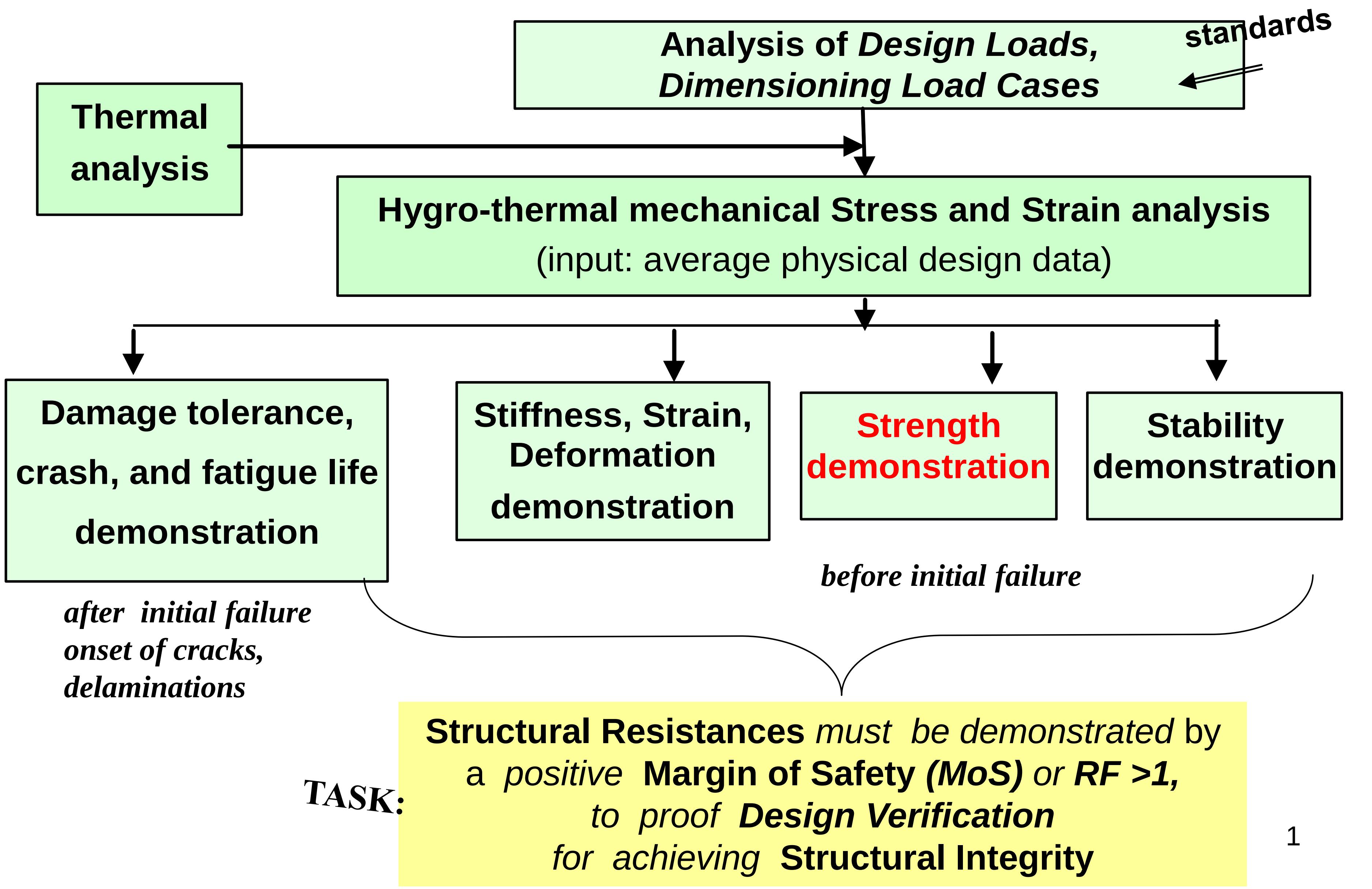
Which Design Verifications are mandatory in Structural Design ?



CONSTRAINTS in Design Development Process : *Cost and Time Reduction*

Industry looks for robust & reliable analysis procedures
in order to replace the expensive 'Make and Test Method'
as far as reasonable.

Virtual tests *shall reduce the amount of* physical tests.

In this context:

Structural Design Development

can be only effective and offer high fidelity

if

qualified analysis tools and necessary test data input are available

for Design Dimensioning and for Manufacturing as well.

A Strength Failure Condition (SFC) is such an Analysis Tool

Consequence for the poor Designer: *To ask*

Is there any Strength Failure Condition ("criterion") he can apply with high fidelity?

Not at all.

Let's do something to partly fill the gap!

Outline of my talk

CARBON COMPOSITES

Reliable Strength Design Verification
- fundamentals, requirements, and some hints -

1 Introduction to Strength Failure Conditions (SFCs)
2 Fundamentals in Modeling when generating SFCs (criteria)
3 Global SFCs versus Modal SFCs
4 Requirements
5 Short Derivation of the Failure-Mode-Concept (FMC)
6 FMC-model applied to an Isotropic Foam (Rohacell 71 G)
7 FMC-model applied to a transversely-isotropic UD-CFRP
Conclusions

Results of a time-consuming „hobby“

Prof. Dr.-Ing. habil. Ralf Georg Cuntze VDI, linked to Carbon Composite e.V.(CCeV) Augsburg

What do we speak about ?

Material: homogenized macromechanical model of the envisaged solid consisting of different constituents

Failure: structural part does not fulfil its functional requirements such as onset of yielding, onset of brittle fracture, Fiber-Failure FF, Inter-Fiber-Failure IFF, leakage, deformation limit, delamination size limit, frequency bound

= project-fixed Limit State with F = Limit State Function

Failure Criterion: F >=< 1 , Failure Condition : F = 1= 100%

F = mathematical formulation of the failure surface (body)

Failure Theory: general tool to predict failure of a structural part, *captures (1) Failure Conditions, (2) Non-linear Stress-strain Curves of a material as input, (3) Non-linear Coding for structural analysis*

Strength Failure Condition (SFC) = subset of a strength failure theory

tool for the assessment of a

'multi-axial failure stress state ' in a critical location of the material.

The **Stresses** are judged by **Strengths !**

Note the Difference: ***Test Data Mapping*** *and* ***Design Verification***

- **Validation of SFCs with Failure Test Data** by

  mapping their course by an average Failure Curve (surface)

For each distinct Load Case with its single Failure Modes a RF must be computed:

- **Delivery of a reliable Design Verification** by

  calculation of a Margin of Safety or a (load) Reserve Factor

  $$MoS > 0 \quad oder \quad RF = MoS + 1 > 1$$

  on basis of a statistically reduced failure curve (surface) .

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Engineering-like SFCs are provided for homogenized (smeared) materials

Shall allow for inserting stresses from the utilized various coordinate systems into stress-formulated failure conditions, - and if possible - invariant-based.

Prediction of: ***Onset of Yielding*** + ***Onset of Fracture*** for non-cracked materials

Assessment of multi-axial stress states in a critical material location,

by **utilizing the uniaxial strength values R** and an **equivalent stress σ_{eq}, representing a distinct actual multi-axial stress state.**

for *** dense & porous,**

*** ductile & brittle behaving materials,**

ductile : $R_{p0.2} \cong R_{c0.2}$ brittle, dense : $R_m^{\,c} \geq 3R_m^{\,t}$

for *** isotropic material**

*** transversally-isotropic material (UD := uni-directional material)**

*** rhombically-anisotropic material (fabrics) + 'higher' textiles etc.**

Material Symmetry used for Homogenized (smeared) Materials

Investigation of the tensorial stress-strain relationships of materials,

6 x 6 stress tensor and 3 x 3 physical properties respecting tensor, **results in**

Material symmetry 'requirements' saying (supported by test evidence)**:**

Number of strengths ≡ number of elasticity properties !

Applicability of Material Symmetry must be checked: Homogenization permitted?

Application of material symmetry, if permitted, then

A *minimum number of properties must be measured, only* (cost + time benefits) !

for more details ….

Test-observed Material Features *(helpful, when generating SFCs)*

1 If a material element can be homogenized to an ideal (= frictionless) crystal,

then, material symmetry demands for the transversely-isotropic UD-material

- 5 *elastic 'constants' , 5 strengths, 5 fracture toughnesses* (CF-lamellen) and
- *2 physical parameters (such as CTE, CME, material friction, etc.)*

(for isotropic materials the respective numbers are 2 and 1)

2 Mohr-Coulomb requires for the real crystal another inherent parameter,

- the *physical parameter 'material friction' : UD* $\mu_{\perp\|}$, $\mu_{\perp\perp}$ *, Isotropic* μ

3 Fracture morphology witnesses:

- Each strength corresponds to a distinct *failure mode*

and to a *fracture type* as Normal Fracture (NF) or Shear Fracture (SF).

⇒ Above Facts and Knowledge gave reason

why the FMC strictly employs single *independent* failure modes

by its failure mode–wise concept.

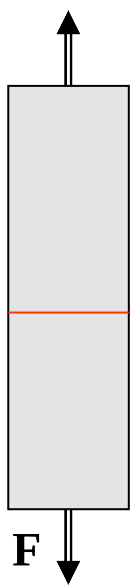
Test-observed Strength Failure Modes of Brittle behaving Isotr. Materials

Normal Fracture (NF)	**Shear Fracture Mode (SF)**	**Crushing Fracture (CrF):**
- no material element change before fracture	- shape change of material element	- volumetric element change before fracture

helpful knowledge for the later choice of invariants

if brittle: failure = fracture failure

Tension



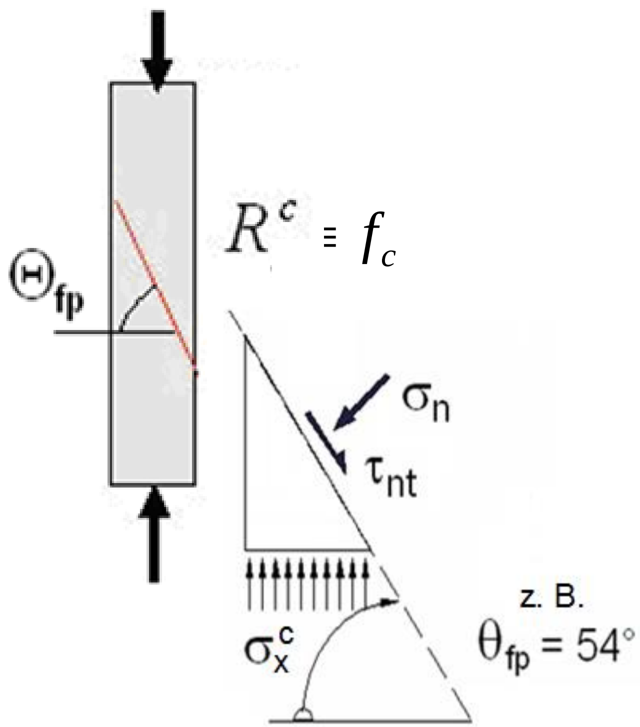
F

$$R^t = f_t$$

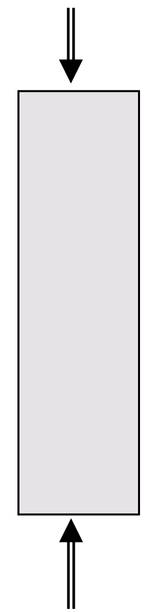
Compression

dense consistency

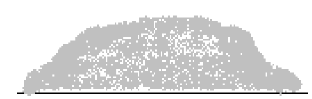
$$R^c \equiv f_c$$



porous consistency



= decomposition of texture



= hill of fragments (crumbs) as result of compression tests

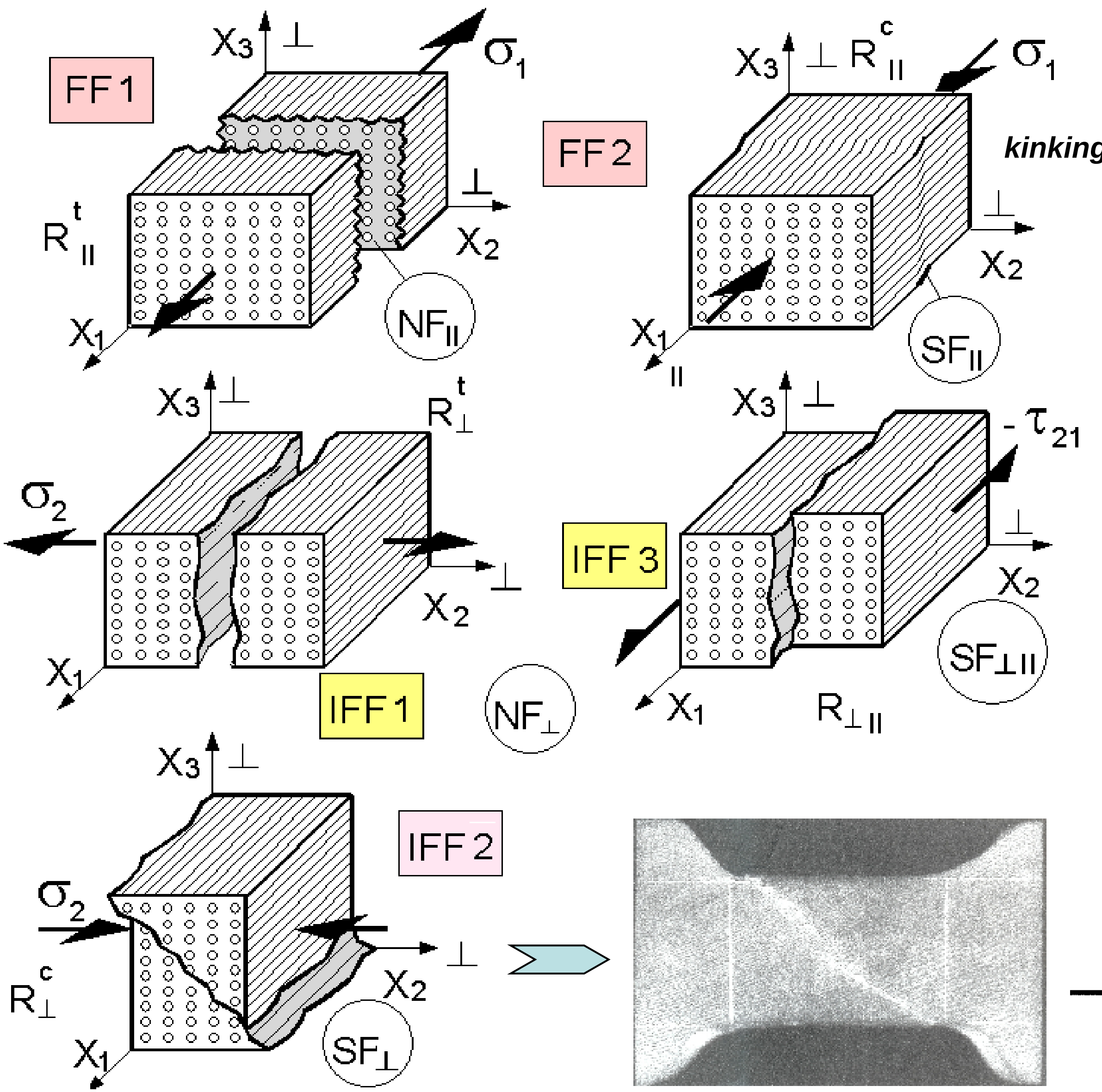
fracture plane angle = measure for friction value

··· needs interaction

t = tension
c = compression
R = strength, resistance
F = Fracture

Observed: ► **Each single Failure Mode is governed by one single strength, only !!**

Test-observed Strength Failure Modes of Brittle behaving UD-Materials



wedge failure type

t = tension

c = compression

▶ **5 Fracture modes exist**

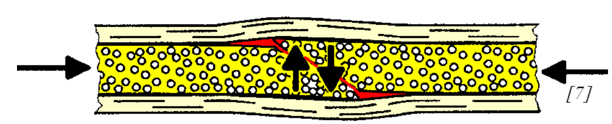
= **2 FF (Fibre Failure)**

+ **3 IFF (Inter Fibre Failure)**

Fracture Types (macroscale-associated):

NF := Normal Fracture
SF := Shear Fracture

Friction occurs in IFF2 and IFF3 !



embedded UD lamina

Mind the difference in UD-analysis : Isolated and embedded UD-behaviour

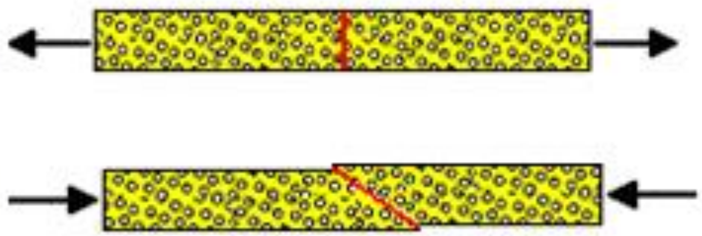
'Isolated' lamina test specimens = weakest link results (series failure system)

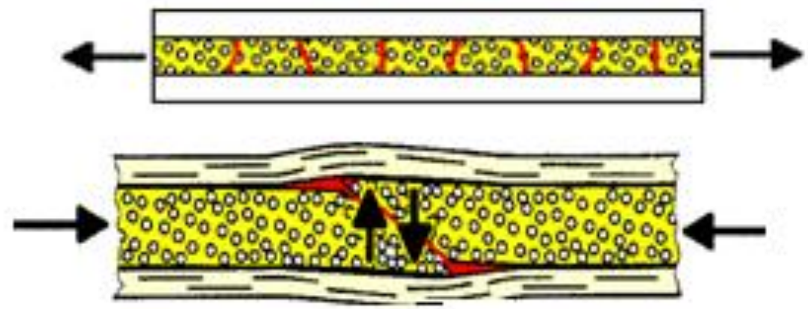
'Embedded' laminas experience in-situ effects = redundancy result (parallel failure system)

Measurement/Determination of strain softening curve ?

IFF 1 :

IFF 2 :





unconstrained lamina

mutually constrained laminas, in laminates

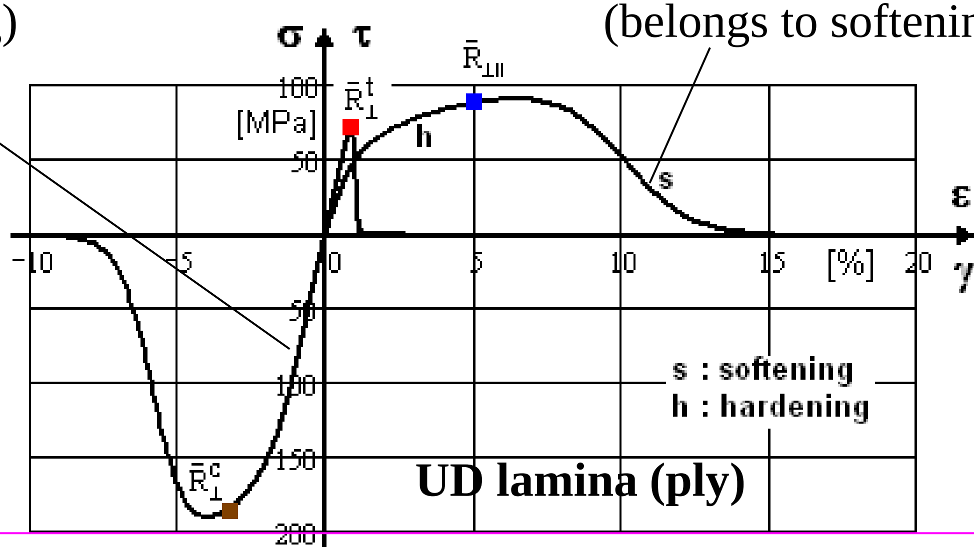
delivers strength property, stress-strain curve

in non-linear laminate analysis

(belongs to hardening)

(belongs to softening)

delivers **basic strength** as analysis input !



h load-c

branch

(i.e. test

s deform

branch

the em

Lesson Learned: *In the Post-IFF regime the embedded lamina experiences no sudden death but still has residual strength and stiffness due to in-situ effect!*

Intention: Creation of Invariant-based SFCs

HELP : Physically-based Choice of Invariants is possible

* **Beltrami : "At 'Onset of Yielding' the material possesses a distinct *strain energy* composed of *dilatational energy* (I_1^2) and *distortional energy* (J_2≡*Mises*) ".**

* **So, from Beltrami, Mises (HMH), and Mohr / Coulomb (friction) can be concluded:**

Each invariant term in the *failure function F* may be dedicated to one physical mechanism in the solid = cubic material element:

- volume change : I_1^2 … *(dilatational energy)* relevant if porous

- shape change : J_2 (Mises) … *(distortional energy)* relevant if brittle behaving

and - friction : I_1 … *(friction energy)*

Mohr-Coulomb

relevant if material element shape changes

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Global and Modal Strength Failure Conditions, General View

Drucker-Prager, Tsai-Wu

1 Global ***strength*** **failure condition** : F ({σ}, {R}) = 1 (usual formulation)

Set of Modal strength failure conditions: F ({σ}, R^{mode}) = 1 (addressed in FMC)

Mises, Puck, Cuntze

Example: UD

vector of 6 stresses (general)

$$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$$

vector of 5 strengths

$$\{R\} = (R_{\parallel}^{t}, R_{\parallel}^{c}, R_{\perp}^{t}, R_{\perp}^{c}, R_{\perp\parallel})^T$$

needs an Interaction of Failure Modes: performed by a

probabilistic-based 'rounding-off' approach (series failure system model)

directly delivering the (material) reserve factor in linear analysis

Mises : Onset of yielding of ductile behaving materials

Cuntze: Onset of fracture of brittle behaving materials

By-the-way, experience with Failure Prediction shows

Strength Failure Condition (SFC) is a necessary but not a sufficient condition to predict Strength Failure (i.e. thin-layer problem).

Interaction of Single Strength Failure Modes in the modal FMC

Interaction of adjacent Failure Modes by a *series failure system* model

= 'Accumulation' of interacting *failure danger portions* Eff^{mode}

$$Eff^{*} = \sqrt[m]{(Eff^{\text{mode }1})^{m} + (Eff^{\text{mode }2})^{m} + \ldots}$$ ***= 1 = 100% , if failure***

with mode-interaction exponent *2.5 < m < 3 from mapping experience*

***as modal* material stressing effort * (in German Werkstoffanstrengung)**

and

$$Eff^{\text{mode}} = \sigma_{eq}^{\text{mode}} \,/\, \overline{R}^{\,\text{mode}}$$

equivalent mode stress

mode associated average strength

later example

* **artificial technical term** created together with QinetiQ in the World-Wide-Failure-Exercise

Facts of so-called Modal SFCs

Modal SFCs (multi-suface domains)

- **Describe one single failure mode in one single mathematical formulation** **(= one part of the failure surface)**

  *** determine all mode model parameters in the respective failure mode domain**

  *** capture a twofold acting failure mode separately, such as** $\sigma_I = \sigma_{III}$ **(isotropic) or** $\sigma_2 = \sigma_3$ **(transversely-isotropic UD material),** mode-wise by the well-known Ansatz $f\,(J_2, J_3)$

- **Re-calculation of the model parameters and of RF just in that failure mode domain where test data must be replaced.**

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Driver for my research work on General Strength Failure Cond. (criteria)

Achievement of practical SFCs under some *pre-requisites* :

- ***physically convincing, numerically robust***
- ***simple, as much as possible***
- ***invariant-based** (like the Mises yield condition)*
- ***allow to compute for each mode an equivalent stress** (very helpful for the designer)*
- ***shall be convex (Drucker postulate) in the hoop plane (isotropic materials), but also in meridional plane (?)***
- ***rigorous indepent treatment of each single failure mode (2 FF + 3 IFF)***
- ***using a material behaviour-linked thinking and not a material-linked one***
- ***engineering approach where all model parameters can be measured*.**

Note on Puck's UD strength failure conditions:

Puck's action plane approach involves some basic differences to Cuntzes Failure-mode-concept-based approach: (1) is not invariant-based, (2) interacts the 3 Inter-Fiber-Failure modes (IFF) by a Mohr-Coulomb-based equation, (3) post-corrects the IFF- influence on FF.

Cuntze provides for each failure mode an equivalent stress, that captures the influence of IFF on FF by his interaction equation, uses less model parameters.

Specifica for the UD-lamina-based High Performance Laminates

Specific Pre-requisites for the establishment of 3D-UD-SFCs:

- **simply formulated from engineering point of view, numerically robust,**
- **physically-based, and therefore need only few information for pre-dimensioning**
- **shall allow for a simple determination of the design driving reserve factor**
- **shall capture failure of the constituents matrix (cohesive), interphase (adhesive), filament**
- **consider residual stresses**

Compliant with John Hart-Smith

- **consider micro-mechanical stress concentration of the matrix around the filaments under transversal stress** (a means: using matrices showing > 6% fracture strain which heps to capture a stress concentration factor of about 6 up to 1% applied transversal strain
- **consider FF, if taking place under bi-axial compression with no external axial stress**

$$\{\sigma\} = (\sigma_1 = 0,\ \sigma_2,\ \sigma_3, 0, 0, 0)^T$$

Example: Assumptions for UD Modelling and Mapping

- **The UD-lamina is macroscopically homogeneous.**

  It can be treated as a homogenized (‘smeared‘) material

  Homogenisation of a solid to a material brings benefits.

  Then Knowledge of Material Symmetry applicable : number of required material properties are minimal, test-costs too

  1 Lamina (ply) = Layer of a Laminate, e.g. UD-laminas = “Bricks“

- The UD-lamina is transversely-isotropic: **On planes, parallel to the fiber direction it behaves orthotropic and on planes transverse to fiber direction isotropic (quasi-isotropic plane)**

- **Mapping creates fidelity, only, if:**

  uniform stress states are about the critical stress location in the material !

  Is very seldom the case.

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Motivation for my non-funded **Investigations**

Existing Links in the Mechanical Behaviour show up: ***Different structural materials***

- *can possess similar material behaviour* ***or***
- *can belong to the same class of material symmetry*

> **similarity aspect**

Welcomed Consequence:

- **The same strength failure function F can be used for different materials**
- **More information is available for pre-dimensioning + modelling from experimental results of a similarly behaving material.**

Background: ***Author's experience with structural material applications, range 4 K - 2000 K***

Basic Features of the author‘s Failure-Mode-Concept (FMC)

- **Each failure mode represents 1 independent failure mechanism and thereby 1 piece of the complete *failure surface***
- **Each failure mechanism is governed by 1 basic strength (is observed !)**
- **Each failure *mode* can be represented by 1 failure *condition.***

  Therefore, equivalent stresses can be computed for each mode !

- **In consequence, this separation requires :**

  An interaction of the Modal Failure Modes !

Failure-Mode-Concept (FMC) Postulate (*example*: UD material

Remember:

- **Each single observed fracture failure modes is linked to one strength**
- **Symmetry of a material showed :** ***Number of strengths =*** $R_{||}^{t}, R_{||}^{c}, R_{\perp||}, R_{\perp}^{t}, R_{\perp}^{c}$

  number of elasticity properties ! $E_{\|}, E_{\perp}, G_{\|\perp}, \nu_{\perp\|}, \nu_{\perp\perp}$

Due to the facts above **Cuntze postulates in his FMC**

> ▶ **Number of failure modes = number of strengths, too !**
>
> **e.g.:   isotropic = 2   or above transversely-isotropic (UD) = 5**

t:= tensile, c: = compression, || : = parallel to fibre,   := transversal to fibre

Formulation of FMC-based Modal SFCs *by using*

- **Invariants**
- **Hypotheses of**
  Beltrami = dedication of invariants to the deformation of the material element, whether it is a shape change (Mises) or a volume change and
  Mohr-Coulomb = internal friction of a brittle behaving solid material
- **Application of the Reqirements of Material Symmetry** = for isotropic brittle behaving materials the characteristic number of quantities is 2 (2 strengths, 2 strength fracture failure modes, 2 basic invariants)
- advantegeous **equivalent stresses** σ_{eq} and of the physically plausible **material stressing effort** (Werkstoffanstrengung) *Eff*

Consequence for needed number of SFC-parameters:

***Tension*: 1 strength parameter. *Compression*: 1 strength + 1 friction parameter. *Interaction*: exponent *m*.**

*** The "requirements" of material symmetry are backed by test observation.**

*** The bi-axial dents in the hoop plane are the consequence of a 2-fold occurring failuremode. The depth of the dent can be either calculated by an effortful probabilistic analysis or by elegantly using J3 as a good shape-giving third invariant to capture the bi-axial additional failure danger.**

*** Explanation of a multifold failure mode of a dense brittle behaving material :**

Uni-axial compression creates one failure mode *but* there are multiple fracture planes possible activated by the spatial flaw distribution with the critical maximum local flaw

<u>Note</u>: Characteristic number of quantities for the transversely-isotropic unidirectional material UD is 5

Cuntzes 3D Modal Strength Failure Cond. (criteria) for Isotropic Foams

Approaches:

$$\frac{\sqrt{4J_2 - I_1^{\,2}/3} - I_1}{2 \cdot \overline{R}_c} = 1$$

Considering bi-axial strength (failure mode occurs twice): **in Effs now**

$$Eff^{NF} = c_{NF} \cdot \frac{\sqrt{4J_2 - I_1^{\,2} \cdot (\Theta_{NF})/3} + I_1}{2 \cdot \overline{R}_t} = \sigma_{eq}^{NF} / \overline{R}_t \ , \qquad Eff^{CrF} = c_{CrF} \cdot \frac{\sqrt{4J_2 - I_1^{\,2} \cdot (\Theta_{CrF})/3} - I_1}{2 \cdot \overline{R}_c} = \sigma_{eq}^{CrF} / \overline{R}_c$$

The two-fold failure danger can be excellently modelled by using the often used invariant J_3 in :

$$\Theta_{NF} = \sqrt[3]{1 + D_{NF} \cdot \sin(3\theta)} = \sqrt[3]{1 + D_{NF} \cdot 1.5 \cdot \sqrt{3} \cdot J_3 \cdot J_2^{\,-1.5}} \qquad \Theta_{CrF} = \sqrt[3]{1 + D_{CrF} \cdot \sin(3\theta)} = \sqrt[3]{1 + D_{CrF} \cdot 1.5 \cdot \sqrt{3} \cdot J_3 \cdot J_2^{\,-1.5}}$$

Mode interaction: $Eff^{NF} = [(Eff^{NF})^m + (Eff^{CrF})^m]^{m^{-1}}$

The failure surface is closed at both the ends: A simple cone serves as closing cap and bottom

$$\frac{I_1}{\sqrt{3} \cdot R_t} = s_{NF} \cdot \left(\frac{\sqrt{2J_2 \cdot \Theta_{NF}}}{R_t}\right) + \frac{\max I_1}{\sqrt{3} \cdot R_t}$$

R_t-normalized Lode-Coordinates

$$\frac{I_1}{\sqrt{3} \cdot R_t} = s_{CrF} \cdot \left(\frac{\sqrt{2J_2 \cdot \Theta_{CrF}}}{R_t}\right) + \frac{\min I_1}{\sqrt{3} \cdot R_t}$$

The slope parameters *s* are determined connecting the respective hydrostatic strength point with the associated point on the shear meridian, *$maxI_1$* must be assessed whereas *$minI_1$* could be measured.

Eff = material stressing effort = Werkstoff-Anstrengung (must be < 1 = 100%)

Cuntzes 3D Modal SFCs (criteria) for Transversely-Isotropic UD-materials

Invariants replaced by their stress formulations

strains from FEA **[Cun04, Cun11]**

FF1 $Eff^{\|\sigma} = \breve{\sigma}_1 / \bar{R}^t_\| = \sigma^{\|\sigma}_{eq} / \bar{R}^t_\| ,$ $\qquad \breve{\sigma}_1 \cong \varepsilon^t_1 \cdot E_\|$ *

FF2 $Eff^{\|\tau} = -\breve{\sigma}_1 / \bar{R}^c_\| = +\sigma^{\|\tau}_{eq} / \bar{R}^c_\| ,$ $\qquad \breve{\sigma}_1 \cong \varepsilon^c_1 \cdot E_\|$

2 filament modes

IFF1 $Eff^{\perp\sigma} = [(\sigma_2 + \sigma_3) + \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}{}^2}\,] / 2\bar{R}^t_\perp = \sigma^{\perp\sigma}_{eq} / \bar{R}^t_\perp$

IFF2 $Eff^{\perp\tau} = [(\frac{\mu_{\perp\perp}}{1-\mu_{\perp\perp}}) \cdot (\sigma_2 + \sigma_3) + \frac{1}{1-\mu_{\perp\perp}} \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}{}^2}\,] / \bar{R}^c_\perp = +\sigma^{\perp\tau}_{eq} / \bar{R}^c_\perp$

3 matrix modes

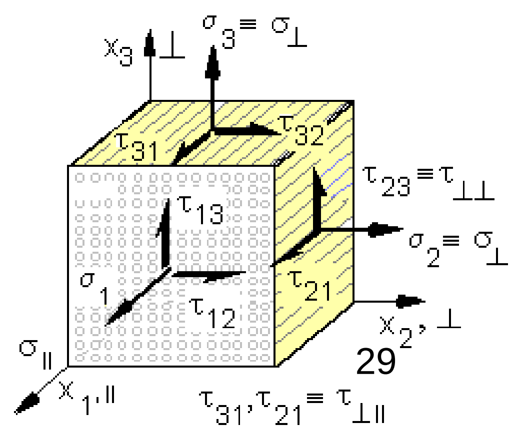
IFF3 $Eff^{\perp\|} = \{[\mu_{\perp\|} \cdot I_{23-5} + (\sqrt{\mu_{\perp\|}{}^2 \cdot I_{23-5}{}^2 + 4 \cdot \bar{R}_{\perp\|}{}^2 \cdot (\tau_{31}^2 + \tau_{21}^2)^2}\,] / (2 \cdot \bar{R}_{\perp\|}{}^3)\}^{0.5} = \sigma^{\perp\|}_{eq} / \bar{R}_{\perp\|}$

with $I_{23-5} = 2\sigma_2 \cdot \tau_{21}^2 + 2\sigma_3 \cdot \tau_{31}^2 + 4\tau_{23}\tau_{31}\tau_{21}$

Modes-Interaction :

$$Eff^{m} = (Eff^{\|\tau})^m + (Eff^{\|\sigma})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\|})^m = 1$$

with mode-interaction exponent $2.5 < m < 3$ **from mapping tests data**

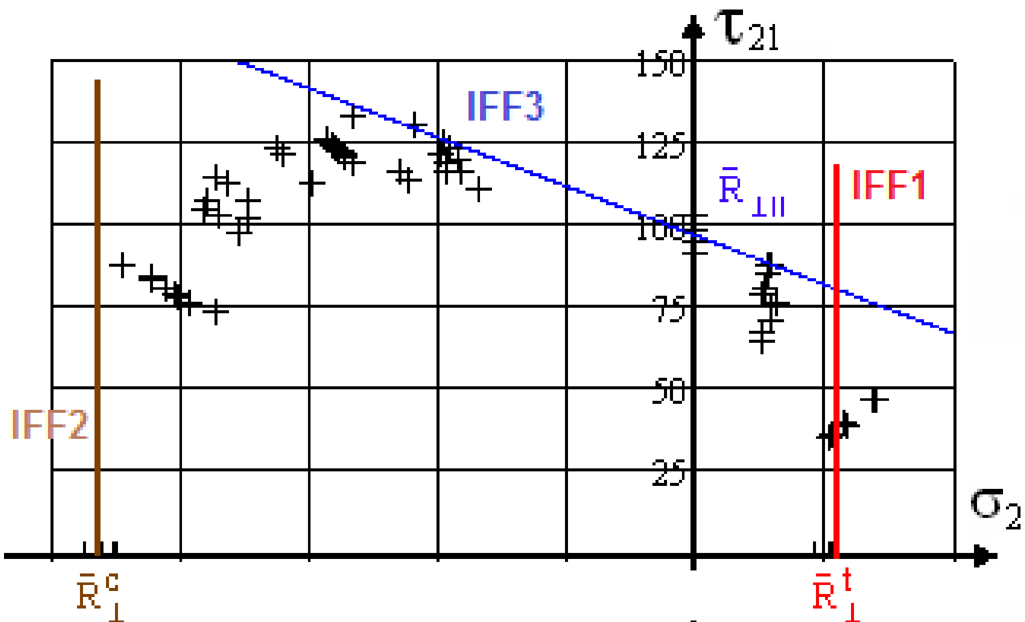


Typical friction value data range: $0.05 < \mu_{\perp\|} < 0.3, \quad 0.05 < \mu_{\perp\perp} < 0.2$

Poisson effect * : bi-axial compression strains the filament without any σ_1

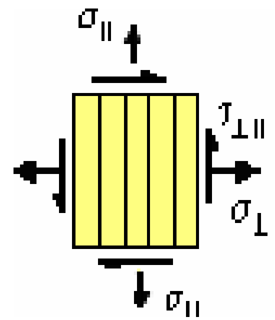
t:= tensile, c: = compression, || : = parallel to fibre, ⊥ := transversal to fibre

Demonstration: Interaction of UD Failure Modes for $\tau_{21}(\sigma_2)$, $\breve{\sigma}_1 = 0$



Mapping of course of IFF test data in a pure mode domain by the associated *Mode Failure Condition.*

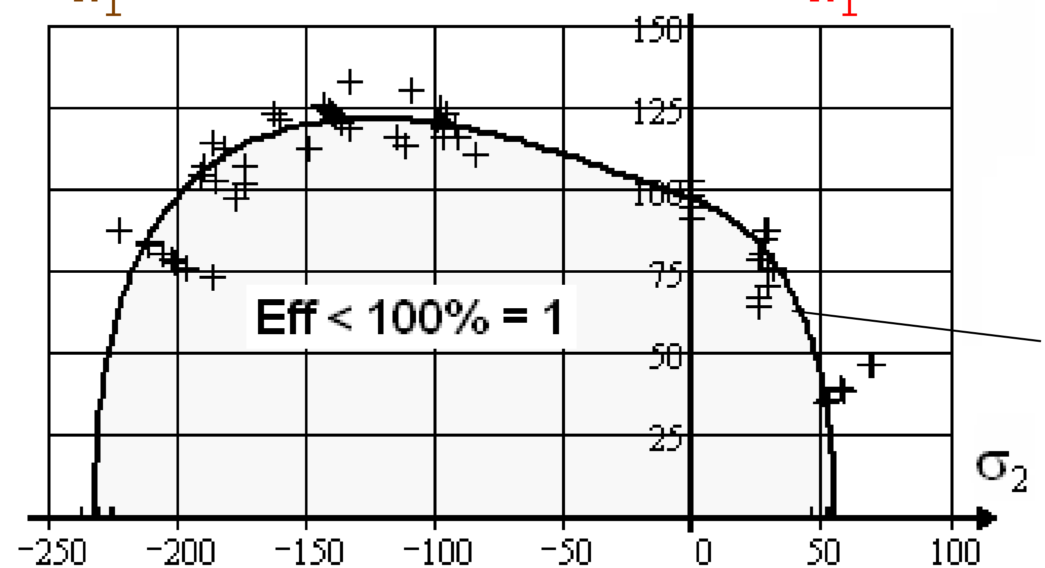
3 IFF pure modes = straight lines !.



IFF 1 : $\dfrac{\sigma_2}{\bar{R}_\perp^t} = 1$

IFF 2 : $\dfrac{-\sigma_2}{\bar{R}_\perp^c} = 1$

IFF 3* *(2D simplified) : $\dfrac{|\tau_{21}|}{\bar{R}_{\perp\parallel} - \mu_{\perp\parallel} \cdot \sigma_2} = 1$

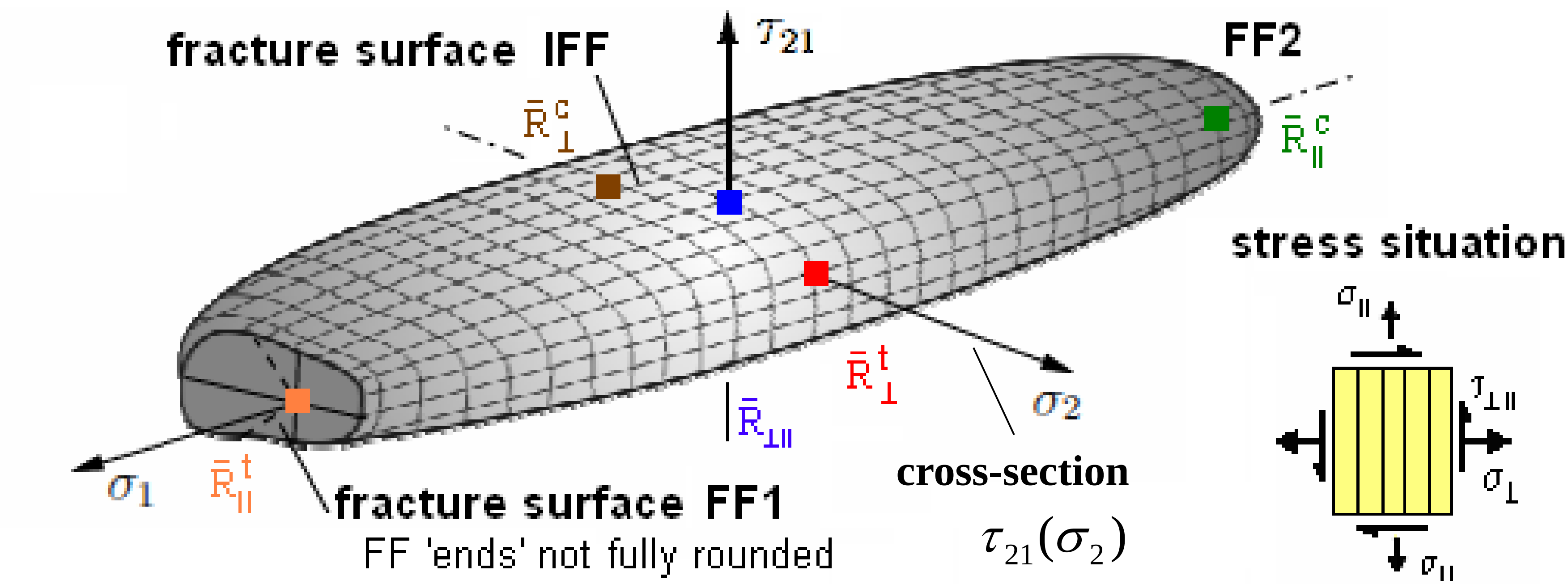


Mapping of course of test data by *Interaction Model*

$$(Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m = 1$$

$$m = 2.5, \ \mu_{\perp\parallel} = 0.3$$

Visualization of 2D-UD-SFCs as Fracture Failure Surface (Body)



$$\{\sigma\} = (\sigma_1, \sigma_2, 0, 0, 0, \tau_{21})^T$$

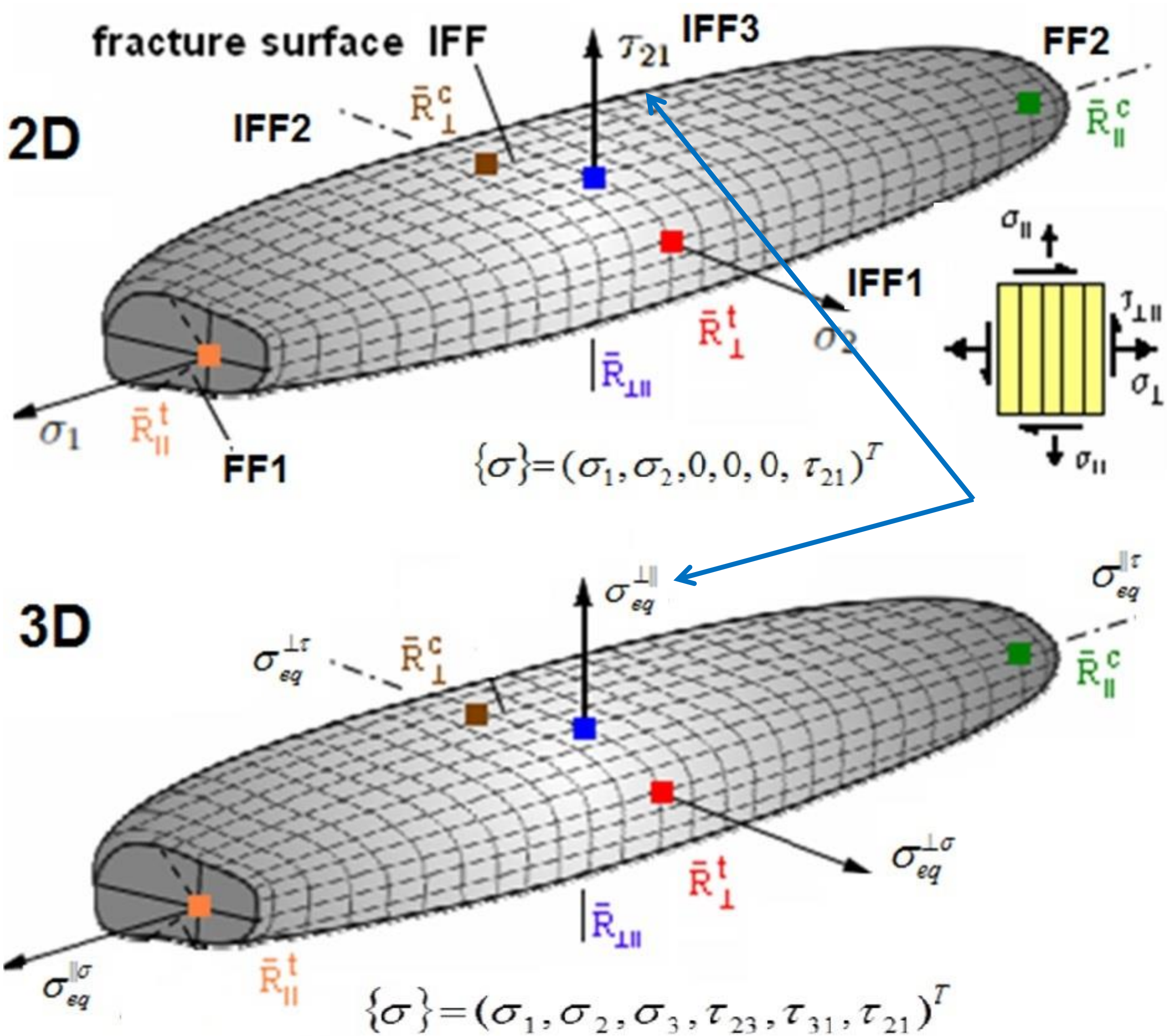
Mode interaction fracture failure surface *of FRP UD lamina*

$$Eff^{\,m} = (Eff^{\,\parallel\tau})^m + (Eff^{\,\parallel\sigma})^m + (Eff^{\,\perp\sigma})^m + (Eff^{\,\perp\tau})^m + (Eff^{\,\perp\parallel})^m = 1$$

(courtesy W. Becker) .

Mapping: Average strengths indicated

2D = 3D Fracture surface if replacing *stress* by *equiv. stress*



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6 FMC-model applied to an Isotropic Foam (Rohacell 71 G)

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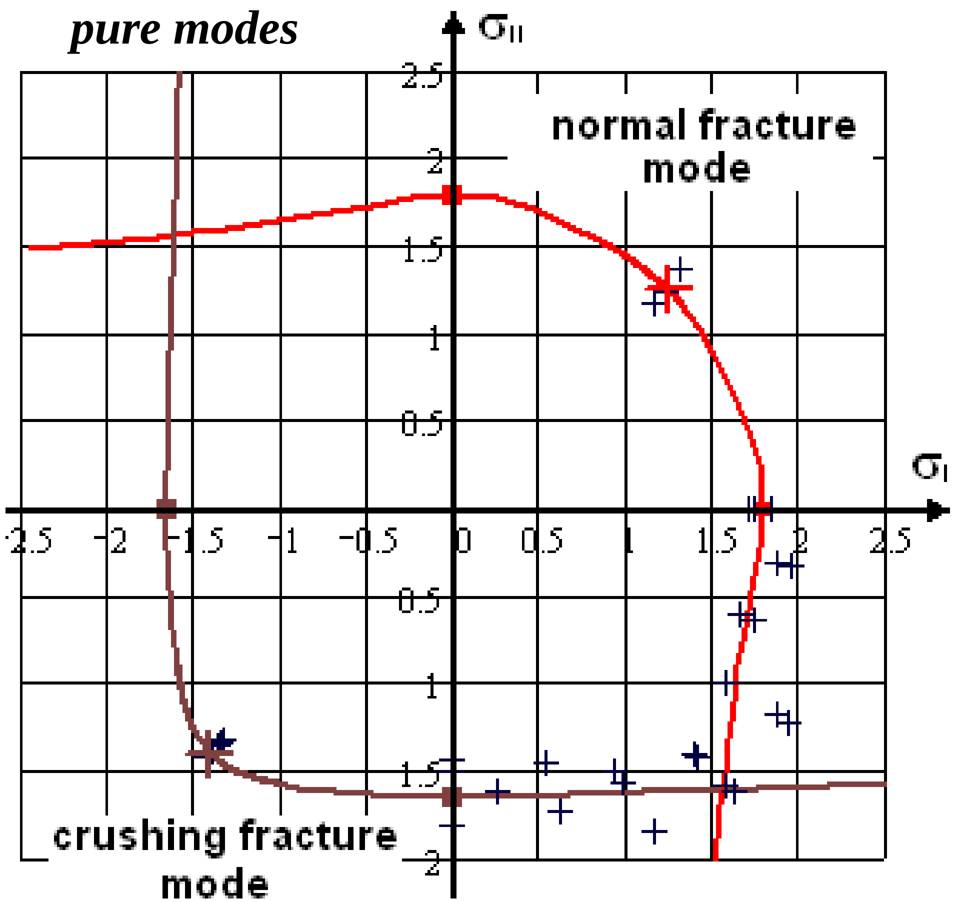
Conclusions

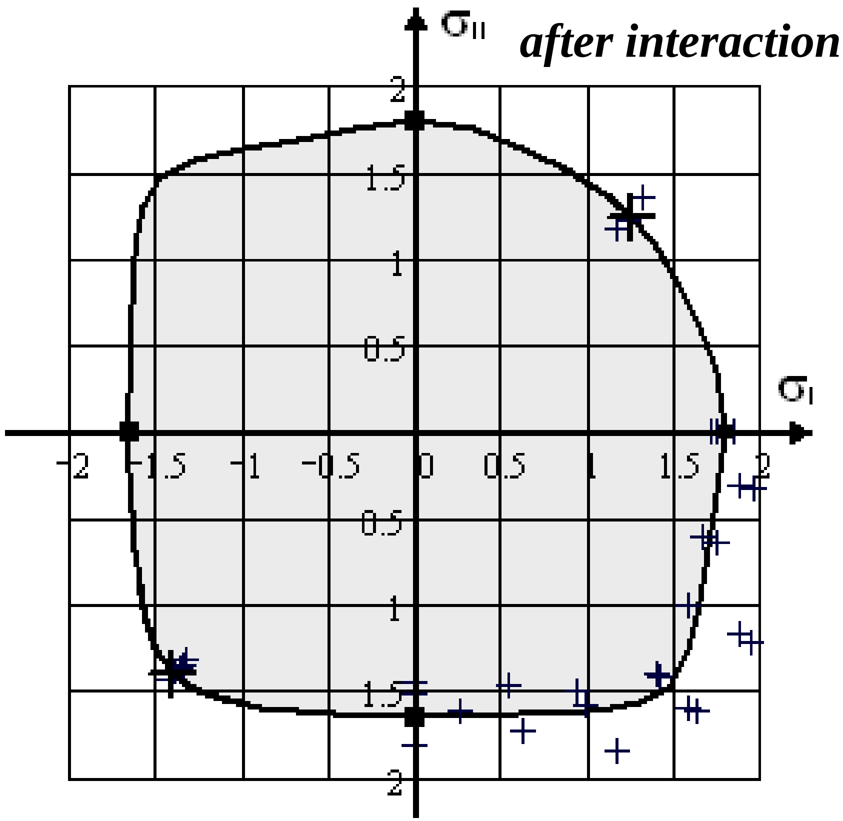
2D - Test Data Set and Mapping in the Principal Stress Plane

Rohacell 71 IG

as similarly behaving material

Principal Plane Cross-section of the Fracture Body (oblique cut)

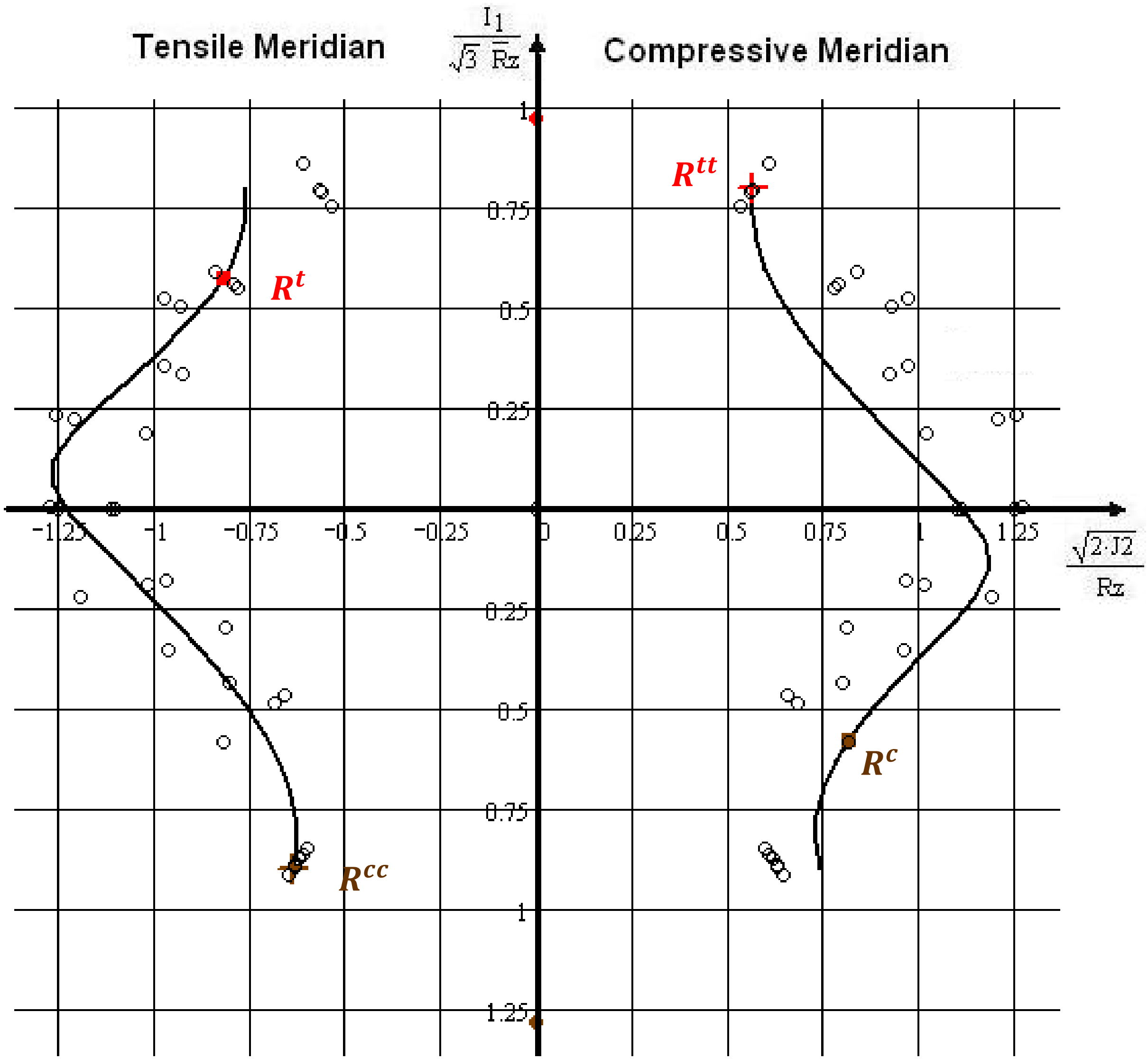




- Mapping must be performed in the 2D-plane because fracture data set is given there
- **The 2D-mapping uses the 2D-subsolution of the 3D-strength failure conditions**
- **The 3D-fracture failure surface (body) is based on the 2D-derived model parameters.**

Courtesy: LBF-Darmstadt, Dr. Kolupaev

Generic Lines of Tensile and of Compressive Meridian



Meridional cross-sections of the Fracture Body

in Lode-Haigh-Westergaard coordinates

bi-axial = +

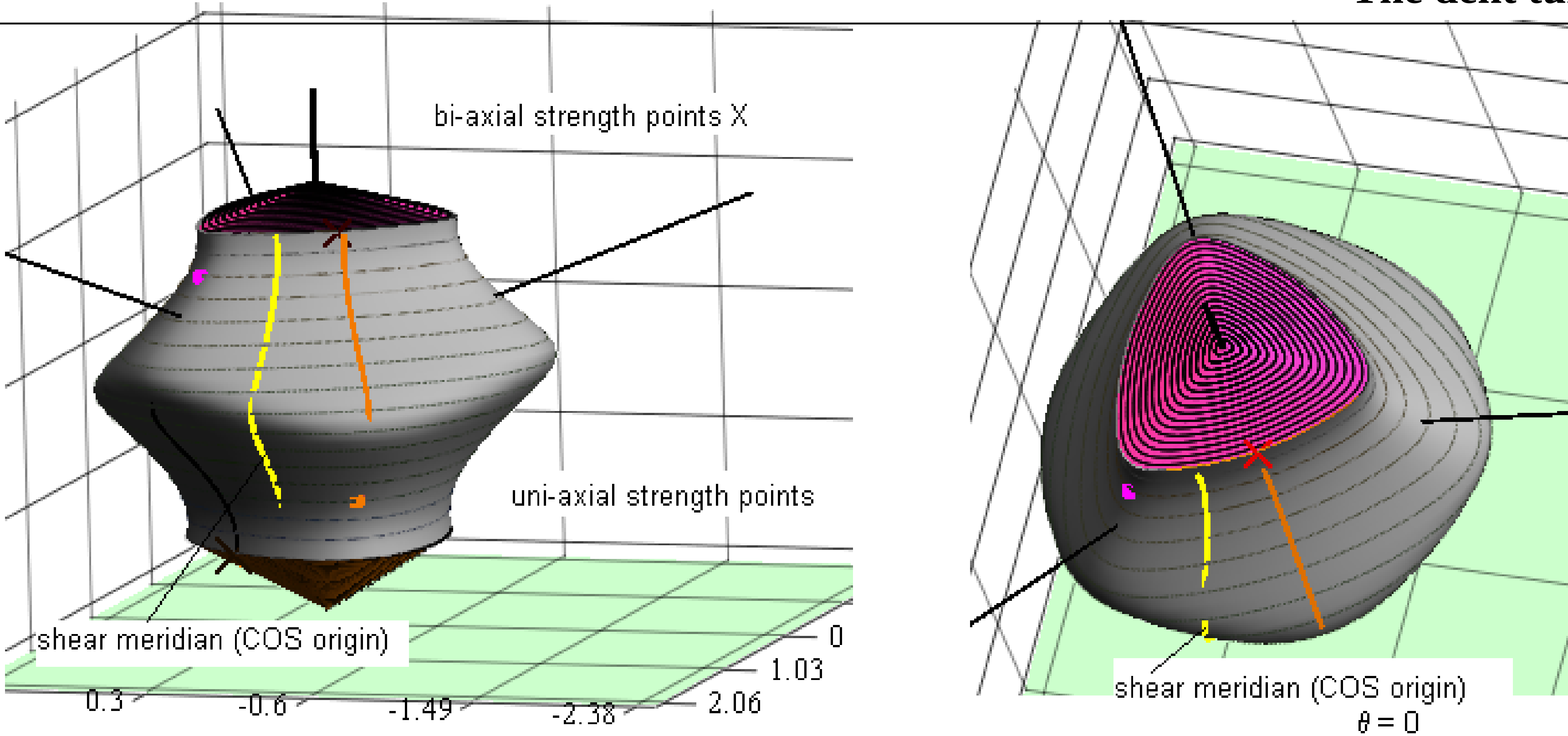
z = tensile, d = compressive

The fracture test data are located at a distinct Lode angle of its associated ring o, 120°-symmetry of the isotropic failure surface (body) .

Cap and bottom are closed by a cone-ansatz, a shape being on the conservative side.

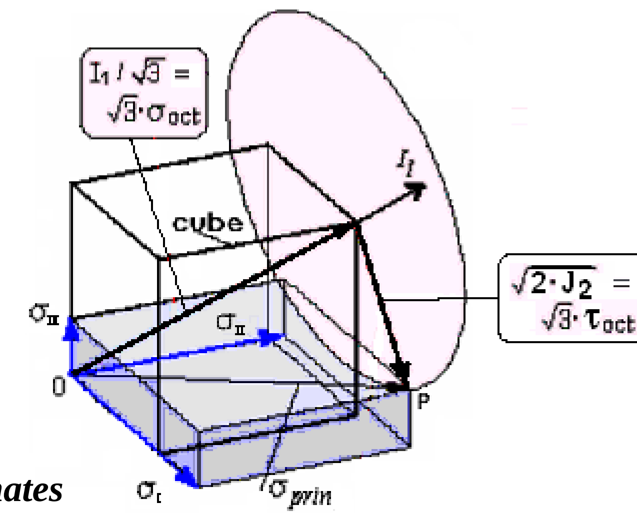
Fracture Failure Surface of *Rohacell 71 IG*

The dent turns !



compressive meridian with tensile meridian form one cross-section shape

The 3D-strength failure condition enables to predict the 120°-symmetric failure body and to judge a 3D- stress state



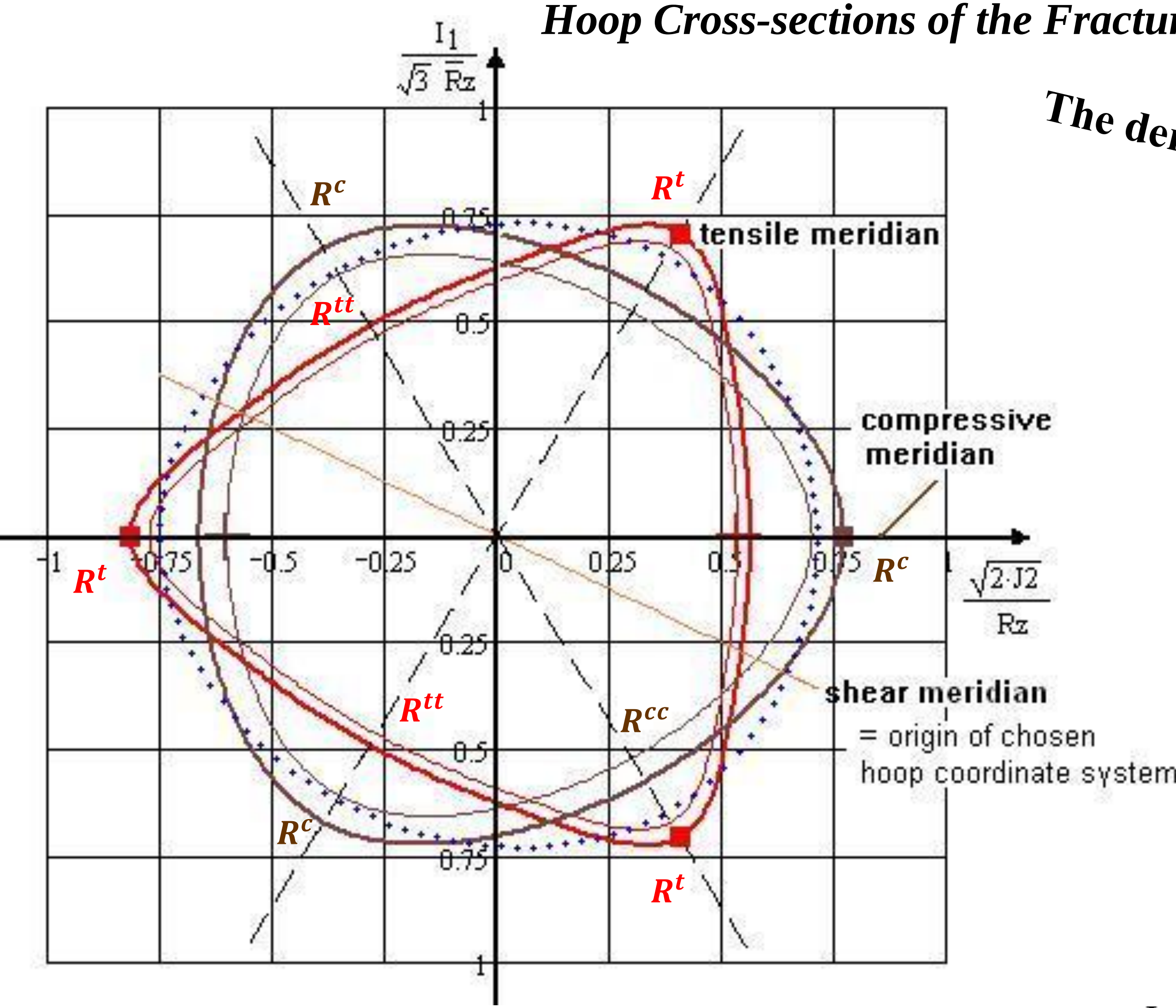
visualization of the Lode-Haigh-Westergaard coordinates

2D Test Data and Mapping in the Octahedral Stress Plane

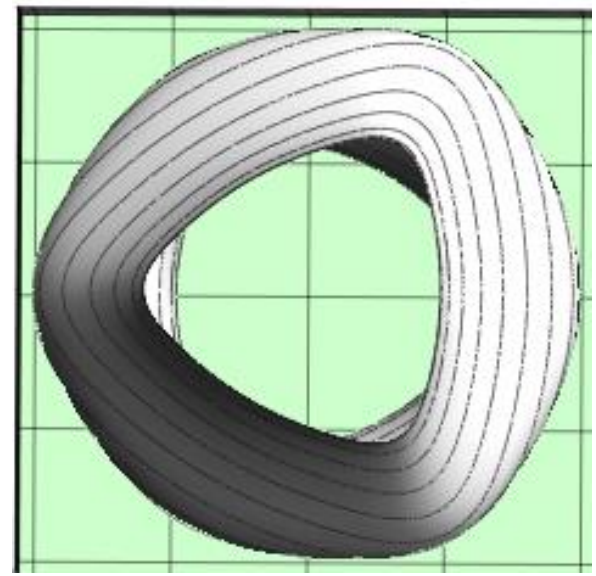
Rohacell 71 IG

as similarly behaving material

Hoop Cross-sections of the Fracture Body



The dent turns !



Caps: No test data, cone was chosen.

Lode-angle, here set as $\sin(3\,\theta)$:

shear meridian angle = 0°

tensile meridian +30° +

compressive meridian -30° +

I1 = 0, is interaction domain: Is about a circle.

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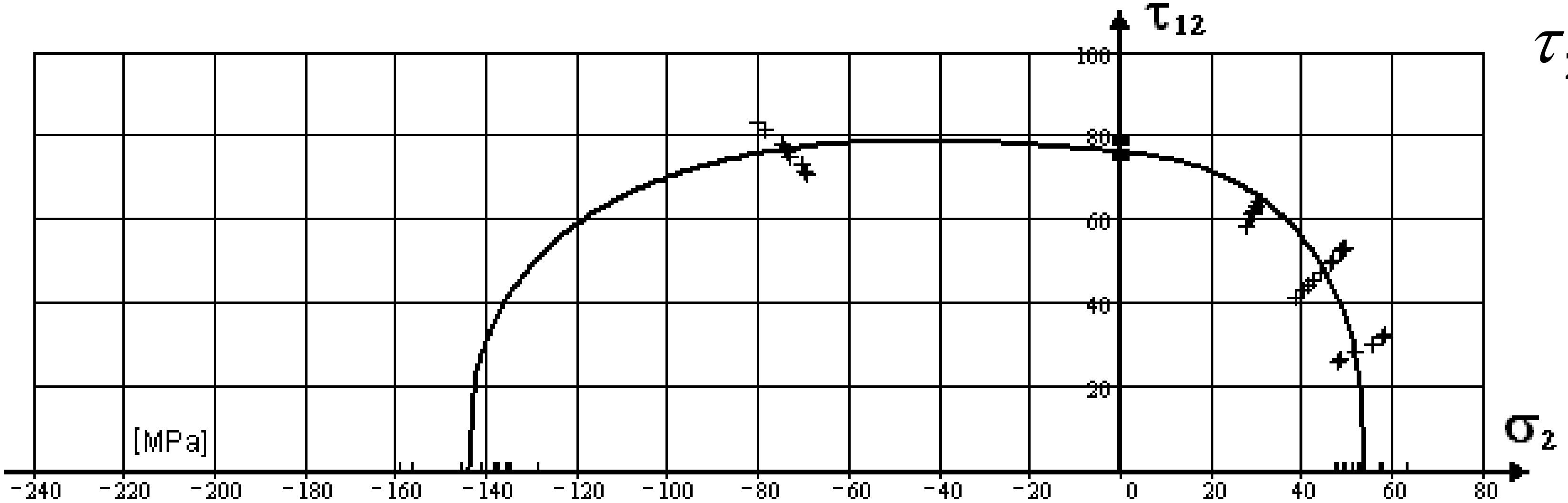
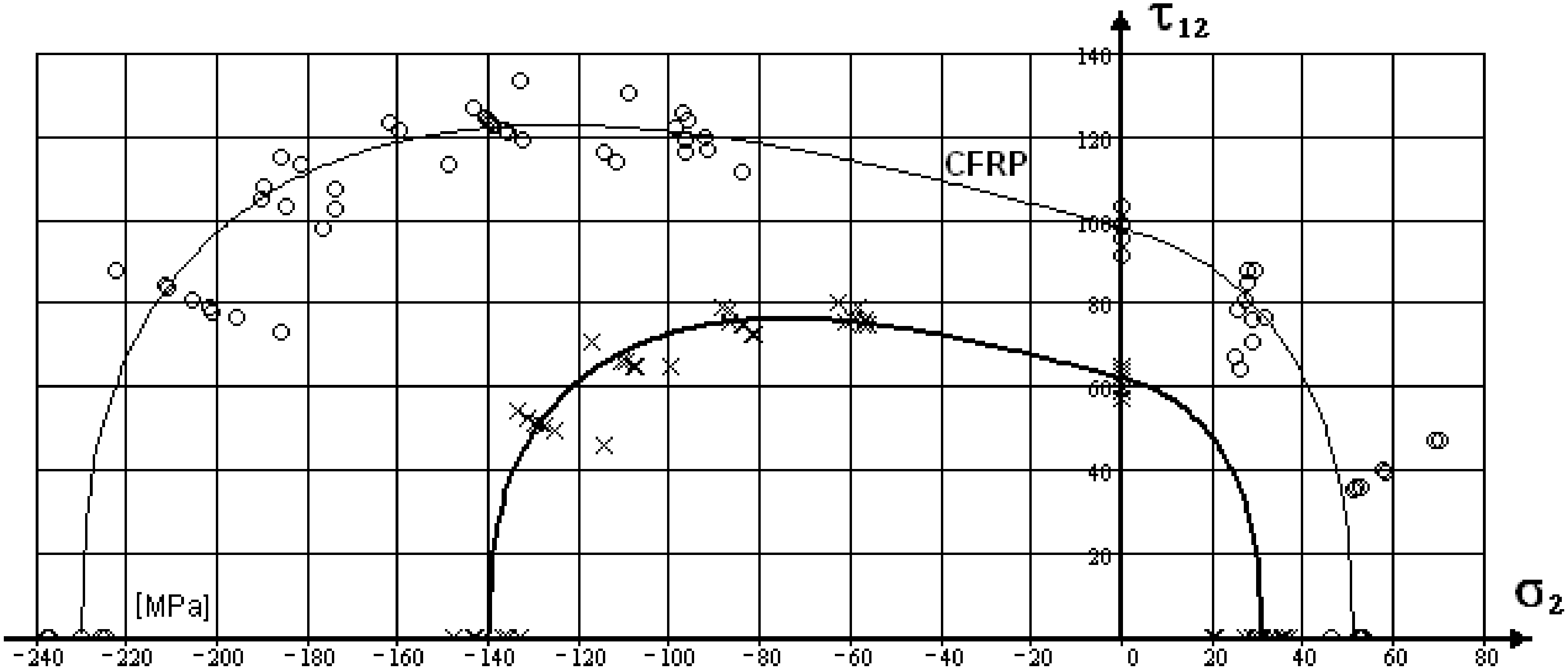
Some own examples and others from the WWFEs-I and –II (1993 – 2013)

GFRP, CFRP examples, mapped by FMC–based UD SCF, 2D stress state

$\tau_{21}(\sigma_2)$

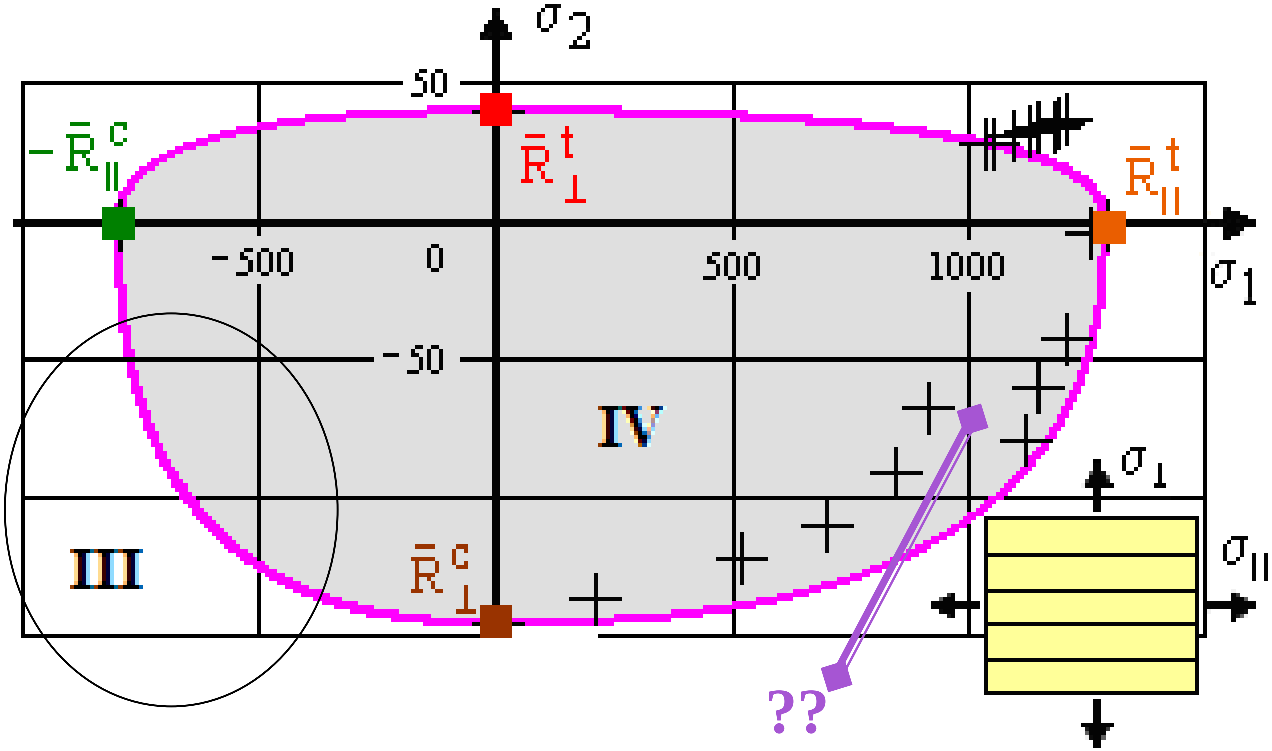
IFF





Test Case 3, WWFE-I $\sigma_2(\breve{\sigma}_1 \equiv \sigma_1)$

$\{\bar{R}\} = (1280,\ 800,\ 40,\ 145,\ 73)^T$



Hoop wound tube UD-lamina. E-glass/MY750epoxy +

$\sigma_1 = \sigma_{hoop}$

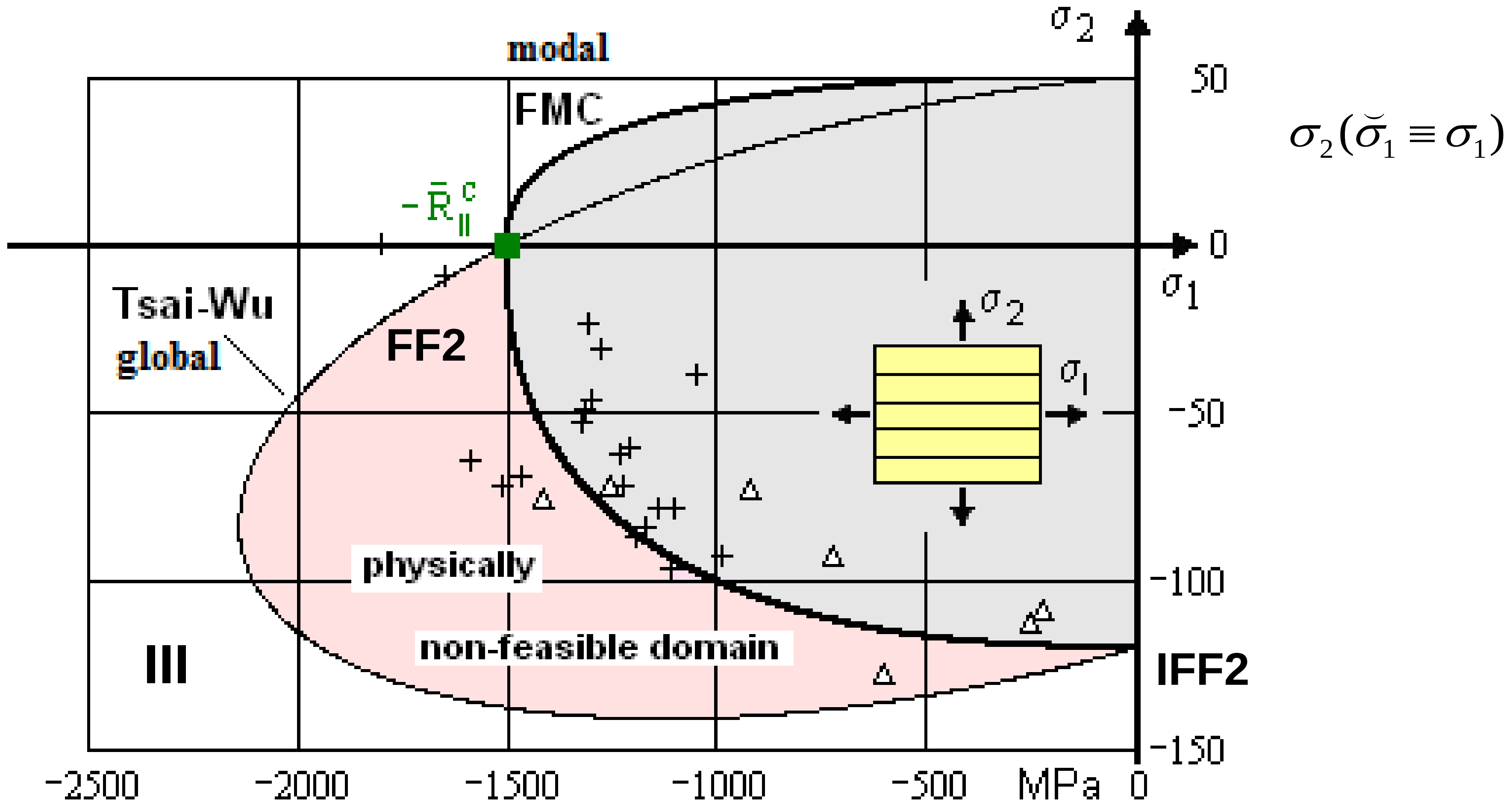
$\sigma_2 = \sigma_{axial}$

Part Prediction: Data of strength points were provided, only, no friction value

Part Test comparison: Test data in quadrant IV show discrepancy. Testing?

No data for quadrants II, III was provided ! But, ..

Mapping in the 'Tsai-Wu non-feasible domain' (quadrant III)



Data: courtesy IKV Aachen, Knops

Lesson Learnt: The modal FMC maps correctly, the *global* Tsai-Wu formulation predicts in quadrant III a non-feasible domain !

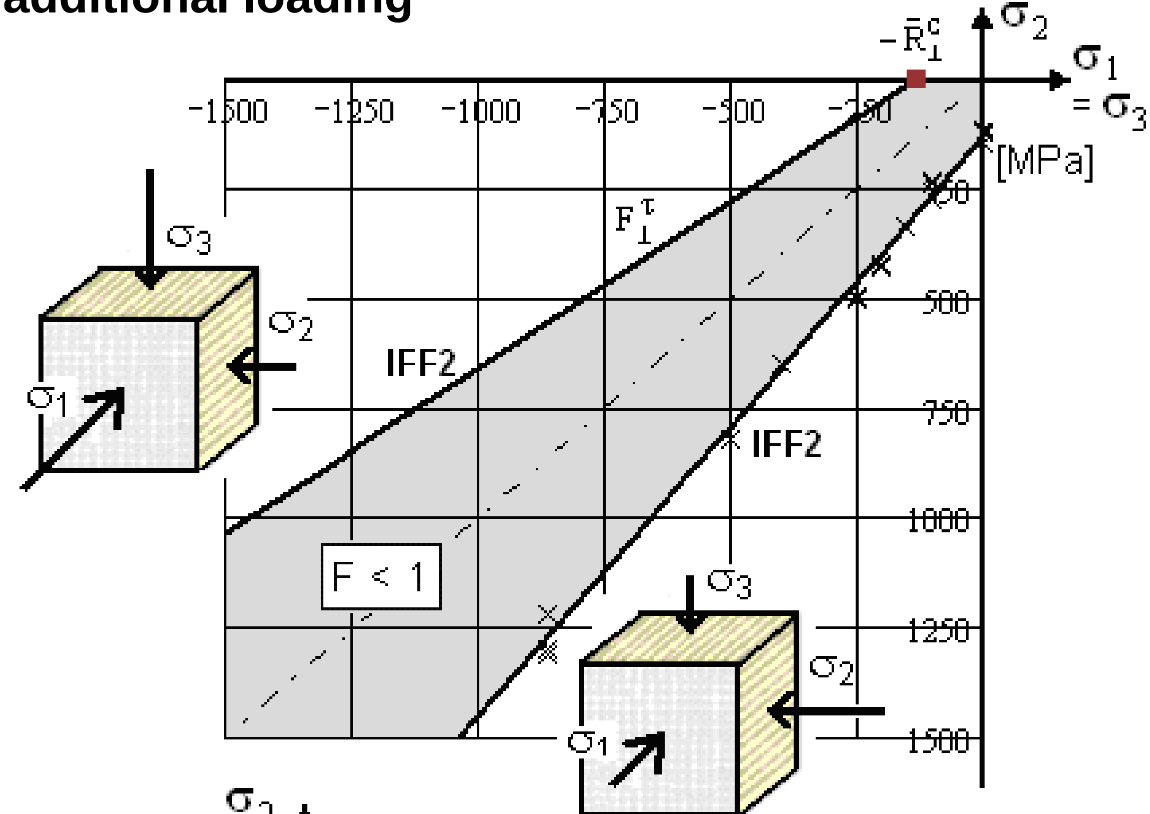
Test Case 5, WWFE-II, UD test specimen, 3D stress state $\sigma_2(\sigma_1 = \sigma_3)$

= hydrostatic pressure with additional loading

UD E-glass/MY750epoxy.

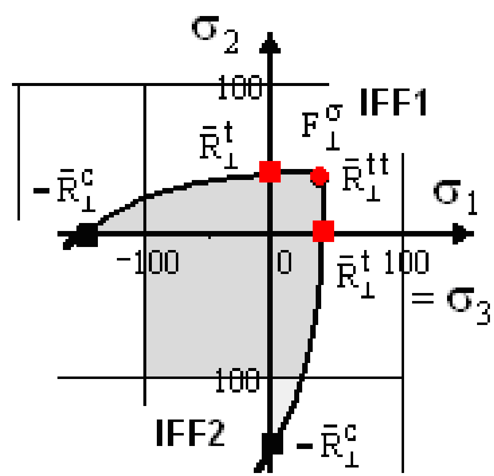
$\nu_{\perp\parallel} = 0.28, \quad \mu_{\perp\perp} = 0.14, \quad m = 2.8 \,,$

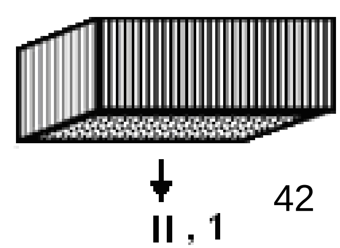
$\{\bar{R}\} = (1280,\ 800,\ 40,\ 132,\ 73)^T \ MPa$



Good Mapping, <u>after</u> QinetiQ re-evaluation of the lower branch test data
Then, the upper branch was fitting other test data, too !

Result: ***Both branches were then reliable and could be used for model validation***





Some Lessons Learnt w.r.t. Reliable Strength Design Verification

- **Validation of SFCs: this requires a uniform stress field at the failure-critical location**
- **All SFC-model parameters must be measurable**
- **Prediction of compressive failure (SF) of brittle behaving materials is not possible, if the physically necessary friction value μ is not available. Some global SFCs do not consider friction and therefore have a significant bottleneck when determining RFs.**
- **Failure is generated pretty locally on the micro-scale, but try to capture failure engineering-like on higher scale formulations !**
- **For pre-design: One may use knowledge from similar behaving materials !**
- **The achievement of a reliable design: This needs an equally well quality of**

  reliable analytical tools, solvers, test data and evaluating engineers !
- **Determination of modal SFC-parameters is performed in each respective pure mode domain. Global SFC-parameters are determined by a global fit over all modes**
- Isotropic materials: the 120°-dents are the probabilistic result of a 2-fold acting of the same failure mode. This shape is usually described by replacing J_2 through $J_2 \cdot \Theta\,(J_3, J_2)$

Theory is the Quintessence of all Practical Experience

A. Föppl

Validation, Verification, and Simulation

Validation:

Verification:

Simulation:

For UD-materials is:

*** Validation of SFCs on UD-coupon level**
*** Verification on laminate level**
*** Simulation on structural element level**

Definitions for Mutual Understanding

TOOLS, needed during the development of a product (full process chain):

Analyses **=** generation of abstract models for the examination of the physical behaviour

Simulation **=** procedure, incl. Analyses *plus* transfer of the simulation results to the system *plus* Adjustment of the (virtual test) simulation results to the physical results.

Special terms:

Damaging portion (Schädigung), *investigated by 'damaging mechanics tools'* ***(Schädigungsmechanik)***

Damage (Schaden) = *accumulation of damaging portions of an engineering critical size. investigated in Damage Tolerance Analysis by fracture mechanics tools* ***(Schadensmechanik)***

Definitions: see Glossary, CCeV-Website !

Design Verification: Achievement of a Reserve against a Design Limit State

For each distinct Load Case with its single Failure Modes must be computed:

Reserve Factor (load-defined !) :

determinisitic or semi-probabilistic

$$RF = \frac{\textbf{Failure Load at } Eff = 100\%}{\textbf{applied Design Load}}$$

valid in linear and non-linear analysis

Material Reserve Factor :

f_{Res} = ***Strength Design Allowable / Applied Stress***

$f_{Res} = RF = 1 / Eff$, valid in linear analysis

Material Stressing Effort :

(Werkstoff-Anstrengung)

$Eff = 100\%$ ***if*** $RF = 1$ ***material exhausted***

applied Design Load = Factor of Safety *j* x Design Limit Load

Some Further Definitions

Material: 'homogenized' model of the envisaged solid or material combination which principally may be a metal, a lamina or a laminate *analysed with effective anisotropic properties*

Composite Material: material made from constituent materials, that when combined, produce a material with characteristics different from the individual component (Fiber Reinforced Plastic, Concrete, Glare, Ceramic Matrix Composites, etc.

Failure: structural part does not fulfil its functional requirements such as *onset of* yielding, brittle fracture, Fiber-Failure FF, Inter-Fiber-Failure IFF, leakage, deformation limit, delamination size limit, frequency bound, ……

= ***project-fixed Limit State*** with F = Limit State Function or Failure Function

Failure Criterion: $F >=< 1$, Failure Condition : $F = 1 = 100\%$ *This is what we write!*

Failure Theory: tool, to predict failure danger of a structural part

Strength Failure Condition (SFC): subset of the strength failure theory

tool, to assess a 'multi-axial failure stress state ' in a **critical** location of the homogenized material. Should consider, that failure occurs at a lower level, e.g. micromechanically.

IFF (Inter-Fiber-Failure) a failure occurring in the matrix, the interphase, or along a non-bonded filament interface

Criticality *depends on the generally required function the composite is designed to, and not only on the inability to carry further loads.*

Conclusions w.r.t. SFCs for Brittle-behaving Materials

- **A modal SFC shall and can only describe a 1-fold occurrence of a mode.**
- **The occurrence of a multi-fold failure mode is considered in the formulas:**

  2-*fold* $\sigma_{II} = \sigma_I$ **(probabilistic effect) is elegantly solved with** J_3

  3-fold $\sigma_{II} = \sigma_I$ **=** σ_{III} **(prob. effect) hydrost. compr., by closing-ansatz**

- **Dents in the *I1<0*–domain are oppositely located to those in the *I1>0*-domain**
- **The Poisson effect, generated by a Poisson ratio *v*, may cause tensile failure under bi-axially stressing (dense material)**

  (analogous to UD material, where filament tensile fracture may occur without any external tension loading)

- **Hoop Planes** (= deviatoric planes = π – planes if *isotropic)* **= convex**
- **Meridian Planes : *not convex !***

Some Literature

[Cun96] Cuntze R.: *Bruchtypbezogene Auswertung mehrachsiger Bruchtestdaten und Anwendung im Festigkeitsnachweis sowie daraus ableitbare Schwingfestigkeits- und Bruchmechanikaspekte*. DGLR-Kongreß 1996, Dresden. Tagungsband 3

[Cun04] Cuntze R.: *The Predictive Capability of Failure Mode Concept-based Strength Criteria for Multidirectional Laminates*. WWFE-I, Part B, Comp. Science and Technology 64 (2004), 487-516

[Cun05] Cuntze R.: *Is a costly Re-design really justified if slightly negative margins are encountered?* Konstruktion, März 2005, 77-82 and April 2005, 93-98 (reliability treatment of the problem)

[Cun12] Cuntze R.: *The predictive capability of Failure Mode Concept-based Strength Conditions for Laminates composed of UD Laminas under Static Tri-axial Stress States. - Part A of the WWFE-II*. Journal of Composite Materials 46 (2012), 2563-2594

[Cun13] Cuntze R.: *Comparison between Experimental and Theoretical Results using Cuntze's 'Failure Mode Concept' model for Composites under Triaxial Loadings - Part B of the WWFE-II*. Journal of Composite Materials, Vol.47 (2013), 893-924

[Cun13b] Cuntze R.: *Fatigue of endless fiber-reinforced composites*. 40. Tagung DVM-Arbeitskreis Betriebsfestigkeit, Herzogenaurach 8. und 9. Oktober 2013, conference book

[Cun14] Cuntze R.: associated paper, see CCeV website http://www.carbon-composites.eu/leistungsspektrum/fachinformationen/fachinformation-2

[Cun15a] Cuntze, R.: *Static & Fatigue Failure of UD-Ply-laminated Parts – a personal view and more.* ESI Group, Composites Expert Seminar, Uni-Stuttgart, January 27-28, keynote presentation, see CCeV website)

[Cun15b] Cuntze, R.: *Reliable Strength Design Verification – fundamentals, requirements and some hints.* 3rd. Int. Conf. on Buckling and Postbuckling Behaviour of Composite Laminated Shell Structures, DESICOS 2015, Braunschweig, March 26 -27, extended abstract , conf. handbook, 8 pages (see CCeV website)

[VDI2014] VDI 2014: German Guideline, Sheet 3 *"Development of Fiber-Reinforced Plastic Components, Analysis".* Beuth Verlag, 2006 (*in German and English, author was convenor).*

Cuntze's Pre-design Input for 3D UD and Isotropic SFCs

Test Data Mapping | **Design Verification**

- **5 strengths :** $\{\overline{R}\} = (\overline{R}_{\parallel}^{t}, \overline{R}_{\parallel}^{c}, \overline{R}_{\perp}^{t}, \overline{R}_{\perp}^{c}, \overline{R}_{\perp\parallel})^{T}$ $\{R\} = (R_{\parallel}^{t}, R_{\parallel}^{c}, R_{\perp}^{t}, R_{\perp}^{c}, R_{\perp\parallel})^{T}$

  average (typical) values | **strength design allowables**

- **2 friction values :** **for 2D** $\mu_{\perp\parallel}$ **, for 3D** $\mu_{\perp\parallel}, \; \mu_{\perp\perp}$

  $\mu_{\perp\parallel} = 0.1$ $\mu_{\perp\perp} = 0.1$

- **1 mode-interaction exponent :** ***m* = *2.6* .**

values, recommended for pre-design

For isotropic brittle behaving material, this means:

* **2 material parameters of the ideal elastic crystal material which determine the orthogonal stress plane** (= π- or hoop plane of the fracture failure body)

* **1 material friction parameter μ of the non-ideal crystal material due to friction inherent to brittle behav. material determining the slope of the meridians** (axial shape of the fracture failure body)

Example: Cuntze's Pre-design Input for 3D UD SFCs

Test Data Mapping | **Design Verification**

- **5 strengths :** $\{\overline{R}\} = (\vec{\overline{R}}_{\parallel}^{t}, \overline{R}_{\parallel}^{c}, \overline{R}_{\perp}^{t}, \overline{R}_{\perp}^{c}, \overline{R}_{\perp\parallel})^T$ $\{R\} = (R_{\parallel}^{t}, R_{\parallel}^{c}, R_{\perp}^{t}, R_{\perp}^{c}, R_{\perp\parallel})^T$

  average (typical) values | **strength design allowables**

- **2 friction values : for 2D** $\mu_{\perp\parallel}$ **, for 3D** $\mu_{\perp\parallel}, \ \mu_{\perp\perp}$

  $\mu_{\perp\parallel} = 0.1$ $\mu_{\perp\perp} = 0.1$

- **1 mode-interaction exponent :** ***m* = 2.6 .**

values, recommended for pre-design

model parameter

Mohr-Coulomb – required 'strength' parameters

Determination of the Load-defined Reserve Factor RF for a foam

Linear elastic problem for this brittle behaving material

Residual stresses = 0

$$\mathbf{RF} = f_{Res} \text{ (material reserve factor)} = Eff^{-1}$$

Stress state:

$\sigma I := 0.9 \quad \sigma II := -0.4 \quad \sigma III := 0.5$

Statistically reduced Strengths:

$Rz := 0.9 \cdot \bar{R}z \quad Rd := 0.85 \cdot \bar{R}d$

Shape parameters: $D\sigma = -0.71 \quad D\tau = 0.21 \quad c1\Theta\sigma = 1.15 \quad c1\Theta\tau = 1.03$

$$I1 := \sigma I + \sigma II + \sigma III \qquad J2 := \frac{\left[(\sigma I - \sigma II)^2 + (\sigma II - \sigma III)^2 + (\sigma III - \sigma I)^2\right]}{6} \qquad J3 := \frac{\left[(2\cdot\sigma I - \sigma II - \sigma III)\cdot(2\cdot\sigma II - \sigma I - \sigma III)\cdot(2\cdot\sigma III - \sigma II - \sigma I)\right]}{27}$$

$I1 = 1 \qquad J2 = 0.44 \qquad J3 = -0.07$

$$Eff\Theta\sigma := c1\Theta\sigma \cdot \frac{\sqrt{4\cdot J2\cdot\sqrt[3]{1 + D\sigma\cdot 1.5\cdot 3^{0.5}\cdot J3\cdot J2^{-1.5}} - \frac{1}{3}\cdot I1^2} + I1}{2Rz}$$

$$Eff\Theta\tau := c1\Theta\tau \cdot \frac{\sqrt{4\cdot J2\cdot\sqrt[3]{1 + D\tau\cdot\left(1.5\cdot 3^{0.5}\cdot J3\cdot J2^{-1.5}\right)} - \frac{1}{3}\cdot I1^2} - I1}{2\cdot Rd}$$

$$Eff := \sqrt[9]{Eff\Theta\sigma^{m_{int}} + Eff\Theta\tau^{m_{int}}} \qquad Eff = 0.802 \qquad RF := \frac{1}{Eff} \qquad RF = 1.25$$

The loading may be monotonically increased by the factor RF !

“ Scientists would rather use someone else's toothbrush than someone else's terminology! “

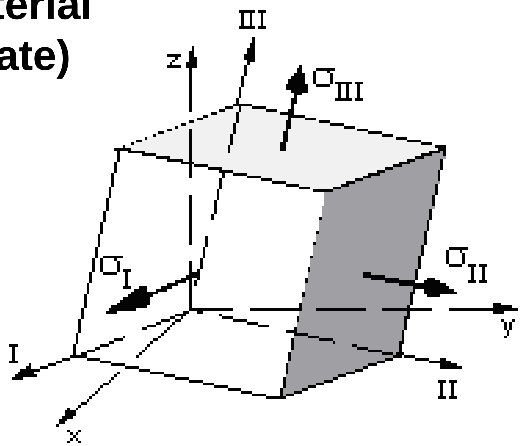
… or *theory*

(Nobel laureate Murray Gell-Mann)

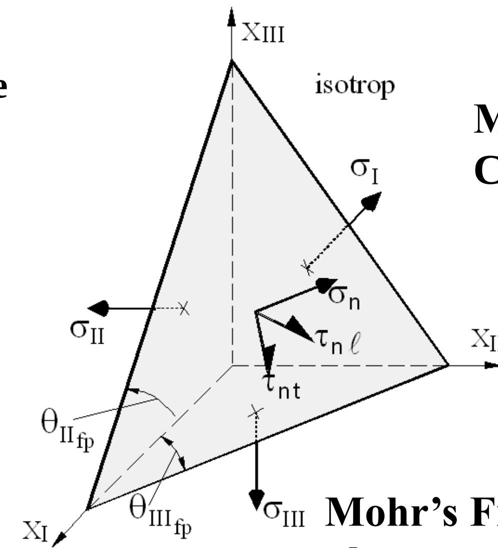
ANHANG

Which are the Stresses & Invariants to be used?

Isotropic Material (3D stress state)



The stress states in the various COS can be transferred into each other



Mohr's COS

Mohr's Fracture plane Stresses

Principal Stresses

$$\{\sigma\}_{principal} = (\sigma_I\,,\ \sigma_{II}\,,\ \sigma_{III}\ )^T$$

Structural Component Stresses

$$\{\sigma\}_{comp} = (\sigma_x\,,\ \sigma_y\,,\ \sigma_z\,,\ \tau_{yz}\,,\ \tau_{xz}\,,\ \tau_{xy}\ )^T$$

$$\{\sigma\}_{Mohr} = (\sigma_\ell\,,\ \sigma_n\,,\ \sigma_t\,,\ \tau_{nt}\,,\ \tau_{t\ell}\,,\ \tau_{\ell n}\ )^T$$

'isotropic' invariants !

$$I_1 = (\sigma_I + \sigma_{II} + \sigma_{III}\ )^T = 3\sigma_{oct} \equiv f(\sigma),$$

$$I_1 = (\sigma_x + \sigma_x + \sigma_z\ )^T$$

$$I_1 = (\sigma_\ell + \sigma_n + \sigma_t\ )^T$$

$$6J_2 = (\sigma_I - \sigma_{II}\ )^2 + (\sigma_{II} - \sigma_{III}\ )^2 + (\sigma_{III} - \sigma_I\ )^2$$
$$= 4(\tau_{III}{}^2 + \tau_{II}{}^2 + \tau_I{}^2\ ) = 9\tau_{oct}{}^2 \equiv f(\tau)$$

$$6J_2 = (\sigma_y - \sigma_z\ )^2 + (\sigma_z - \sigma_x\ )^2 + (\sigma_x - \sigma_y\ )^2$$
$$+6(\tau_{yz}{}^2 + \tau_{zx}{}^2 + \tau_{xy}{}^2\ )\ \ (Mises, HMH\ )$$

$$6J_2 = (\sigma_n - \sigma_t\ )^2 + (\sigma_t - \sigma_\ell\ )^2 + (\sigma_\ell - \sigma_n\ )^2$$
$$+6(\tau_{nt}{}^2 + \tau_{t\ell}{}^2 + \tau_{n\ell}{}^2\ )$$

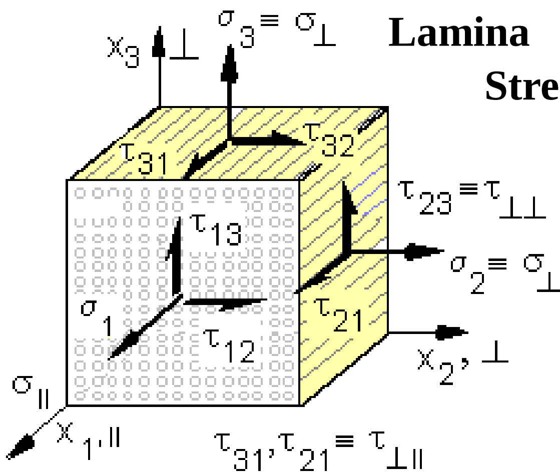
$$27J_3 = (2\sigma_I - \sigma_{II} - \sigma_{III}\ )(2\sigma_{II} - \sigma_I - \sigma_{III}\ )(2\sigma_{III} - \sigma_I - \sigma_{II}\ ),\quad I_\sigma = 4J_2 - I_1{}^2/3,\quad \sigma_{mean} = I_1/3$$

Invariant := Combination of stresses –powered or not powered- the value of which does not change when altering the coordinate system.

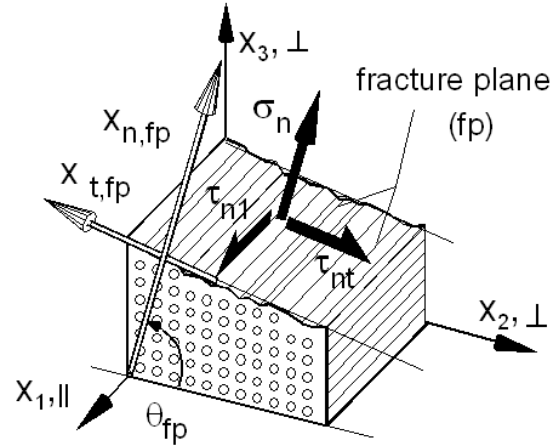
Stress States and Invariants:

Transversely-Isotropic Material (◄ Uni-Direct. Fibre-Reinforced Plastics)

Lamina Stresses



Transformation of lamina stresses into the quasi-isotropic plane stresses



Mohr, Puck, Hashin: Fracture is determined by the (Mohr) stresses in the fracture plane !

$\{\sigma\}^{quasi-isotropic\ plane}_{principal} = (\sigma_1, \sigma_2^p, \sigma_3^p, 0, \tau_{31}^p, \tau_{21}^p)^T$	$\{\sigma\}_{lamina} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$	$\{\sigma\}_{Mohr} = (\sigma_\ell, \sigma_n, \sigma_t, \tau_{nt}, \tau_{t\ell}, \tau_{\ell n})^T$
$I_1 = \sigma_1, \quad I_2 = \sigma_2^p + \sigma_3^p$ $I_3 = \tau_{31}^{p\,2} + \tau_{21}^{p\,2}$	$I_1 = \sigma_1, \quad I_2 = \sigma_2 + \sigma_3$ $I_3 = \tau_{31}^2 + \tau_{21}^2$ **'UD invariants'!** *[Boehler]*	$I_1 = \sigma_1, \quad I_2 = \sigma_n + \sigma_t$ $I_3 = \tau_{t\ell}^2 + \tau_{n\ell}^2$
$I_4 = (\sigma_2^p - \sigma_3^p)^2 + 0$ $I_5 = (\sigma_2^p - \sigma_3^p)(\tau_{31}^{p\,2} - \tau_{21}^{p\,2}) + 0$	$I_4 = (\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2$ $I_5 = (\sigma_2 - \sigma_3)(\tau_{31}^2 - \tau_{21}^2) - 4\tau_{23}\tau_{31}\tau_{21}$	$I_4 = (\sigma_n - \sigma_t)^2 + 4\tau_{nt}^2$ $I_5 = (\sigma_n - \sigma_t)(\tau_{t\ell}^2 - \tau_{n\ell}^2) - 4\tau_{nt}\tau_{t\ell}\tau_{n\ell}$

Self-explaining Notations for Strength Properties (homogenised material) neu !!!!

	loading	Fracture Strength Properties: tension			compression			shear			*required by material symmetry*
	direction or plane	1	2	3	1	2	3	12	23	13	
9	general orthotropic	R_1^t	R_2^t	R_3^t	R_1^c	R_2^c	R_3^c	R_{12}	R_{23}	R_{13}	comments
5	UD, ≅ non-crimp fabrics	$R_{\parallel}^{\ t}$ NF	$R_{\perp}^{\ t}$ NF	$R_{\perp}^{\ t}$ NF	$R_{\parallel}^{\ c}$ SF	$R_{\perp}^{\ c}$ SF	$R_{\perp}^{\ c}$ SF	$R_{\parallel\perp}$ SF	$R_{\perp\perp}$ NF	$R_{\parallel\perp}$ SF	$R_{\perp\perp} = R_{\perp}^{\ t} / \sqrt{2}$ (compare Puck's modelling)
6	fabrics	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	*Warp = Fill*
9	fabrics general	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	*Warp ≠ Fill*
5	mat	R_{1M}^t	R_{1M}^t	R_{3M}^t	R_M^c	R_{1M}^c	R_{3M}^c	R_M^τ	R_M^τ	R_M^τ	$R_M^\tau (R_M^t)$
2	isotropic	R_m SF	R_m SF	R_m SF	*deformation-limited*			R_M^τ	R_M^τ	R_M^τ	*ductile, dense* $R_M^\tau = R_m / \sqrt{2}$
		R_m NF	R_m NF	R_m NF	R_m^c SF	R_m^c SF	R_m^c SF	R_m^σ NF	R_m^σ NF	R_m^σ NF	*brittle, dense* $R_M^\sigma = R_m^t / \sqrt{2}$

NOTE: *As a consequence to isotropic materials (European standardisation) the letter R has to be used for strength. US notations for UD material with letters X (direction 1) and Y (direction 2) confuse with the structure axes' descriptions X and Y . *Effect of curing-based residual stresses and environment dependent on hygro-thermal stresses. *Effect of the difference of stress-strain curves of e.g. the usually isolated UD test specimen and the embedded (redundancy) UD laminae. R_m := 'resistance maximale' (French) = tensile fracture strength (superscript t here usually skipped), R:= basic strength. Composites are most often brittle and dense, not porous! SF = shear fracture

Elasticity Properties *(homogenised material)* *(self-explaining denotations)*

considers VDI 2014,

proposed to ESA-Hdbk

	direction or plane	Elasticity Properties									
		1	2	3	12	23	13	12	23	13	
9	***general orthotropic***	E_1	E_2	E_3	G_{12}	G_{23}	G_{13}	ν_{12}	ν_{23}	ν_{13}	**comments**
5	***UD, ≅ non-crimp fabrics***	$E_{\parallel}$	$E_{\perp}$	$E_{\perp}$	$G_{\parallel\perp}$	$G_{\perp\perp}$	$G_{\parallel\perp}$	$\nu_{\parallel\perp}$	$\nu_{\perp\perp}$	$\nu_{\parallel\perp}$	$G_{\perp\perp} = E_{\perp} / (2 + 2\nu_{\perp\perp})$ $\nu_{\perp\parallel} = \nu_{\parallel\perp} \cdot E_{\perp} / E_{\parallel}$ *quasi-isotropic 2-3-plane*
6	***fabrics***	E_W	E_F	E_3	G_{WF}	G_{W3}	$G_{\vert W3}$	ν_{WF}	ν_{W3}	ν_{W3}	*Warp = Fill*
9	***fabrics general***	E_W	E_F	E_3	G_{WF}	G_{W3}	G_{F3}	ν_{WF}	ν_{F3}	ν_{W3}	*Warp ≠ Fill*
5	***mat***	E_M	E_M	E_3	G_M	G_{M3}	G_{M3}	ν_M	ν_{M3}	ν_{M3}	$G_M = E_M/(2+2\nu_M)$ *1 is perpendicular to quasi-isotropic mat plane*
2	***isotropic*** *for comparison*	E	E	E	G	G	G	ν	ν	ν	$G = E/(2+2\nu)$

Lesson Learned: - *Unique, self-explaining denotations are mandatory*
- *Otherwise, expensively generated test data cannot be interpreted and go lost*

Hygrothermal Properties *(homogenised material)*

		Hygro-thermal properties					
	direction	1	2	3	1	2	3
9	general orthotropic	α_{T1}	α_{T2}	α_{T3}	α_{M1}	α_{M2}	α_{M3}
5	UD, $\cong$ non-crimp fabrics	$\alpha_{T\|\|}$	$\alpha_{T\perp}$	$\alpha_{T\perp}$	$\alpha_{M\|\|}$	$\alpha_{M\perp}$	$\alpha_{M\perp}$
6	fabrics	α_{TW}	α_{TW}	α_{T3}	α_{MW}	α_{MW}	α_{M3}
9	fabrics general	α_{TW}	α_{TF}	α_{T3}	α_{MW}	α_{MF}	α_{M3}
5	mat	α_{TM}	α_{TM}	α_{TM3}	α_{MM}	α_{MM}	α_{MM3}
2	isotropic for comparison	α_T	α_T	α_T	α_M	α_M	α_M

.. analogous for λ, c

material friction μ as strength property

NOTE: Despite of annoying some people, I propose to rethink the use of α for the CTE and β for the CME. Utilizing α_T and α_M automatically indicates that the computation procedure will be similar.

WWFE Assumptions for UD Modelling

- **The UD-lamina is macroscopically homogeneous.**

  It can be treated as a homogenized (‘smeared‘) material

- **The UD-lamina is transversely-isotropic:**

  On planes, parallel to the fiber direction it behaves orthotropic and on planes transverse to fiber direction isotropic (quasi-isotropic plane)

- **Uniform stress state about the critical stress ‘point‘ (location)**

Some well-known Developers which formulated isotropic 3D Strength Failure Conditions (SFCs)

Willam-Warnke, Ottosen etc.

Hencky-Mises-Huber

Richard von Mises
1883-1953
Mathematician

Eugenio Beltrami
1835-1900
Mathematician

Otto Mohr
1835-1918
Civil Engineer

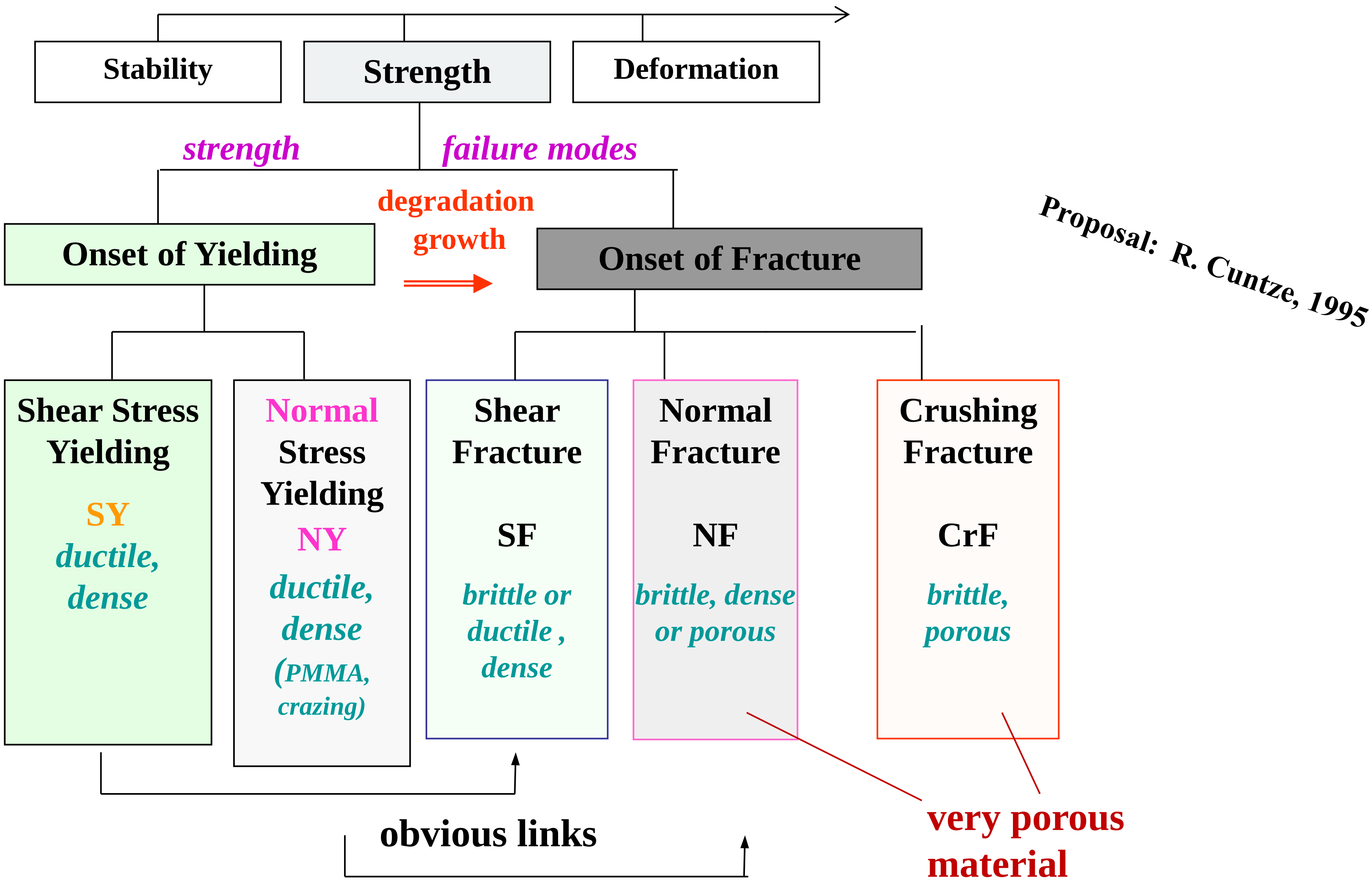
Charles de Coulomb
1736-1806
Physician

'Onset of Yielding'

'Onset of Cracking'

Hence again, a civil engineer may proceed

Scheme of Strength Failures Types for *isotropic materials*



Note: The growing yield body (SY or NY) is confined by the fracture surface (SF or NF)!

Global and Modal Strength Failure Conditions *General View*

Drucker-Prager, Tsai-Wu

1 Global *strength* failure condition : F ({σ}, {R}) = 1 (usual formulation)

Set of Modal strength failure conditions: F ({σ}, R^{mode}) = 1 (addressed in FMC)

Mises, Puck, Cuntze

Example: UD

vector of 6 stresses (general)

$$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$$

vector of 5 strengths

$$\{R\} = (R_{\parallel}^{t}, R_{\parallel}^{c}, R_{\perp}^{t}, R_{\perp}^{c}, R_{\perp\parallel})^T$$

needs an Interaction of Failure Modes: performed by a

probabilistic-based 'rounding-off' approach (series failure system model)

directly delivering the (material) reserve factor in linear analysis

Note: In the quasi-isotropic plane of the UD material just 5 stresses are active: $\{\sigma\}_{principal}^{quasi\text{-}isotropic\,plane} = (\sigma_1, \sigma_2^p, \sigma_3^p, 0, \tau_{31}^p, \tau_{21}^p)^T$

By-the-way: Experience with Failure Prediction prove

A Strength Failure Condition (SFC) is a necessary but not a sufficient condition to predict Strength Failure (example: thin-layer problem).
On top, an energy condition may be to fulfill.

Industrial Requirements for Improved Designing of Composite Parts

Static loading:

- **Validated 3D strength failure conditions for isotropic (foam), transversely-isotropic UD materials, and orthotropic materials (e.g. textiles) to determine 'Onset of fracture' and 'Final fracture'**
- **Standardisation of material test procedures, test specimens, test rigs, and test data evaluation for the structural analysis input**

Cyclic (dynamic) loading : fatigue

- **Development of practical, physically-based lifetime-prediction methods**
- **Generation of S-N curve test data for the verification of prediction models**
- **Consideration of manufacturing imperfections (tolerance width of uncertain design variables) in order to achieve a production cost minimum by „Design to Imperfections“ includes defects**
- **Delamination growth models: for duroplastic and thermoplastic matrices**
- **Consideration of media, temperature, creeping, aging**
- **Provision of more damping because parts become more monolithic.**

Design Verification = Achievement of a Reserve against a Limit State

For each distinct Load Case with its single Failure Modes must be computed:

Reserve Factor (is load-defined) : ***RF = Failure Load / applied Design Load***

Material Reserve Factor : f_{Res} = ***Strength / Applied Stress***

if linear analysis: $f_{Res} = RF = 1 / Eff$

Material Stressing Effort : ***Eff = 100% if RF = 1*** **(Anstrengung)**

(Werkstoff-Anstrengung)

is applicable in linear and non-linear analysis.

Determination of the load-defined Reserve Factor *RF*

Linear elastic problem for the envisaged brittle behaving CFRP

then simplified $\boldsymbol{RF = f_{Res}}$ **(material reserve factor)** $= \boldsymbol{Eff^{-1}}$

Residual stresses : 0 (effect vanishes with increasing micro-cracking) in $MPa = N/mm^2$

Stress state vector : $\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T = (0, -60, 0, 0, 0, 50)^T$

Strengths vector: $\{R\} = (R_{\parallel}^t, R_{\parallel}^c, R_{\perp}^t, R_{\perp}^c, R_{\perp\parallel})^T = (1200, 850, 35, 100, 80)^T$

Mode interaction exponent: $m = 2.7$

Friction value: $\mu_{\perp\parallel} = 0.3$

Roughly estimated from average values

$\{\bar{R}\} = (1378, 950, 40, 125, 97)^T$

WWFE-I: UD T300/PR319EP

Calculation: negative *Effs* are nonsense and are to be bypassed

$$Eff^{\perp\sigma} = \frac{\sigma_2 - |\sigma_2|}{\bar{R}_{\perp}^t} = 0 \qquad Eff^{\perp\tau} = \frac{-\sigma_2 + |\sigma_2|}{\bar{R}_{\perp}^c} = 0.60 \qquad Eff^{\perp\parallel} = \frac{|\tau_{21}|}{\bar{R}_{\perp\parallel} - \mu_{\perp\parallel} \cdot \sigma_2} = 0.51$$

$$Eff^m = (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m$$

$$Eff = 0.72, \ \boldsymbol{RF = 1/Eff = 1.39}, \ MoS = RF - 1 = 0.39$$

Loading may be increased by the factor *RF* until obtaining fracture limit state $Eff = 100\% \equiv RF = 1$.

Facts of so-called Global SFCs

Global SFCs (one failure surface)

- **Regard all failure modes of the material by one single mathematical formulation. This might even capture a (simplified view) * 2-fold acting failure mode (such as $\sigma_I = \sigma_{II}$: *is a joint failure probability*) or a * 3-fold acting failure mode (such as $p_{hyd} = \sigma_I = \sigma_{II} = \sigma_{III}$)**

- **Requires a re-calculation of all model parameters in the case that a test data change must be performed in a distinct failure mode domain of the multi-fold failure surface (body).** **Consequence: A change in one failure domain deforms the failure surface in all other – physically independent – failure domains. There is a big chance that a Reserve Factor, to be determined in the independent domain, might be not on the conservative side**

- **There are global SFCs that just use basic strengths as model parameters. This is physically not permitted because Mohr-Coulomb friction acts in the case of brittle behaving materials.**

  Note: a distinct failure mode can cause different failure “planes“ , is maximum flaw driven

Lode angle J3

Conclusions wrt. Beltrami-based *Failure Mode Concept*

- **The FMC – applied to UD material - is an efficient concept,**

  that improves prediction + simplifies design verification.

  Formulation basis is whether the material element experiences a *volume* change, a *shape change* and *friction* .

- **Delivers a combined formulation of *independent modal failure modes,***

  without the well-known drawbacks of global SFC formulations

  (which *mathematically combine in-dependent failure modes)* .

- **The FMC-based 3D UD Strength Failure Conditions are simple but describe physics of each single failure mechanism pretty well.**

Conclusions w.r.t. Failure Mode Concept

- **The FMC is an efficient concept,**

  that improves prediction + simplifies design verification

  is applicable to brittle and ductile, dense and porous,
  isotropic, transversely-isotropic and orthotropic materials

  if clear failure modes can be identified and if the material element can be homogenized.

  Formulation basis is whether the material element experiences
  a *volume* change, a *shape change* and *friction* .

  Builds* not on the *material* but on *material behaviour !

- **Delivers a combined formulation of *independent modal failure modes*,**

  without the well-known drawbacks of global SFC formulations

  (which *mathematically combine in-dependent failure modes)* .

- **The FMC-based Failure Conditions are simple but**
  describe physics of each single failure mechanism pretty well.

- **Mapping of a brittle behaving isotropic porous foam and of a transversely-isotropic UD material was successful, thereby validating the SFC models. Some new findings were provided !**

Keep in mind !

All is difficult prior to becoming simple!

[Moslik Saadi]