\sigma\} = (\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{xz}, \tau_{xy})^T$$

$$\{R\} = (R^t, R^c)^T \text{ with } \mu$$

Normal Fracture NF for $I_1 > 0$ $\leftrightarrow$ Crushing Fracture CrF for $I_1 < 0$

$$F^{NF} = c^{NF} \cdot \Theta^{NF} \cdot \frac{\sqrt{4J_2 \cdot \Theta^{NF} - I_1^2/3} + I_1}{2 \cdot \bar{R}^t} = 1 \leftrightarrow F^{CrF} = c^{CrF} \cdot \Theta^{CrF} \cdot \frac{\sqrt{4J_2 \cdot \Theta^{CrF} - I_1^2/3} + I_1}{2 \cdot \bar{R}^c} = 1$$

$$Eff^{NF} = c^{NF} \cdot \frac{\sqrt{4J_2 \cdot \Theta^{NF} - I_1^2/3} + I_1}{2 \cdot \bar{R}^t} = \frac{\sigma_{eq}^{NF}}{\bar{R}^t} \leftrightarrow Eff^{CrF} = c^{CrF} \cdot \frac{\sqrt{4J_2 \cdot \Theta^{CrF} - I_1^2/3} + I_1}{2 \cdot \bar{R}^c} = \frac{\sigma_{eq}^{CrF}}{\bar{R}^c}.$$

If a failure body is rotationally symmetric, then $\Theta = 1$ like for the neutral or shear meridian, respectively.
A 2-fold acting mode makes the rotationally symmetric fracture body 120°-symmetric and is modelled by using the invariant J_3 and Θ as non-circularity function with d as non-circularity parameter

$$\Theta^{NF} = \sqrt[3]{1 + d^{NF} \cdot \sin(3\vartheta)} = \sqrt[3]{1 + d^{NF} \cdot 1.5 \cdot \sqrt{3} \cdot J_3 \cdot J_2^{-1.5}} \leftrightarrow \Theta^{CrF} = \sqrt[3]{1 + d^{CrF} \cdot 1.5 \cdot \sqrt{3} \cdot J_3 \cdot J_2^{-1.5}}$$

Lode angle ϑ, here set as $\sin(3 \cdot \vartheta)$ with 'neutral' (shear meridian) angle $\vartheta = 0°$ ($\rightarrow \Theta = 1$);
tensile meridian angle 30° $\rightarrow \Theta^{NF} = \sqrt[3]{1 + d^{NF} \cdot (+1)}$; compr. mer. angle -30° $\rightarrow \Theta^{CrF} = \sqrt[3]{1 + d^{CrF} \cdot (-1)}$.

Mode interaction $\rightarrow$ Equation of the fracture body: $Eff = [(Eff^{NF})^m + (Eff^{SF})^m]^{m^{-1}} = 1 = 100\%$

$$Eff = \sqrt[m]{\left(c^{NF} \cdot \frac{\sqrt{4J_2 \cdot \Theta^{NF} - I_1^2/3} + I_1}{2 \cdot \bar{R}^t}\right)^m + \left(c^{CrF} \cdot \frac{\sqrt{4J_2 \cdot \Theta^{CrF} - I_1^2/3} + I_1}{2 \cdot \bar{R}^c}\right)^m} = 1.$$

Curve parameter relationships obtained by inserting the compressive strength point (0, $-\bar{R}^c$, 0):

* 120°-rotat. symmetric $\Theta \neq 1$: $c_{1\Theta}^{SF} = 1 + c_{2\Theta}^{SF} \cdot \sqrt[3]{1 + d^{SF} \cdot (-1)}$, with

c^{NF}, Θ^{NF} from the 2 points ($\bar{R}^t$, 0, 0) and ($\bar{R}^{tt}, \bar{R}^{tt}$, 0) or by minimum error fit, if data available,

c^{CrF}, Θ^{CrF} from the 2 points ($-\bar{R}^c$, 0, 0) and ($-\bar{R}^{cc}$, $-\bar{R}^{cc}$, 0) or by minimum error fit.

The failure surface is closed at both the ends! A paraboloid serves as closing cap and bottom

$$\frac{I_1}{\sqrt{3} \cdot \bar{R}^t} = s^{cap} \cdot \left(\frac{\sqrt{2J_2 \cdot \Theta^{NF}}}{\bar{R}^t}\right)^2 + \frac{\max I_1}{\sqrt{3} \cdot \bar{R}^t}, \quad \frac{I_1}{\sqrt{3} \cdot \bar{R}^t} = s^{bot} \cdot \left(\frac{\sqrt{2J_2 \cdot \Theta^{CrF}}}{\bar{R}^t}\right)^2 + \frac{\min I_1}{\sqrt{3} \cdot \bar{R}^t}$$

Slope parameters s are determined connecting the respective hydrostatic strength point with the associated point on the tensile and compressive meridian, max I_1 must be assessed whereas min I_1 can be measured. $\bar{R}^t$ works as normalization strength. [CUN §5]).

Isotropic Material: Stresses and Invariants used in Numerical Applications

* Structural Stresses and Invariants:

$$I_1 = (\sigma_x + \sigma_y + \sigma_z)\ ,\ \ I_2 = \sigma_x \cdot \sigma_y + \sigma_z \cdot \sigma_y + \sigma_x \cdot \sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{xz}^2$$

$$I_3 = \sigma_x \cdot \sigma_y \cdot \sigma_z + 2\tau_{xy} \cdot \tau_{yz} \cdot \tau_{xz} - \sigma_x \cdot \tau_{yz}^2 - \sigma_z \cdot \tau_{xy}^2 - \sigma_y \cdot \tau_{xz}^2$$

Main Invariants I_1 , $J_2 = I_1^{\,2}/3 - I_2 = \left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2\right]/6,\ \ J_3 = 2 \cdot I_1^{\,3}/27 - I_2 \cdot I_1/3 + I_3.$

* Lode angle ϑ on the hoop plane measured from the chosen point zero (here) the shear meridian reads where $\vartheta = 0$

$$\Theta = \sqrt[3]{1 + d \cdot \left(1.5 \cdot 3^{0.5} \cdot J_3 \cdot J_2^{\,-0.5}\right)} = \sqrt[3]{1 + d \cdot \sin(3\vartheta)} \quad \text{using}$$

$$Sek = 1.5 \cdot 3^{0.5} \cdot J_3 \cdot J_2^{\,-1.5},\ \ \vartheta = \mathrm{Re}\left(a\sin(Sek)/3\right),\ \ \vartheta° = \vartheta \cdot 180°/\pi$$

with d = non-circularity parameter, quantifying the isotropic 120°-symmetry (denting).

* Principal Stresses and Invariants:

Principal Stresses are the components of the stress tensor if the shear stresses become zero

$$3\sigma_I = I_1 + 2\sqrt{I_1^{\,2} - 3I_2} \cdot \cos\tau,\ \ 3\sigma_{II} = I_1 + 2\sqrt{I_1^{\,2} - 3I_2} \cdot \cos\left(\vartheta - 2\pi/3\right),\ \ 3\sigma_{III} = I_1 + 2\sqrt{I_1^{\,2} - 3I_2} \cdot \cos\left(\vartheta - 4\pi/3\right)$$

$\sigma_I, \sigma_{II}, \sigma_{III}$ principal stresses, $\sigma_I > \sigma_{II} > \sigma_{III}$ mathematical stresses (> means more positive).

$$I_1 = (\sigma_I + \sigma_{II} + \sigma_{III}) = f(\sigma),\ \ 6J_2 = (\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2 = f(\tau)$$

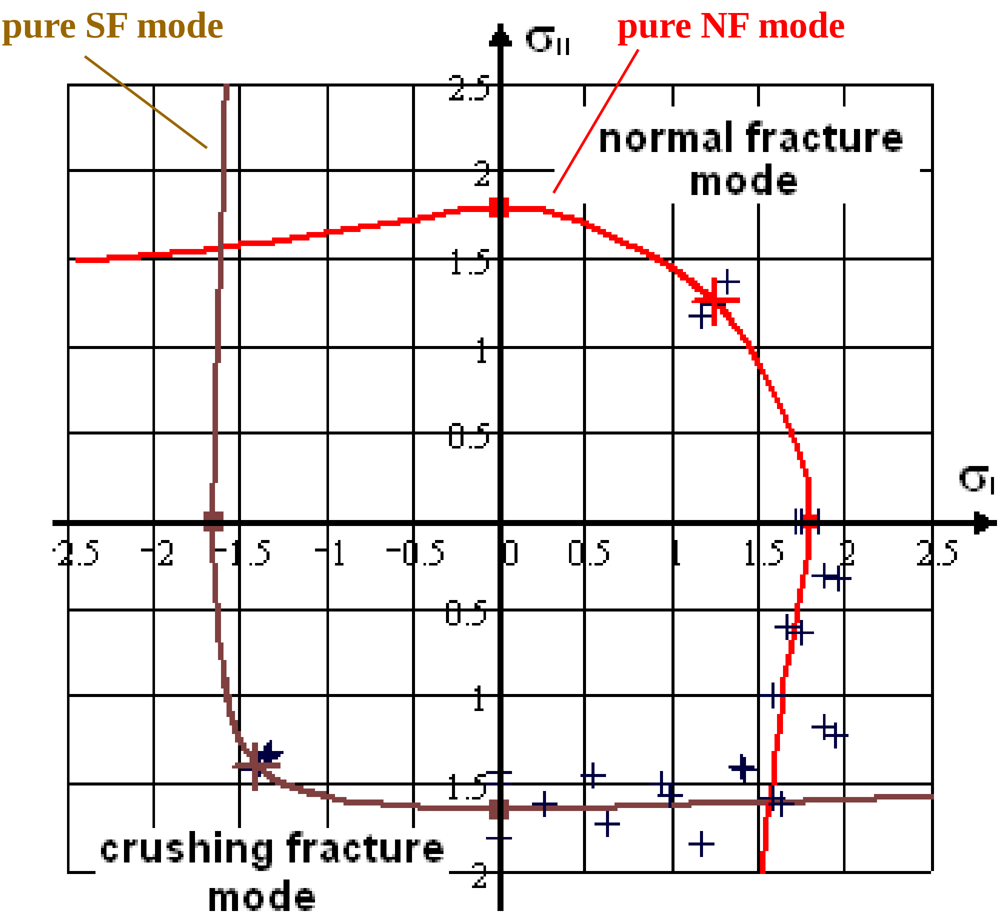
$$27J_3 = (2\sigma_I - \sigma_{II} - \sigma_{III}) \cdot (2\sigma_{II} - \sigma_I - \sigma_{III}) \cdot (2\sigma_{III} - \sigma_I - \sigma_{II}),$$

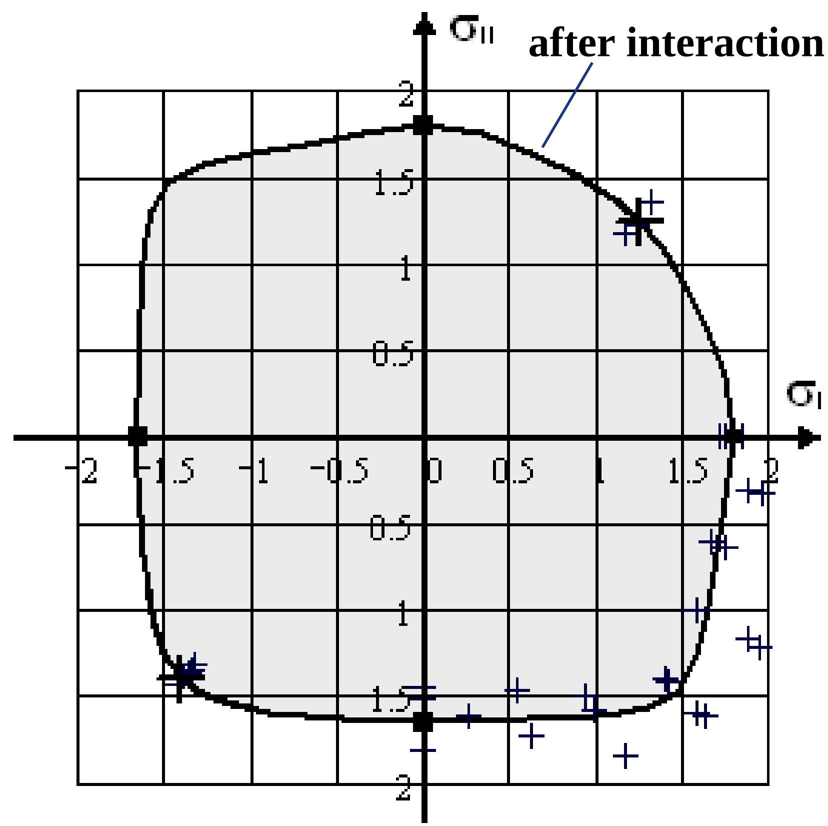
Mapping examples for very different isotropic (homogenized) materials follow ▶

<u>Foam:</u> Mapping of the course of <u>2D-Test Data</u> in the Principal Stress Plane

'Principal Plane Cross-section' of the Fracture Body (oblique cut)

Rohacell 71 IG





- **Mapping is to base on average Strengths** $\bar{R}$
- **Mapping <u>must</u> be performed in the 2D-plane because fracture data set is given there**

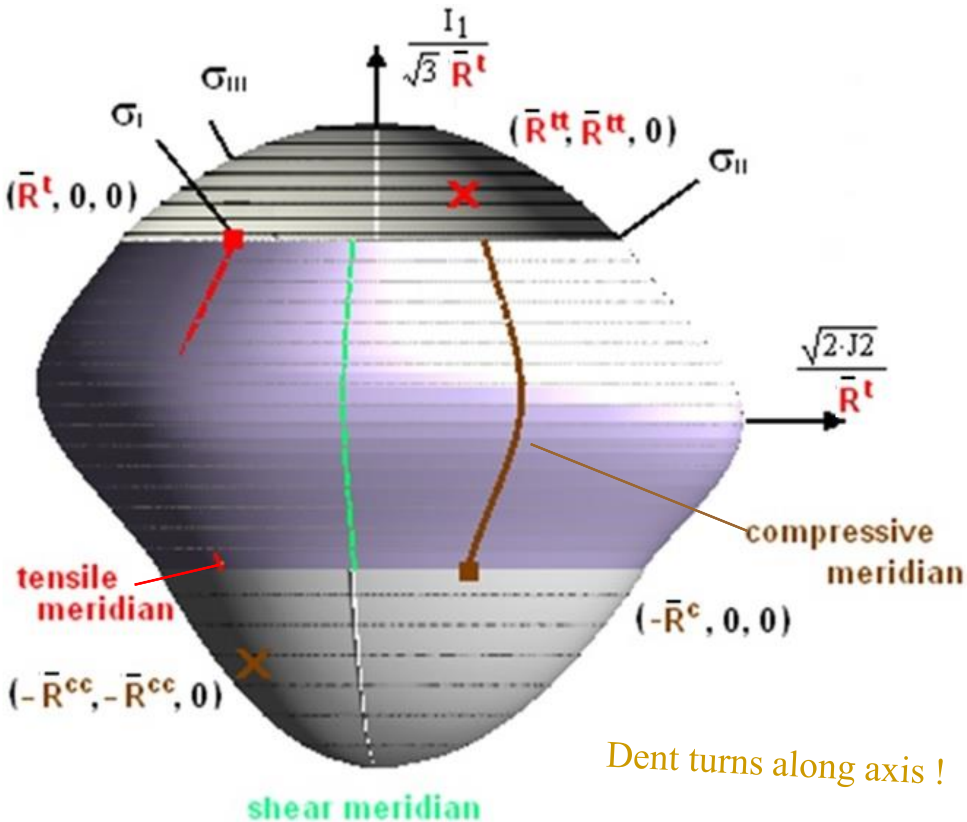
  2D-mapping uses the 2D-subsolution of the 3D-SFC
- **The 3D-fracture failure surface (body) is then given on basis of the 2D-derived model parameters.**

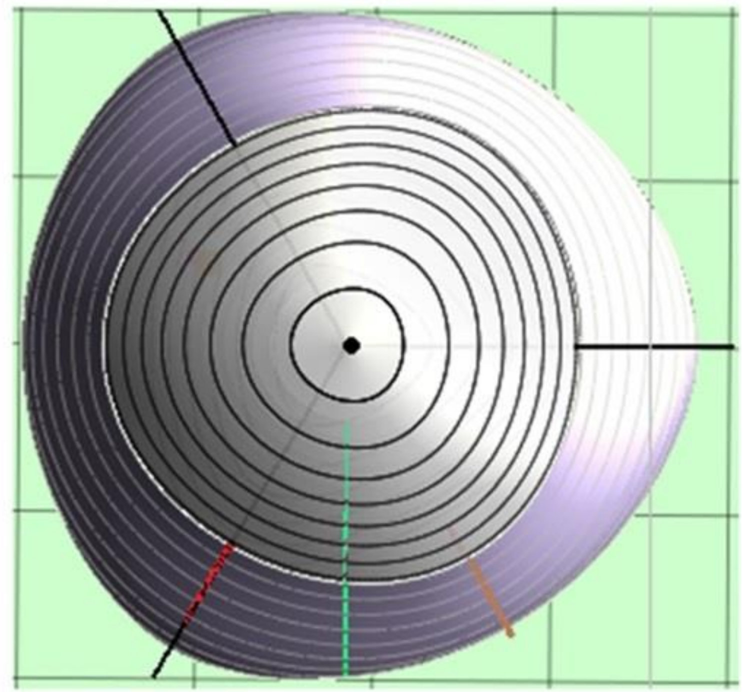
Application isotropic

Courtesy: LBF-Darmstadt, Dr. Kolupaev

Foam: Mapped Surface of not rotationally-symmetric Fracture Body *(novel)*

Isotropic *Rohacell 71 IG* [*CUN*]

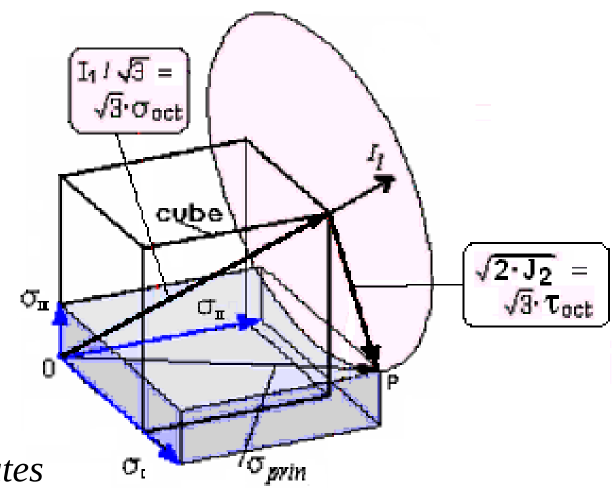




The 3 axes can be exchanged due to 120° symmetry of isotropic bodies!

$$Eff^{NF} = c^{NF} \cdot \frac{\sqrt{4J_2 \cdot \Theta^{NF} - I_1^{\,2}/3} + I_1}{2 \cdot \bar{R}^t} = \frac{\sigma_{eq}^{NF}}{\bar{R}^t},$$

$$Eff^{CrF} = c^{CrF} \cdot \frac{\sqrt{4J_2 \cdot \Theta^{CrF} - I_1^{\,2}/3} + I_1}{2 \cdot \bar{R}^c} = \frac{\sigma_{eq}^{CrF}}{\bar{R}^c}.$$

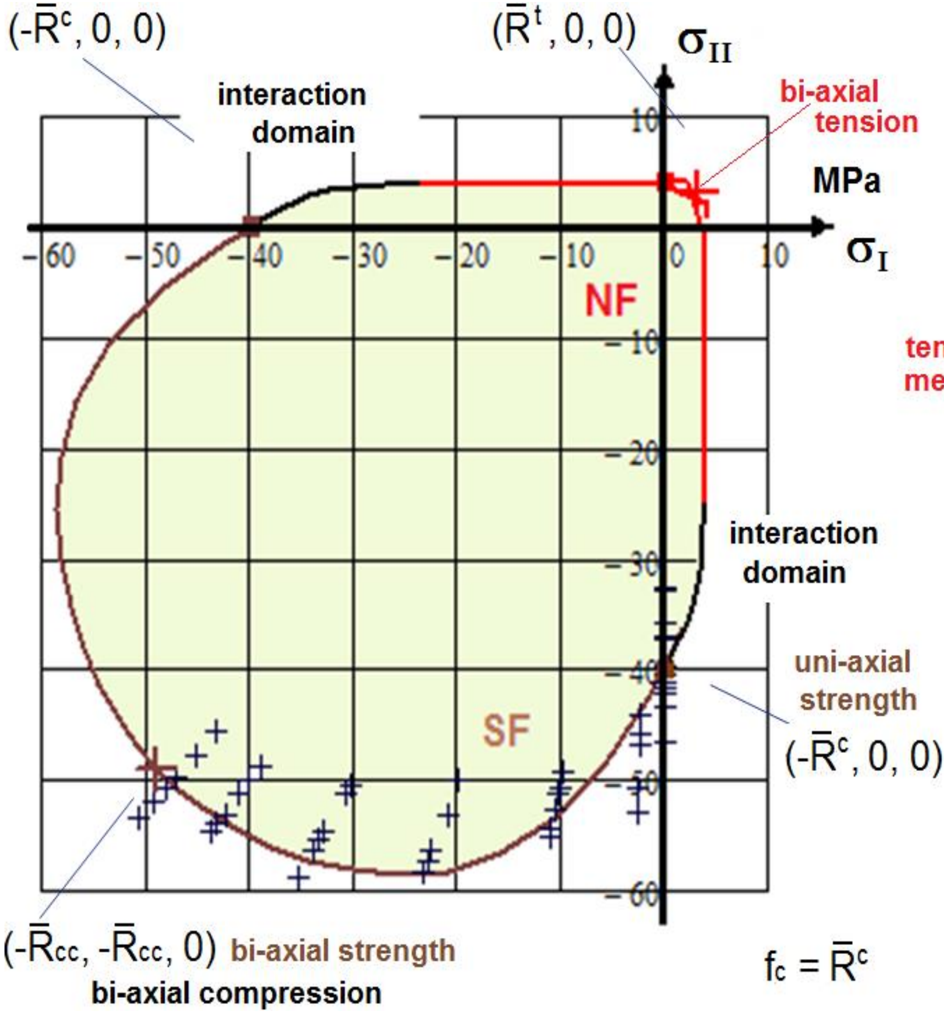


This visualization required a 40-page MATHCAD calculation !!!

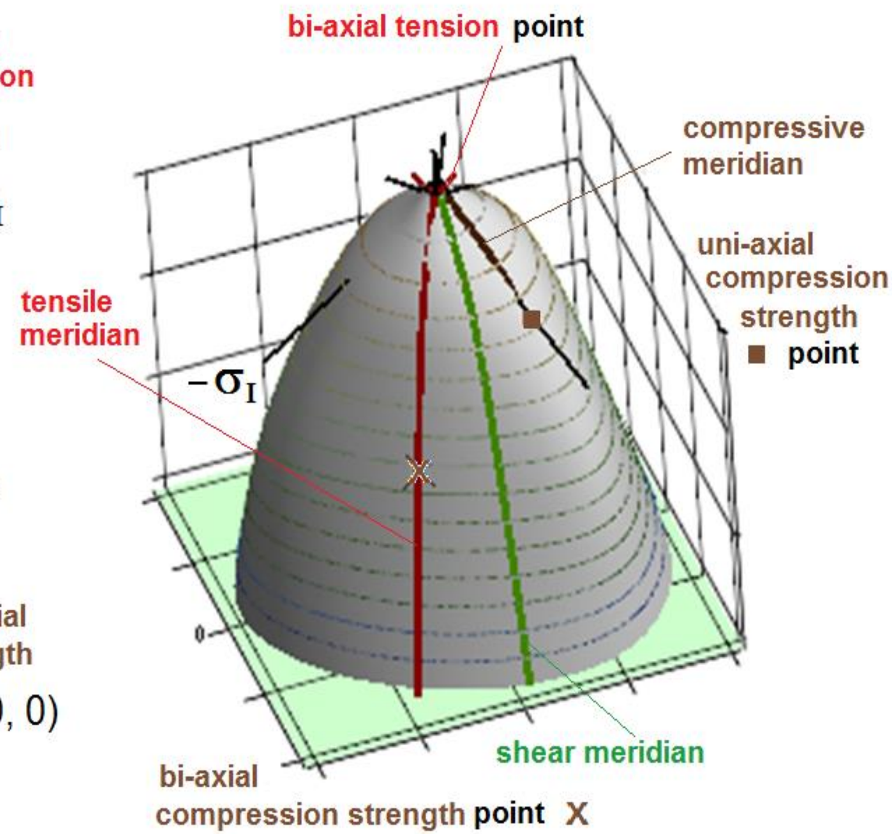
Visualization of the Lode-(Haigh-Westergaard) coordinates

Application isotropic

Normal Concrete: 3D test data with 3D-Body and 2D-Fracture Failure Envelope



$f_c = \bar{R}^c$



$$Eff^{NF} = c^{NF} \cdot \frac{\sqrt{4J_2 \cdot \Theta^{NF} - I_1^2/3} + I_1}{2 \cdot \bar{R}^t} = \frac{\sigma_{eq}^{NF}}{\bar{R}^t},$$

$$Eff^{SF} = \frac{c_{2\Theta}^{SF} \cdot I_1 + \sqrt{(c_{2\Theta}^{SF} \cdot I_1)^2 + 12 \cdot c_{1\Theta}^{SF} \cdot 3J_2 \cdot \Theta^{SF}}}{2 \cdot \bar{R}^c} = \frac{\sigma_{eq}^{SF}}{\bar{R}^c}$$

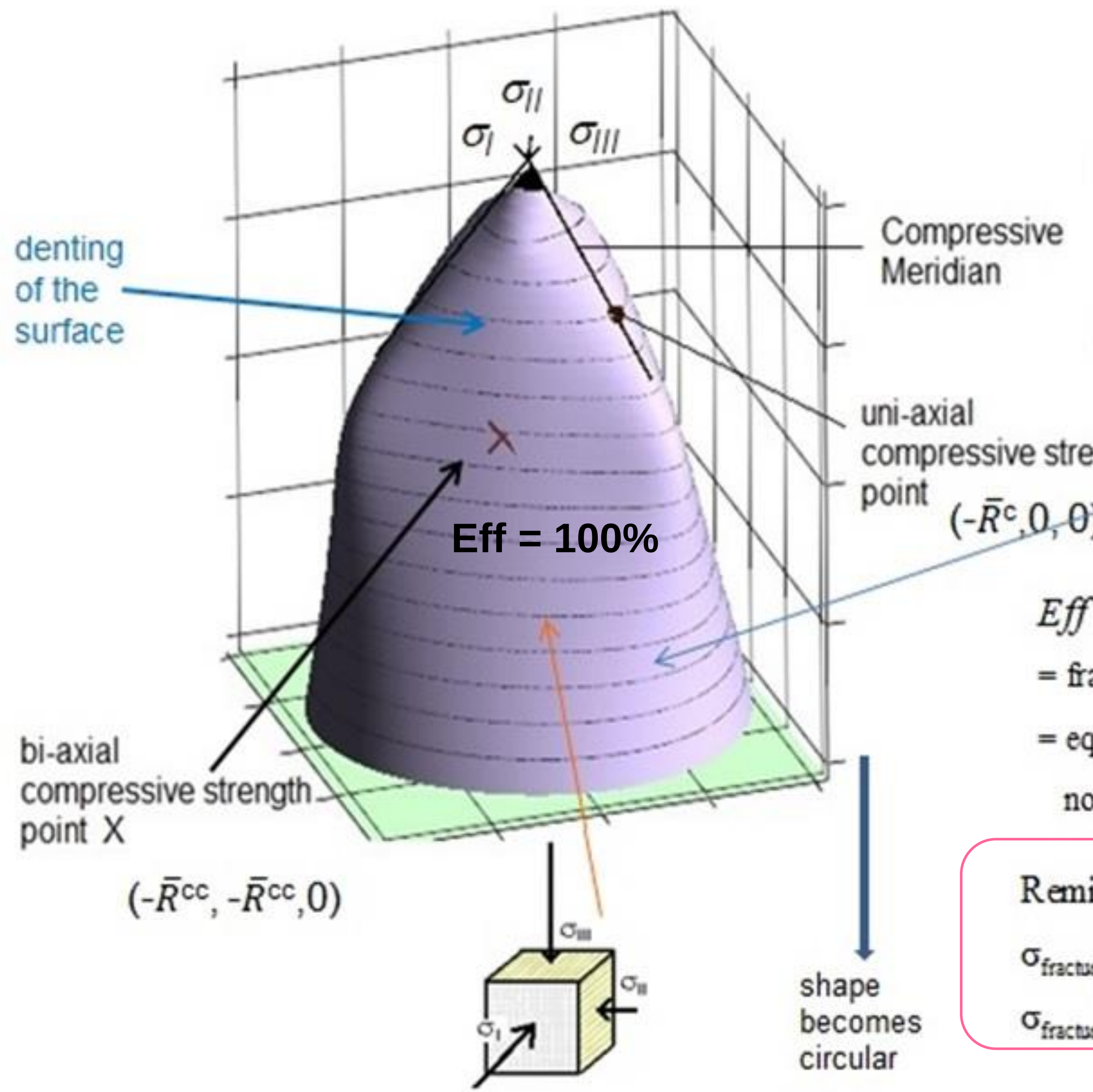
$$Eff = (Eff^{NF})^m + (Eff^{SF})^m = 1$$

Normal Concrete, mapping of 2D-test data in the Principal Stress Plane (bias cross-section of fracture body). R:= strength ≡ f;:.t:=tensile, c:=compressive; bar over means mean value. μ = 0.2

Application isotropic . (test data, courtesy Dr. S. Scheerer, IfM Dresden).

<u>Ultra High Performance Concrete</u> : <u>3D test data</u> with *Novel* 3D Fracture Body

Dent turns along axis !



$$Eff^{NF} = c_{\Theta}^{NF} \cdot \frac{\sqrt{4J_2 \cdot \Theta^{NF} - I_1^2/3} + I_1}{2 \cdot \bar{R}^t}$$

$$Eff^{SF} = \frac{c_{2\Theta}^{SF} \cdot I_1 + \sqrt{(c_{2\Theta}^{SF} \cdot I_1)^2 + 12 \cdot c_{1\Theta}^{SF} \cdot 3J_2 \cdot \Theta^{SF}}}{2 \cdot \bar{R}^c}$$

$$Eff = [(Eff^{NF})^m + (Eff^{SF})^m]^{m^{-1}} = 1 = 100\%$$

= fracture surface definition

= equation of surface of 120°-symmetric non-circular fracture body

[CUN§4]

Remind: $\bar{R}^c = 160$ MPa

$\sigma_{fracture} = (\sigma_I, \sigma_{II}, \sigma_{III})^T = (-160, 0, 0)^T \rightarrow 100\ \%$

$\sigma_{fracture} = (-230, -6, -6)^T \rightarrow 100\ \%$

- The size of denting reduces with negatively increasing I_1.
- The cross-section becomes more and more circular.

Against usual citations:
There does not exist a material strength increase !

Application isotropic

PMMA (plexiglass): SFCs for Normal Yielding and Shear Yielding (*for direct use*)

Normal Yielding NY (hyperboloid) $I_1 > 0$ Shear Yielding SY (paraboloid) $I_1 < 0$

$$F^{NY} = \frac{x^2}{(c_2^{NY})^2} - \frac{(y - c_1^{NY})^2}{c_3^{NY\,2}} = 1 \text{ with } x = \frac{\sqrt{2 \cdot J_2 \cdot \Theta^{NY}}}{\bar{R}^t_{NY}}, \; y = \frac{I_1}{\sqrt{3} \cdot \bar{R}^t_{NY}} \Leftrightarrow F^{SY} = c_1^{SY} \cdot \frac{3J_2 \cdot \Theta^{SY}}{\bar{R}^{c\,2}_{0.2}} + c_2^{SY} \cdot \frac{I_1}{\bar{R}^c_{0.2}} = 1$$

Considering bi-axial strength (failure mode occurs twice, $\Theta \neq 1$). In Effs now, index Θ dropped.

$$Eff^{NY} = \frac{c_3^{NY} \cdot \sqrt{-c_2^{NY\,2} \cdot y^2 + (\Theta^{NY})^2 \cdot (c_3^{NY\,2} + c_1^{NY\,2}) \cdot x^2} + c_2^{NY} \cdot c_1^{NY} \cdot y}{c_2^{NY} \cdot (c_3^{NY\,2} + c_1^{NY\,2})}, \quad Eff^{SY} = \frac{c_2^{SY} \cdot I_1 + \sqrt{(c_2^{SY} \cdot I_1)^2 + 12 \cdot c_1^{SY} \cdot 3J_2 \cdot \Theta^{SY}}}{2 \cdot \bar{R}^c_{0.2}}$$

Onset of Crazing = Normal Yielding NY (for fracture similar)

c^{NY}, d^{NY} from the two points $(\bar{R}^t_{NY}, 0, 0)$ and $(\bar{R}^{tt}_{NY}, \bar{R}^{tt}_{NY}, 0)$ d^{SY} from the point $(-\bar{R}^{cc}_{0.2}, -\bar{R}^{cc}_{0.2}, 0)$

Two-fold failure danger can be modelled by using the well known invariant J_3 including d = non-circularity parameter

$$\Theta^{NY} = \sqrt[3]{1 + d^{NY} \cdot \sin(3\vartheta)} = \sqrt[3]{1 + d^{NY} \cdot 1.5 \cdot \sqrt{3} \cdot J_3 \cdot J_2^{-1.5}} \text{ and } \Theta^{SY} = \sqrt[3]{1 + d^{SY} \cdot \sin(3\vartheta)} = \sqrt[3]{1 + d^{SY} \cdot 1.5 \cdot \sqrt{3} \cdot J_3 \cdot J_2^{-1.5}}$$

Lode angle ϑ , here set as $\sin(3 \cdot \vartheta)$ with ‘neutral ‘shear meridian angle 0°; compressive meridian angle -30°.

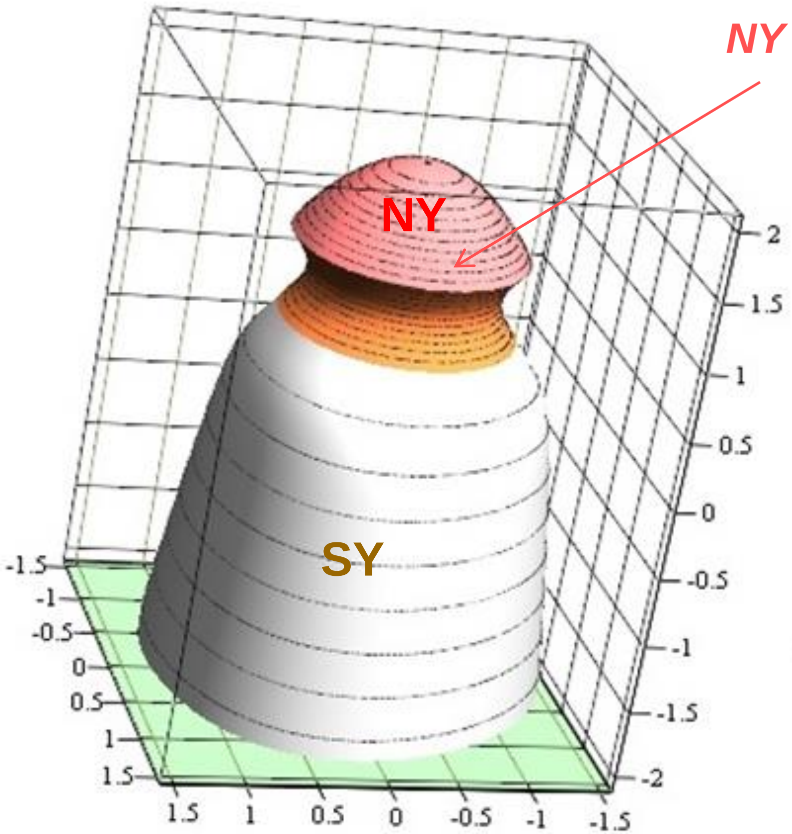
A failure body is rotationally-symmetric if $\Theta = 1$

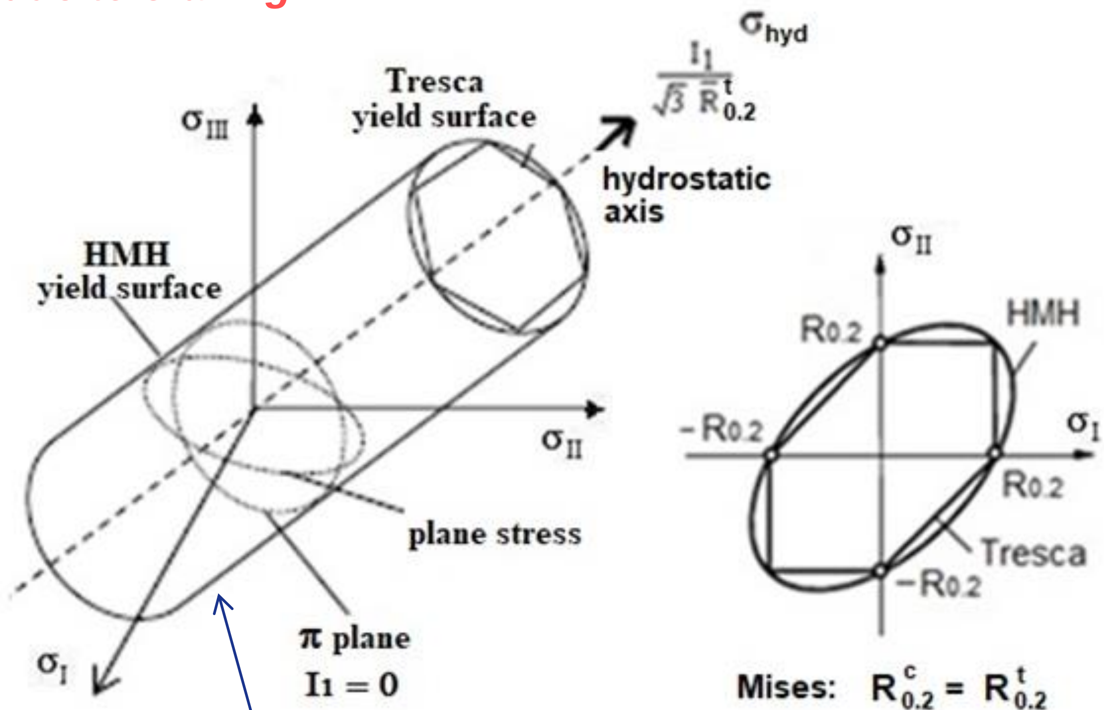
Equation of the yield failure body: $Eff = [(Eff^{NY})^m + (Eff^{SY})^m]^{m^{-1}} = 1 = 100\%$ total effort, interaction

$0 < d^{NY} < 0.5$, $0 < d^{SY} < 0.5$, meridian angles $\vartheta°$: $\bar{R}^t_{0.2}$ at 30° ; $\bar{R}^{tt}_{0.2}$, -30°; $\bar{R}^c_{0.2}$, -30°; $\bar{R}^{cc}_{0.2}$, 30°

PMMA: (left) **Onset-of-Yield surface (novel NY with SY) and** (right) **for comparison Hencky-Mises-Huber with Tresca yield surface** (engineering yield strengths are used)

NY due to crazing !





Mises Cylinder $\frac{3J_2}{\bar{R}_{0.2}^{\,2}} = 1$

Crazing failure **occurs which shows an increase in volume due to the formation of tension-elongated fibrils [*CUN§4.1*] and *shear yielding SY* does not.**

$\bar{R}^t = 37;\ \bar{R}^{tt} = 36;\ \bar{R}^{ttt} = 42;\ \bar{R}^c = 60;\ \bar{R}^{cc} = 69;\ \sigma_{Itm} = 34,\ \sigma_{IItm} = 18,\ \sigma_{It0} = 48,$
$\sigma_{IIt0} = -19.\ c_1^{NY} = 0.83,\ c_2^{NY} = 0.66, c_3^{NY} = 0.41, c_1^{SY} = 1.21, c_2^{SY} = 0.24, s^{cap} = -0.81,$
$d^{NF} = -0.26;\ d^{SF} = -0.08;\ m = 2.6,\ set \max I_1 = 3 \cdot R^{ttt} = 8.43;\ \min I_1 = -4.58.$

Check of identical hoop curve at the Cap-NF contact I_1 performed.

Main Conclusions w.r.t. Isotropic Strength Failure Conditions (SFCs)

similar for UD

- **A SFC can only describe a 1-fold occurring failure mode.**
- **A multi-fold occurrence must be additionally considered in the formulas:**

  2-fold $\sigma_{II} = \sigma_I$ (**probabilistic effect**), **is elegantly solved with** J_3

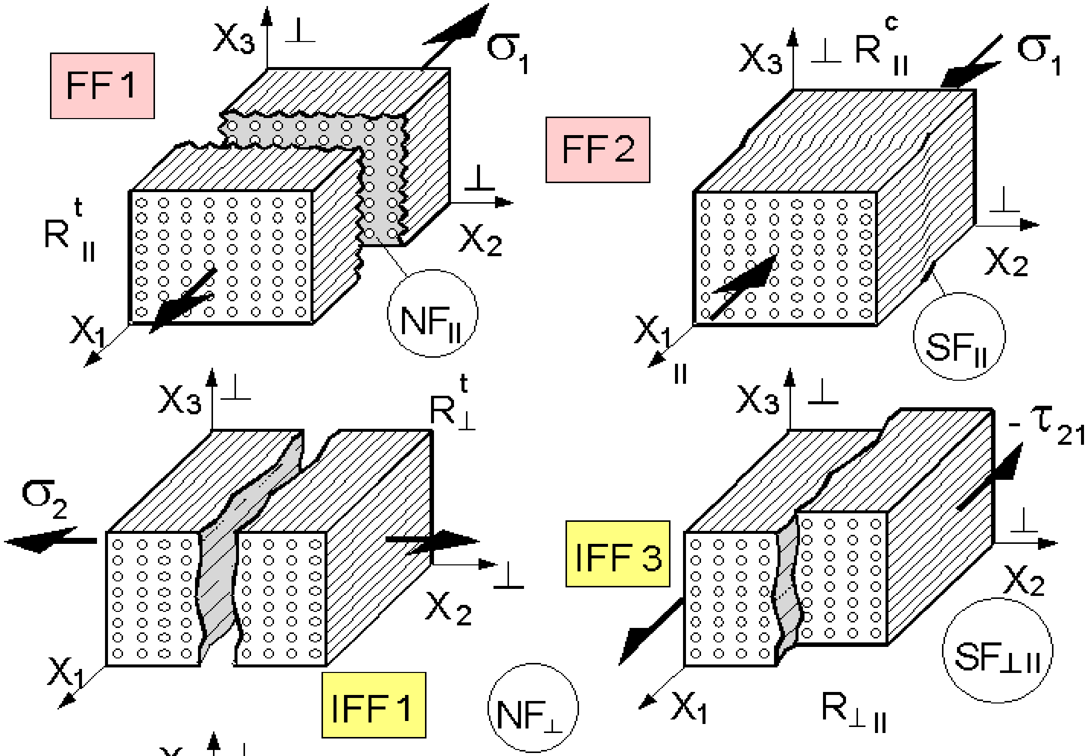
  3-fold $\sigma_{II} = \sigma_I = \sigma_{III}$ (**prob. effect**) **hydrost. compression, closing cap**

- **Failure Bodies of *brittle* isotropic materials are non-rotational and *ductile* ones also** → no Mises cylinder**. They are just '120°-symmetric' with differently pronounced dents being the probabilistic result of a 2-fold acting of the same failure mode.** This shape is usually described by replacing J_2 through $J_2 \cdot \Theta(J_3, J_2)$. Dents, located in the domain $I_1 < 0$ *are* oppositely to those in the domain $I1 > 0$ (tension)
- **The Poisson effect, generated by a Poisson ratio *v*, may cause tensile failure under bi-axially compressive stressing (dense concrete and analogous UD material, where filament tensile fracture may occur without any external tension loading** σ_1**) !**
- **Hoop Planes = deviatoric planes,** π**-planes: *convex***
- **Meridian Planes** for 'Onset of Crazing', NY**: *are not convex*** *for positive* I_1 ***!***

Drucker's Stability Criterion is violated!

26

UD: Which Strength Failure Modes are observed with these brittle Materials?



kinking

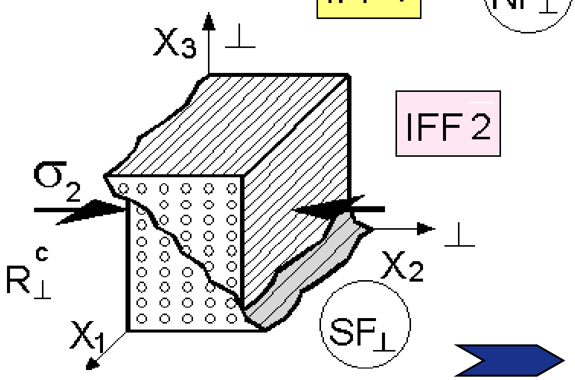
t = tension

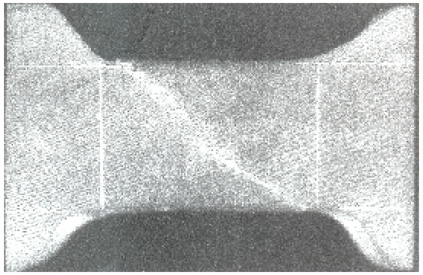
c = compression

▶ **5 Fracture modes exist**

= 2 FF (Fibre Failure)

\+ 3 IFF (Inter Fibre Failure)





wedge failure type

Fracture Types:

NF := Normal Fracture

SF := Shear Fracture

A UD Strength Failure Criterion captures the fracture of the fiber, the matrix, fiber-matrix interface and of the delamination of a layer as a subpart of the laminate.

Dense

IFF2 to replace

$$Eff^{\perp\tau} = [a_{\perp\perp}\cdot(\sigma_2+\sigma_3) + b_{\perp\perp}\sqrt{\sigma_2^{\,2} - 2\sigma_2\sigma_3 + \sigma_3^{\,2} + 4\tau_{23}^{\,2}}\,] / \bar{R}_\perp^c = \sigma_{eq}^{\perp\tau} / \bar{R}_\perp^c$$

$$I_1 = \sigma_1,\ I_2 = \sigma_2+\sigma_3,\ I_3 = \tau_{31}^{\,2} + \tau_{21}^{\,2},\ I_4 = (\sigma_2-\sigma_3)^2 + 4\cdot\tau_{23}^2$$

$$I_5 = (\sigma_2-\sigma_3)\cdot(\tau_{31}^{\,2} - \tau_{21}^{\,2}) - 4\tau_{23}\tau_{31}\tau_{21}\cdot$$

Invariant SFC-formulas now replaced by their stress formulations

FF1: $Eff^{\|\sigma} = \sigma_1 / \bar{R}_\|^t = \sigma_{eq}^{\|\sigma} / \bar{R}_\|^t$

FF2: $Eff^{\|\tau} = -\sigma_1 / \bar{R}_\|^c = +\sigma_{eq}^{\|\tau} / \bar{R}_\|^c$

IFF1: $Eff^{\perp\sigma} = \frac{1}{2}\cdot[(\sigma_2+\sigma_3) + \sqrt{\sigma_2^{\,2} - 2\sigma_2\cdot\sigma_3 + \sigma_3^{\,2} + 4\tau_{23}^{\,2}}\,] / \bar{R}_\perp^t = \sigma_{eq}^{\perp\sigma} / \bar{R}_\perp^t$

IFF2: $F_{porosity} = \frac{1}{2}\cdot\sqrt{a_{\perp\perp por}^{\,2}\cdot I_2^{\,2} + b_{\perp\perp por}^{\,2}\cdot I_4} - a_{\perp\perp por}\cdot I_2\ ] / \bar{R}_\perp^c = 1$

FF3: $Eff^{\perp\|} = \{\frac{1}{2}\cdot[b_{\perp\|}\cdot I_{23-5} + (\sqrt{b_{\perp\|}^{\,2}\cdot I_{23-5}^{\,2} + 4\cdot\bar{R}_{\perp\|}^{\,2}\cdot(\tau_{31}^2+\tau_{21}^2)^2}\,] / \bar{R}_{\perp\|}^{\,3}\}^{0.5}$.

$\{\sigma_{eq}^{\text{mode}}\} = (\sigma_{eq}^{\|\sigma},\ \sigma_{eq}^{\|\tau},\ \sigma_{eq}^{\perp\sigma},\ \sigma_{eq}^{\perp\tau},\ \sigma_{eq}^{\|\perp})^T$, $I_{23-5} = 2\sigma_2\cdot\tau_{21}^2 + 2\sigma_3\cdot\tau_{31}^2 + 4\tau_{23}\tau_{31}\tau_{21}$.

Insertion: Compressive strength point $(0,\ -\bar{R}_\perp^c)$ + bi-axial fracture stress $\bar{R}_\perp^{tt}$ (porosity effect)

delivers $a_{\perp\perp por} \cong \mu_{\perp\perp} / (1-\mu_{\perp\perp})$, $b_{\perp\perp por} = a_{\perp\perp por} + 1 = 1/(1-\mu_{\perp\perp})$, $b_{\perp\|} \cong 2\cdot\mu_{\perp\|}$.

From mapping experience obtained typical FRP-ranges: $0 < \mu_{\perp\|} < 0.3$, $0 < \mu_{\perp\perp} < 0.2$.

Failure Surface (failure body) = interaction equation for porous UD ceramics:

$$Eff^{m} = (Eff^{\|\tau})^m + (Eff^{\|\sigma})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\|})^m = 100\%\ \text{if failure}$$

Two-fold failure danger in the σ_2-σ_3-domain stands for a failure surfce closing, modelled by

$$Eff^{m} = (Eff^{\|\tau})^m + (Eff^{\|\sigma})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\|})^m + (Eff_\perp^{\text{MfFd}})^m = 1$$

with $Eff_\perp^{\text{MfFd}} = (\sigma_2^t + \sigma_3^t) / 2\bar{R}_\perp^{tt}$, and $\bar{R}_\perp^{tt} \approx \bar{R}_\perp^t / \sqrt[m]{2}$ after [*Awa*78]

considering $\sigma_2^t = \sigma_3^t$ and $\sigma_2^c = \sigma_3^c$; $\bar{R}_\perp^{tt} \le \bar{R}_\perp^t$, $\bar{R}_\perp^{cc} \le \bar{R}_\perp^c$ if porous.

detailed view →

From mapping experience obtained typical range of interaction exponent $2.5 < m < 2.9$.

The superscripts σ, τ mark the failure driving stress!

5 Modal 3D UD SFCs *(is the simple 'Mises' amongst the 3D UD criteria)*

capturing micro-tensile failure of fibers under bi-axial compression within the macro-mechanical SFC

strains from FEA [*Cun04*, *Cun11*]

FF1 $Eff^{\|\sigma} = \breve{\sigma}_1 / \bar{R}_{\|}^{t} = \sigma_{eq}^{\|\sigma} / \bar{R}_{\|}^{t}$, $\breve{\sigma}_1 \cong \varepsilon_1^t \cdot E_{\|}$ *

FF2 $Eff^{\|\tau} = -\breve{\sigma}_1 / \bar{R}_{\|}^{c} = +\sigma_{eq}^{\|\tau} / \bar{R}_{\|}^{c}$, $\breve{\sigma}_1 \cong \varepsilon_1^c \cdot E_{\|}$

2 filament modes

IFF1 $Eff^{\perp\sigma} = [(\sigma_2 + \sigma_3) + \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^{\ 2}}] / 2\bar{R}_{\perp}^{t} = \sigma_{eq}^{\perp\sigma} / \bar{R}_{\perp}^{t}$

IFF2 $Eff^{\perp\tau} = [(\frac{\mu_{\perp\perp}}{1-\mu_{\perp\perp}}) \cdot (\sigma_2 + \sigma_3) + \frac{1}{1-\mu_{\perp\perp}} \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^{\ 2}}] / \bar{R}_{\perp}^{c} = +\sigma_{eq}^{\perp\tau} / \bar{R}_{\perp}^{c}$

3 matrix modes

IFF3 $Eff^{\perp\|} = \{[\mu_{\perp\|} \cdot I_{23-5} + (\sqrt{\mu_{\perp\|}^{\ 2} \cdot I_{23-5}^{\ 2} + 4 \cdot \bar{R}_{\perp\|}^{\ 2} \cdot (\tau_{31}^2 + \tau_{21}^2)^2}] / (2 \cdot \bar{R}_{\perp\|}^{\ 3})\}^{0.5} = \sigma_{eq}^{\perp\|} / \bar{R}_{\perp\|}$

with $I_{23-5} = 2\sigma_2 \cdot \tau_{21}^2 + 2\sigma_3 \cdot \tau_{31}^2 + 4\tau_{23}\tau_{31}\tau_{21}$

Interaction of modes:

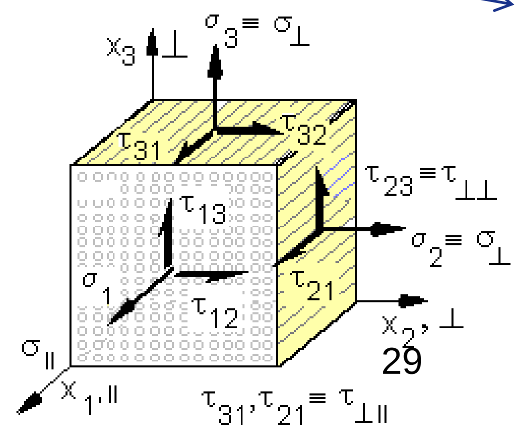
$$Eff^m = (Eff^{\|\tau})^m + (Eff^{\|\sigma})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\|})^m = 1$$

interaction example

with mode-interaction exponent $2.5 < m < 3$ from mapping tests data

Typical friction value data range: $0.05 < \mu_{\perp\|} < 0.3, \quad 0.05 < \mu_{\perp\perp} < 0.2$

see [*Pet16*] for measurement



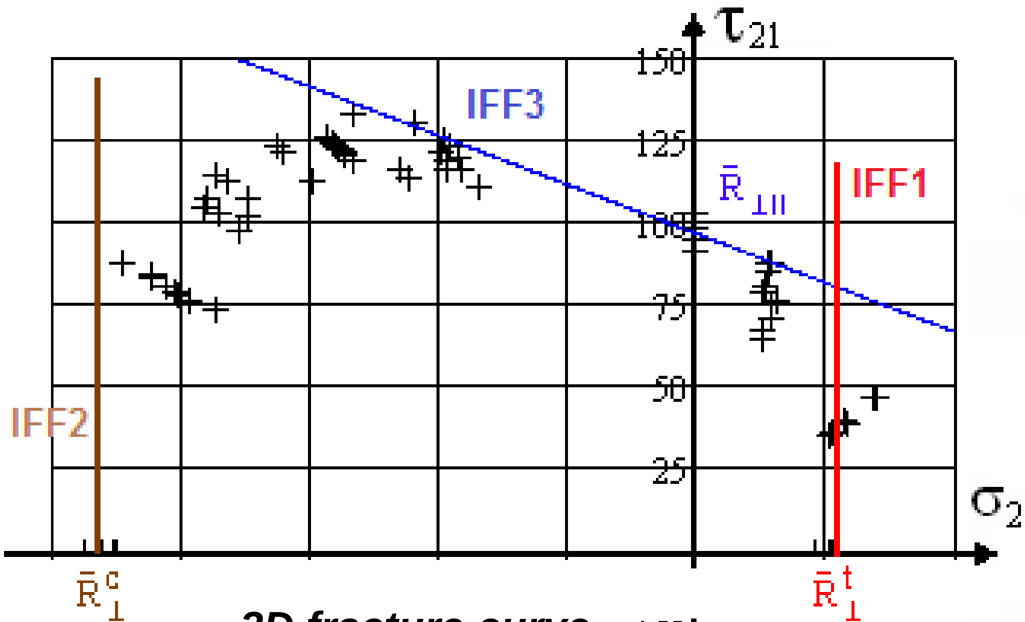
Poisson effect * : bi-axial compression strains the filament without any σ_1

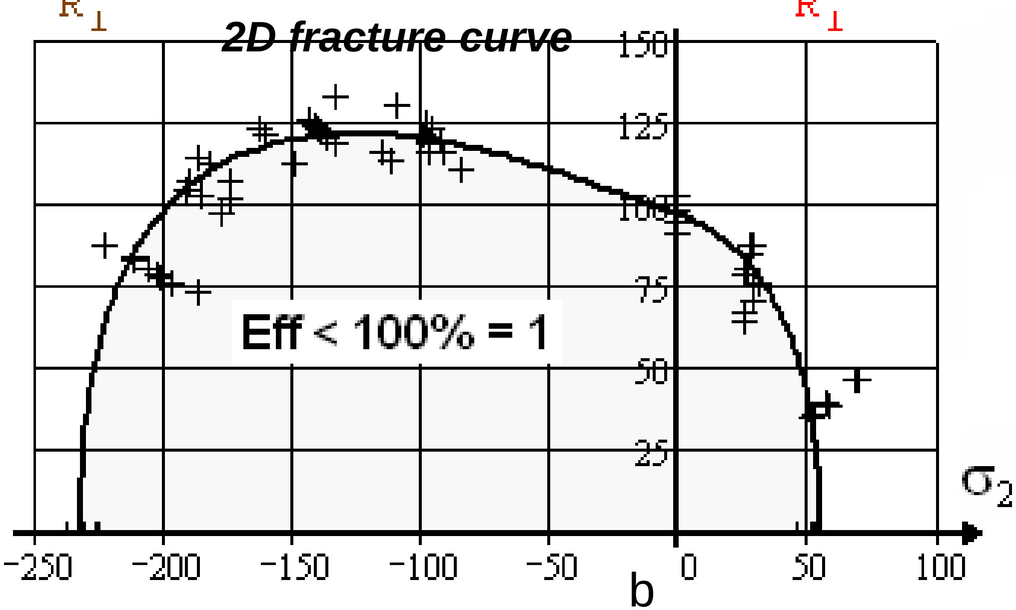
t:= tensile, c: = compression, || : = parallel to fibre, ⊥ := transversal to fibre

UD: Visualization of Interaction of UD Failure Modes in the Mode Transition Zones

$\breve{\sigma}_1 = 0$

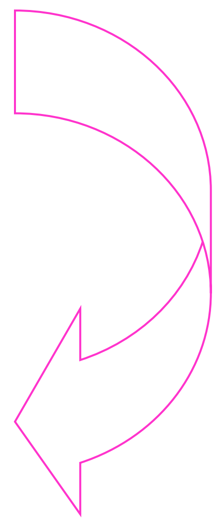
$\tau_{21}(\sigma_2)$ or $\{\sigma\} = (0, \sigma_2, 0, 0, 0, \tau_{21})^T$

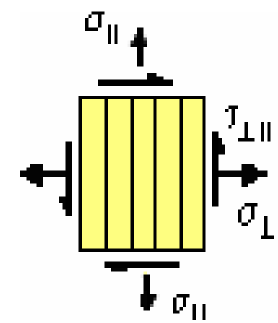




Mapping of course of IFF test data in a pure mode domain by the *single Mode Failure Conditions.* *m = 2.7*

3 IFF pure modes = 3 piecewise straight lines !





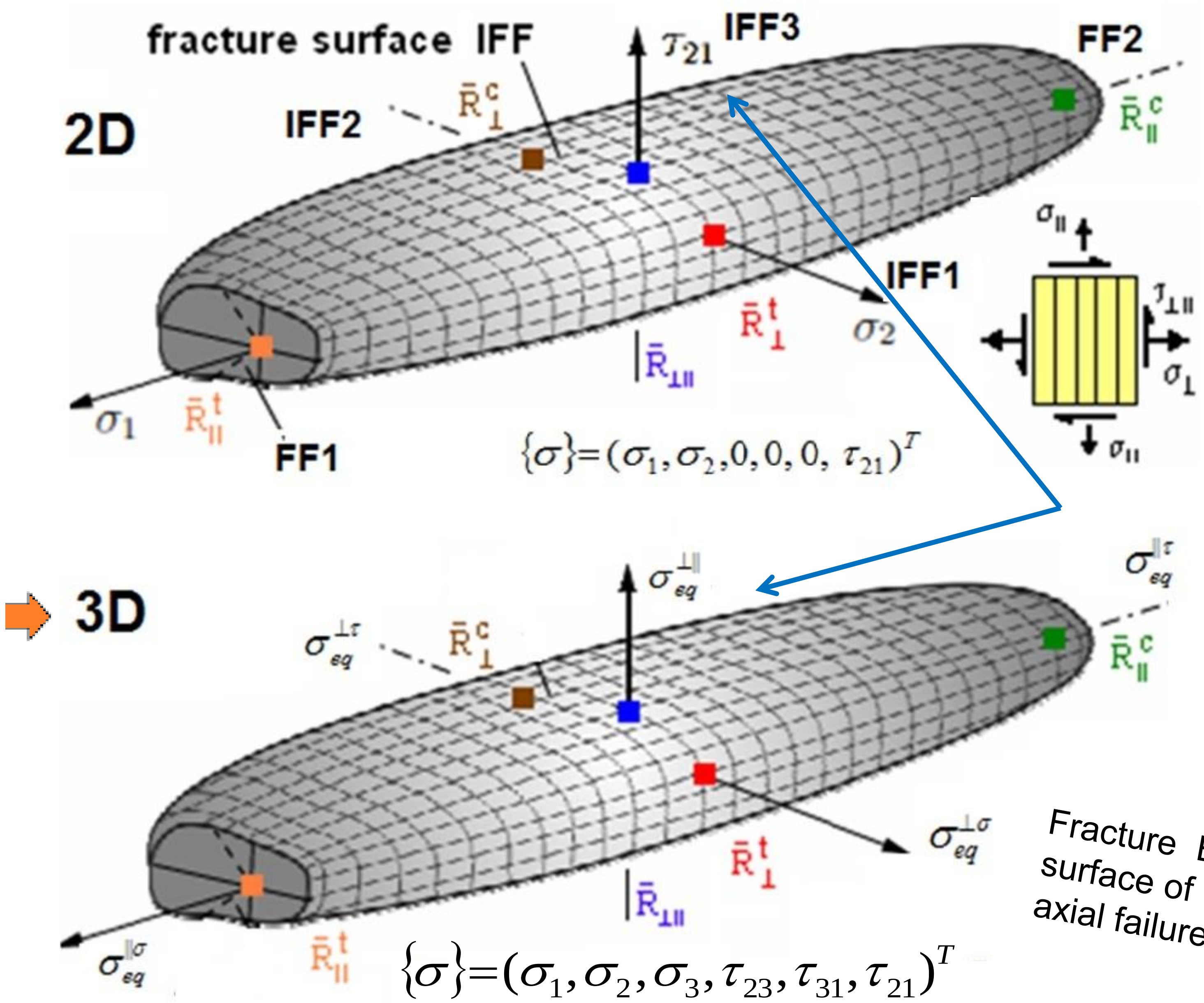
Mapping of course of test data by *the Interaction Model*

$$(Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp||})^m = 1$$

$$Eff = \left(\frac{\sigma_2^{\ t}}{\bar{R}_2^t}\right)^m + \left(\frac{-\sigma_2^{\ c}}{\bar{R}_2^c}\right)^m + \left(\frac{|\tau_{12}|}{\bar{R}_{\perp||} - \mu_{\perp||} \cdot \sigma_2^c}\right)^m = 1$$

Application to UD

UD: 2D ➔ 3D Fracture Body after Replacement of σ, τ by σ_{eq}^{mode}



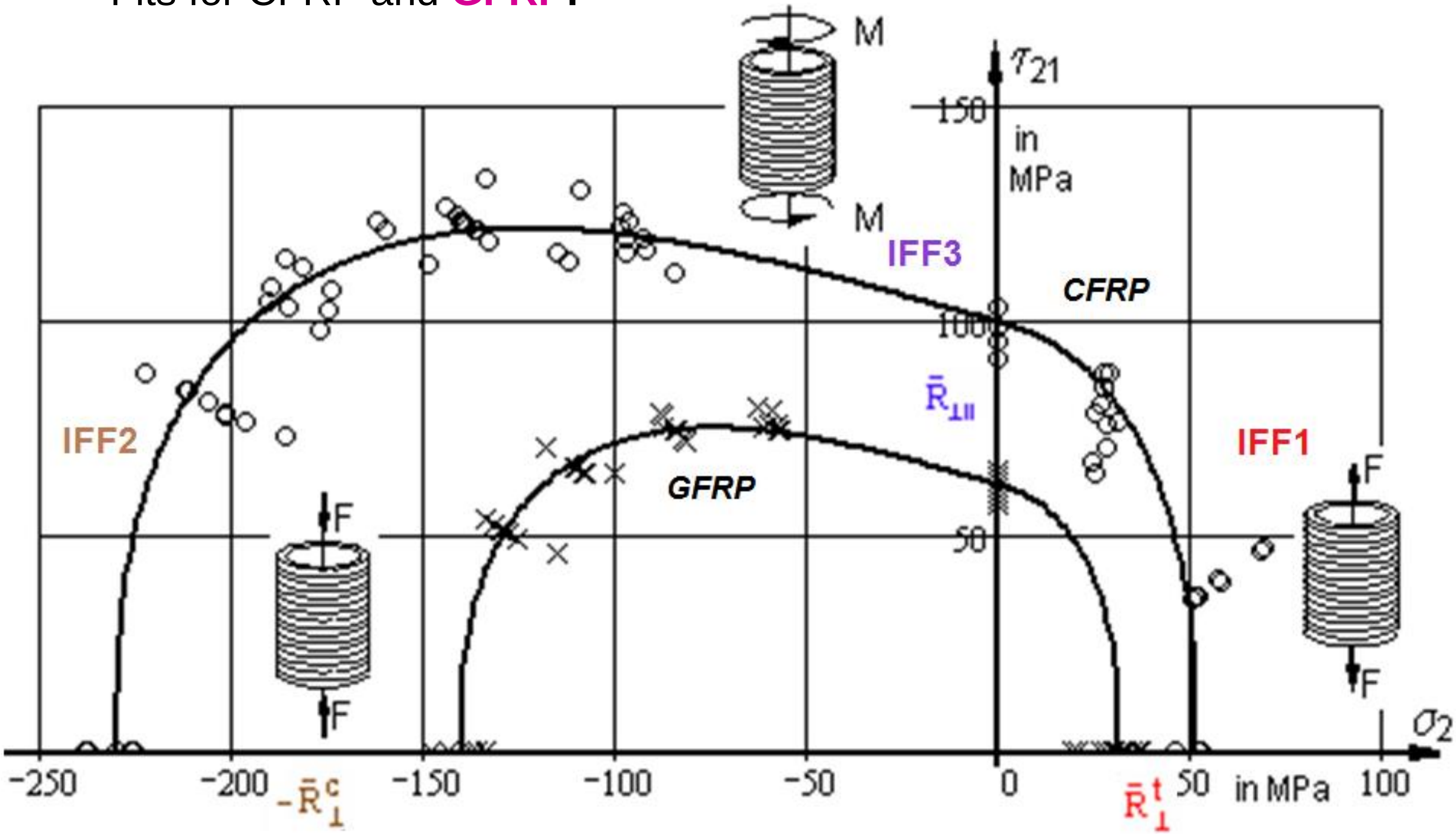
$\nu_{\perp\perp} = 0.4,$
$\nu_{\perp\|} = 0.3$

'Puck's cigar

Delamination onset predictable

Fracture Body = smoothed surface of the ends of the multi-axial failure stress vectors

Fits for CFRP and **GFRP!**



$$Eff = \left(\frac{\sigma_2{}^t}{\bar{R}_2^t}\right)^m + \left(\frac{-\sigma_2{}^c}{\bar{R}_2^c}\right)^m + \left(\frac{|\tau_{12}|}{\bar{R}_{\perp\parallel} - \mu_{\perp\parallel} \cdot \sigma_2^c}\right)^m = 1$$

Organizer : *QinetiQ , UK* *(Hinton, Kaddour, Soden, Smith, Shuguang Li)*

Aim: ***'Testing Predictive UD Failure Theories***

= SFC + non-linearity treatment + programming

Fiber–Reinforced Polymer Composites to the full !'

Procedure of the WWFE-I (2D test data**) and WWFE-II (3D test data):**

Part A : ***Blind Predictions*** **with average strength data $\bar{R}$ only.**
(Necessary friction value information μ was not provided !)

Part B : ***Comparison Theory-Test*** **with Test data sets, which were partly not applicable or even involved false failure points. More than 50% could not be used without specific care!**

Autrhor's WWFE-contribution:

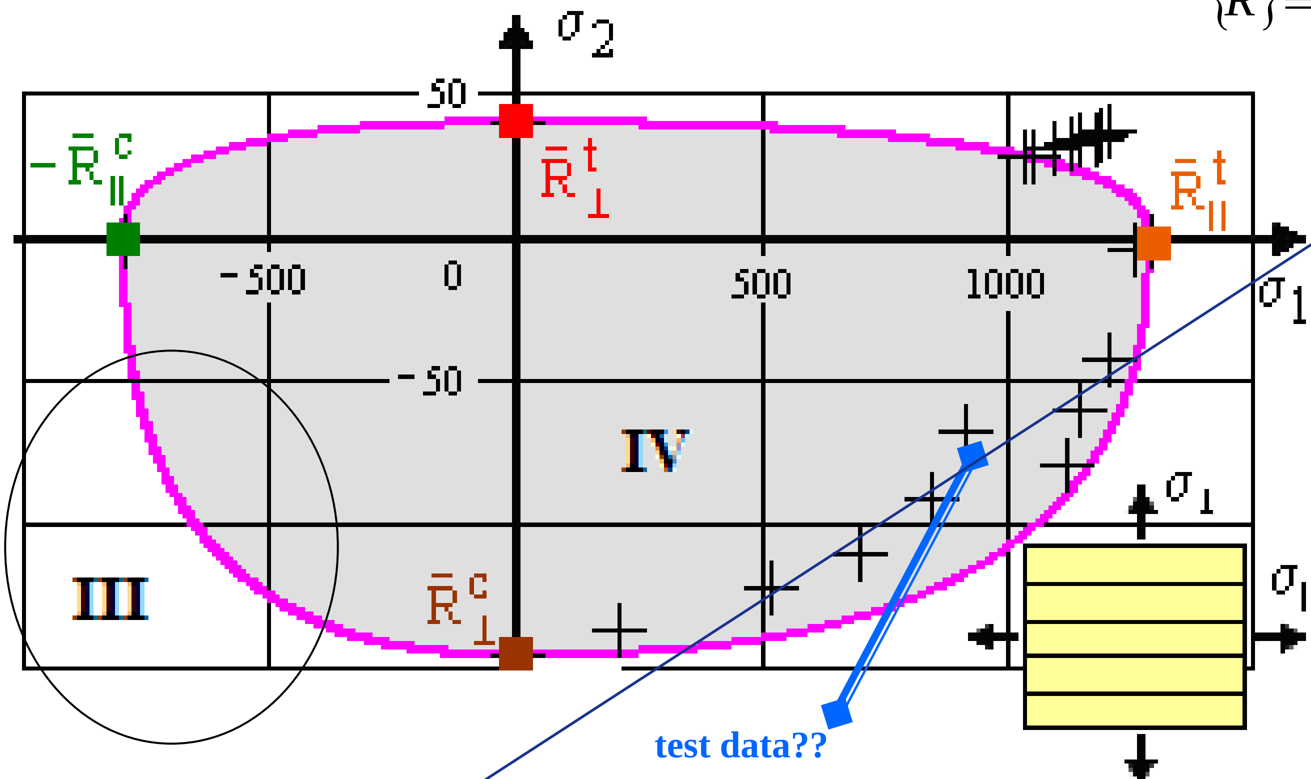
Cuntze's invariant-based strength criteria mapped the provided accurate test data sets best.
One third of the provided test data sets were not usable !

In WWFE-I author was official winner and ranked best in WWFE-II !

UD: Mapping of Test Case 3, WWFE-I, data

$\sigma_2(\breve{\sigma}_1 \equiv \sigma_1)$

$\{\bar{R}\} = (1280,\ 800,\ 40,\ 145,\ 73)^T$



Hoop wound tube
UD-lamina.
E-glass/MY750epoxy

$\sigma_1 = \sigma_{hoop}$

$\sigma_2 = \sigma_{axial}$

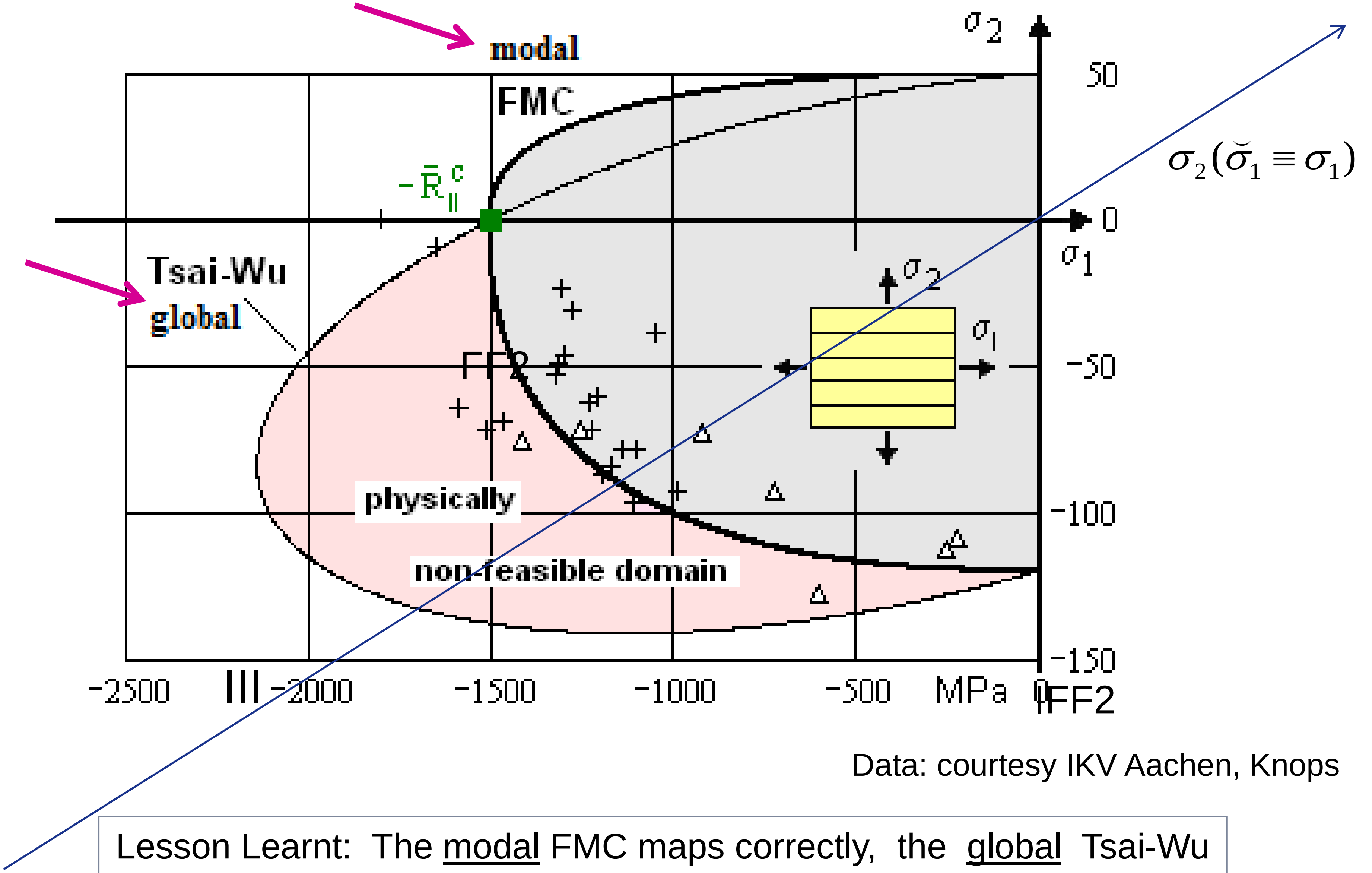
Part A: Data of strength points were provided, only

Part B: Test data in quadrant IV show discrepancy, testing?

No data for quadrants II, III was provided !

Further Test Cases are assessed in [CUN 22 Life work]

UD: Mapping in the 'Tsai-Wu non-feasible domain', quadrant III $\sigma_2(\sigma_1)$



Data: courtesy IKV Aachen, Knops

Lesson Learnt: The modal FMC maps correctly, the global Tsai-Wu formulation predicts a non-feasible domain !

UD: What is really required for the Pre-design using Cuntze's 3D UD SFCs ?

Test Data Mapping — Design Verification

(statistical mean to use, indicated by a bar over)

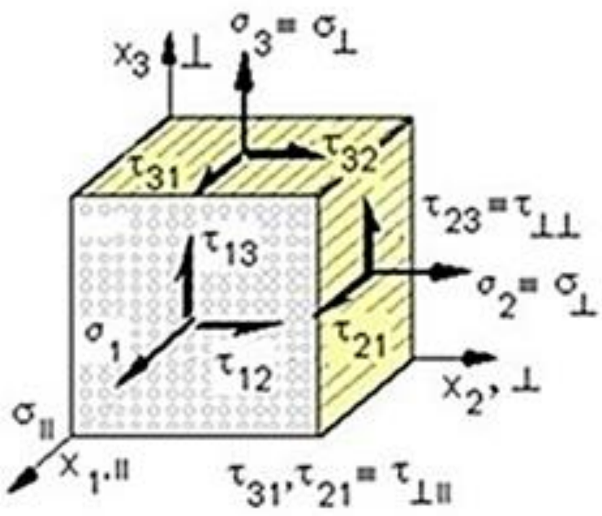
- 5 strengths : $\{\overline{R}\} = (\overline{R}_{\|}^{t}, \overline{R}_{\|}^{c}, \overline{R}_{\perp}^{t}, \overline{R}_{\perp}^{c}, \overline{R}_{\perp\|})^T$ $\{R\} = (R_{\|}^{t}, R_{\|}^{c}, R_{\perp}^{t}, R_{\perp}^{c}, R_{\perp\|})^T$

  average (typical) values — strength design allowables

- 2 friction values : for 2D $\mu_{\perp\|}$ for 3D $\mu_{\perp\|}$, $\mu_{\perp\perp}$

  $\mu_{\perp\|} = 0.15$ $\mu_{\perp\perp} = 0.2$ ← friction values, recommended for pre-design

- 1 mode-interaction exponent : $m = 2.6$. ← recommended for pre-design



5 measurable parameters !
Not 75 Parameters
as cited in the WWFE-II conclusions !

Numerical example UD Design Verification by *RF* > 1

2D Design Verification of a critical UD lamina in a distinct laminate wall design

Asssumption: *Linear analysis permitted, *design FoS $j_{ult} = 1.25$

* Design loading (action): $\{\sigma\}_{design} = \{\sigma\} \cdot j_{ult}$

* 2D-stress state: $\{\sigma\}_{design} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T \cdot j_{ult} = (0,\ -75,\ 0,\ 0,\ 0,\ 52)^T$ MPa

* Residual stresses: 0 $(\textit{effect vanishes with increasing micro-cracking})$

* Strengths (resistance) : $\{\bar{R}\} = (1378,\ 950,\ 40,\ 125,\ 97)^T$ MPa averages from mesurement

strength design allowable $\{R\} = (R_{\|}^t, R_{\|}^c, R_{\perp}^t, R_{\perp}^c, R_{\perp\|})^T = (1050,\ 725,\ 32,\ 112,\ 79)^T$ MPa

* Friction values : $\mu_{\perp\|} = 0.3,\ (\mu_{\perp\perp} = 0.35)$, Mode interaction exponent: $m = 2.7$

$$\{Eff^{\,mode}\} = \left(Eff^{\|\sigma},\ Eff^{\|\tau},\ Eff^{\perp\sigma},\ Eff^{\perp\tau},\ Eff^{\|\perp}\right)^T = \left(0.88,\ 0,\ 0,\ 0.21,\ 0.20\right)^T$$

$$Eff^{\,m} = (Eff^{\|\sigma})^m + (Eff^{\|\tau})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\|})^m = 100\% \ .$$

The results above deliver the following material reserve factors $f_{RF} \rightarrow RF$

$$* \ Eff^{\perp\sigma} = \frac{\sigma_2 - |\sigma_2|}{2 \cdot \bar{R}_{\perp}^t} = 0, \quad Eff^{\perp\tau} = \frac{-\sigma_2 + |\sigma_2|}{2 \cdot \bar{R}_{\perp}^c} = 0.60, \quad Eff^{\perp\|} = \frac{|\tau_{21}|}{\bar{R}_{\perp\|} - \mu_{\perp\|} \cdot \sigma_2} = 0.51$$

$$Eff = [(Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\|})^m]^{1/m} = 0.72.$$

$\Rightarrow \quad f_{RF} = 1 / Eff = 1.39 \rightarrow$ RF $= f_{RF}$ (if linearity permitted) $\rightarrow$ MoS $= RF - 1 = 0.39 > 0$!

$\rightarrow$ *Laminate wall design is verified !*

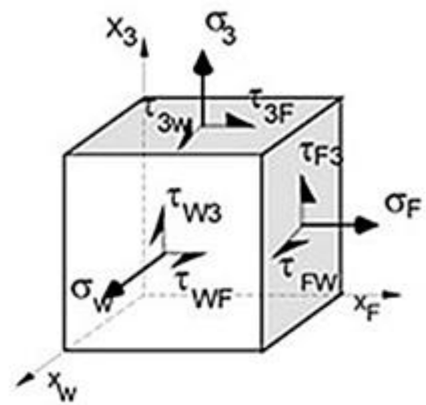
The stress-based Strength Criteria set reads:

$$\left(\frac{\sigma_W + |\sigma_W|}{2\cdot \bar{R}_W^t}\right)^m + \left(-\frac{\sigma_W + |\sigma_W|}{2\cdot \bar{R}_W^c}\right)^m + \left(\frac{\sigma_F + |\sigma_F|}{2\cdot \bar{R}_F^t}\right)^m + \left(\frac{-\sigma_F + |\sigma_F|}{2\cdot \bar{R}_F^c}\right)^m + \left(\frac{|\tau_{WF}|}{\bar{R}_{WF} - \mu_{WF}\cdot(\sigma_W + \sigma_F)}\right)^m$$

$$+\left(\frac{\sigma_3 + |\sigma_3|}{2\cdot \bar{R}_3^t}\right)^m + \left(\frac{-\sigma_3 + |\sigma_3|}{2\cdot \bar{R}_3^c}\right)^m + \left(\frac{|\tau_{3W}|}{\bar{R}_{3W} - \mu_{3W}\sigma_3}\right)^m + \left(\frac{|\tau_{3F}|}{\bar{R}_{3F} - \mu_{3F}\sigma_3}\right)^m = 1 \ .$$

This set matches with a 'generic' number 'assumed' 9 for orthotropic materials !

W = warp
F = fill (weft)



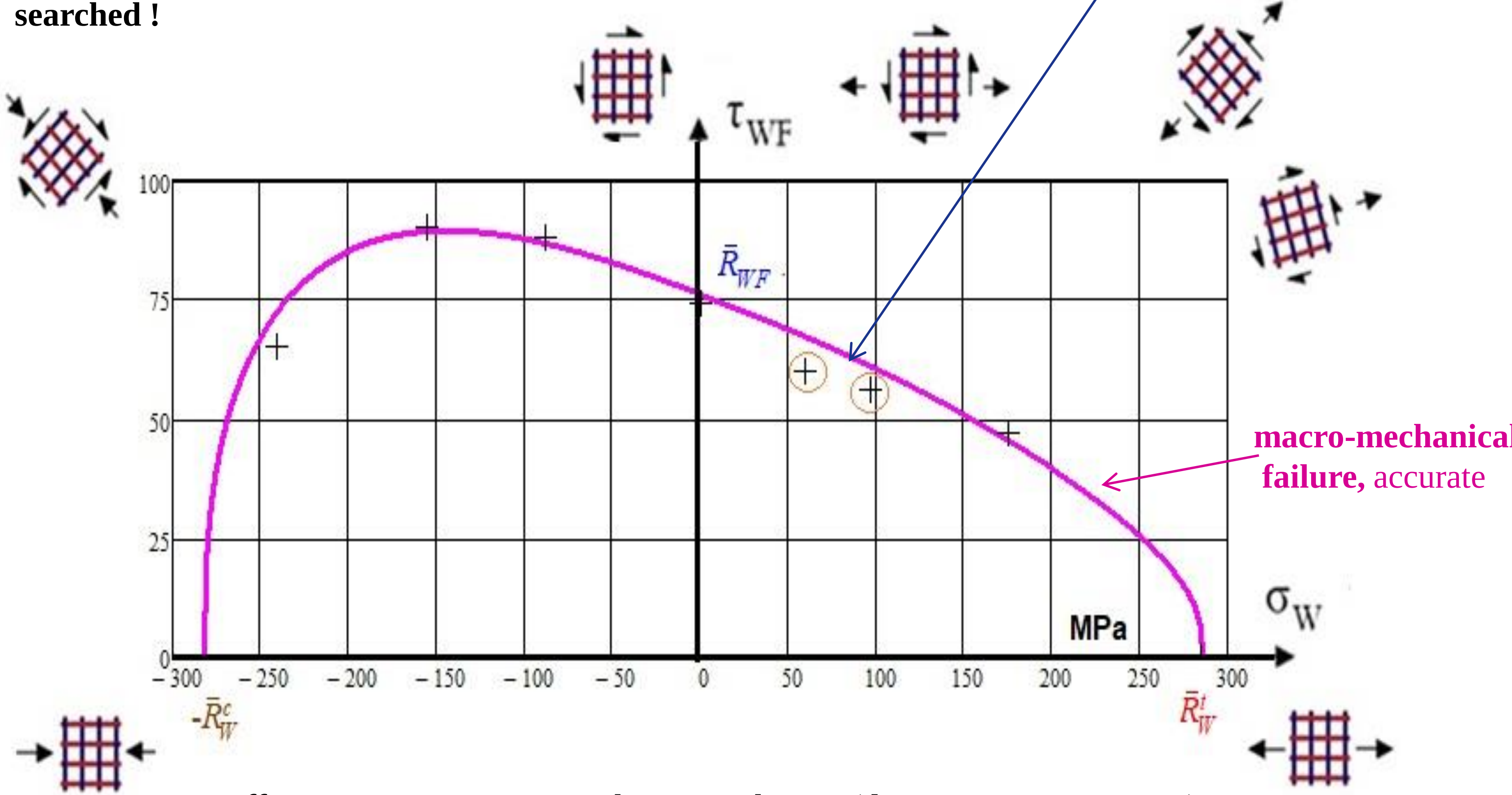
rhombically-anisotropic Orthotropic:

$$\{\sigma\} = (\sigma_W, \sigma_F, \sigma_3, \tau_{3F}, \tau_{3W}, \tau_{FW})^T$$

$$\{R\} = (R_W^t,\ R_W^c,\ R_F^t,\ R_F^c,\ R_{WF},\ R_3^t,\ R_3^c,\ R_{3F},\ R_{3W})^T$$

with $\mu_{WF}, \mu_{3W}, \mu_{3F}$

Lesson Learned for testing: The used inclined, off-axis coupon test specimen are not anymore applicable if the result belongs to a micro-mechanical failure, however macro-mechanical failure stress states are searched !

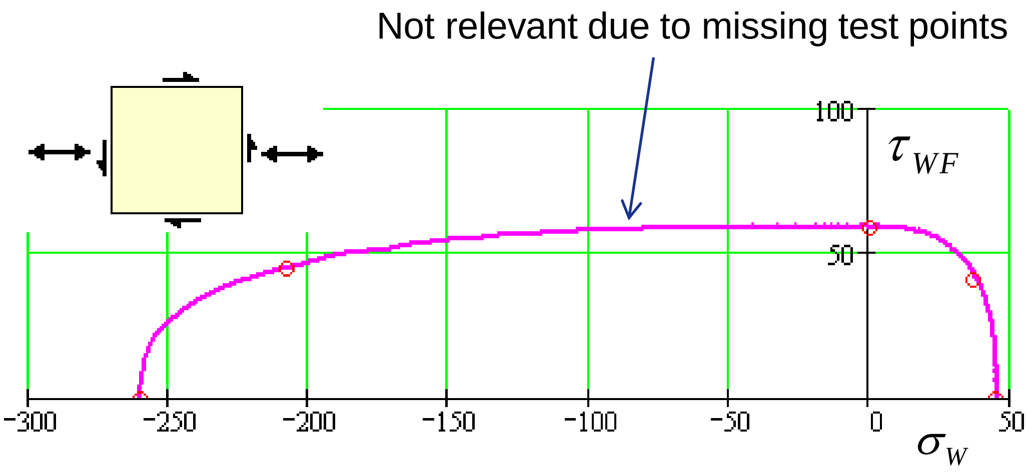


Off-axis coupon tests, Failure envelope . (data set Siemens AG). m = 2.6

Plain weave fabric laminate. RT=23°C. μ_{WF} = 0.14

$$\text{2D: } Eff = \left(\frac{\sigma_W + |\sigma_W|}{2\cdot\bar{R}_W^t}\right)^m + \left(-\frac{\sigma_W + |\sigma_W|}{2\cdot\bar{R}_W^c}\right)^m + \left(\frac{\sigma_F + |\sigma_F|}{2\cdot\bar{R}_F^t}\right)^m + \left(\frac{-\sigma_F + |\sigma_F|}{2\cdot\bar{R}_F^c}\right)^m + \left(\frac{|\tau_{WF}|}{\bar{R}_{WF} - \mu_{WF}\cdot(\sigma_W + \sigma_F)}\right)^m = 1\ .$$

<u>Orthotropic Fabric</u> : **Fibre-Reinforced Ceramics** (brittle, porous)



$\tau_{WF}(\sigma_W)$

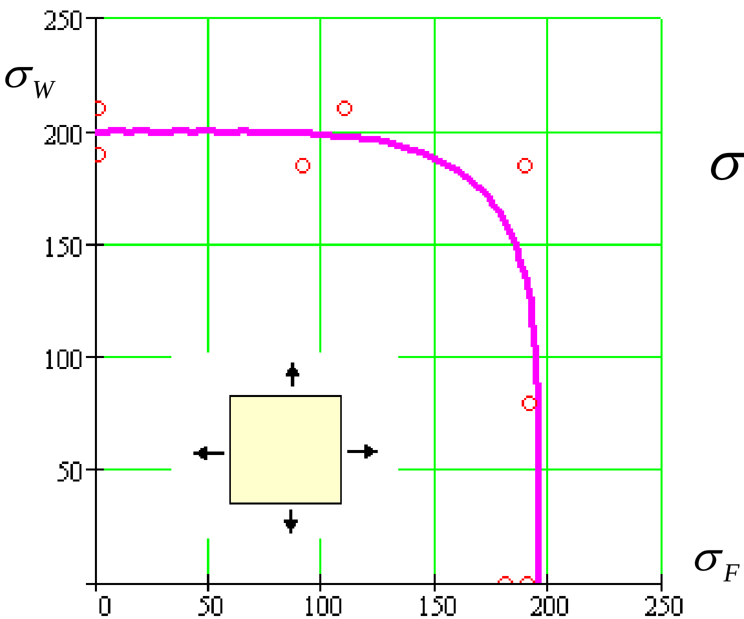
C/C-SiC, T= 1600°C

[Geiwitz/Theuer/Ahrendts 1997] , tension/compression-torsion, ***tube??***

$$\{\bar{R}\} = (\bar{R}_{\parallel}^t, \bar{R}_{\parallel}^c, \bar{R}_{\perp}^t, \bar{R}_{\perp}^c, \bar{R}_{\perp\parallel}) = (-, -, 45, 260, 59)^T$$

$m = 2.8$

$$\left(\frac{\sigma_W}{\bar{R}_W^t}\right)^m + \left(\frac{-\sigma_W}{\bar{R}_W^c}\right)^m + \left(\frac{\tau_{WF}^{\ 2}}{\bar{R}_{WF}^{\ 2}}\right)^m = 1$$



$\sigma_F(\sigma_W)$

$$\{\bar{R}\} = (\bar{R}_W^t, \bar{R}_W^c, \bar{R}_F^t, \bar{R}_F^c, \bar{R}_{WF}, \bar{R}_3^t, \bar{R}_3^c, \bar{R}_{3F}, \bar{R}_{3W})^T$$

$\{\bar{R}\}$ = *vector of mean strength values*

C/SiC, ambient temperature *[MAN-Technologie, 1996], tension/tension, tube*

$$\{\bar{R}\} = (200, -, 195, -, -,,,,)^T, \ m = 5$$

$$\left(\frac{\sigma_W}{\bar{R}_W^t}\right)^m + \left(\frac{\sigma_F}{\bar{R}_F^t}\right)^m = 1$$

NOTE: For <u>woven fabrics</u> enough test information for a <u>real</u> validation is not yet available!

Conclusions & Findings

In the frame of his material symmetry-driven thoughts the author could test-proof some ideas that help to complete and simplify the Strength Mechanics Building by finding missing links and by providing engineering-practical strength criteria for the 3 material families on basis of measurable parameters, only.

- **Confirmed 'Generic' numbers found will simplify theoretical and test tasks: Isotropic (2), UD (5), Orthotropic (9)**
- **Beside standard Shear Yielding SY also Normal Yielding NY exists (*analogous to the fracture failure modes Shear Fracture SF and Normal Fracture NF*)**
- **A SFC can only describe a one-fold occurring failure mode. Multi-fold failure ($\sigma_{II} = \sigma_{III}$, $\sigma_2 = \sigma_3$) must be additionally considered in each global and modal SFC**
- **The fracture failure surface terminates the growing yield surface, if applicable**
- **The common effect of neighboring modes was probabilistically considered by the mapping experience-based mode interaction exponent *m***
- **From experiments is known, that *brittle* isotropic materials possess a 120°-axially symmetric failure body in the compressive domain. However, d*uctile* materials in the tensile domain also possess a so-called '*120°-axially symmetric yield loci surface'* instead of a rotationally symmetric 'Mises cylinder'?**
- **Based on test results, first ever visualizations of the derived 3D failure surfaces have been performed**
- **First direct use of the measurable friction value *μ* in a SFC** **(possible after effortful Mohr transformation work)**
- **Explanation-possibility by *Eff:* Technical strength *R* is a Standard-fixed value, concrete** $\sigma_{ax} = -R^c = 160$ MPa **and cannot change. Under a slight hydrostatic pressure of 6 MPa the a distinct 'strength capacity' increases** $\sigma_{ax} = -224$ MPa**, however *Eff* (*Werkstoffanstrengung)* remains 100% !!**
- **Clear notations identify the material properties of the 3 families**
- **Available multi-axial fracture test data have been mapped to best possible 3D-validate the derived SFCs.**

On Gaps between Theory and Experiment:

- *Experimental results can be far away from the reality like a bad theoretical model.*
- *Theory creates a model of the reality, 'only', and*
  1 Experiment is 'just' 1 realization of the reality.

However, "Theory is the Quintessence of all Practical Experience" *A. Föppl*

Dazu ergänzend meine persönliche Erfahrung,

nach **1 Mannjahr** Freizeit zum Checken der WWFE-Testdaten auf Brauchbarkeit mit Korrekturbitten (*teilweise erfolgreich*) an die Veranstalter,

„Die Erzeugung <u>zuverlässiger</u> 3D-Testdaten und Probekörper
ist noch herausfordernder als die
Aufstellung einer zugehörigen , auf physikalischen Überlegungen beruhenden Theorie"

"Why not applying Cuntze's

test-validated Strength Failure Criteria (SFC) ?"

Dank fürs Zuhören und Zusehen.

Es wäre schön, falls ich Sie für neue Ansätze *Ihrerseits* etwas begeistern konnte.

Ihr Ralf Cuntze

Curriculum Vitae: CUNTZE, Ralf

- 1964, Dipl.-Ing. Civil Engineering (structural engineering, TU Hannover)
- 1968, Dr.-Ing. Structural Dynamics (TU Hannover)
- 1968 - 1970, DLR FEA-programming
- **1970** - **2004, MAN-Technologie: Head 'Structural and Thermal Analysis'**
  ARIANE 1-5, GROWIAN, Uranium Enrichment centrifuges, Solar Plants, Pressure Vessels, etc.
- 1978, Dr.-Ing. habil. Mechanics of Lightweight Structures (TUM)
- 1980 – 200 Lecturer UniBw Fracture Mechanics (construction), Lightweight (mech. eng.)
- 1980 - 2011: **Surveyor/Advisor** for German BMFT (MATFO, MATEC), BMBF (LuFo), DFG
- 1987, Full Professorship, *not started in favor of interesting industry tasks*
- 1998, Honorary Professorship at **Universität der Bundeswehr München UniBw**
- **1972 – 2018 contributor to the German** Aerospace **Hdbk HSB**
- **2006, VDI Guideline 2014 „Development of FRP-Components" (***editor, sheet 3***)**
- 2019, GLOSSAR "Technical terms for composite parts". Springer
- 1972 - 2004 working on multiple ESA/ESTEC Standards and
  2004 - 2009 heading the „Stability Handbook" Working Group
- **since 2009 with Carbon Composites e.V. and CU Bau (carbon concrete)**
- 2019-2023 „Life-Work Cuntze - a compilation" (about 850 pages, [CUN], downloadable
  from https://www.carbon-connected.de/Group/Prof.Ralf.Cuntze)

Attachment

basically on Terminology

Common working over the engineering disciplines has become mandatory !

* Spelled Criterion: $F \leq 1$, $F \geq 1$ ⇔ Written $F = 1$ (mathematically *a Limit State Condition*)

* Stress: component of the stress tensor, not a stress component (*the word* tensor *is unfortunately skipped*)

* Stress component:given as tensile and compressive stress component of a shear stress

* Civil Engineering (CE) basically works with brittle materials: Tension is indexed

* Mechanical Engineering basically works with ductile materials: Compression is indexed

* Strength : internationally *R* from Resistance (*in CE partly still f from Festigkeit*)

downloadable from https://www.carbon-connected.de/Group/Prof.Ralf.Cuntze

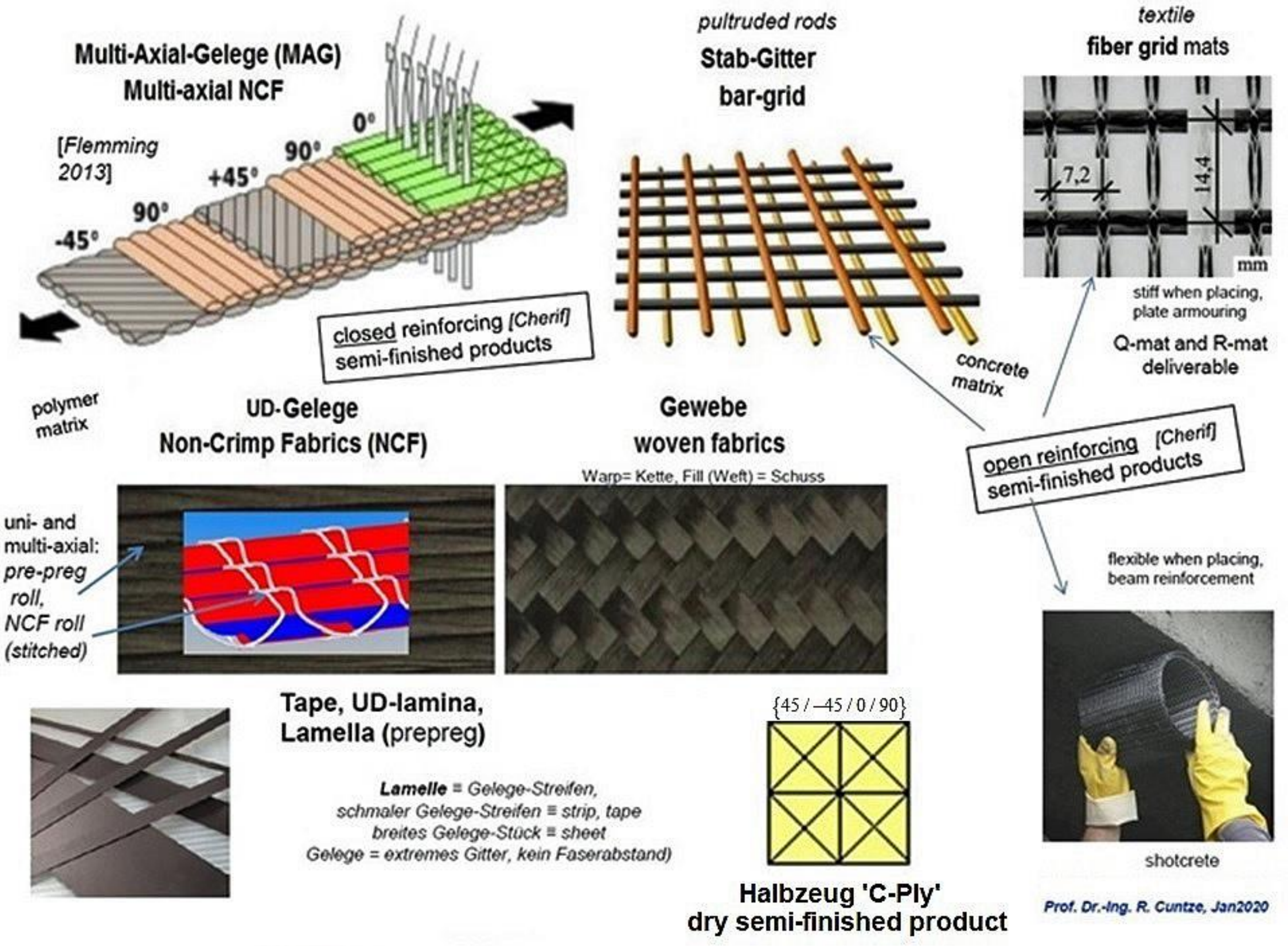

Multi-Axial-Gelege (MAG)
Multi-axial NCF
[Flemming 2013]
0°
90°
+45°
90°
-45°
closed reinforcing [Cherif]
semi-finished products
pultruded rods
Stab-Gitter
bar-grid
textile
fiber grid mats
7,2
14,4
mm
stiff when placing, plate armouring
Q-mat and R-mat deliverable
concrete matrix
polymer matrix
UD-Gelege
Non-Crimp Fabrics (NCF)
Gewebe
woven fabrics
Warp= Kette, Fill (Weft) = Schuss
open reinforcing [Cherif]
semi-finished products
uni- and multi-axial:
pre-preg roll,
NCF roll (stitched)
flexible when placing, beam reinforcement
Tape, UD-lamina,
Lamella (prepreg)
Lamelle ≡ Gelege-Streifen,
schmaler Gelege-Streifen ≡ strip, tape
breites Gelege-Stück ≡ sheet
Gelege = extremes Gitter, kein Faserabstand)
{45 / –45 / 0 / 90}
Halbzeug 'C-Ply'
dry semi-finished product
shotcrete
Prof. Dr.-Ing. R. Cuntze, Jan2020

Fig.1, Construction reinforcement products: (left) 'open-reinforcing' fiber grid, pultruded round bars (CF, GF, AF, BsF); (center) so-called rebars in a bar grid ; (right) 'Closed –reinforcing' UD lamella strips (tape, sheet)

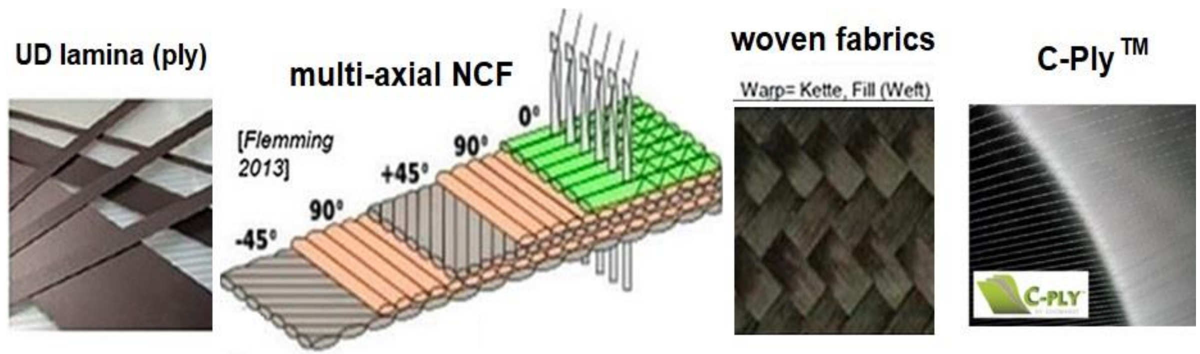


Fig.2, Visualization of applicable closed fiber reinforcing semi-finished products:(left) UD-layer (ply, lamella in CE), composing traditional laminates, stitched Non-Crimped Fabrics (NCF) and woven fabric, (right) novel deliverable C-ply^{TM} = balanced angle ply (see [CUN *§3*]

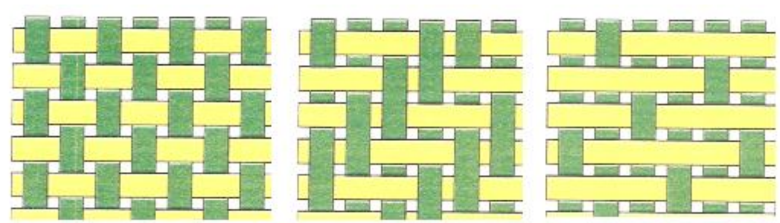


Fig.3: (up) Differently woven fabrics [IKV Aachen]. (center) Plain weave (Leinwandbindung) → Twill weave (Köperbindung) 2/2 → Atlas or Satin weave1/4 [Wikipedia 2023]; (down) Different fracture failure due to ceramic pockets impacting progressive failure

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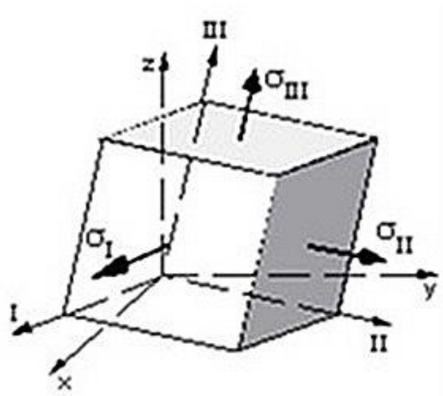
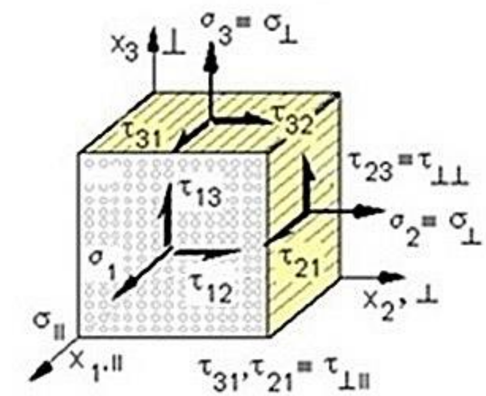
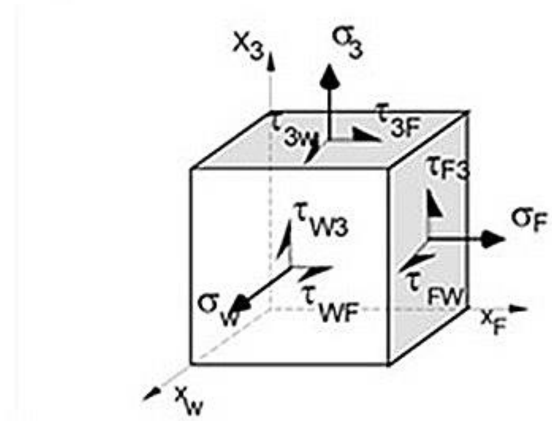


isotropic



transversely-isotropic



rhombically-anisotropic

$\{\sigma\} = (\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{xz}, \tau_{xy})^T$ $\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{13}, \tau_{21})^T$ $\{\sigma\} = (\sigma_W, \sigma_F, \sigma_3, \tau_{3F}, \tau_{3W}, \tau_{FW})^T$

Isotropic: $\{R\} = (R^t, R^c)^T$ with μ

Transversely-isotropic: $\{R\} = (R^t_{\parallel}, R^c_{\parallel}, R^t_{\perp}, R^c_{\perp}, R_{\perp\parallel})^T$ with $\mu_{\perp\parallel}, \mu_{\perp\perp}$

Orthotropic: $\{R\} = (R^t_W, R^c_W, R^t_F, R^c_F, R_{WF}, R^t_3, R^c_3, R_{3F}, R_{3W})^T$ with $\mu_{WF}, \mu_{3W}, \mu_{3F}$

Figure: 3D-stress states and strengths employed in ceramic analyses Warp (W, Kette), Fill (F, Schuss, weft). Rhombically-anisotropic = orthotropic

Self-explaining, symbolic Notations for Strength Properties

prepared by the author for ESA - Material Handbook

		Fracture **Strength Properties**									
	loading	**tension**			**compression**			**shear**			
	direction or **plane**	1	2	3	1	2	3	12	23	13	
9	**general orthotropic**	R_1^t	R_2^t	R_3^t	R_1^c	R_2^c	R_3^c	R_{12}	R_{23}	R_{13}	**friction properties**
5	**UD**	$R_{\parallel}^t$ NF	$R_{\perp}^t$ NF	$R_{\perp}^t$ NF	$R_{\parallel}^c$ SF	$R_{\perp}^c$ SF	$R_{\perp}^c$ SF	$R_{\parallel\perp}$ SF	$R_{\perp\perp}$ NF	$R_{\parallel\perp}$ SF	$\mu_{\perp\perp}$, $\mu_{\perp\parallel}$,
6	**fabrics**	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	*Warp = Fill*
9	**fabrics general**	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	μ_{W3}, μ_{F3}, μ_{WF}
5	**mat**	R_{1M}^t	R_{1M}^t	R_{3M}^t	R_M^c	R_{1M}^c	R_{3M}^c	R_M^τ	R_M^τ	R_M^τ	*(UD, turned direction)*
2	**isotropic matrix**	R_m SF	R_m SF	R_m SF	*deformation-limited*			R_M^τ	R_M^τ	R_M^τ	μ
		R_m NF	R_m NF	R_m NF	R_m^c SF	R_m^c SF	R_m^c SF	R_m^σ NF	R_m^σ NF	R_m^σ NF	μ

NOTE: *As a consequence to isotropic materials (European standardisation) the letter R has to be used for strength. US notations for UD material with letters X (direction 1) and Y (direction 2) confuse with the structure axes' descriptions X and Y . *Effect of curing-based residual stresses and environment dependent on hygro-thermal stresses. *Effect of the difference of stress-strain curves of e.g. the usually isolated UD test specimen and the embedded (redundancy) UD laminae. R_m:= 'resistance maximale' (French) = tensile fracture strength (superscript t here usually skipped), R:= basic strength. Composites are most often brittle and dense, not porous! SF = shear fracture

Elasticity Properties of the homogenized material

		Elasticity Properties									
	direction or **plane**	1	2	3	12	23	13	12	23	13	
9	***general orthotropic***	E_1	E_2	E_3	G_{12}	G_{23}	G_{13}	ν_{12}	ν_{23}	ν_{13}	**comments**
5	***UD, ≅ non-crimp fabrics***	$E_{\parallel}$	$E_{\perp}$	$E_{\perp}$	$G_{\parallel\perp}$	$G_{\perp\perp}$	$G_{\parallel\perp}$	$\nu_{\parallel\perp}$	$\nu_{\perp\perp}$	$\nu_{\parallel\perp}$	$G_{\perp\perp} = E_{\perp}/(2+2\nu_{\perp\perp})$ $\nu_{\perp\parallel} = \nu_{\parallel\perp} \cdot E_{\perp} / E_{\parallel}$ *quasi-isotropic 2-3-plane*
6	***fabrics***	E_W	E_F	E_3	G_{WF}	G_{W3}	$G_{\vert W3}$	ν_{WF}	ν_{W3}	ν_{W3}	*Warp = Fill*
9	***fabrics general***	E_W	E_F	E_3	G_{WF}	G_{W3}	G_{F3}	ν_{WF}	ν_{F3}	ν_{W3}	*Warp ≠ Fill*
5	***mat***	E_M	E_M	E_3	G_M	G_{M3}	G_{M3}	ν_M	ν_{M3}	ν_{M3}	$G_M = E_M/(2+2\nu_M)$ *1 is perpendicular to quasi-isotropic mat plane*
2	***isotropic*** *for comparison*	E	E	E	G	G	G	ν	ν	ν	$G=E/(2+2\nu)$

<u>Lesson Learned:</u> - *Unique, self-explaining denotations are mandatory*
- *Otherwise, expensively generated test data cannot be interpreted and go lost*

Hygrothermal Properties of homogenized materials

		Hygro-thermal properties						
	direction	1	2	3	1	2	3	
9	general orthotropic	α_{T1}	α_{T2}	α_{T3}	α_{M1}	α_{M2}	α_{M3}	comments
5	UD, $\cong$ non-crimp fabrics	$\alpha_{T\|\|}$	$\alpha_{T\perp}$	$\alpha_{T\perp}$	$\alpha_{M\|\|}$	$\alpha_{M\perp}$	$\alpha_{M\perp}$	
6	fabrics	α_{TW}	α_{TW}	α_{T3}	α_{MW}	α_{MW}	α_{M3}	$Warp = Fill$
9	fabrics general	E_W	E_F	E_3	α_{MW}	α_{MF}	α_{M3}	$Warp \neq Fill$
5	mat	α_{TM}	α_{TM}	α_{TM3}	α_{MM}	α_{MM}	α_{MM3}	
2	isotropic for comparison	α_T	α_T	α_T	α_M	α_M	α_M	

NOTE: Despite of annoying some people, I propose to rethink the use of α for the CTE and β for the CME.
Utilizing α_T and α_M automatically indicates that the computation procedure will be similar.

Note on Use: UD-Micro-mechanical Properties

Some lamina analyses require a micro-mechanical input, but not all micro-mechanical properties can be measured :

Solution: *Micro-mechanical equations are calibrated by macro-mechanical test results (lamina level) = an inverse parameter identification*

Condition: *Micro-mechanical properties can be only applied together with the equations they have been determined with!*

Micro-mechanical formulas applied in:

Elasticity domain: may be helpful tools (new formulas)

Strength domain : attempted, but not yet successful.

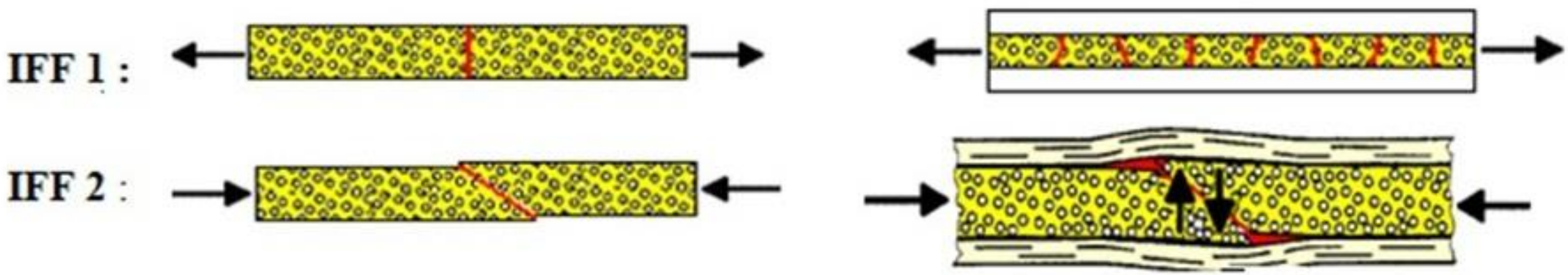
Isolated UD-material (generates hardening curve) and embedded (softening curve)

Isolated‘ lamina test specimens

= weakest link results (series failure system)

‘Embedded‘ laminas experience in-situ effects

= redundancy result (parallel failure system)



unconstrained lamina

delivers strength property, stress-strain curve

(belongs to hardening)

delivers **basic strength** as analysis input !

mutually constrained laminas, in laminates

in non-linear laminate analysis

(belongs to softening)