Why not Designing multidirectional Laminates with In-plane Strength Design Sheets applying the UD Criteria of Tsai-Wu and Cuntze?

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Abstract:

Novel simulation-driven product development shifts the role of physical testing to virtual testing. This requires High Fidelity concerning material models such as a Strength Failure Criterion (SFC). Usual assumption for the material model is an homogenized, ideally homogeneous solid material. Nowadays, reliable SFCs must be 3D-validated, because the Finite Element Analysis output gives spatial stresses, which for instance is necessary to design joints etc. Due to the fact, that many laminates are just in-plane loaded it is nevertheless very helpful to have 2D Design Sheets, at least for checking FEA laminate results.

A structural engineer must design to several Design Dimensioning Load cases and in each case to all the activated strength failure modes. As the task here is the plane loading of the envisaged multi-stack laminates composed of transversely-isotropic unidirectional UD layers the depicted 3D-SFCs are reduced to the necessary 2D-SFCs.

Since strength failure envelopes in Design Sheets significantly depend on the applied SFC exemplarily two SFCs, those of Tsai-Wu and Cuntze will be depicted in short. Thereby, a comparison Tsai-Wu versus Cuntze is a concern which requires a transformation of the Tsai-Wu SFC into the notation of the [*VDI 2014*].

In this paper, an idea of Stephen Tsai [*Tsa22*] is followed in order to get a deeper mechanical feeling for laminates when designing them to First-Ply Failure (*FPF*), which includes Fiber Failure (FF) and Inter-Fiber-Failure (IFF) and marks 'Onset-of-fracture'. This would enable to reduce the effort for Design Dimensioning regarding optimization and finally for Design Verification considering testing. Thereby, the idea of Tsai was to compare his results with Cuntze's FMC-based ones. In order to perform this failure stress-based 'Omni-(principal strain) failure envelopes' (*term from S. Tsai*) are derived for different materials. These are the envelopes of the intact 'Non-FPF area' and can serve as master envelopes for building Strength Design Sheets.

In practical composite laminate development are to capture the so-called 'Quad-stacked' laminates of the (0 / 45/90/-45)-family (*usually non-stitched prepregs*) and the novel 'Double-Double-stacked' laminates of the $\{\varphi/\text{-}\psi/\text{-}\varphi/\psi\}$-family (*NCF, stitched layers product).* For them, above envelopes are searched which practically is the same procedure applicable for all laminates.

It is of highest interest *How far different materials possess different envelopes*? To be able to respond five differently stiff CFRPs and one GFRP were investigated. Essential for the designer is the computation of a Reserve Factor f_{RF}. This is approximately possible by using Tsai-Melo's envelope-internal circle radius *r*. Main conclusion of this work is: The envelopes can be used to establish valuable 'Strength Pre-design Sheets', however, these depend on the SFC choice.

1 Introduction

1.1 Motivation

One basic task in structural component development is the Static Design. This involves Design Dimensioning and finally Design Verification of the chosen design.

The variety of new materials in engineering needs much knowledge about the failure state in order to enable verification of the designed structural component, and this much more since lightweight design requires a higher exertion of the material and thereby contributes to sustainable engineering. Despite of all the high performance numerical tools Design Sheets for Dimensioning are essentially additional tools which enable as dimensioning controlling elements to perform a promising reliable component development. Steve Tsai initiated Ralf Cuntze to a Design Sheet idea and to apply the FMC-based SFC in order to compare the results.

> *Dear Ralf,*
>
> *"We have mentioned earlier that for strength, material and laminate are equivalent, and can be interchanged by their respective factors. We all know that for a given material we can predict the strength of its laminates. The laminates' strengths have the same ratios for all materials. Conversely, for a given laminate the relative strength of various materials are also fixed. That applies to all laminates. As you can see in [Tsa22], we showed such rigid relations between materials and laminates, the first for DD, and the second for Quad. You can see that the lamination and material factors can be interchangeable. Optimization and testing for data can be done totally differently from what we do today. There is a lot that we can learn but wish to share with you our progress. Hopefully you can critique what we have done and move forward to other advances with your own strength failure criterion and its possible implications". Steve*

Design Verification demands for reliable reserve factors *RF* and these - beside a reliable structural analysis - demand for reliable SFCs. Such a SFC is the mathematical formulation $F = 1$ of a failure curve or of a failure surface (body). Generally required are a yield condition and fracture strength conditions. The *yield* SFC usually describes just one mode, i.e. for isotropic materials the classical 'Mises' describes shear yielding SY. *Fracture* SFCs usually must describe two independent fracture modes, shear fracture SF and normal fracture NF in the simple isotropic case. For the here focused transversely-isotropic UD material a so-called material-inherent 'generic' number 5 for fracture seems to be given [*Cun23a, Cun22*]. This means for UD altogether 3 Inter Fiber failure (IFF) and 2 Fiber Failure (FF) modes and further 5 strengths. Considering the design with brittle UD material this means a set of Strength (*fracture*) Failure Criteria (SFC) has to be provided.

Principally, in order to avoid either to be too conservative or too un-conservative, a separation is required of the always needed 'analysis of the average structural behavior' in Design Dimensioning (*using average properties and average stress-strain curves*) in order to obtain best information (= 50% expectation value) from the mandatory single Design Verification analysis of the final design, where statistically minimum values for strength and minimum, mean or maximum values for the task-demanded other properties are applied as Design Values. There it is to demonstrate that 'No relevant Limit State is met'. The paper at hand focuses mapping of the curves of test data by the SFCs. In these formulations each strength is an average strength $\bar{R}$, due to statistics consequently indicated by a bar over. The letter R is used in a general formulation and further for the Strength Design Allowables.

Design verification with respect to Static Strength is performed here on material level by stresses in the critical location of undisturbed areas such as uniform material areas.

For performing an accurate designing it is to note:

* Whereas modelling is performed with average properties and average stress-strain curves, in the verification of the chosen final laminate design - task-required - upper or lower or average properties are to insert in the analysis, like $\bar{R}$.
* The present stress-based design verifications - i.e. in Aerospace - requires stress criteria and as input A- or B-strength Design Allowables R.
* A strain-based design verification as precondition for certification, would firstly need permission of the FAA including authority-accepted strain criteria coupled to *Strain* Design Allowables (*also statistically reduced*), which are not available as official values in material data sheets and this is the objection here. A special Strain-based Design makes just sense if the material has some ductility and if the part is just a few cycles submitted to an extreme loading beyond the 'plastic' limit of the material such as a pipe under earthquake loading. On top this would require a Damage Tolerance Proof.

1.2 Basics for UD-SFC formulations

Desired as models are 'homogeneous' solids, however, reality is much more complicate. Practically, all materials are composites. One distinguishes two structural composite types: Material Composites and Composite Materials. A structural material usually is the model on the considered scale of a homogenized complex solid that became 'smeared' to usually obtain an engineering-like macro-model. A Material Composite [*Cun22*] is structural-mechanically a composite 'construction of different materials' whereas a Composite Material is a combination of constituent materials, different in composition (constituents retain their identities in the composite). Usually a Composite Material can be modelled as a smeared material and this is the case in the following investigation.

Physical experience makes to consider some aspects:

- If a material element can be homogenized to an ideal crystal (= frictionless), material symmetry requires for the isotropic and the transversely-isotropic UD material a distinct minimum number of properties. This is witnessed by tests
- A real solid material model is represented by a description of the ideal crystal + a description of its friction behavior. Mohr-Coulomb asks for the real crystal another physical parameter, namely the inherent material friction value μ with one value for isotropic and two values for UD materials. → Unfortunately SFCs often employ just strengths. This is physically not accurate because Mohr-Coulomb acts in the case of compressed brittle materials! The computed *RF* may not be on the safe side. *Failure envelopes are not just an empirical fit of uniaxial tensile and compressive strength tests as it was still assumed in the World Wide Failure Exercises !*
- Invariants are a combination of stresses – powered or not powered – the value of which does not change when altering the coordinate system CoS. This attribute is used when looking for an optimum formulation of a usually desired scalar SFC
- Direct use of the measurable, physically clear friction value μ in the SFC formulation instead of using fictitious friction model parameter. This matches with the engineer's thinking in physical properties. A good guess for isotropic and UD materials is $\mu = 0.2$

- A usual SFC just describes a 1-fold occurring failure mode or mechanism! A multi-fold occurrence of a failure with its joint probabilistic effects must be additionally considered in the SFC formulas of each author.

1.3 Terminology, Specific Terms

Modeling the variety of laminates is a challenge. In this context, essential for the interpretation of the failures faced after testing with potential property reduction, is the knowledge about the lay-up (stack) of the envisaged laminate, because crimped fabrics and non-crimped NCF-materials behave differently. It is further extremely necessary to provide the material-modeling design engineer and his colleague in production (*for his Ply Book*) with a clear, distinguishing description of UD-lay-ups, of Non-Crimp-Fabrics NCFs (*stitched multi-UD-layer tack*) and of Fabric layers (*crimped*). Due to unclear descriptions unfortunately one can often not use valuable test results of fiber-reinforced materials. One could distinguish the various types a clear optical designation in order to enable a realistic material modelling, namely by a square bracket [...] and a wavy bracket {...}, which optically helps to distinguish NCF **{**stitched UD-stack**}** from those woven fabrics where one practically cannot mechanically separate the single woven layers within one fabric layer as in the case of *plain weave* binding, $\begin{bmatrix} 0 \\ 90 \end{bmatrix}$, which is symmetric in itself. Applied this means:

* Single UD-layers-*deposited* stack $[0/90]_S = [0/90/90/0]$-lay-up, prepreg
* Semi-finished product, *stitched* NCF: $\{0/90\} + \{90/0\}$ symmetrically stacked, dry deliverable 'building blocks' : $\{0/45/\text{-}45/90\}$, *novel* C-ply™ $\{\varphi/\text{-}\psi/\text{-}\varphi/\psi\}$, Double-Double $\{75/-75/-15/15\}_r$ with r = repetition.

Some terms for a better common understanding shall be further added here, see also [*Cun19*]:

<u>Design Dimensioning</u>: static and cyclic sizing

<u>A-Basis (strength) Design Allowable (or "A"-Value):</u> statistically-based material property, above which at least with a probability P = 99% of the population of values is expected to fall, with a confidence level of C = 95%. For failure-redundant laminates often the higher "B"-value is permitted with P = 90%, C= 95%

<u>Design Load</u>: maximum amount (of loading) a (load-carrying) system is to be designed to

<u>Delamination:</u> separation of material layers within a laminate or also in a textile reinforced concrete (may be local or may cover a large area of the laminate)

<u>Design Verification</u> (from Latin, veritas facere): fulfillment of a design requirement <u>data set</u> (for a deformation, a frequency, design load, etc)

<u>Double-</u>Double laminates: Two angle-plies of different fiber angles form a four-ply sub-laminate or building-block, respectively, (for instance C-ply™ from Chomarat)

<u>Equivalent stress</u>: (1) Equivalent to a multi-axial stress state combining the effects of those stresses that are active in a distinct failure mode. (2) The uni-axial scalar σ_{eq}-value can be compared to the mode-'reigning' associated uni-axial 'basic' strength R

(strength) Failure Condition: Condition on which a failure becomes effective, meaning F = 1 for one limit state. Mathematical formulation of the failure surface that takes the form F = 1 = 100 % . *Most often meant is a strength failure condition SFC. Aim of a SFC is to assess multi-axial states of stresses* (strength) Failure Criterion (SFC): Distinctive feature defined as a condition for one of the 3 states, taking the form F > 1, F = 1, F < 1

Failure function F : mathematical formulation of the failure event, $F = 1$

Failure Mode Concept (FMC)**:** invariant, failure mode-based general concept to generate strength failure conditions for single failure modes. It is a 'modal' formulation in contrast to 'global' concepts where all failure modes are mathematically linked and a concept for materials that can be homogenized (smeared). The applicability of a SFC ends if homogenization as pre-requisite of modeling is violated

First-Ply-Failure (FPF): First Inter-Fiber-Failure IFF in a lamina of the laminate capturing FF and IFF

Lamina: analytical designation of the single UD ply as computational element of the laminate, used as laminate subset or building block for modelling. It might capture several equal physical layers (plies)

Layer: deposit from winding, tape-laying process etc.

Margin of Safety *MoS*: $MoS = RF - 1 > 0$

Material Properties: 'Agreed' values to achieve a common and comparable design basis. Must be provided with average value and coefficient of variation cov

Material Stressing Effort *Eff* (not material utilization): artificial term, generated in the UD World Wide Failure Exercises in order to get an English term for the excellent, meaningful German term Werkstoff-Anstrengung. Tsai's *(otherwise still fixed letter R chosen)* so-called Strength Ratio R corresponds to *Eff*

Omni failure envelope: envelope of the intact non-FPF area and is based on all conceivable ply orientations

PAN-CF: precursor PolyAcrylNitril-based CF (base CF type)

PAN-UHM-CF: higher graphitized PAN-CF

Ply: physical fiber-reinforced material layer part

'Quad laminates': (0°, 45°,-45°, 90°) sub-laminate family as laminate building block in aerospace

Reserve Factor *RF*: load-defined value $RF_{\mathrm{ult}} = \textit{final failure load} \ / \ \textit{design load DL}, RF > 1$

(material) Reserve factor f_{RF}: $f_{RF} = \textit{strength design allowable } R \,/\, \textit{stress at design load DL}$

Strength Design Allowables: statistically reduced average values such as A- and B-values or, respectively, 5% fractiles in civil engineering. $\{R\} = (R_{||}^t, R_{||}^c, R_{\perp}^t, R_{\perp}^c, R_{\perp\parallel})^{\mathrm{T}}$ [*Cun22*]

Transversely-isotropic material (UD, uni-directional): material model assumption, where the plane 2-3 is quasi-isotropic and due to that UD is termed transversely-isotropic

Validation of a model (from validus = strong): 'qualification' of a created model by well mapping physical test results with the derived model (here a material failure model).

1.4 Basic Tasks: Test Data Mapping and Design Dimensioning with Design Verification

Validation of the SFC model is obtained, if the courses of test data points are well mapped. This delivers an average strength set compiled exemplarily as in $\{\bar{R}\} = (1378,\ 950,\ 40,\ 125,\ 97)^T \mathrm{MPa}$. If shear or compression occurs a typical friction value μ is required on top. *Validation* of UD lamina-material SFCs models can be only achieved by 2D-test results together with 3D-lamina test specimen results. Any laminate test case serves for the *verification* of the laminate design.

In Design Verification of the finally chosen laminate stack the average strength values $\{\bar{R}\}$ are statistically reduced to the so-called strength Design Allowables ('A- or B-values'$\{R\}$). This shrinks the failure envelope obtained during mapping.

The verification of the design requires a Reserve Factor $RF \geq 1.00$ (*Fig.1-2*). In order to be able to correctly classify the importance of the following elaboration in the entire design process, the design verification procedure will be briefly described.

If linear analysis is sufficient solution (presumption): $\sigma \sim$ load $\rightarrow RF \equiv f_{RF} = 1/\mathrm{Eff}$

material reserve factor $$f_{\mathrm{RF},\,ult} = \frac{\text{Strength Design Allowable } R}{\text{Stress at } j_{ult} \cdot \text{Design Limit Load}} > 1,$$

Non-linear analysis required: σ not proportional to load

reserve factor (load-defined) $$RF_{ult} = \frac{\text{Predicted Failure Load at } Eff = 100\%}{j_{ult} \cdot \text{Design Limit Load}} > 1.$$

A very simple example shall depict the RF-calculation as most essential task in design which streamlines every procedure when generating a design tool in the following chapters:

Asssumption: Linear analysis permitted, design FoS $j_{\mathrm{ult}} = 1.25$

- Design loading (action): $\{\sigma\}_{\mathrm{design}} = \{\sigma\} \cdot j_{\mathrm{ult}}$
- 2D-stress state: $\{\sigma\}_{\mathrm{design}} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T \cdot j_{\mathrm{ult}} = (0,\ -76,\ 0,\ 0,\ 0,\ 52)^T \mathrm{MPa}$
- Residual stresses: $0\ (\textit{effect vanishes with increasing micro-cracking})$
- Strengths (resistance) : $\{\bar{R}\} = (1378,\ 950,\ 40,\ 125,\ 97)^T \mathrm{MPa}$ average from mesurement

  statistically reduced $\{R\} = (R_{\parallel}^t, R_{\parallel}^c, R_{\perp}^t, R_{\perp}^c, R_{\perp\parallel})^T = (1050,\ 725,\ 32,\ 112,\ 79)^T \mathrm{MPa}$
- Friction values : $\mu_{\perp\parallel} = 0.3,\ (\mu_{\perp\perp} = 0.35)$, Mode interaction exponent: $m = 2.7$

$$\{Eff^{\mathrm{mode}}\} = \left(Eff^{\parallel\sigma},\ Eff^{\parallel\tau},\ Eff^{\perp\sigma},\ Eff^{\perp\tau},\ Eff^{\parallel\perp}\right)^T = (0.88,\ 0,\ 0,\ 0.21,\ 0.20)^T$$

$$Eff^m = (Eff^{\parallel\sigma})^m + (Eff^{\parallel\tau})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m = 100\%\ .$$

The results above deliver the following material reserve factors f_{RF}

- $$Eff^{\perp\sigma} = \frac{\sigma_2 + |\sigma_2|}{2 \cdot R_\perp^t} = 0, \quad Eff^{\perp\tau} = \frac{-\sigma_2 + |\sigma_2|}{2 \cdot R_\perp^c} = 0.68, \quad Eff^{\perp\parallel} = \frac{|\tau_{21}|}{R_{\perp\parallel} - \mu_{\perp\parallel} \cdot \sigma_2} = 0.55$$

$$Eff = [(Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m]^{1/m} = 0.80.$$

$$\Rightarrow\ f_{\mathrm{RF}} = 1/Eff = 1.25 \rightarrow RF = f_{\mathrm{RF}}\ (\text{if linearity permitted}) \rightarrow MoS = RF - 1 = 0.25 > 0\ !$$

Fig.1-2: Example for a Design Verification of an applied stress state in a critical UD lamina location of a distinct laminate wall design

The certification–relevant load-defined Reserve Factor RF corresponds in the given linear case to the material reserve factor f_{RF}, the value of which is 1.39 > 1 → *Laminate wall design is verified!*

Steve Tsai's hope for the future: *"Materials and laminates are equivalent and the same entity with different views. They are interchangeable through their single parameters all locked in through their* *transformation and interpolation properties in a compact, elegant, continuous field, totally different from a collection of discrete quad laminates. Lack of data can no longer derail innovations".*

1.5 'Quad'-Laminates and Double-Double (DD) Laminates Lay-ups

Beside so-called 'Quad-laminates' (*standard laminates with 0°, 90°, 45°, -45° fiber orientations)* Tsai investigated a novel semi-finished product, termed C[TR]-Ply, and created the promising 'Double-Double laminate (see [*Kap22*] and [*Cun23a*]). In the latter document the not simply to perform transfer of Tsai's notation on stresses and strengths has been executed compatible to the German Standard VDI 2014.

The idea is: *Laminate parameter plots can replace Tsai's former carpet plots. However, now all laminates can be portrayed in one plot. It is helpful to assess what laminates can and cannot do and which one is the best delivering a significant decision support?*

Whereas the 'Quad' family is well known the novel 'DD' family has to be presented.

Double-Double (DD) means a sub-laminate of two angle-plies or two Doubles, respectively. Two angle-plies of different fiber angles will form a four-ply sub-laminate. This is a multi-ply semi-fished product identified by the brackets {..} to discriminate it from [..] for the UD-layer prepreg stacks.

DD is automatically balanced, needs no ten percent rule, no stacking sequence Homogenization makes mid-plane symmetry unnecessary. Of-course, stitching of the C-ply harms the UD material, however this is captured in the material tests determining the material design values in test data evaluation. In strength analysis the repeated double angle-ply sub-laminate and the full laminate can be modelled ply-wise as $\{\varphi / -\varphi / \psi / -\psi\}$ in each sub-laminate stack. A stack $\pm\varphi,\ \pm\psi$ corresponds to the ω-angle in net-theory $\pm\omega_1, \pm\omega_2$, where $\alpha_1 = \omega_1,\ \alpha_2 = -\omega_1,\ \alpha_3 = \omega_2\ ,\ \alpha_4 = -\omega_2$.

2 Choice of UD Strength Fracture Criteria viewing 'Modal' and 'Global' SFCs

Present SFCs can be basically separated into two groups, global and modal SFC ones, which is of concern for a comparison of Tsai-Wu (global) and Cuntze (modal). The *Fig.2-1* presents the main differences between them (*The author chose the term "Global" as a 'play on words' to "modal" and to being self-explaining*). Global SFCs describe the full failure surface by one single mathematical equation. This means that for instance a change of the UD *tensile* strength $\bar{R}_{\perp}^{t}$ affects the failure curve in the *compression* domain, where no physical impact can be! Hence, the computed Reserve Factor *RF* may not be on the safe side in this domain. This is physically not accurate. Mohr-Coulomb acts in the case of compressed brittle materials! The undesired consequence in Design Verification is: The computed *Reserve Factor RF* may be not on the safe side. The same happens to be if the SFC employs just strengths and no friction value

However, Modal SFCs need an interaction of the failure modes. This is performed by a probabilistic approach (series failure system) in the transition zone of neighboring modes.

The following table depicts the advantages and disadvantages of global and modal SFCs.

Table 2-1: Pros (+, for) and Cons (-, contra) of 'Global' and 'Modal' SFCs

<u>Global SFCs</u> like Tsai-Wu, Drucker-Prager

- (+) Describe the full failure surface by one single mathematical equation ('single-value criterion')
- (-) Usual global SFCs do not capture a multi-fold acting failure mode, i.e. $\sigma_I = \sigma_{II}$ or $\sigma_2 = \sigma_3$ or a 3-fold acting failure mode under σ_{hyd} with tension or compression
- (-) Re-calculation: In the case of a test data change in a distinct mode domain re-calculation of model parameters is mandatory. Any change in one of the 'forcibly married' modes requires a new global mapping which also changes the failure curve in a physically independent failure domain, see *<u>Fig.2-2</u>*. In consequence, the material reserve factor has to be determined again
- (-) The determination of *RF* for multi-axial stress states seems to be questionable for the simple Drucker-Prager model (*conical failure body*) - still often used in civil engineering.

<u>Modal SFCs</u> like Mises, Hashin, Puck, Cuntze

The 'Mises' (HMH) yield failure condition was the model of the author. It is a modal SFC that captures just one failure mode. Later, Hashin with his 4 modes supported the author's modal thinking.

- (+) Describe each failure mode-associated part of the full failure surface by one single equation. Therefore, modal SFCs are more physically-based than global SFCs
- (+) A change within one mode just hits this mode, *see Fig.2-2*. *RF* is just to re-determine in the affected failure mode domain!
- (+) Equivalent stresses σ_{eq} are always determinable for isotropic UD materials
- (+) Cuntze's SFCs capture multi-fold occurring failure modes by an additional term
- (+) Cuntze's SFCs directly use the well to estimate parameter friction value μ
- (-, +) Cuntze FMC-based set affords an interaction of the SFCs to capture all activated failure modes. This delivers information about the mode's design-driving size via $Eff^{\,mode}$
- (+) By using the interaction equation $Eff\,(Eff^{\,modes}) = 1$ the modal SFC-set formally becomes a quasi-global SFC but without the bottlenecks of a global SFC and no more numerical effort.

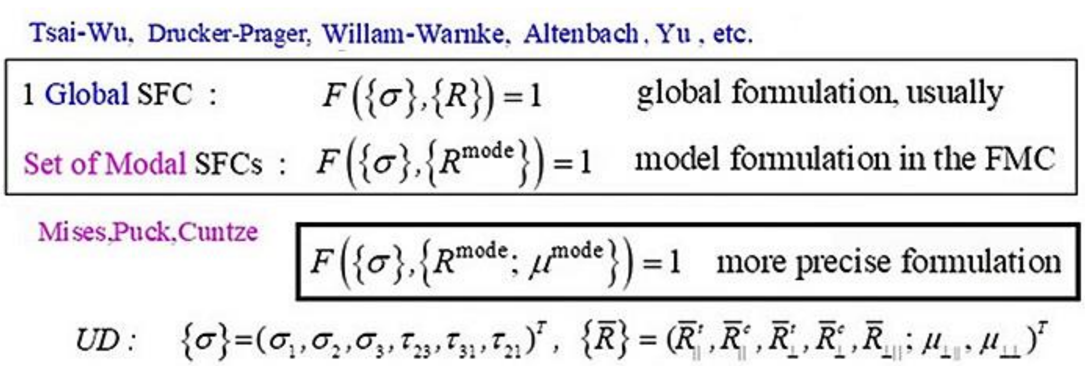
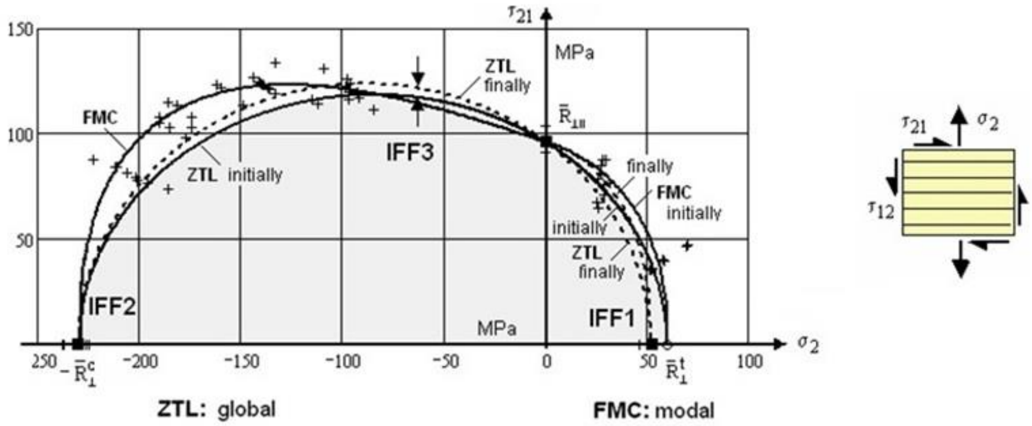


Fig.2-1: 'Global' and 'Modal' SFCs

Fig.2-2 visualizes for a distinct global SFC, used in a German guideline, how dramatically a change of the tensile strength $\bar{R}_{\perp}^{t}$ affects the failure curve in the compression domain, although no physical impact can be!



$$\frac{\sigma_2^{\,2}}{\bar{R}_{\perp}^{t}\cdot\bar{R}_{\perp}^{c}}+\sigma_2\cdot\left(\frac{1}{\bar{R}_{\perp}^{t}}-\frac{1}{\bar{R}_{\perp}^{c}}\right)+\frac{\tau_{21}^{\,2}}{\bar{R}_{\perp\parallel}}=1 \qquad \left(\frac{\sigma_2}{\bar{R}_{\perp}^{t}}\right)^m+\left(\frac{-\sigma_2}{\bar{R}_{\perp}^{c}}\right)^m+\left(\frac{|\tau_{21}|}{\bar{R}_{\perp\parallel}-\mu_{\perp\parallel}\cdot\sigma_2)}\right)^m=1$$

Fig.2-2: Effect of global modelling, ZTL-SFC, *still used in the HSB*

Before tackling the design sheet task the 3D SFCs of Tsai-Wu and Cuntze shall be displayed which generally capture spatial stress states in a laminate. Because the envisaged Design Sheet can be just the 3D SFCs will be later reduced to 2D ones.

3 Cuntze's Failure Mode Concept-based modal SFC set, FF1, FF2, IFF1, IFF2, IFF3

$$F\left(\{\sigma\},\{\bar{R}\},\mu_{\perp\perp},\mu_{\perp\parallel}\right)=1 \ \text{ with } \ \{\sigma\}=(\sigma_1,\sigma_2,\sigma_3,\tau_{23},\tau_{31},\tau_{21})^T, \ \{\bar{R}\}=(\bar{R}_{\parallel}^{t},\bar{R}_{\parallel}^{c},\bar{R}_{\perp}^{t},\bar{R}_{\perp}^{c},\bar{R}_{\perp\parallel})^T$$

3.1 Material Symmetry and 'Generic' Number

Under the design-simplifying presumption "Homogeneity is a permitted assessment for the material concerned", and regarding the respective material tensors, it follows from material symmetry that the number of strengths equals the number of elasticity properties!
Fracture morphology gives further evidence: Each strength property corresponds to a distinct strength failure mode and to a distinct strength failure type, to Normal Fracture (NF) or to Shear Fracture (SF). This means, a characteristic number of quantities is fixed: 2 for isotropic material and 5 for the transversely-isotropic UD lamina (≡ lamellas in civil engineering). Hence, the applicability of material sym*metry involves that in general just a minimum number of properties needs to be measured* (cost + time benefits) which is helpful when setting up strength test programs. This is beneficial regarding material modelling and the amount of testing, see literature of Christensen [*Chr98*].

Witnessed material symmetry knowledge seems to tell: There might exist a 'generic' (*term was chosen by the author*) material inherent number for the UD brittle materials, namely:

- 5 e*lastic 'constants', 5 strengths, 5 strength failure modes fracture* (NFs with SFs)

- *2 physical parameters such as the 2 coefficients of thermal expansion CTE, the 2 coefficients of moisture expansion CME, and the 2 friction values* $\mu_{\perp\perp}, \mu_{\perp||}$

3.2 Basic Features

The basic features of the FMC, derived about 1995, are: see [*Cun06, 12, 13, 17, 23a, 23b*]

- Each failure mode represents 1 independent failure mechanism and thereby represents 1 piece of the complete failure surface.
- A failure mechanism at the lower micro-scopic mode level shall be considered in the applied desired macro-scopic SFC
- Each failure mechanism or failure mode is governed by 1 basic strength *R*, only (*witnessed!*)
- Each failure mode can be represented by 1 SFC. Therefore, equivalent stresses can be computed for each mode. This is of advantage when deriving S-N curves and generating Haigh diagrams in fatigue with minimum test effort in order to relatively effortless obtain Constant Fatigue Life curves, see [*Cun23b*] for lifetime estimation. Modal SFCs lead to a *clear* mode strength-associated equivalent stress
- Of course, the modal FMC-approach requires an interaction of all modes reading

$$Eff = \sqrt[m]{(Eff^{\text{mode 1}})^m + (Eff^{\text{mode 2}})^m + \ldots} = 1 = 100\% \quad \textit{for} \quad \text{Onset-of-Failure} .$$

It employs the so-called 'material stressing effort' (*artificial term, generated in the WWFE in order to get an English term for the German term Werkstoffanstrengung*)

$$Eff^{\text{yield mode}} = \sigma_{eq}^{\text{Mises}} / R_{0.2} \;\rightarrow\; Eff^{\text{fracture mode}} = \sigma_{eq}^{\text{fracture mode}} / R \ .$$

To apply is a mode interaction exponent *m,* also termed rounding-off exponent, the size of which is high in case of low scatter and vice versa. The value of *m* is obtained by curve fitting of test data in the transition zone of the interacting modes. FRP mapping experience delivered that $2.5 < m < 3$. A lower value chosen for the interaction exponent is more on the safe *RF* side or more 'design verification conservative'. For CFRP m = 2.6 is recommended from mapping experience. From engineering reasons, the 'out-smoothing' *m* is chosen the same in all transition zones of adjacent mode domains. Using the interaction equation in the mode transition zones is leading again to a pseudo-global failure curve or surface. In other words, a '*single surface failure description*' is obtained again, such as with Tsai-Wu but without the shortcomings of the global SFCs.
Above interaction of adjacent failure modes is modelled by the 'series failure system'. That permits to formulate the total material stressing effort *Eff* generated by all activated failure modes as 'accumulation' of Eff^{modes} ≡ sum of the single mode failure danger proportions. *Eff* = 100% = 1 represents the mathematical description of the complete surface of the failure body! In practice, i.e. in thin UD laminas, at maximum, 3 modes of the 5 modes (*2 FF + 3 IFF*) will physically interact. Considering 3D-loaded thick laminas embedded in laminates, there, all 3 IFF modes might interact. In order to only use experimentally derivable material quantities, Cuntze directly introduced in his 3D-SFCs the internal material friction μ as a SFC model parameter. The direct introduction of the measurable friction value is possible for the modal shear fracture SFCs and could be achieved after an effortful transition of the SFC formulated in structural stresses into a Mohr stresses formulated one (see *Chapter 6 in Cun22*).

3.3 *Cuntze's FMC-based Set of the 3D-mapping Modal SFCs*

Table 3-1 collects the FMC-derived 5 SFC formulations. Therein, the used invariants have been still inserted into the stress formulations.

Table 3-1 'Dense' UD materials: Cuntze's 3D SFC formulations for FF1, FF2 and IFF1, IFF2, IFF3

FF1: $Eff^{\|\sigma} = \breve{\sigma}_1 / \bar{R}^t_{\|} = \sigma^{\|\sigma}_{eq} / \bar{R}^t_{\|}$ with $\breve{\sigma}_1 \cong \varepsilon^t_1 \cdot E_{\|}$ (matrix neglected)

FF2: $Eff^{\|\tau} = -\breve{\sigma}_1 / \bar{R}^c_{\|} = +\sigma^{\|\tau}_{eq} / \bar{R}^c_{\|}$ with $\breve{\sigma}_1 \cong \varepsilon^c_1 \cdot E_{\|}$

IFF1: $Eff^{\perp\sigma} = [(\sigma_2 + \sigma_3) + \sqrt{\sigma_2{}^2 - 2\sigma_2 \cdot \sigma_3 + \sigma_3{}^2 + 4\tau_{23}{}^2}\,] / 2\bar{R}^t_{\perp} = \sigma^{\perp\sigma}_{eq} / \bar{R}^t_{\perp}$

IFF2: $Eff^{\perp\tau} = [a_{\perp\perp} \cdot (\sigma_2 + \sigma_3) + b_{\perp\perp}\sqrt{\sigma_2{}^2 - 2\sigma_2\sigma_3 + \sigma_3{}^2 + 4\tau_{23}{}^2}\,] / \bar{R}^c_{\perp} = \sigma^{\perp\tau}_{eq} / \bar{R}^c_{\perp}$

IFF3: $Eff^{\perp\|} = \{[b_{\perp\|} \cdot I_{23-5} + (\sqrt{b_{\perp\|}{}^2 \cdot I_{23-5}{}^2 + 4 \cdot \bar{R}_{\perp\|}{}^2 \cdot (\tau_{31}^2 + \tau_{21}^2)^2}\,] / (2 \cdot \bar{R}_{\perp\|}{}^3)\}^{0.5} = \sigma^{\perp\|}_{eq} / \bar{R}_{\perp\|}$

$$\{\sigma^{\text{mode}}_{eq}\} = (\sigma^{\|\sigma}_{eq},\ \sigma^{\|\tau}_{eq},\ \sigma^{\perp\sigma}_{eq},\ \sigma^{\perp\tau}_{eq},\ \sigma^{\|\perp}_{eq})^T, \quad I_{23-5} = 2\sigma_2 \cdot \tau_{21}^2 + 2\sigma_3 \cdot \tau_{31}^2 + 4\tau_{23}\tau_{31}\tau_{21}$$

Inserting the compressive strength point $(0, -\bar{R}^c_{\perp}) \rightarrow a_{\perp\perp} \cong \mu_{\perp\perp} / (1 - \mu_{\perp\perp})$, $b_{\perp\perp} = a_{\perp\perp} + 1$

from a measured fracture angle $\rightarrow \mu_{\perp\perp} = \cos(2 \cdot \theta^c_{fp}{}^{\circ} \cdot \pi / 180)$, for $50° \rightarrow \mu_{\perp\perp} = 0.174$.

$b_{\perp\|} = 2 \cdot \mu_{\perp\|}$. Typical friction value ranges: $0 < \mu_{\perp\|} < 0.25$, $0 < \mu_{\perp\perp} < 0.2$.

$$Eff = [(Eff^{\|\sigma})^m + (Eff^{\|\tau})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\|})^m + (Eff^{\perp\tau})^m]^{m^{-1}}$$

with the mode portions

$$Eff^{\|\sigma} = \frac{(\varepsilon_1 + |\varepsilon_1|) \cdot E_{\|}}{2 \cdot R^t_{\|}},\quad Eff^{\|\tau} = \frac{(-\varepsilon_1 + |\varepsilon_1|) \cdot E_{\|}}{2 \cdot R^c_{\|}},\quad Eff^{\perp\sigma} = \frac{\sigma_2 + |\sigma_2|}{2 \cdot \bar{R}^t_{\perp}},$$

$$Eff^{\perp\tau} = \frac{-\sigma_2 + |\sigma_2|}{2 \cdot \bar{R}^c_{\perp}},\quad Eff^{\perp\|} = \frac{|\tau_{21}|}{\bar{R}_{\perp\|} + 0.5 \cdot \mu_{\perp\|} \cdot (-\sigma_2 + |\sigma_2|)}\ .$$

Above interaction equation, even just in the in-plane loading case, includes all mode material stressing efforts Eff^{modes} and each of them represents a portion of load-carrying capacity of the material. In thin laminas in practice at maximum 3 modes of the 5 modes will physically interact. The superscripts shall indicate the failure active σ- or τ-stress.

From friction parameters b to friction values μ: The structural stresses-formulated UD-fracture curve $\sigma_2(\sigma_3)$ could be transferred into a Mohr-Coulomb one obtaining $\tau_{nt}(\sigma_n)$, [*Cun22*]. This novel, mathematically pretty challenging transformation enabled the author to link the fictitious friction parameters b of the respective SFCs via a determined fracture angle with the measurable physical friction value μ, see also [*Pet15*]. The author's FF1- and FF2-formulations for instance take care, that transversal equi-biaxial compression might cause FF1. The two FF formulations correspond to a maximum stress SFC, however the macro-mechanical FFs capture micro-mechanical failure of the constituent fiber under bi-axial compressive stressing with strains. The invariants in the originally invariant-formulated failure functions F are replaced by the associated stresses and then Eff is inserted and for the Eff s resolved.

Fig.3-1 presents the 2D-failure surface of the lamina, showing the pure IFF modes in the upper part and the interaction envelope Eff = 1 of the 3 IFF modes in the lower part.

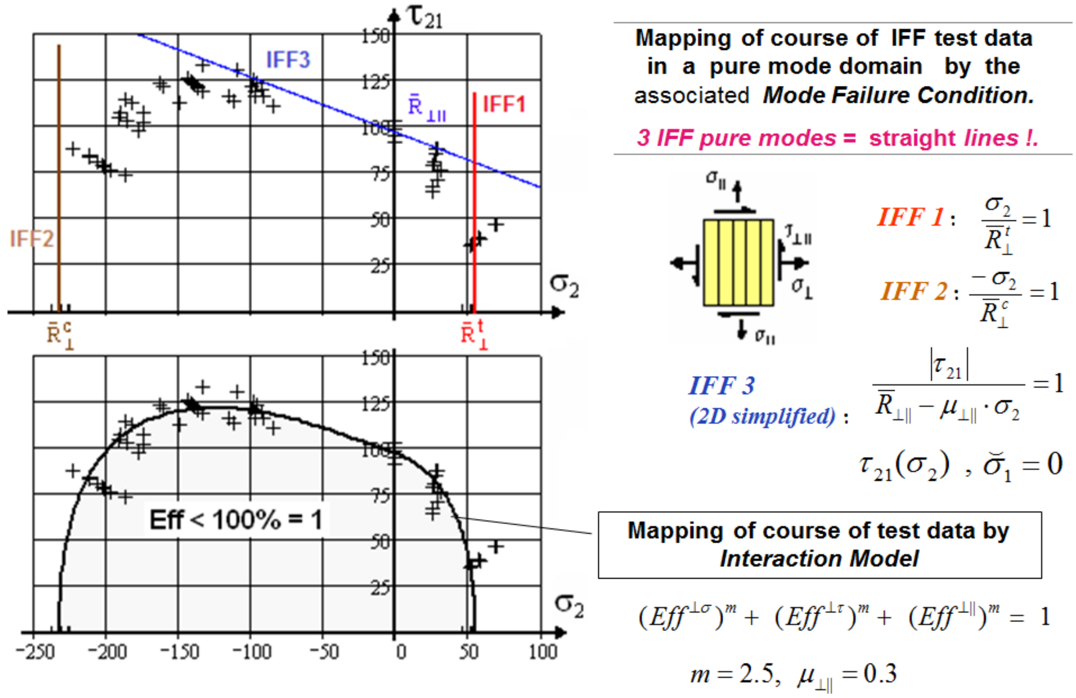


Fig.3-1: Visualization of the IFF interaction of a UD-material

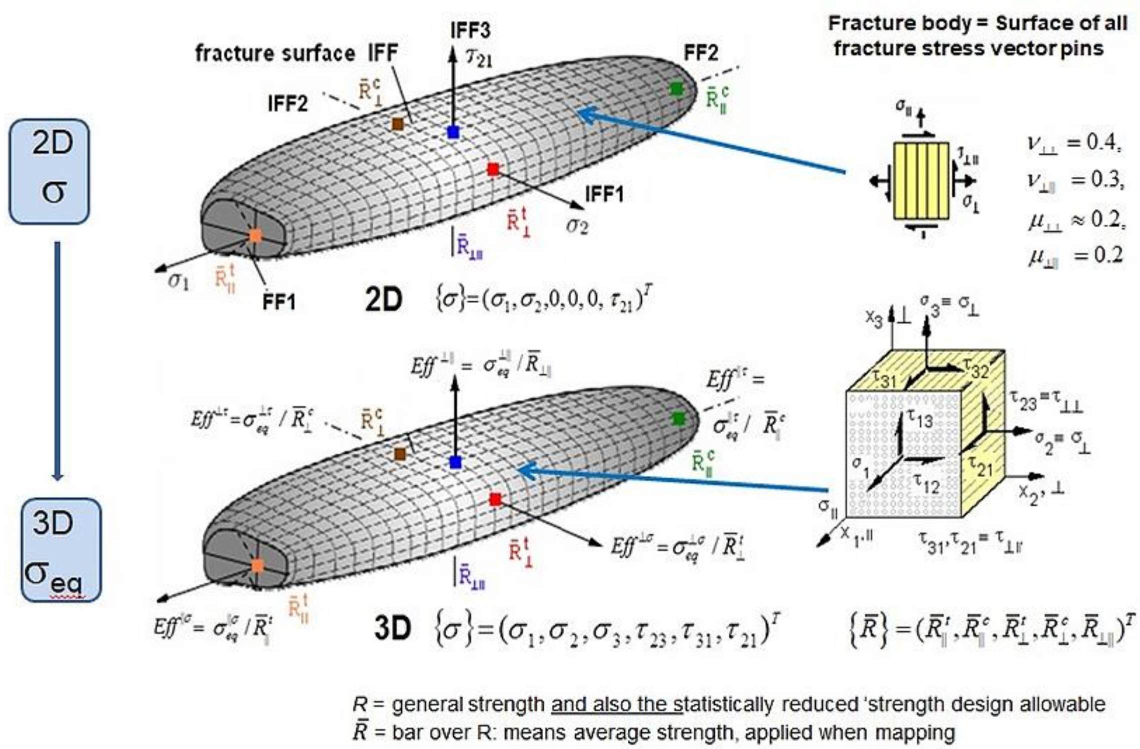


Fig.3-2: From a 2D failure body to a 3D failure body by replacing stresses by equivalent stresses

Fig.3-2 depicts the fracture failure body of UD materials. The upper picture contains the failure body of the plane 2D stress state and the lower picture the body of the 3D stress state. These look the same and are the same. One must only replace the UD-lamina stresses of the 2D-case by equivalent stresses to obtain the 3D-fracure failure body. Only some years ago Cuntze sorted out *If one replaces the lamina stresses by the associate equivalent mode stresses then the 2D-failure body becomes a general 3D-failure body.*

Of interest is not only the interaction of the fracture surface parts in the discussed mixed failure domains or interaction zones of adjacent failure modes, respectively, but additive failure danger is faced in a multi-fold failure domain (superscript MfFD). There the associated mode stress effort acts twofold. It activates failure in two directions which is considered by adding a multi-fold failure term, proposed in [Awa78] for isotropic materials, $\sigma_{II}(\sigma_{III})$. It can be applied to brittle UD material in the transversal (quasi-isotropic) plane as well, $\sigma_2(\sigma_3)$.

For completion, *Fig.3-3* presents a failure curve for Inter-Fiber-Failure of a GFRP and a CFRP. The test rig for the tube test specimen was a dedicated Tension-Compression/Torsion machine.

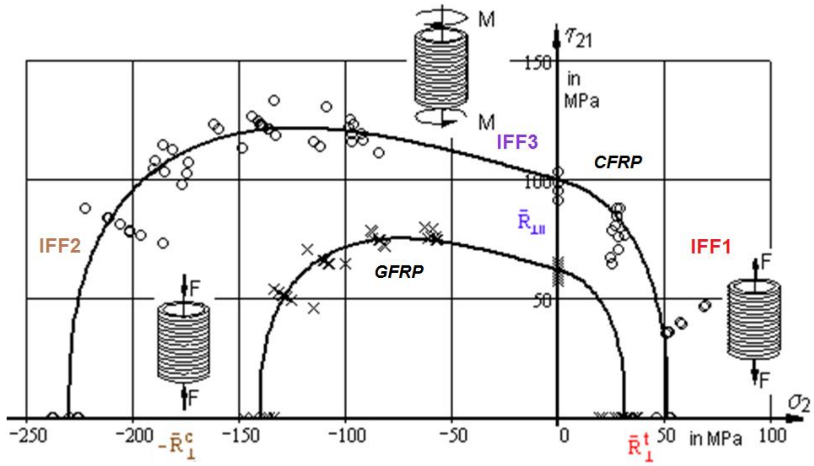


Fig.3-3, IFF test results: 2 GFRP, 1 CFRP test series (from MAN Technologie research project on Puck's IFF criterion), m =2.7 Fig. E-glass / LY556, HT976, DY070; CFRP: T300 / LY556, HT976

4 Short Presentation of the 3D-capable UD SFC of Tsai-Wu

$F(\{\sigma\},\{\bar{R}\}) = 1: \{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^{\mathrm{T}}, \{\bar{R}\} = (\bar{R}_{||}^t, \bar{R}_{||}^c, \bar{R}_{\perp}^t, \bar{R}_{\perp}^c, \bar{R}_{\perp||}, \bar{R}_{\perp\perp})^{\mathrm{T}} = (\mathrm{X}, X', Y, Y', S, \mathrm{S}_{23})^{\mathrm{T}}$

'Onset-of Fracture' limit has been termed by Tsai 'First Ply Failure (FPF) and includes FF and IFF.

A short presentation of the Tsai-Wu SFC is helpful to remind that different SFCs map failure differently and will lead to some difference regarding Design Sheets indicating 'Onset-of-(Fracture) Failure'. The applied strengths in the formulas are intentionally marked by a bar over in order to remind that model validation is the subject.

A general anisotropic tensor polynomial expression of Zakharov and Goldenblat-Kopnov with the parameters F_i, F_{ij} as strength model parameters was the basis of the Tsai-Wu SFC, see [*Tsai71*],

$$\sum_{i=1}^{6}\left(F_i \cdot \sigma_i\right) + \sum_{j=1}^{6}\sum_{i=1}^{6}\left(F_{ij} \cdot \sigma_i \cdot \sigma_j\right) = 1 .$$

In *Fig.4-1* the general types of stress of a UD-composite element causing FF and IFF. Shown are the oblique (inclined) planes in which brittle fracture occurs and the sketches of the stressed 2D and 3D material elements.

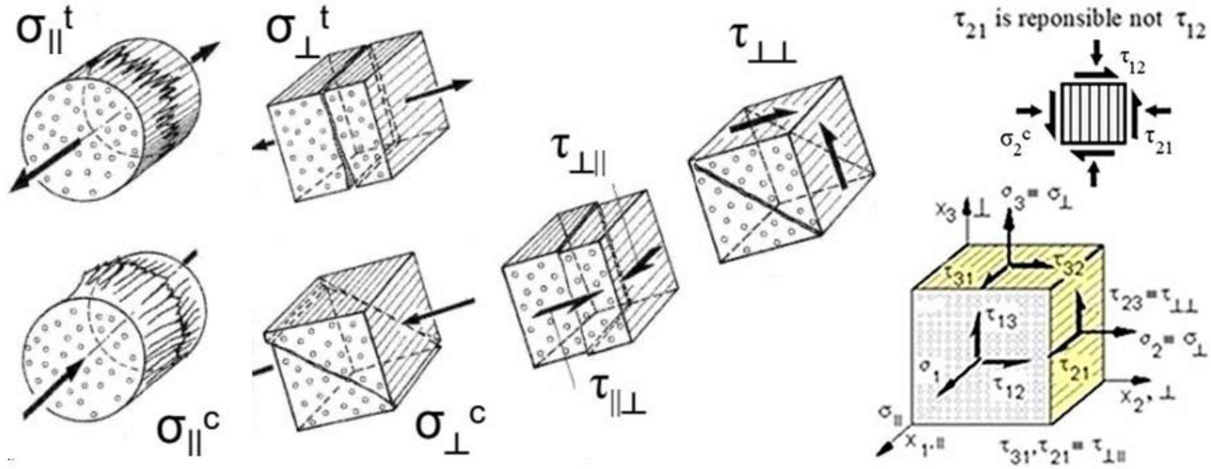


Fig. 4-1: The 6 stresses acting at a UD material element [Lut6, Puck]

From this tensor formulation Tsai-Wu used the linear with the quadratic terms, see *Table 4-1*:

*Table 4-1: Tsai-Wu **3D SFC** [Tsa71, Tsa22]*

$$F_i \cdot \sigma_i + F_{ij} \cdot \sigma_i \cdot \sigma_j = 1 \quad \text{with} \quad (\text{i,j} = 1,2..6) \quad \text{or executed}$$

$$F_{11} \cdot \sigma_1^2 + F_1 \cdot \sigma_1 + 2F_{12} \cdot \sigma_1 \cdot \sigma_2 + 2F_{13} \cdot \sigma_1 \cdot \sigma_3 + F_{22} \cdot \sigma_2^2 + F_2 \cdot \sigma_2 +$$
$$+ 2F_{23} \cdot \sigma_2 \cdot \sigma_3 + F_{33} \cdot \sigma_3^2 + F_{33} \cdot \sigma_3^2 + F_3 \cdot \sigma_3 + F_{44} \cdot \tau_{23}^2 + F_{55} \cdot \tau_{13}^2 + F_{66} \cdot \tau_{12}^2 = 1$$

with the strength parameters

$$F_1 = 1/\bar{R}_{||}^t - 1/\bar{R}_{||}^c, \; F_{11} = 1/(\bar{R}_{||}^t \cdot \bar{R}_{||}^c), \; F_2 = 1/\bar{R}_{\perp}^t - 1/\bar{R}_{\perp}^c, \; F_{22} = 1/(\bar{R}_{\perp} \cdot \bar{R}_{\perp}^c) = F_{33},$$
$$F_{13} = F_{12}, \; F_{55} = F_{66} = 1/\bar{R}_{\perp||}^2, \; 2F_{23} = 2F_{22} - 1/\bar{R}_{\perp\perp}^2, \; F_{44} = 2 \cdot (F_{22} + F_{23})$$

and - *in order to avoid an open failure surface* - the interaction term

$$F_{12} = \bar{F}_{12} \cdot \sqrt{F_{11} \cdot F_{22}} \quad \text{with} \quad -1 \le \bar{F}_{12} \le 1 \quad \text{is used.}$$

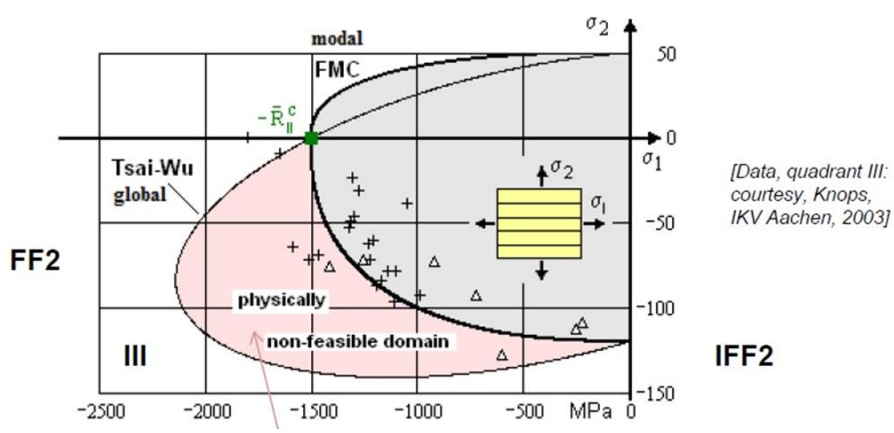


Fig.4-2, WWFE-II: Mapping of $\sigma_2(\sigma_1)$ test data (test results: M. Knops, IKV Aachen, [Kno3]

The bi-axial material parameter F_{12} is 'principally' obtained from bi-axial compression tests. Usually it is applied F_{12} = - 0.5. The inter-laminar (3D) strength quantity $\bar{R}_{\perp\perp}$ is the result of the formalistic evaluation of the polynomial model. For 2D-applications it is not necessary and thus a determination problem is not given. The FMC does not need this quantity but just the 5 physically necessary measurable strengths.

A difference between Tsai-Wu and Cuntze is seen in *Fig.4-2* where a non-feasible 2D area is predicted, whereas the modal versions of Puck and Cuntze map the course of test data.

Some special comments on the interpolative 'global' SFC of Tsai-Wu:

(1) *The formulation is mathematically elegant*
(2) *For $F_{12} \neq 0$ the predicted bi-axial failure stress values are higher than the strengths $R_{\parallel}^{c}, R_{\perp}^{c}$ in the (σ_1^c, σ_2^c) domain*
(3) Treatment of (σ_2, τ_{21}) like (σ_2, τ_{31}), which is not accurate but model inevitably
(4) Cannot map for instance the $(\sigma_2{}^c, \tau_{21})$-humb in *Fig.3-3*, because the material inherent internal friction cannot be directly considered in the global SFC. Hence, the computed Reserve Factor may not be on the safe side in this domain
(5) For application just strength values are necessary, but this is not sufficient!
(6) No information on the prevailing failure mode FF or IFF is received.

For enabling a direct comparison of the two SFCs, the 2D-Tsai-Wu SFC will be rewritten using the material stressing effort *Eff* = *σ* / *R*

$$\frac{\sigma_1^2 / Eff^2}{\bar{R}_{\parallel}^t \cdot \bar{R}_{\parallel}^c} + \frac{\sigma_1}{Eff} \cdot (\frac{1}{\bar{R}_{\parallel}^t} - \frac{1}{\bar{R}_{\parallel}^c}) + \frac{2F_{12}}{\sqrt{\bar{R}_{\parallel}^t \cdot \bar{R}_{\parallel}^c \cdot \bar{R}_{\perp}^t \cdot \bar{R}_{\perp}^c}} \cdot \frac{\sigma_1 \cdot \sigma_2}{Eff^2} + \frac{\sigma_2^2 / Eff^2}{\bar{R}_{\perp}^t \cdot \bar{R}_{\perp}^c} + \frac{\sigma_2}{Eff} \cdot (\frac{1}{\bar{R}_{\perp}^t} - \frac{1}{\bar{R}_{\perp}^c}) + \frac{\tau_{12}^2 / Eff^2}{\bar{R}_{\perp\parallel}^2} = 1 \, .$$

5 Input for the Derivation Procedure of In-plane (2D) Laminate 'Design Sheets'

In the isotropic case the stress state in the CoS (x,y) reads in structural laminate stresses and associate principal stresses

$$3D: \{\sigma\} = (\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{zx}, \tau_{xy})^T \equiv (\sigma_I, \sigma_{II}, \sigma_{III})^T$$
$$2D: \{\sigma\} = (\sigma_x, \sigma_y, 0, 0, 0, \tau_{xy})^T \equiv (\sigma_I, \sigma_{II}, 0)^T \, .$$

Principal stresses are those components of the stress tensor when the basis is changed in such a way that the shear stress becomes zero. In the case of the transversely-isotropic UD lamina material principal internal stresses have only a practical sense in the quasi-isotropic plane σ_3 (σ_2). In the lamina plane they are not useful. However, using them for the external stresses makes sense in order to enable 2D plots which allow just two variables at the two coordinates.

5.1 External Loading of the Laminate

Loading by Forces: External principal loads (*stresses*)

It is a good idea to work, if possible and reasonable, with the principal normal forces (accurate *principal normal force fluxes*). The corresponding transformation angle δ, which rotates the structural laminate CoS (x,y) in mathematically positive direction into the CoS (x_I, x_{II}), is calculated, so that the principal normal forces of the envisaged laminate building block can be determined from the section forces, see *Fig.5-1*.

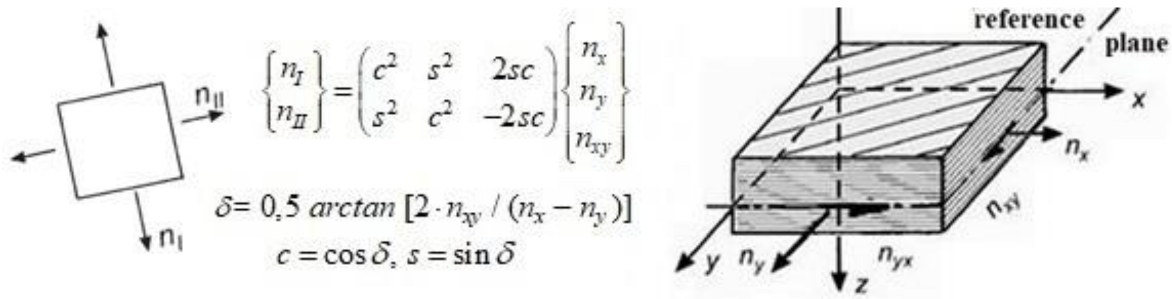


Fig.5-1: (left) Transformation into the principal section forces; (right) Plate designations, coordinates and section forces. t is laminate thickness

Because the results of the Cuntze's modal 2D-SFC set shall be compared with Tsai's global SFC it is to inform about essential CoS differences regarding Tsai's notation, see *Fig.5-2*

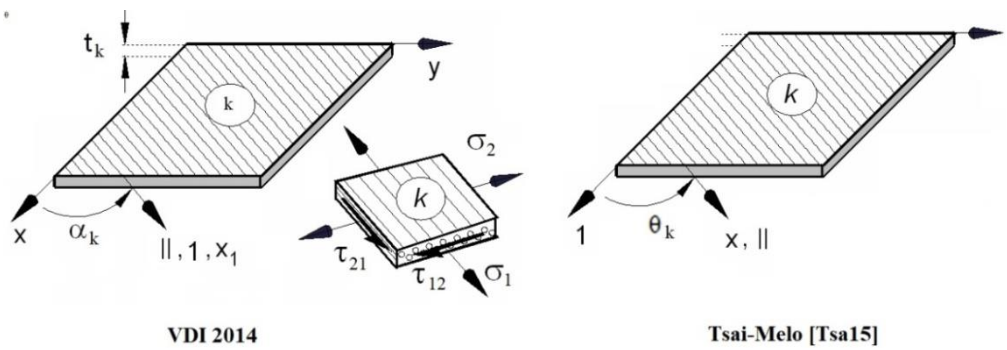


Fig.5-2: The differently applied suffix 1 for the coordinates and the different positive angle direction

Loading by Straining: external principal strains, derived from force loading

Principal strains are determined from $\varepsilon_I, \varepsilon_{II} = 0.5 \cdot [(\varepsilon_x + \varepsilon_y) \pm \sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}\,]$

(*Mind: the engineering shear strain is related to the tensor shear strain by* $\gamma_{xy}^2 = 4 \cdot \varepsilon_{xy}^2$*).*

Strains are proportional to stress quantities if linearity is given.

All required relationships are listed (*t is laminate thickness*) in the following subchapter.

Laminate plate equations:

These equations of the Classical Laminate Theory (*CLT*) read

$$\begin{Bmatrix} n^\circ \\ m \end{Bmatrix} = [K] \cdot \begin{Bmatrix} \varepsilon \\ \kappa \end{Bmatrix} = \begin{bmatrix} A & B \\ B^T & D \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon \\ \kappa \end{Bmatrix} \quad \Rightarrow \text{ no bending: } \{n^\circ\} = [A] \cdot \{\varepsilon\}, \{\varepsilon\} = [A]^{-1} \cdot \{n^\circ\} \ ,$$

where the superscript ^{0} denotes the reference plane and ε, κ represent strain, curvature.

Following *Fig.5-1* one could choose between a loading and a thickness-normalized stressing

$$n_I, n_{II} = 0.5 \cdot \left[(n_x + n_y) \pm \sqrt{(n_x - n_y)^2 + 4 \cdot n_{xy}^2} \right] \text{ (after trigonometric reformulation) or}$$

$$\sigma_I, \sigma_{II} = 0.5 \cdot \left[(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4 \cdot \tau_{xy}^2} \right] \text{ in principal external stresses .}$$

Usually principal stresses and strains are understood as *internal* stresses and strains however these internal principal stress and strains do not have any meaning in the case of anisotropic materials. The required relationships are listed in the following subchapter (*t is laminate thickness*).

5.2 Calculation of Internal Lamina (ply) Stresses and Strains

5.2.1 Relationships of the k[th] Lamina strains and stresses

* In the lamina (ply) CoS:

$$[S]_k = \begin{bmatrix} \frac{1}{E_\parallel} & \frac{-\nu_{\parallel\perp}}{E_\perp} & 0 \\ \frac{-\nu_{\perp\parallel}}{E_\parallel} & \frac{1}{E_\perp} & 0 \\ (symm) & & \frac{1}{G_{\perp\parallel}} \end{bmatrix}_k , \quad [Q] = \begin{bmatrix} \frac{E_\parallel}{1-\nu_{\parallel\perp}\nu_{\perp\parallel}} & \frac{\nu_{\perp\parallel}E_\perp}{1-\nu_{\parallel\perp}\nu_{\perp\parallel}} & 0 \\ \frac{\nu_{\parallel\perp}E_\parallel}{1-\nu_{\parallel\perp}\nu_{\perp\parallel}} & \frac{E_\perp}{1-\nu_{\parallel\perp}\nu_{\perp\parallel}} & 0 \\ (symm) & & G_{\perp\parallel} \end{bmatrix} = [S]^{-1}, \ \nu_{\perp\parallel}\cdot E_\perp = \nu_{\parallel\perp}\cdot E_\parallel$$

$$\{\varepsilon\}_k = \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}_k = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix}_k \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{21} \end{Bmatrix}_k = [S]_k \cdot \{\sigma\}_k , \ \{\sigma\}_k = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{21} \end{Bmatrix}_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{21} \end{Bmatrix}_k$$

where [Q] is denoted 'reduced 3D stiffness matrix' [C].

* In the 'rotated' laminate CoS:

Applying the transformation matrices, $[T_\sigma]^T = [T_\varepsilon]^{-1}, [T_\sigma]_k^{-1} = [T_\varepsilon]$,

$$[T_\sigma]_k = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix}_k , \ [T_\varepsilon]_k = \begin{bmatrix} c^2 & s^2 & -sc \\ s^2 & c^2 & sc \\ 2sc & -2sc & c^2 - s^2 \end{bmatrix}_k \quad \text{using } c = \cos\gamma, \ s = \sin\gamma$$

and using the strain condition $\{\varepsilon'\}_k = \{\varepsilon'\}$ of the k[th] lamina embedded in the laminate stack, the 'rotated' lamina stresses $\{\sigma'\}_k$ can be derived

$$\{\sigma'\}_k = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{yx} \end{Bmatrix}_k = \begin{bmatrix} Q'_{11} & Q'_{12} & Q'_{16} \\ Q'_{12} & Q'_{22} & Q'_{26} \\ (symm) & & Q'_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yx} \end{Bmatrix}_k = [Q']_k \cdot \{\varepsilon'\}_k \quad \text{with } [Q'] = [T_\sigma]\cdot[Q]\cdot[T_\sigma]^T$$

and from them $\{\sigma\}_k = [T_\sigma]_k^{-1} \cdot \{\sigma'\}_k$ as input for the SFC insertion in order to compute *Eff*.

5.2.2 Plane Laminate loading and External Principal Laminate Strains

From the force loadings the external laminate strains are determined

$$\{\varepsilon'\} = [A]^{-1} \cdot \{n\} \rightarrow \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [A]^{-1} \cdot \begin{Bmatrix} n_x \\ n_y \\ n_{xy} \end{Bmatrix} \quad \text{with} \quad [A] = \sum_{k=1}^{n} [Q']_k \cdot \mathrm{t}_k \; .$$

Taking the external principal stresses as loading it reads with *t* as laminate thickness

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [A]^{*-1} \cdot \begin{Bmatrix} \sigma_I \\ \sigma_{II} \\ 0 \end{Bmatrix} \quad \text{with} \quad [A]^* = [A]/t \;\; \text{and} \;\; \begin{Bmatrix} \varepsilon_I \\ \varepsilon_{II} \\ 0 \end{Bmatrix} = [T_\varepsilon] \cdot \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [T_\varepsilon] \cdot \{\varepsilon'\}$$

5.3 Stress-based 2D Strength Failure Criteria of Tsai-Wu and Cuntze

For obtaining the envisaged Design Sheets just the 2D versions of the two stress-strength-based SFCs are of interest. Strain-based SFCs (procedure: strain ε < failure strain *e*) are generally not permitted regarding present authority regulations which require strength design allowables (procedure: stress σ < failure stress = strength *R*).

2D Tsai-Wu: After the insertion of the parameters F_{ij} the reduced 'global SFC: reads

$$\{\sigma\} = (\sigma_1, \sigma_2, \tau_{12})^{\mathrm{T}}, \quad \{\bar{R}\} = (\bar{R}_{||}^t, \bar{R}_{||}^c, \bar{R}_{\perp}^t, \bar{R}_{\perp}^c, \bar{R}_{\perp||})^T,$$

$$\frac{\sigma_1^2 / Eff^2}{\bar{R}_{||}^t \cdot \bar{R}_{||}^c} + \frac{\sigma_1}{Eff} \cdot \left(\frac{1}{\bar{R}_{||}^t} - \frac{1}{\bar{R}_{||}^c}\right) + \frac{2F_{12}}{\sqrt{\bar{R}_{||}^t \cdot \bar{R}_{||}^c \cdot \bar{R}_{\perp}^t \cdot \bar{R}_{\perp}^c}} \cdot \frac{\sigma_1 \cdot \sigma_2}{Eff^2} + \frac{\sigma_2^2 / Eff^2}{\bar{R}_{\perp}^t \cdot \bar{R}_{\perp}^c} + \frac{\sigma_2}{Eff} \cdot \left(\frac{1}{\bar{R}_{\perp}^t} - \frac{1}{\bar{R}_{\perp}^c}\right) + \frac{\tau_{12}^2 / Eff^2}{\bar{R}_{\perp||}^2} = 1$$

For the parameter F_{12} , in order to bypass an open failure surface, the value -0.5 is applied. Here, the *Eff* corresponds to the so-called 'Tsai strength ratio' R.

2D Cuntze:

$$\{\sigma\} = (\sigma_1, \sigma_2, \tau_{12})^{\mathrm{T}}, \quad \{\bar{R}\} = (\bar{R}_{||}^t, \bar{R}_{||}^c, \bar{R}_{\perp}^t, \bar{R}_{\perp}^c, \bar{R}_{\perp||})^T, \;\; \mu_{\perp||}.$$

$$Eff = [(Eff^{||\sigma})^m + (Eff^{||\tau})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp||})^m + (Eff^{\perp\tau})^m]^{m^{-1}}$$

$$Eff^{||\sigma} = \frac{(\sigma_1 + |\sigma_1|) \cdot E_{||}}{2 \cdot R_{||}^t}, \quad Eff^{||\tau} = \frac{(-\sigma_1 + |\sigma_1|) \cdot E_{||}}{2 \cdot R_{||}^c}, \quad Eff^{\perp\sigma} = \frac{\sigma_2 + |\sigma_2|}{2 \cdot \bar{R}_{\perp}^t}, \quad Eff^{\perp\tau} = \frac{-\sigma_2 + |\sigma_2|}{2 \cdot \bar{R}_{\perp}^c}, \quad Eff^{\perp||} = \frac{|\tau_{21}|}{\bar{R}_{\perp||} + 0.5 \cdot \mu_{\perp||} \cdot (-\sigma_2 + |\sigma_2|)}$$

<u>Note</u>: When automatically inserting the FEA stress output $\{\sigma\}$ into the *Eff*-equations some *Effs* may become negative which mechanically means zero *Eff*. In order to make an automatic use of the FMC-based fracture SFCs also in a 3D state of stresses possible and to avoid complicate queries in the computer program absolute values are used in order to avoid a sign query. Due to successful comparison with the 3D-reduced SCF (*suffix 3 dropped*) the 3D-reduced shear failure $Eff^{\perp||}$ could be further simplified to the above Mohr-Coulomb formulation. Negative Effs are physical nonsense and are to make zero. The interaction exponent is taken *m* = 2.6. For the friction value the same value is inserted for all materials with $\mu_{\perp||} = 0.2$.

A reminder for the numeric procedure:

Determination of material Stressing Effort $Eff \neq 1$: $Eff = [\Sigma\, (Eff^{\text{modes}})^m]^{m^{-1}}$

Determination of failure curve, surface of failure body $Eff = 1$: $1 = \Sigma\, (Eff^{\text{modes}})^m$.

6 Establishment of the Program and Material Data Input

A failure strain envelope seems to give a better normalized result for all laminate stacks than a failure stress (strength) envelope. Within the obtained FPF-envelope an internal area is obtained, which seems to be generally representative for each chosen stack and CFRP material. From this finding design sheets can be derived.

6.1 Loading from principal external loads or - normalized – from principal stresses

For the achievement of the stress-based SFC linked FPF strain failure envelope the steps include loops over the 3 load components n_i in the domain $-1 < n_i < 1$ or for simplification loops over the 2 principal loads n_I, n_{II} or the 2 principal stresses σ_I, σ_{II} (*is a normalization*), respectively. The visualization of the calculation results can be performed in $\varepsilon_{II}(\varepsilon_I)$-plots or in $\sigma_{II}(\sigma_I)$-plots.

Visualization in external principal strains $\varepsilon_{II}(\varepsilon_I)$ *by* $\begin{Bmatrix} \varepsilon_I \\ \varepsilon_{II} \end{Bmatrix} = \left[T_\varepsilon^{-1}\right] \cdot \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \mathrm{f} \begin{Bmatrix} \sigma_I \\ \sigma_{II} \end{Bmatrix}$.

6.2 Solution Procedure to Determine Failure States

The procedure takes a single layer, $k = 1$, under arbitrary load ratios into account

$$\text{No bending: } \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \{\varepsilon'\} = [A]^{-1} \cdot \{n^\circ\} = [A]^{-1} \cdot \begin{Bmatrix} n_x \\ n_y \\ n_{xy} \end{Bmatrix} \quad \text{with} \quad [A] = \sum_{k=1}^{n} [Q']_k \cdot \mathrm{t}_k$$

$$\{\sigma'\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q'_{11} & Q'_{12} & Q'_{16} \\ Q'_{12} & Q'_{22} & Q'_{26} \\ (symm) & & Q'_{66} \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [Q'] \cdot \{\varepsilon'\} \quad \text{with} \quad [Q'] = [T_\sigma] \cdot [Q] \cdot [T_\sigma]^T$$

$$\text{and finally from} \quad \{\sigma\} = [T_\sigma]^{-1} \cdot \{\sigma'\}$$

the lamina stresses as required input for the SFC insertion are obtained in order to iteratively compute for $Eff_{\text{FPF}} = 1$ the *failure stress* state $\{\sigma\} \Rightarrow \{\sigma\}_{\text{FPF}}$. From these values the failure loading state can be derived and the associated principal loading strains. Taking the external principal stresses as loading it reads with *t* as laminate thickness

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [A]^{-1} \cdot \begin{Bmatrix} \sigma_I \\ \sigma_{II} \\ 0 \end{Bmatrix} \quad \Rightarrow \begin{Bmatrix} \varepsilon_I \\ \varepsilon_{II} \\ 0 \end{Bmatrix} = [T_\varepsilon] \cdot \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [T_\varepsilon] \cdot \{\varepsilon'\}$$

or with Tsai's normalized stiffness matrix $[A]^* = [A]/t$.

For the envisaged lamina the following steps are to go for each principal loading ratio (*force, strain preferred*). Before all steps - as guiding parameter input - the determination of the relationship of the forces-representative principal strains is to perform.

Then it is to perform:

- Take the external lamina (ply) principal strains (*laminate, a single ply k =1*) ε_I, ε_{II} as varying representatives of the force loading and as coordinates of the envisaged graph 'non-FPF area'
- Determine values of Eff^{modes} for each lamina, oriented under the angle α, and of the strain ratio angle ξ, regarding *Fig. 6-2*
- Determine FPF failure strains $\varepsilon_{I,\text{FPF}}, \varepsilon_{II,\text{FPF}}$ from applying

$$Eff_{\text{FPF}} = [(Eff^{\parallel\sigma})^m + (Eff^{\parallel\tau})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\parallel})^m + (Eff^{\perp\tau})^m]^{m^{-1}} = 1$$

- For the all the *i* (ξ,α)-combinations compute from $Eff_{\text{FPF},i}$ the factor $f_{RF,\,i} = 1/Eff_{\text{FPF},i}$
- Store data and determine strain FPF-envelope points and map the full envelope.

Fig.6-1: Derivation of the non-FPF area

Fig.6-2 visualizes the ***i*** combinations of (ξ, α) to be executed. 361 strain states are evaluated and the corresponding material reserve factors $f_{\text{RF, i}}$ are stored. The smallest reserve factor determines the FPF limit and one point on the envelope.

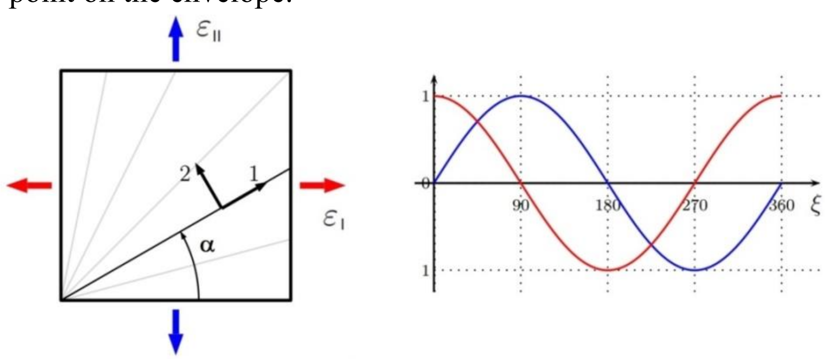


Fig.6-2: FPF procedure for each ply-orientation 0° ≤ α ≤ 90° and principal strain loading ratio angle ξ

6.3 Test Data Sets for the building blocks 'Quad' and 'DD'

Guided from another investigation Cuntze could sort out, that carbon fibers – due to the graphitization-caused stiffness size – and thereby CFRP could be divided into three stiffness grades: Standard PAN-CFRP, UHM PAN-CFRP and mPitch-CFRP. Of practical interest is whether essential differences will arise between the low CFRP grade and the UHM CFRP grade.

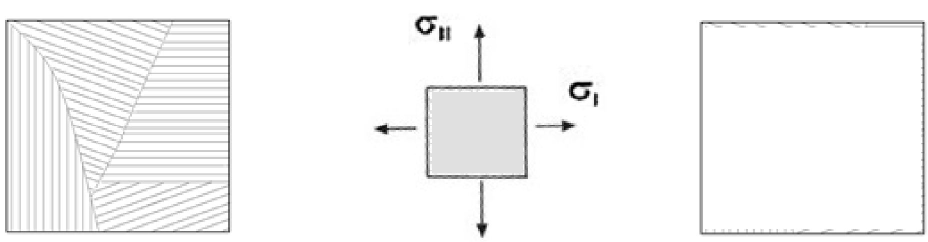
Table 6-1 includes the set of materials treated in this document. Here, available average values have been used as it is a comparison of materials. For UMS, a full public data set was not available and missing properties had to be estimated, which might be sufficient for this elaboration.

Table 6-1: Average properties of applied FRP materials used

Fibers	$E_{\parallel}$ GPa	$E_{\perp}$ GPa	$G_{\parallel\perp}$ GPa	$\nu_{\perp\parallel}$	$\bar{R}_{\parallel}^{t}$ MPa	$\bar{R}_{\parallel}^{c}$ MPa	$\bar{R}_{\perp}^{t}$ MPa	$\bar{R}_{\perp}^{c}$ MPa	$\bar{R}_{\perp\parallel}$ MPa	$\mu_{\perp\parallel}$
1 Toray T300/Ep, 7	135	5.6	1.3	0.32	1850	1470	40	125	95	0.2
2 T800/ Cytec, 7	162	9.0	5.0	0.32	2700	1570	63	145	98	0.2
3 IM7/ 977-3, 7	191	9.9	7.8	0.35	3250	1600	62	98	75	0.2
4 T700/M21GC, 7	126	8.3	4.1	0.3	2230	1537	71	202	78	0.2
5 Toray M60J, 5µm	365	6	4	0.3	2010	785	32	168	>39	0.2
6 E-glass/MY750,	46	16.2	5.8	0.28	1280	800	40	145	73	0.2

6.4 Programming of the two Criteria Tsai-Wu and Cuntze

Usual Task: Search of the most critical material location in the critical lamina of a stack. Viewing just one building block (*a prepreg or a stitched NCF*) of a laminate the loading state this task is presented in the *Fig.6-3* for two stack or lay-up families:



'Quad' [0/45/90/-45] (prepreg) DD $\{\varphi/\text{-}\psi/\text{-}\varphi/\psi\}$ (stitched NCF, e.g. C-ply™)

Fig.6-3 Building block: (left) Quad (minimum fiber orientation angle difference of 45°); (right) C-ply™ with other C-ply Bild fehlt noch Eps or Sigs schreiben

Tsai's idea was to derive on basis of a generally loaded single ply a strain-based non-FRP area and using this area to check whether the principal strains of the critical lamina (ply) of a designed laminate lies within this area. Such an application works for all lay-ups. The procedure could be seen as theoretically determined FPF failure stress states using normally average strength properties. However, here designing is the task and strength Design Allowables *R* are to apply.
The application of these force loading-representative principal strains seems to be more generally practicable.

For three UD plies out of an arbitrary stack *Fig.6-4* presents the associate 3 FPF principal strain envelopes according to the associated principal FPF-stresses. This means that the failure strains are elastically derived from the failure stresses. In the figure some principal stress state points $(\sigma_{I}, \sigma_{II})$ are attached onto the principal strain state points curve $\varepsilon_{II}(\varepsilon_{I})$.

In the isotropic case the magnitude of the stress normal to the principal plane (at zero shear stress) is termed principal stress and the associated strain is called principal strain. In the cases of anisotropy this does not work anymore.

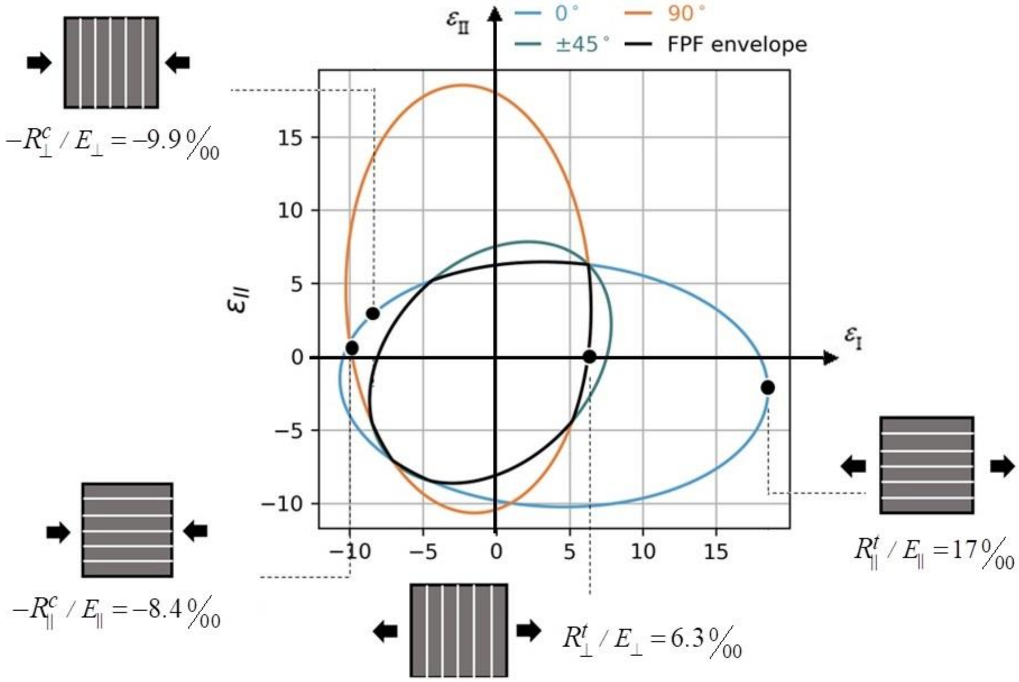


Fig.6-4, FPF, Tsai-Wu: FPF-envelopes Eff = 100% of 3 single UD-laminas under 4 different stress states potentially leading to FPF in terms of FPF failure stresses-linked equivalent principal strains. ε in ‰. IM7/ 977

The internal area of the 3 plies (0°,45°,90°) can be termed non-FPF failure area and is limited by an envelope which was termed 'Omni-failure envelope' by Tsai. This area becomes a general one if all *i* combinations are treated and is the focus now.

6.5 'Omni-failure envelopes' embedding a 'Non-FPF area'

In the following chapter some different CFRP materials are investigated and one GFRP. The obtained results are using the 2D-SFCs of Tsai-Wu and Cuntze.

Fig.6-5 presents the numerical results of the FPF-linked principal strain curves. The associate Tsai-Wu envelope has been implemented and shows a significant effect of the strength failure criterion (SFC) used. The different lateral properties determine the shape of the obtained symmetrical 'butterfly'.

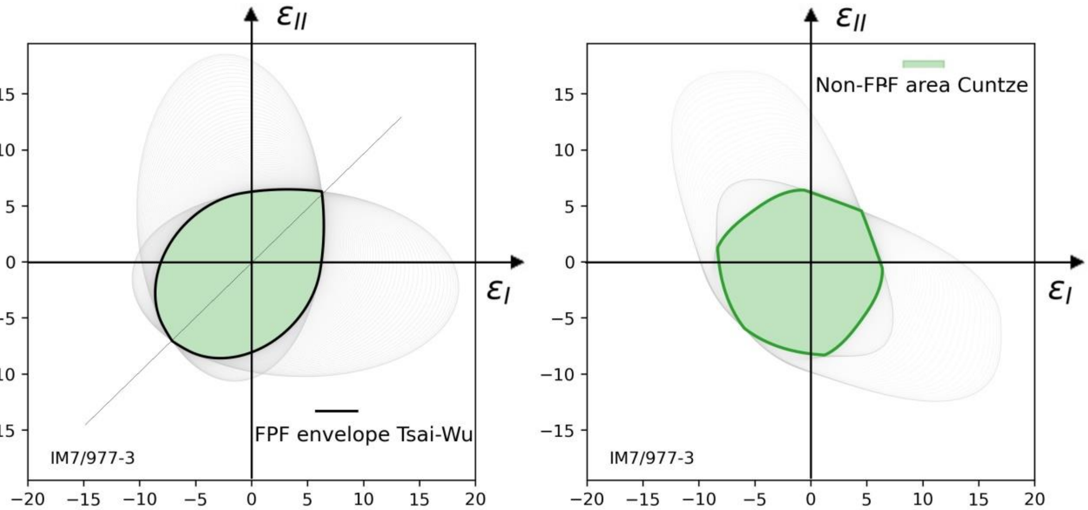


Fig.6-5 Bundle of all FPF envelopes = 'butterflies': All ply FPF-envelopes enclosing a non-FPF failure area; 0°< α < 90° (91 ply angles). Principal strain in ‰, suffix FPF is skipped. CFRP IM7/977-3. In all pictures: (left) Tsai-Wu with $\mu_{\perp\parallel} = 0,\ F_{12} = -0.5$ *and (right) Cuntze with* $\mu_{\perp\parallel} = 0.2,\ m = 2.7$ *.*

principal strain curves. *Fig.6-5* displays different 'butterflies' (*name, how the author Cuntze termed the bunch of **i** FPF-curves,* derived with the SFCs of Tsai-Wu and Cuntze).

Fig.6-6 presents the 'butterflies' of two CFRP. The 'Non-FPF area' due to the two SFCs is pretty different. This might be caused by the different mapping quality potential of the SFCs.

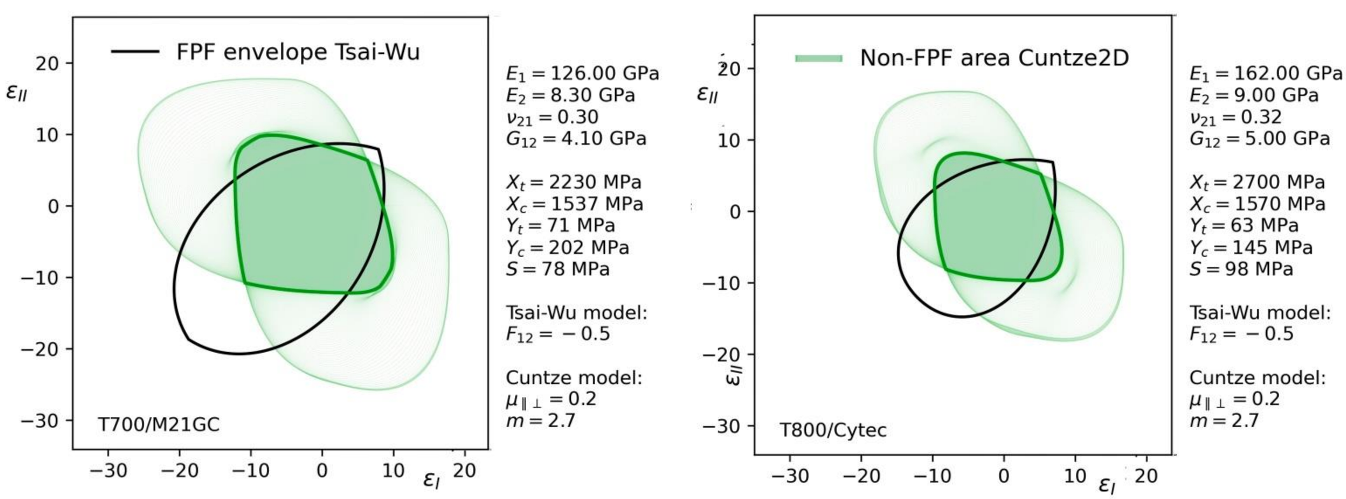


Fig.6-6 FPF envelope–butterflies' of 2 CFRPs, Tsai-Wu's 'Omni failure envelope' (black) with Cuntze's Non-FPF-area (green): (left)T700/M21GC and (right)T800/Cytec, ε in ‰

The following part figures in *Fig.6-7* show the butterflies of two more CFRPs and finally *Fig.6-8* presents the very large 'butterfly' of a GFRP.

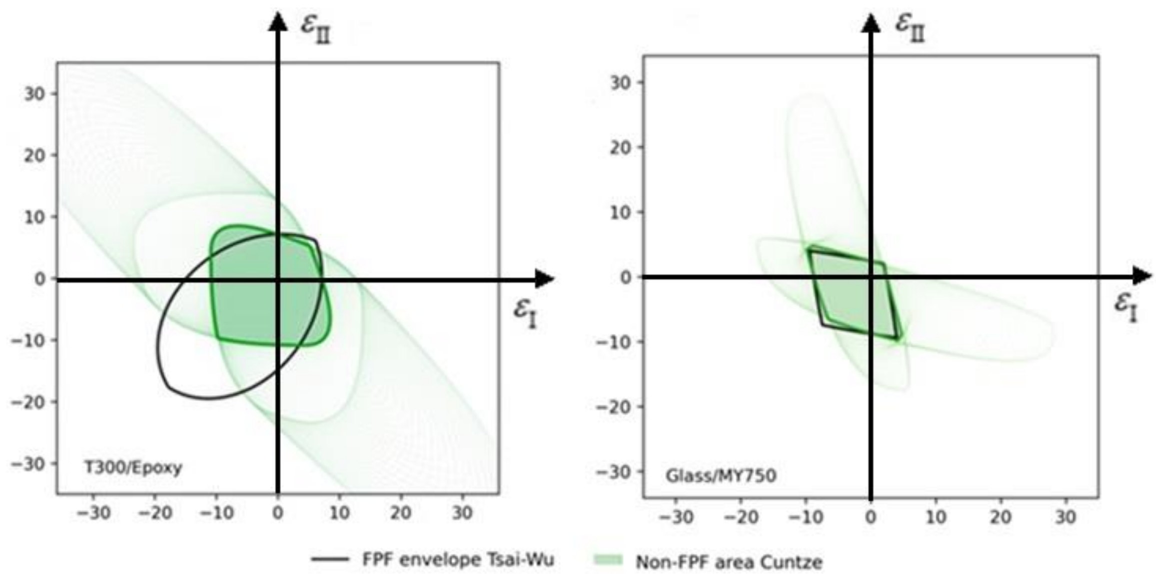


Fig.6-7 'FPF envelope–butterflies', Tsai-Wu versus Cuntze: (left) T300/Ep and (right) ***E****-Glass /MY750*

Meanwhile E. Kappel succeeded with another determination of the 'Omni-failure envelope'. Also the other author R. Cuntze sorted out that a function for the diagonally symmetric envelope can be achieved (*however no clear single numerical solution is given, but must be found by queries*), simplifying the procedure and directly delivering the reserve factor f_{RF}. [*Cun24*]. The reason to do this was the finding that the stress state ($\sigma_1, \sigma_2, \tau_{21} = 0$) characterizes the worst scenario.

Of high interest is the shape of the Ultra-High-Modulus M60J/Ep 'butterfly'. The very stiff fibers turn the shape in *Fig.6-8* to the principal strain coordinates.

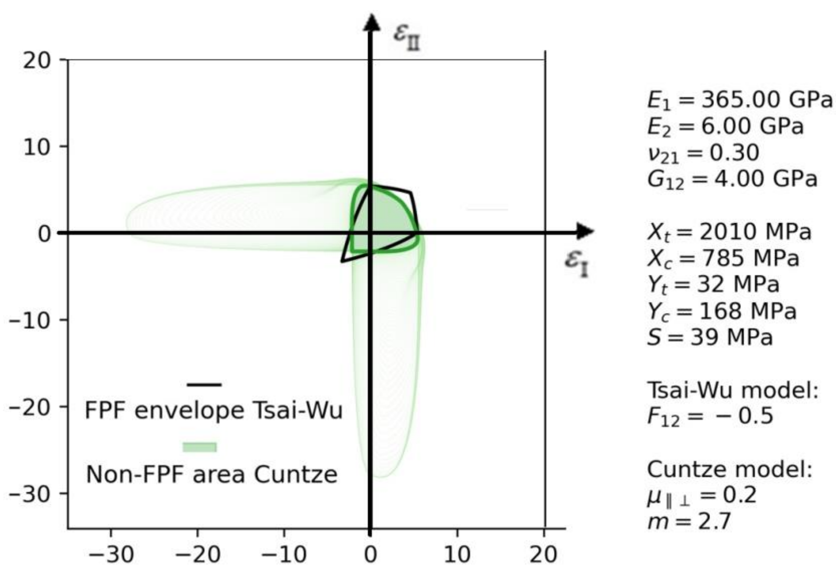


Fig.6-8: 'FPF envelope–butterflies', Tsai-Wu envelope and 'Non-FPF area Cuntze

7 Application of the Non-FPF area within the 'Omni failure envelope' by a Design Sheet

7.1 General on Applicability limits of SFCs and Use of Analysis Results

Fabrication:

Before any application of a UD-SFCs some pre-requisites and validity limits are to check to really achieve a *reliable Design Verification process*:

- Good fiber placement and alignment
- '*Fabrication signatures'* such as fabrication-induced fiber waviness and wrinkles are small and do not vary in the test specimens

Validity Limits for the applicability of SFCs:

- The UD-lamina is homogenized to a macroscopically homogeneous solid or the lamina is treated as a 'smeared' material
- The UD-lamina is transversely-isotropic: On planes transverse to the fiber direction it behaves quasi-isotropically
- For validation of the model a uniform stress distribution about the critical stress 'point' location is mandatory.
- One can conclude that laminates usually have smaller CoVs. This is due to the favorable compensation of the effect of the flaws across the laminate thickness
- When applying test data from 'isolated lamina' test specimens (*like tensile coupons*) to an embedded lamina of a laminate one should consider that coupon test deliver tests results of 'weakest link' type. An embedded or even an only one-sided constrained lamina, however, possesses redundant behavior
- A SFC usually describes only a one-fold occurrence of a mode or of a failure mechanism, respectively!
- As a SFC is a necessary but not a sufficient condition to predict failure *[Leg02, Wei15]* a fracture mechanics-based energy condition may be to fulfill, too. Even in plain (smooth) stress regions a SFC can be only a necessary condition which may be not sufficient for the prediction of 'onset of fracture', i.e. the in-situ lateral strength in an embedded lamina. Example: thick layers fail earlier than thin ones under the same 2D stress state see e.g. Due to being strain-controlled, the material flaws in a *thin* lamina cannot grow freely up to micro-crack size in the thickness direction (*this is sometimes called 'thin layer effect'*), because the neighboring laminas act as micro-crack-stoppers. Considering fracture mechanics, the strain energy release rate, responsible for the development of damage in the 90° plies from flaws into micro-cracks and larger, increases with increasing ply thickness. Therefore, the actual absolute thickness of a lamina in a laminate is a driving parameter for initiation or onset of micro-cracks, i.e. [*Fla82*].

Delamination and Edge Effect:

Delamination within a laminate may occur in tensile-shear cases and compression-shear cases (*remember the so-called wedge failure of Puck with its inclined fracture plane* [*VDI97*]). Considering such a delamination a 3D stress state is to regard. This is especially the case if bends in the structure are stretched or compressed which also generates stresses across the wall thickness, too. These stresses are activated by the delamination-critical stresses including inter-laminar stresses (index $_3$): $\{\sigma\}_{\text{lamina}} = (0, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$. Delamination is a failure of the 'low-scale structure' laminate.

At the edges delamination is termed edge effect. Within the laminate it can be predicted by the application of the inter-laminar stresses-associated 3D-SFCs. At the edges it is – due to the stress singularity – a task of fracture mechanics tools using a cohesive zone model [*Wei15*].

7.2 *Procedure according to a Design Sheet substantiated by a Numerical Example*

The procedure to determine strain-based 'Omni-failure envelopes' takes a single layer, k = 1, under arbitrary load ratios into account.

7.2.2 In-plane Loading with Bending

$$\begin{Bmatrix} n^\circ \\ m \end{Bmatrix} = [K] \cdot \begin{Bmatrix} \varepsilon \\ \kappa \end{Bmatrix} = \begin{bmatrix} A & B \\ B^T & D \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon \\ \kappa \end{Bmatrix} \quad \rightarrow (\varepsilon_x, \varepsilon_y, \gamma_{xy})_{\text{load case}} \Rightarrow (\varepsilon_I, \varepsilon_{II})_{\text{load case}}$$

In the case of bending the maximum strained lamina is to search and its principal strains to determine. The procedure is identical to the following full membrane loading case.

7.2.2 In-plane Loading

The forces-strain relation can be formulated as follows: (see *§ 5.2.1*)

$$\text{No bending:} \quad \{\varepsilon'\} = [A]^{-1} \cdot \{n^\circ\} \quad \text{with} \quad [A] = \sum_{k=1}^{n} [Q']_k \cdot \mathrm{t}_k$$

From this at first, as guiding parameters which represent the loading,

the external principal strains are derived $\begin{Bmatrix} \varepsilon_I \\ \varepsilon_{II} \\ 0 \end{Bmatrix} = [T_\varepsilon] \cdot \{\varepsilon'\}$, and then their maximum values

which are reached at FPF. These values are achieved after determination of the 'rotated' lamina stresses

$$\{\sigma'\} = [Q'] \cdot \{\varepsilon'\} \quad \text{with} \quad [Q'] = [T_\sigma] \cdot [Q] \cdot [T_\sigma]^T$$ and then calculating the

decisive lamina stresses $\{\sigma\} = [T_\sigma]^{-1} \cdot \{\sigma'\}$

required as input for the insertion into the FPF-SFC in order to compute for $Eff_{\mathbf{FPF}}$ =1 the *failure stress* state $\{\sigma\}_{\text{FPF}}$. From these values the failure load level can be derived and the associated activated principal loading strains $(\varepsilon_I, \varepsilon_{II}) \Rightarrow (\varepsilon_I, \varepsilon_{II})_{\text{FPF}}$.

For pre-design it would be sufficient to check whether the FEA-obtained principal strain point $\Rightarrow (\varepsilon_I, \varepsilon_{II})_{\text{load case}}$ lies within the envelope. This has been performed in *Fig.7-1* for a 3-ply stack. In *Fig.7-1* for three single plies the FPF *failure strain envelopes* are displayed. Four 'loading' points are added to visualize some uni-axial failure stress-based principal strain points $(\varepsilon_I, \varepsilon_{II})$ on the FPF-envelopes. The right part of the figure presents the area which is free of FPF (intact), termed 'Omni failure envelope' by Tsai. In addition, for a chosen load level in order to outline the different reserves a strain-based material Reserve Factor f_{RF} are marked. The Reserve Factors are given by the vector length ratio = *failure point value* divided by the *load point value*. According to the assumed linearity load-strain or stress-strain the load-defined *RF* can be determined linearly and reads $RF = f_{\text{RF}}$.

Table 7-1: Design Verification procedure in short

(1) Determination of the principal strain state of the critical lamina of the laminate applying structural analysis with Classical Laminate Theory
(2) Proof that the principal strain state lies within the 'Non-FPF area'. Otherwise redesign.

Challenge is the automatic determination of a number for the material reserve factor f_{RF}.

For three plies the individual FPF-envelopes are found in *Fig.7-1* . This figurevisualizes for a given principal strain loading the computation procedure to get the ratio of the two vectors (length $\sqrt{\varepsilon_I{}^2 + \varepsilon_{II}{}^2}$), the loading vector and straightly elongated the FPF-envelope vector. Thereby one has to keep in mind "The procedure is based on linear-elasticity".

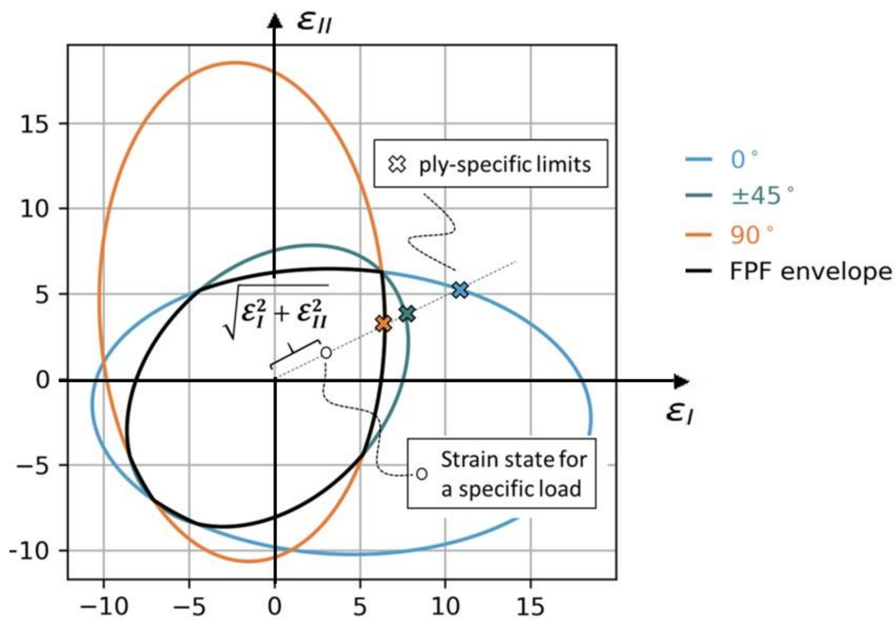


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Fig.7-1 FPF: FPF-strain envelopes of 0°, 90°, ± 45° plies with a chosen lamina design load point • and associate FPF points. ε in ‰, IM7/ 977-3

Determining the material reserve factor in a critical location of a distinct ply of the laminate for a specific load state requires the comparison of the actual principal strain state with the ply-specific limits, as *Fig.7-2* illustrates. The process is not straightforward for arbitrary envelope shapes, as an intersection point of a scaled load state with the envelope needs to be determined. *Fig.7-2,left* illustrates the difficulty. Of course, in consequence a simplification was searched.

The idea of Tsai-Melo is to use the equivalent radius *r* of an 'Omni failure envelope-internal circle' as so-called 'Unit Circle Criterion'. Mind, please: Using the radius $r = \sqrt{Non - FPFarea}$ as an area-equivalent circle will violate the basic requirement for the radius choice 'Remaining on the safe side in Design Verification'. Meanwhile all the various fitting proposals apply as circle origin (0, 0), *Fig.7-2,right*.

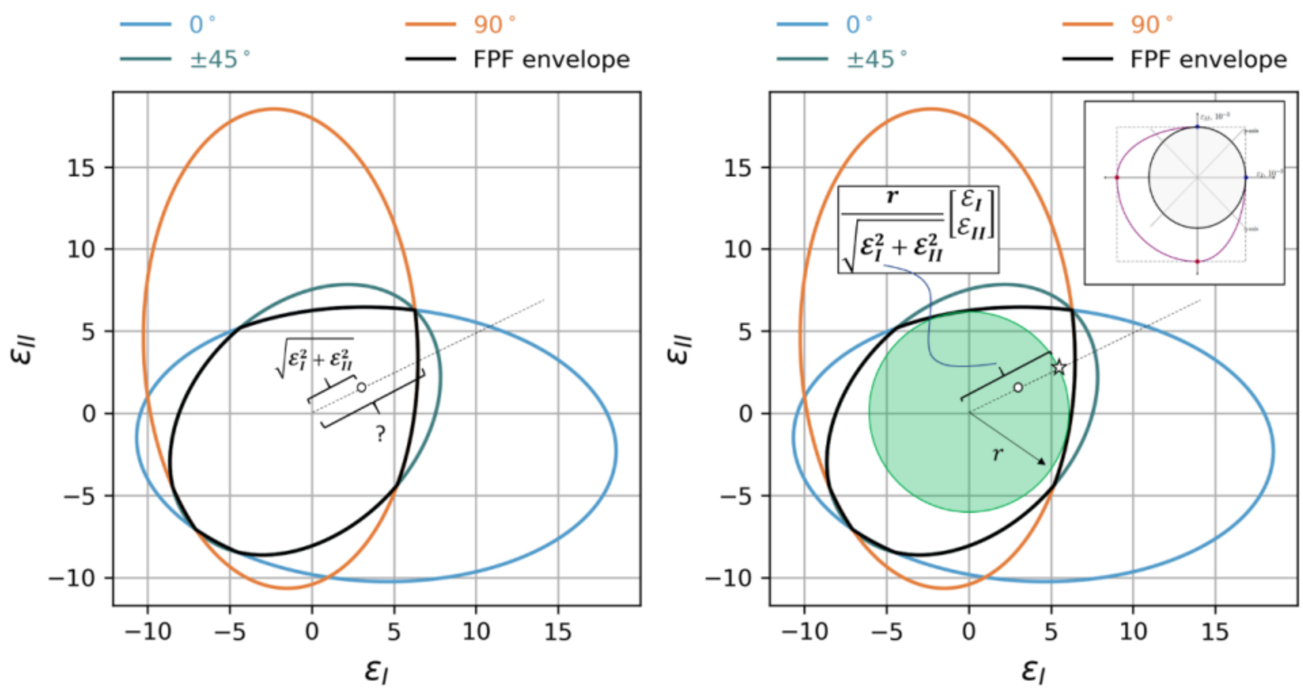

Fig.7-2 FPF: FPF-strain envelopes of 0°, 90°, ± 45° plies with (left) a chosen lamina design load point • and an associate FPF –envelope point. ε in ‰, IM7/ 977-3, (right) Display of the Tsai-Melo circle radius r. The bold black line is envelope surrounding the Non-FPF area

Tsai and Melo proposed the unit-circle criterion (UCC) as an approximation of the complex envelope shape.

Nettles proposes his Nettles-circle (NC) as a simplification of the UCC (see [*Kap22b*]). Its radius is defined by the tensile-anchor point of the envelope $r_{NC} = |(\varepsilon_1, 0)|$. Introducing the NC simplifies the strain-state assessment. *Fig.7-2* shows the NC in green color. The comparison of the NC radius and the current strain-state magnitude allows for the direct determination of the material reserve factor $f_{RF} = \frac{r}{\sqrt{\varepsilon_I^2+\varepsilon_{II}^2}}\begin{bmatrix}\varepsilon_I\\ \varepsilon_{II}\end{bmatrix}$.

The resulting radius r_{NC} = ?.?‰.

= A similar strategy can be applied for the Cuntze Non-FPF areas. However, this requires a little modification of the process. The inner circle in a Cuntze Non-FPF area requires the determination of the minimum inner radius. Therefore, the minimum of all radii is used, which is determine form all k points of the area circumference, recognized in the modification $r = \min\left(\left[\sqrt{\varepsilon_I^2+\varepsilon_{II}^2}\right]_k\right)$ and the resulting radius is r_{CC}= 5.8 ‰.

Fig.7-3 hereafter visualizes the process.

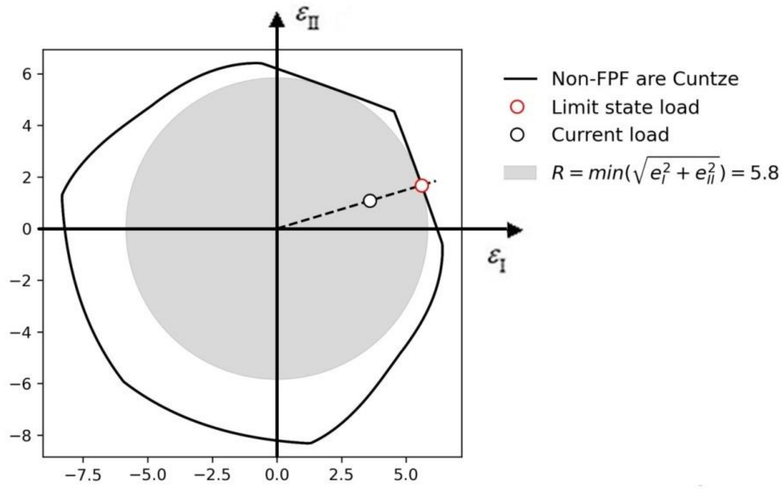

Fig.7-3: Inner circle in Cuntze's Non-FPF area.ε in ‰, IM7/ 977-3

7.3 Collection of four obtained 'Non-FPF areas' of Tsai-Wu and Cuntze

The full variety of 'Omni failure envelopes' of all the investigated materials are compiled in this chapter. All these 'Omni (principal strains) failure envelopes' surround a 'Non-FPF area'. They can serve as a basis for an associate 'Strength Pre-Design sheet'. Fig.7-4 depicts the NON-PDF areas for two 'better' CFRP materials.

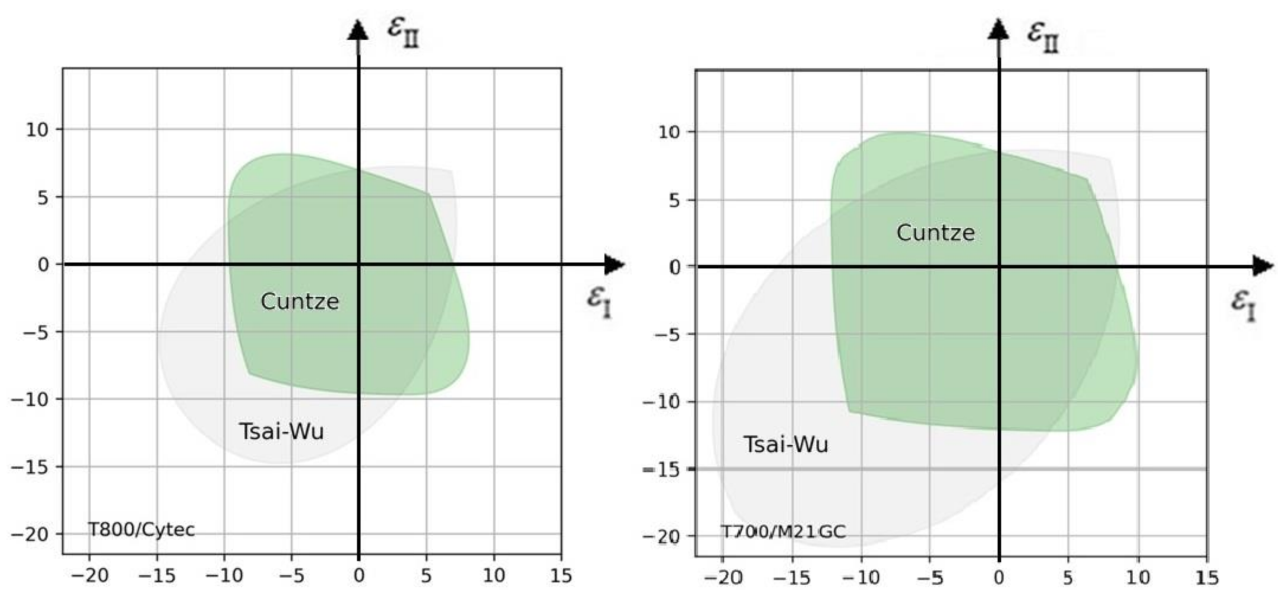

Fig.7-4 'Non-FPF area' of two UD materials, Tsai-Wu (grey) versus Cuntze (green): (left) T800/Cytec, (right) T700/M21GC, ε in ‰

The difference of the shapes in *Fig.7-5*, standard modulus CFRP with GFRP, seems to come from the fact that the GFRP is less anisotropic. It is further obvious that the difference Tsai-Wu to Cuntze becomes smaller with decreasing anisotropy as it is the case with GFRP.

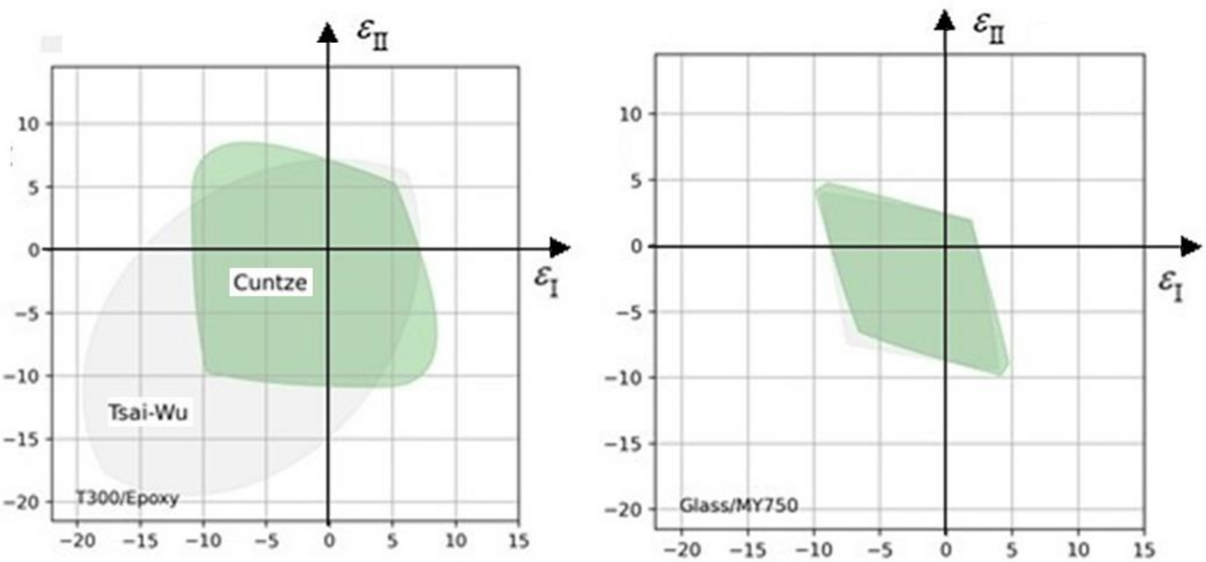

Fig.7-5 'Non-FPF areas', Tsai-Wu (grey) versus Cuntze (green): (left) T300/Ep, (right) Glass/MY750, ε in ‰

Finally *Fig.7-6(left)* comprises the Non-FPF areas of five materials and *Fig.7-4 (right)* intentionally provides for comparison reasons the area of a very stiff CFRP. Drawing the right conclusions here is a task that still needs to be done later.

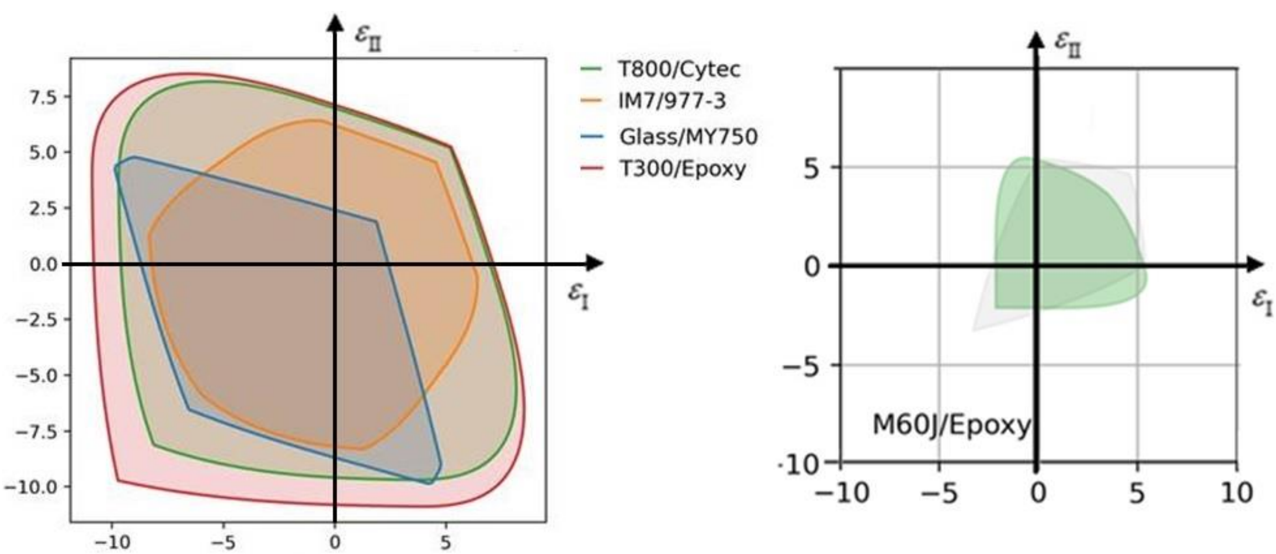

Fig.7-6 'Non-FPF areas: (left Cuntze) Compilation T300+ IM7 +T800 + glass, (right Tsai-Wu (grey) with Cuntze (green)) very stiff PAN-UHM CFRP (Toray M60J/Ep); ε in ‰

7.4 General Conclusions and Specific ones on Cuntze's SFC set

The more than 50 year's old 'global' Tsai-Wu strength criterion needs not to be assessed here more further, just conclusions on the half that old 'modal' Failure-Mode-Concept (FMC)-based one shall be listed. This knowledge is necessary for the interpretation of the application results of the Non-FPF area or the enveloping 'Omni failure envelope', respectively, used in a Design Sheet:

- The FMC is a material symmetry-driven, invariant-linked basis to optimally generate SFCs

- FMC-based ‘modal’ SFCs deliver a combined formulation of independent modal failure modes, without facing the shortcomings of ‘global’ SFC formulations, which mathematically combine *in-dependent* failure modes. A SFC just describes a 1-fold occurring failure mode or mechanism
- Failure envelopes are not just an empirical fit through uniaxial tensile and compressive strength points as it was still assumed in the WWFE-I, –II and further [*Kad13*]! Friction is acting.
- The determination of model parameters is to perform by mapping test data in each pure failure domain, and of the interaction exponent *m* by mapping the transition zone between the modes. A good guess is $m = 2.7$ for all mode transition domains and all material families
- The experience of Cuntze shows: Similarly behaving materials possess the same shape of a fracture body and the same *F* can be used
- The use of the entity ‘material stressing effort’ *Eff* excellently supports ‘understanding the multi-axial strength capacity of materials’. 3D-compression stress states have a higher bearing capacity, but the value of *Eff* nevertheless remains at 100%. This has nothing to do with an increase of a (uniaxial) technical strength *R* which is the result of a Standard-fixed, generally welcomed common agreement that offers the chance to compare materials!
- The size of each $Eff^{\text{mode}} = \sigma_{eq}^{\text{mode}} / R^{\text{mode}}$ informs the designing engineer about the mode’s failure importance, thereby giving a hint which mode is the critical one for the redesign. Clear equivalent stresses $\sigma_{eq}^{\text{mode}}$ can be calculated for a modal SFC!
- Effective strengths of embedded laminas depend on ply-‘thinness’ (*advantage of the thin* C_{ply}) and stress rate. Beyond IFF the embedded ply, strain-controlled by the vicinity, still contributes to the strength and stiffness capacity. In this context, a SFC is a necessary but not a sufficient condition to predict failure, a fracture mechanics-based energy condition may be to fulfill.

7.5 ‘Omni (principal strain) failure envelope’ viewing SFC-Differences Tsai-Wu $\Leftrightarrow$ Cuntze

Tsai was right with “*As we will see, such simplicity does not exist for failure envelopes in stress space. Each laminate must be evaluated individually. It is remarkable that the envelopes in (principal) strain space do not vary very much among the CFRPs*”.

There are natural differences of the depicted two 3D-SFCs which cannot become fully obvious in the here tackled 2D loading case. However, also here differences of the Non-FPF areas are recognized. These may basically depend on a not sufficiently well mapping of test results in the compressive domain $\sigma_2^c(\sigma_1^c)$.

These differences affect the quality of using the ‘Omni failure envelope’ as a desired valuable basis for a practical ‘Strength Pre-Design Sheet’ for instance for the Airbus Aerospace Handbook [*HSB*].

7.6 Application of the ‘Omni (principal strain) failure envelope’ in Strength Design Sheets

Central aim of this investigation was the generation of a ‘Strength Pre-Design Sheet’.

The significant results are collected by the following lines:

- With the FPF-stress-based failure a generic basis is obtained for a distinct composite material which covers all its potential laminate stacks containing all possible ply orientations. The internal 'Omni intact area' covers that 'Non-FPF area', where all laminates are not FPF-failed.
- The use of the radius *r* of the internal circle within the 'Non-FPF area' gives a conservative assessment of the load-determined principal strains computed for each load case.

Final conclusion:
Only a validation-qualified SFC material-model leads to a reliable 'Non-FPF area' and thereby determines the quality of the Reserve Factor number, when using the 'Non-FPF area' as a 'Strength Design Sheet'. A validated SFC is always a standard precondition of any reliable Design Verification with and without using a 'Strength Design Sheet'.

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