Tackling Uncertainties in Design
- uncertain design parameters, safety concept, modelling, analysis –

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Modern light-weight structures are the result of an optimisation compromise between all the product's functional requirements such as stiffness, strength and the operational requirements such as lifetime. Design driving are the material properties and the failure conditions for fracture and yielding. Responsible for the quality of the structure achieved under minimum mass requirement are a qualified analysis procedure, the input of reliable data including the dimensioning load cases and the safety concept. At present, usually a deterministic safety concept is applied that employs factor of safety values to implement reliability in the structure. Special task of the designer is the development of a so-called robust structure that does not essentially change its behaviour under the usual scatter of the stochastic design parameters. Thereby, the aerospace engineer relies on the existence of qualified processes such as qualified analysis, test, manufacturing and NDI procedures as pre-requisites for the application of a safety concept. Their goodness has to be checked and information carefully monitored.

1. Problem Description

Industry looks for robust, reliable (uncertainty-tackling) prediction methods in order to replace the expensive 'Make and Test Method' as far as reasonable. Such methods are required for the full process chain where design and design verification are an essential part of (*Fig. 1*). The designers task is to sort out weakest links in the design process which involve highest uncertainty. Just then a qualified prediction is achievable. Pre-conditions for this achievement are: Excellent technical specifications of the to be developed structural product, a consistent design philosophy incl. safety concept, margins, nonlinear analyses; accurate modelling, strength design allowables, $\sigma - \varepsilon$ curve, testing and test data measurement and evaluation, geometry, choice of yield condition and fracture conditions, structural analysis procedure, damage detection and damage assessment, imperfection monitoring, and finally the determination of the margin of safety value itself. Simultaneous Engineering and a practical risk judging are mandatory. The final design result must be reliable.

However, there are a lot of uncertainties to be tackled in order to succeed. Uncertainties can be found in all of the areas above, recalling analysis, the whole modelling process, and manufacture. The nature of uncertainty of scattering design parameters might be of *mechanical* type (e.g. solution procedure, meshing as well as in the results provided by testing, evaluation of 'raw test data') but also of *statistical* type such as with the measurements performed, the lack of accurate information due to insufficient sample size in measurements of a specific design parameter, and limited observations or tests used when estimating the statistical distributions for stochastic modelling.

All these uncertainty sources contribute to the overall *Structural Risk* defined here arbitrarily as '*amount of costs* (incurred in the case of later failure) *times the probability that the distinct failure occurs in the structural part*'.

The new German collaborative research centre SFB 805 has set up a working hypothesis for load-carrying structures in mechanical engineering: "Uncertainty comes up when the process attributes of such a structure cannot be fully determined." In order to achieve more certainty and thereby higher structural reliability it must be transferred the lack of knowledge or doubtfulness or uncertainty into a *stochastic* uncertainty because then, a quantitative assessment of a load-carrying situation is possible.
Processes, applied from concept phase in the development till the manufacturing process determine the structural product's attributes. Operational processes turn out the behaviour of the structure. The assessment of the effect of uncertainties and of scatter is an essential task for increasing structural reliability and guaranteeing structural integrity.
Responsible for a reliable structure life accompanying predictive method is the quantitative measurement of properties by NDI means, for instance of micro-damaging of laminated walls. All measures that reduce the uncertainty of the design parameters help to reduce the risk.

2. Design Dimensioning

Design must fulfil many of the following design requirements: mass, production cost and life cycle cost, geometry, environmental loadings (static, cyclic and impact loads, temperature, moisture, chemical) limits of deformation, lifetime, leakage, eigen-frequency, strength, stiffness, dimensional stability, buckling, connections, interfaces, support conditions, manufacturability, repairability, testability, inspectability, reliability, availability, maintainability, and safety.
Essential topic is the establishment of all external with internal loadings of the structure being of the type: hygro-thermal, mechanical, acoustical environment as well as of the corresponding lifetime requirements (duration, number of cycles). Loadings are often specified by a Technical Specification from the customer, or an authority (e.g. ESA,, FAA) or a Standard (EN, DIN, Betonkalender). Result is a set of combinations of loadings termed l*oad cases*, including the design driving d*imensioning load cases.* Thinking about the loadings (very often the design parameter of highest uncertainty) improves understanding of structural behaviour, helps engineering judgement and reduces uncertainty.
The usual deterministic optimisation procedure for a structure is an optimisation in respect of the different actual failure modes. A distinct set of design parameters is optimized in the design space with respect to an optimum state such as for the failure modes buckling, fracture, limited strains or a natural frequency. All the possible failure limit states are not met by the deterministic set of optimal design parameters by a certain distance due to the required factors of safety (FoS, not applicable for stress concentrations) which are usually used as load-increasing factors. Unfortunately, this distance is not quantifiable. However, a probabilistic optimisation provides the designer with a measure for the distance by giving him a number for a *reliability* $\Re$ *= 1- failure probability.* Of course, this number is a fictitious one because it depends on the quality of the used model and the input. But, it considers the probability of the combined appearance (joint probability of failure) of the design parameters. This means of all the values, the stochastic design parameters may take in the design space. Therefore usually, the set of deterministically derived optimal design parameters may be different

to that of a probabilistically derived one with for instance the consequence that the nominal values in the drawing will be different.

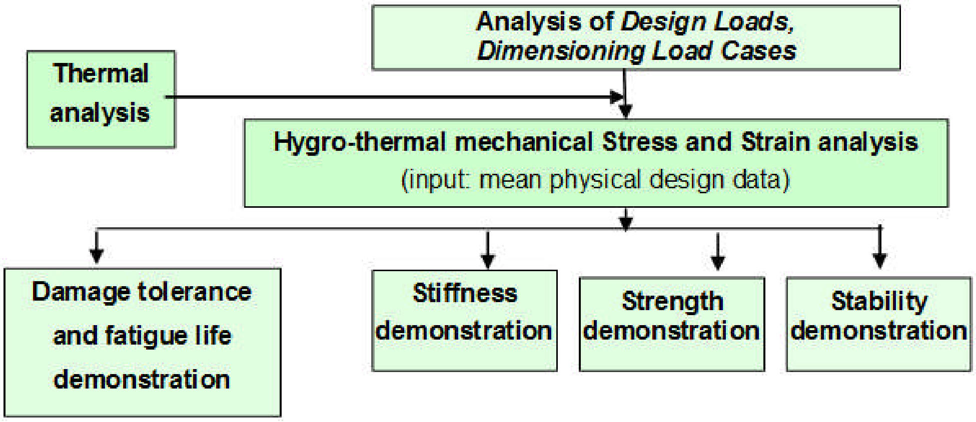

Fig.1: Structural Design-Analysis Flow Chart

3. Safety Concept applied and Design Factor of Safety

A safety concept is a concept, that implements structural reliability in design (safety is actually a wrong term but used, see Annex). Two formats are available for considering design uncertainties: 1) The deterministic format, which accounts for design uncertainties in a lumped or global manner by enlarging a design load by multiplication with a FoS, *Table 1*. 2) The probabilistic format maps each single design parameter's uncertainty into a probability density function. Thereby, the joint probability of failure caused by a combination of design parameters can be considered.

A bridge between a fully probabilistic safety concept and the traditional deterministic safety concept is the Partial Safety Factor concept being the simplest probabilistic safety factor concept! Partial FoS are dedicated there where the highest uncertainties are located. The actual safety concept in aerospace use this improved deterministic format. It discriminates at its lowest level (two parameters instead of one global one) load uncertainties considering factors (e.g. in spacecraft such factors KM, KP have the same size as the FoS, [2]) from the design uncertainties considering factors FoS.

Purpose of these design FoS is to guaranty quality of the design and of the test as well in order to achieve a certain level of Structural Reliability for the hardware. FoS are used to decrease the chance of failure by covering the uncertainties (which affect the risk of structural failure) of all the given variables outside the control of the designer which are primarily uncertainties in the statistical distribution of loads, uncertainties in manufacturing processes, material strength properties. However mind, missing accuracy in modelling, computing, manufacturing processes, or test data determination cannot be covered by a FoS value! Values for the FoS are different as for cases such as manned, un-manned spacecraft and 'design verification by analysis only', *Table 1*. Furthermore, different industry has different risk acceptance attitudes and apply differently high FoS values !

Additionally FoS are utilized in design, when the sizing approach is complex. Such a factor accounts for specific uncertainties linked to analysis difficulties. Such factors are *fitting factor* (often 1.2 in spacecraft), *welding factor, casting factor* etc. FoS values are usually based on long term experience with structural testing and manufacturing processes. This experience, for instance, is not available for glass and ceramic structures, see [2]. A possible high scatter of a design parameter is still respected by statistics (e.g. when computing a strength design allowable).

Table 1: Some minimum design factors of safety (FoS) in spacecraft [2]: Y:=yield, U:=ultimate

Structure type / Sizing case	**FoS Y**	**FoS U**	**FoS Y** verification by analysis only	**FoS U** verification by analysis only	**FoS U**	**FoS burst**
	external loadings incl. external pressure				**controlled** internal pressure	
Metallic structures	1.1	1.25	1.25	2.0	1.0	1.5
Fibre Reinforced Plastics	?	1.25	–	2.0	1.0	1.5
Glass/ Ceramics	–	2.5	–	5.0	–	–

It must remain in the responsibility of the designer to tackle the uncertainties in design and test, and this depends on the actual branch of industry. The use of qualified design procedures simplifies design verification and benefits final product certification [5].
Whether FoS may be reduced in classical mechanical engineering depends on the attitude of the discipline, and how the severity of a probable failure is assessed together with its occurrence probability. The assessment includes whether costly inspection should be avoided or not. If an operational life extension is planned then the application of a Damage Tolerance Concept does not make always sense. It may be concluded: A reduction of a FoS requires to think about another design policy that includes all parts of a product process chain. For aerospace is valid, in order to compete, just additional 'pocket' FoS on top of the minimum FoS values may be skipped.

4. Modelling and Analysis

Analysis aims to predict and therefore to accurately model the response of a structure or just the material, subjected to a set of mechanical and environmental constraints. An essential aspect is the utilization of statistical properties. *Fig.2* depicts some guideline which input data should be used to finally obtain reliable analysis results.
Aspects at model choice: The accuracy of the model can be only as good as the *input values* are. These must be adequately defined, and the *scatter* expected for each *design parameter* has to be estimated, at least. Modelling is often confirmed by testing to ensure that the predicted response and the actual tested performance are as expected and as required. This adds confidence to the use of the applied software and leads to model *validation*. Task and deadline determine the model choice! *Think first, analyse then!* This is the more valid if the usual worst-case load scenario is applied.
Note: To achieve design verification the so-called (strength) *Design Allowables are* to be applied. These are values, beyond which at least 99% (“A” value) or 90% (“B” value) of the population of values is expected to fall, with a 95% confidence (reflects test data quantity basis), see MIL-Hdbk 17.

1 Input: ***DESIGN*** **Stress & Strain Analysis** *(Struktur-Analyse)*

** Mean elasticity properties and geometry (thickness, length) to represent mean structural behaviour. Is economic as well as a necessity in case of (usual) redundant behaviour of the structure.*

** Choice of code-dependent + problem-dependent stress-strain curve*

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2 Input: Strength Demonstration (verification) *(Nachweis)*

One-sided (static and fatigue strength), and two-sided tolerance bands (thickness, E-modulus) have to be considered …

3 Input: Stiffness Demonstration

Due to stiffness requirements → upper and/or lower tolerance limits

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4 A-and B-value ***Design Allowables*** **(Aerospace) (statistics–based, Mil Hdbk)**

A-values: In application of the military Safe Life Concept

B-values: In application of a Damage Tolerance Concept (multiple load paths).

advantage for composite laminates

Fig.2: Input in structural analyses (aerospace)

5. Deterministic and Probabilistic Design

The design requirements determine the tasks which the design has to fulfil and subsequently the necessary types of analyses. Dependent on the type of analysis that is to be carried out (i.e. strength, stiffness, deformation, stability), it firstly has to be checked which type of stress-strain curve has to be used. Doing this, the design requirements for strength design verification can be contradictory, for example, to the requirements for deformation design verification. On one side the various tasks cannot be solved in a 'one-shot' structure analysis and on the other side the efforts of many analyses and associated evaluations should be kept small. Probabilistics, however, support the effort-cutting compromise of applying a typical stress-strain curve, and it is therefore recommended to generally use typical stress-strain curves to optimally predict structural behaviour in the analyses, *Fig.3*.

In probabilistics-based optimisation no FoS are utilized but statistical distributions of the stochastic design parameters are applied (assumed are: for loads usually an extreme value distribution, for strength a Weibull or a log-normal one). Essential aspect of this type of optimisation is the sensitivity that means the influence of a design parameter on the objective function such as a collapse load or a mass value. The lower the change of the sensitivity measures is, in case of a change of the scatter of a design parameter, the more robust the design is. The knowledge of such sensitivity measures helps with management decisions, such as: Which of the geometrical tolerances have less influence and can therefore be met simpler and fulfilled cheaper, however, with keeping the same reliability?

Goals of Structural Analysis are a mass minimisation of the structure (mathematical formulation) in the prescribed design space w.r.t. side constraints such as cost, project deadlines, manufacturing and NDI needs, risk; prediction of structural behaviour and strength analysis (design verification). Deterministic optimisation of a structural model provides with a set of nominal (mean) design parameters with which the (failure) limit states are not reached by a not quantifiable distance in the frame of the scatter covering deterministic FoS. Stochastic (probabilistic) optimisation however delivers a set of mean design parameters (coordinates of the so-called most probable failure point') with a measure for this distance and directly considering the scatter of all design parameters.

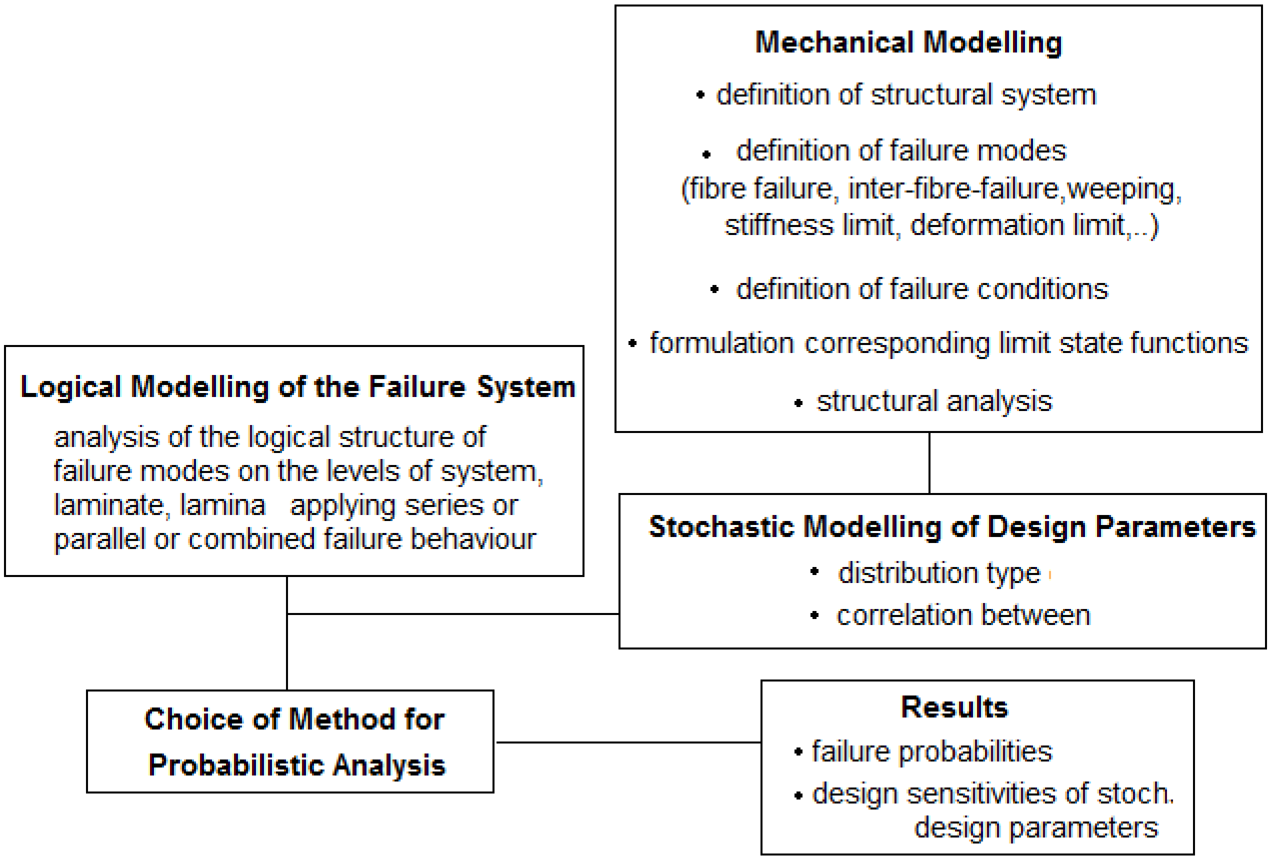

Fig.3: Modelling in case of probabilistic structural analysis

Example: The advantageous use of the probabilistic method shall be demonstrated by a simple example the influence of reduction of a manufacturing tolerance: The thickness tolerance could be reduced from ± 0.20 mm to ± 0.05 mm. How much may the nominal thickness of the Booster of the ARIANE 5 reduced keeping the same theoretical structural reliability $\Re = 1 - 5 \cdot 10^{-6}$? As the result from a relatively simple probabilistic model the wall could be changed from t = 8.2 +- 0.20 mm to t = 8.1 +- 0.05 mm. This lead to a mass reduction (250 kg) and an additional benefit of fuel savings.
The applicability of a probabilistic method is mandatory if a reliability target or a failure probability has to be met. For instance, for the full failure system of the A5 launcher this consists of two different types of failure probabilities. One type is *failure rate* dependent (e.g. failure of launcher valves and batteries with the well-known bath-tub failure distribution) and the other the here treated *failure state* type, used for the other sub-failure system of the launcher, for the structure.
A probabilistic method tackles the combined uncertainties and regards its probability of occurrence. It adds technical information that cannot be obtained by the FoS concept. It enables to disclose the risk characteristics caused by the design-driving parameters.
The use of probabilistic analyses in design can be recommended because it provides a quantitative 'feeling' for the influence of the scatter of the design parameter.

6. Deterministic and probabilistic Design Verification

The usual way to "verify a design" is to show by computation of a MoS $\geq$ 0, that the resistance of a structure is higher than the loading. The determination of the load-based reserve factor or of the margin of safety follows the equations

$$f_{Res} = \frac{Final\ Failure\ Load}{Design\ Load} = MoS + 1 = \frac{design\ allowable\ 'load\ \text{resistance}'}{Design\ Load}$$

and if linear analysis is permitted (σ proportional load) above equation reduces to

$$f_{Res} = \frac{design\ allowable\ \ "strength"}{\sigma(Design\ Load)}\ .$$

In structural reliability analysis (*Fig.3*), this way is more complex. It's objective is the evaluation of a probability of occurrence of a given failure state or of a survival probability ($\equiv$reliability) $\Re > \Re_{required}$ or $\Re = 1 - p_f$ with $p_f < p_{f,\ admissible}$. For the computation of $\Re$ efficient numerical procedures are available.

Some Lessons Learnt w.r.t increasing reliability of results

1. Physics have to be modelled accurately in the analysis part *Mechanical Modelling, Fig.3.* All dimensioning load cases and failure modes have to be accounted for.
2. A *robust, reliable design* or the robustness to later changes of the design parameters with identification of the most sensitive design parameters is a need
3. Failure probability does not dramatically increase if a MoS turns slightly negative. A local safety measure of e.g. *-1 %* should be no problem in *design development* as the MoS value does not outline the risk or the failure probability. Therefore, no overreaction by re-designing but application of a *'Think (about) Uncertainties'* attitude by recognizing the main driving design parameters and by reducing the scatter (uncertainty) of them ! This highly pays off. Both, an increasing mean value and a decreasing standard deviation increase MoS and $\Re$. Essential question w.r.t. all uncertainties is whether these increase the risk (criticality) to an unacceptable level or not. Thereby, the interdependence of risk with severity of failure and its probability of occurrence is always to be considered.
4. *Theory,* 'only' creates a model of the reality, and *experiment* is 'just' one realisation of the reality. Experimental results can be far away from the reality like a non-accurate theoretical model. Find a compromise to cost-optimally achieve a satisfying analysis-test verification procedure for a robust design.
5. Quantitative measurement of degradation by NDI. This means for instance the measurement of sub-sequent micro-damaging in laminated walls. Controlling and monitoring are uncertainty tackling measures.

Literature

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Annex. For an accurate understanding some definitions shall be mentioned that are of interest when generating reliable load-carrying structures:

• Design Value: value of a property used in design that is assumed to consider its uncertainty
• Design Verification: demonstration that the design fulfils the requirements
• Dimensioning Load Case: physically possible, driving design load case which is of a certain probability of occurrence
• Factor of Safety (FoS) or Safety Factor : deterministic factor (based on long experience) which increases the level of the given loading
• Failure mode: observable effect of the mechanism through which the failure occurs. Strength (addressed here, mainly), fracture mechanics, and other failure modes. Examples: normal fracture and shear fracture, local buckling, leakage, given deformation limit, excessive wear, corrosion, initiation of yielding, etc.
• Margin of Safety (MoS) or Safety Margin: decimal fraction by which the failure load exceeds the design load
• Non-redundant (single load path) and redundant (multiple load path) structures: If non-redundant the applied loads are distributed through a single member within an assembly, the failure of which would result in the loss of structural integrity of the component involved. If redundant the applied loads are safely distributed to other load-carrying members in case of failure.
• Reliability $\Re$: aptitude of a product to perform the required functions at certain performance levels under specific conditions and for a given period of time, expressed in terms of probability
• Reserve factor (RF): load-defined factor as ratio of a resistance value and an action value.
• Resistance (R): material property or a structural property counteracting the applied loadings
• Robustness: 1/ (uncertainty • complexity) [definition of Ontonix] . Robustness and reliability are cross-linked to some extent.
• Safety (security): in engineering used if human beings might be excessively endangered
• Safety Concept: deterministic or probabilistic concept or format, respectively. Note: considers the uncertainties of the design variables in a different manner
• Stochastic design parameter (uncertain basis variable): design parameter which is uncertain (before realization) and random (after realization).
• Structural Integrity: characteristic of a structure that enables it to withstand the load environment and the usage imposed during service
• Structural Reliability $\Re = 1 - p_f$ with p_f as failure probability: ability of a structure to fulfil during a distinct lifetime with a distinct reliability the functional requirements. Note: considers the probability of combinations of the significant scattering (stochastic, random) design parameters (better design variables)
• Uncertainty : 'unclearness (fuzzy)' in loadings, strengths, and other design parameters such as geometrical parameters, applied engineering models