AG-Structural Integrity + UAG Fatigue,
4. April 2019, Steyr, 25 min + 5 min

CARBON
COMPOSITES

Novel Fatigue Lifetime Prediction for Brittle Materials

by using

Strength Failure Mode-linked Modeling of Loading,
a Basic S-N Curve,
and Application of a Strength Mode-linked variation of
Kawai's 'Modified Fatigue Strength Ratio' for estimating further S-N curves

- brittle material behavior such as UD (here), grey cast iron, and concrete -

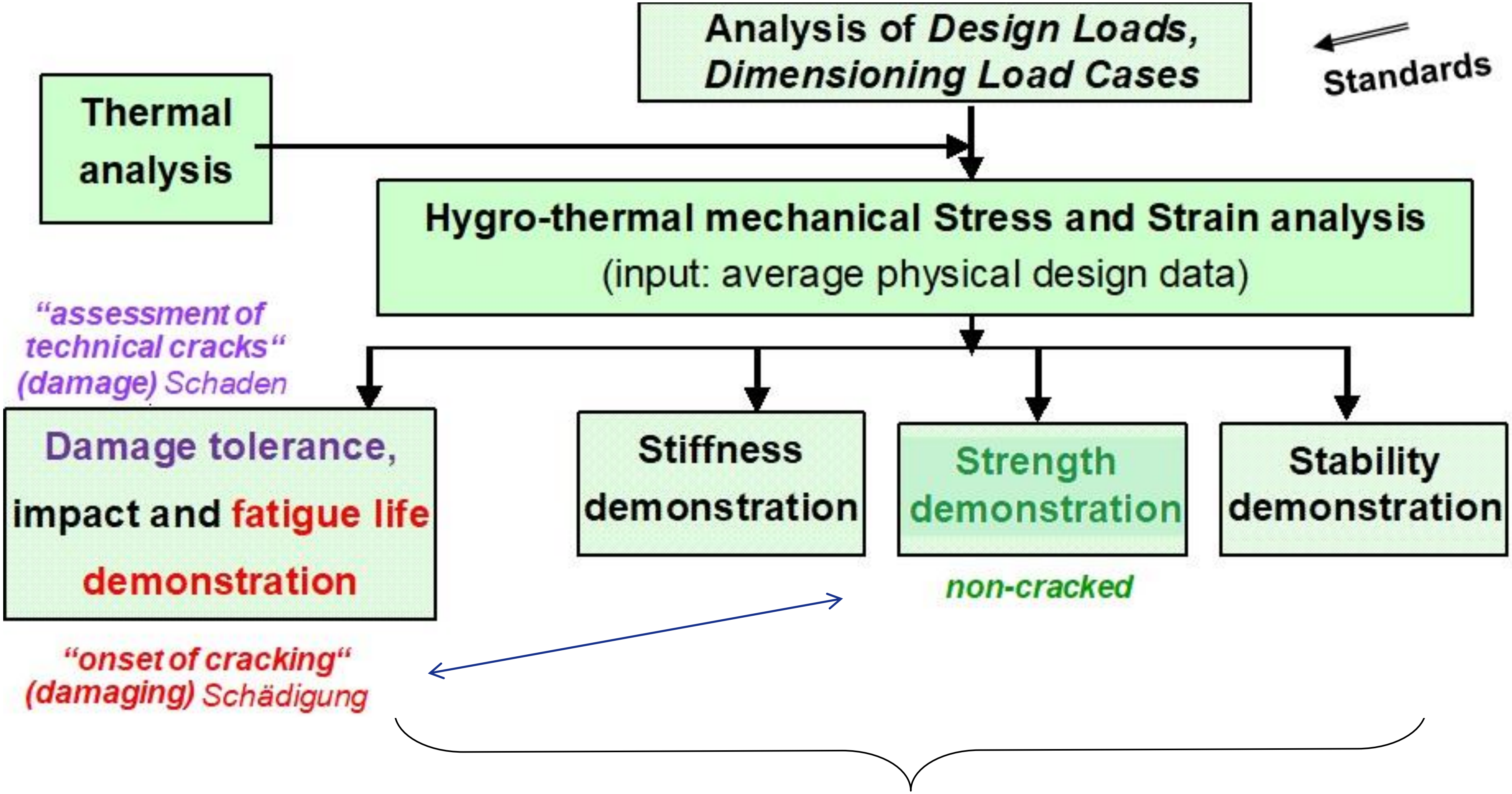
1 **Introduction** to Static and Fatigue Design
2 Cuntze's ***Failure Mode Concept-based*** Strength Criteria
3 Cuntze's ***Fatigue Lifetime Prediction Concept***
4 Generation and ***Novel Interpretation of*** *UD* ***Haigh Diagrams***
5 **Steps** of the Proposed *Fatigue Lifetime Prediction Concept*

Presentation of never funded hobby-investigations.

Prof. Dr.-Ing. habil. Ralf Cuntze VDI, formerly MAN-Technologie AG
linked to Carbon Composite e.V. (CCeV) Augsburg, heading the WGs
"Engineering" (Mechanical Engineering, since 2009) and
"" Dimensioning and design verification of composite parts" (Civil Engineering, since 2011)
Since 1970 in CFRP composite business

Flow Diagram: Structural Design and Design Verification

Nachweise



Analysis of *Design Loads, Dimensioning Load Cases*

Standards

Thermal analysis

Hygro-thermal mechanical Stress and Strain analysis

(input: average physical design data)

"assessment of technical cracks" *(damage)* Schaden

Damage tolerance, impact and fatigue life demonstration

Stiffness demonstration

Strength demonstration

non-cracked

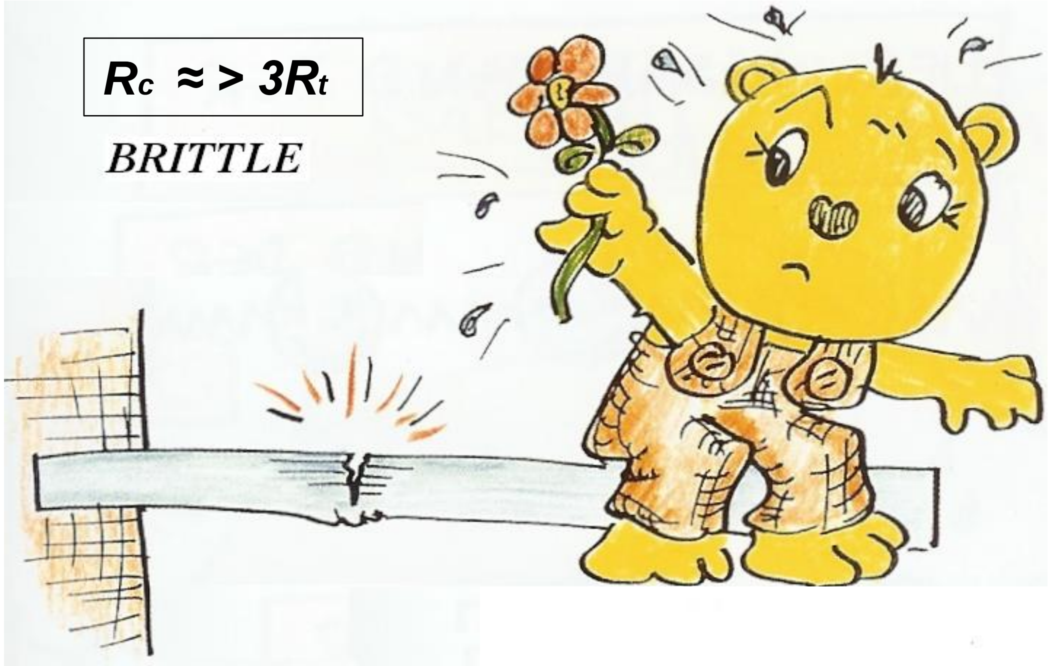
Stability demonstration

"onset of cracking" *(damaging)* Schädigung

'Resistances', to be demonstrated
by a Reserve factor *RF* ≥ 1 or a *positive Margin of Safety MoS ≥ 0*
in order to achieve Structural Integrity *!*

How may one principally discriminate *Material Behaviour* ?

$R_c \approx > 3R_t$



One feels good until sudden fracture occurs

Courtesy: Prof. C. Mattheck

Ductile Fracture =
type of failure in a material or a structure generally preceded by a large amount of plastic deformation

Design Verification by theoretical prediction

STATIC :

- Reserve Factor is load-defined :

$$RF = \frac{\text{Predicted Failure Load}}{j \cdot \text{Design Limit Load}} > 1.$$

Material Reserve Factor :

$$f_{\mathrm{Res}} = \frac{\text{Strength Design Allowable}}{\text{Stress at } j \cdot \text{Design Limit Load}} > 1.$$

CYCLIC :

- *$RF_{\text{life, Predicted Lifetime}}$*
- Determination of **Inspection time**
- Determination of **Replacement time**

$$RF_{life} \approx \frac{\text{Predicted Lifetime}}{j_{life} \cdot \text{Design Limit Lifetime}} > 1.$$

j = design factor of safety

Global versus Modal Strength Failure Conditions SFCs (criteria)

zwangsverheiratet

1 Global ***strength*** **failure condition : F ({σ}, {*R*}) = 1 (usual formulation)**

Set of Modal **strength failure conditions : F ({σ}, R^{mode}) = 1 (addressed in FMC)**

Example: UD

vector of 6 stresses (general)

$$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$$

vector of 5 strengths

$$\{R\} = (R_{||}^t, R_{||}^c, R_{\perp}^t, R_{\perp}^c, R_{\perp||})^T$$

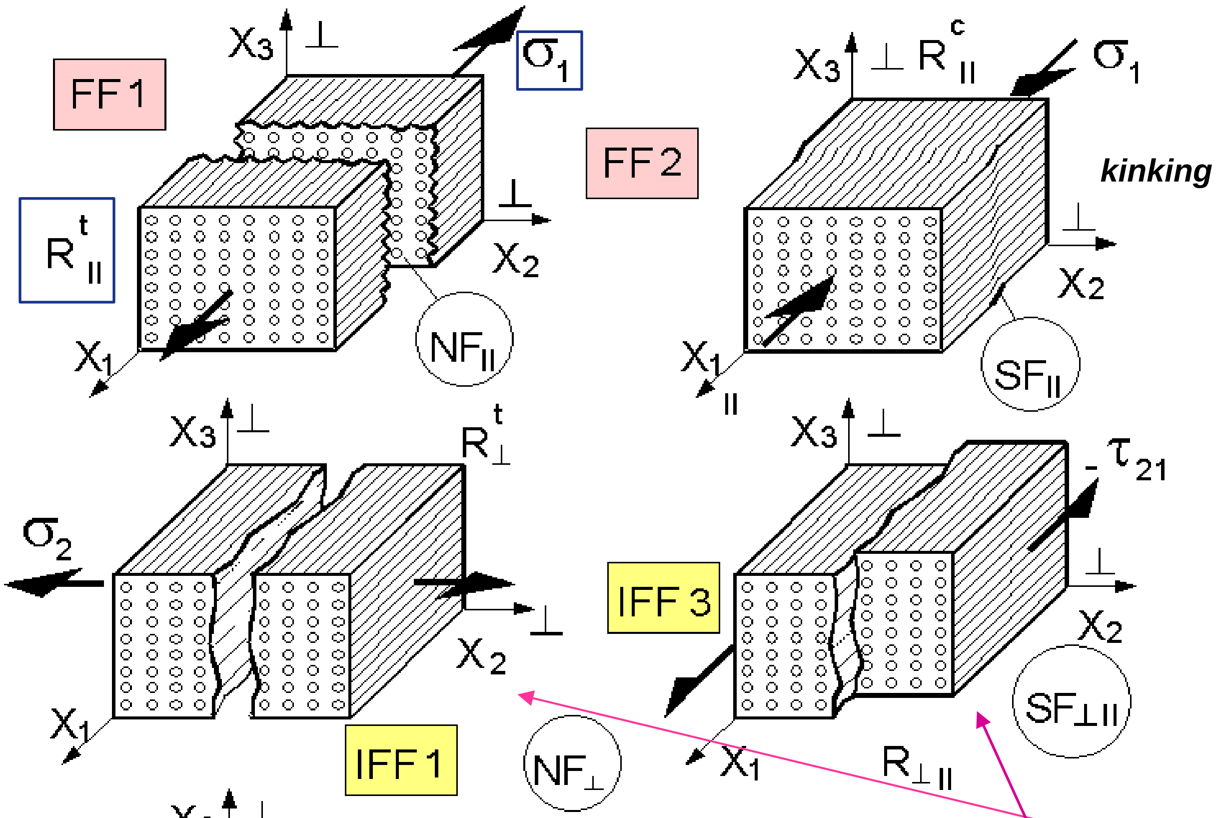
needs an Interaction of Failure Modes:
performed here by a *series failure system model*

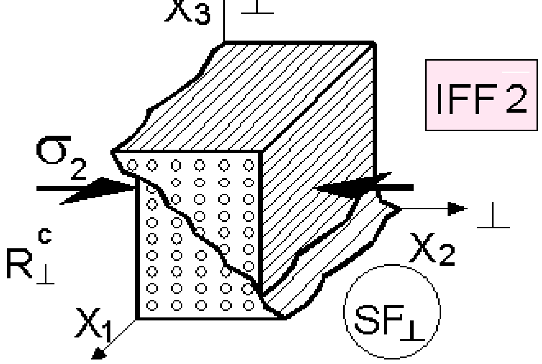
each failure mode is reigned
by one associated strength

Observed ! Fracture Morphology of Transversely-isotropic UD Material

[Cun04]

t = tension
c = compression





wedge type

▶ 5 Fracture modes exist

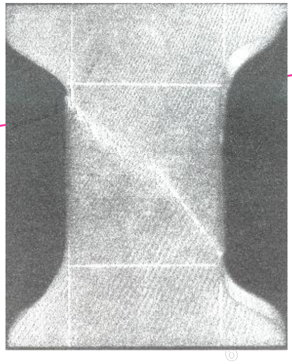
= 2 FF (Fiber Failure)
+ 3 IFF (Inter Fiber Failure)

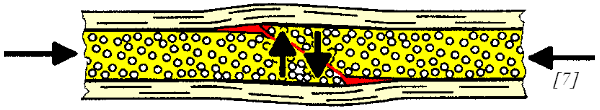
critical in a loaded laminate:
FF1, FF2 + possibly IFF2 !

from the observations follow ..

NF := Normal Fracture

SF := Shear Fracture





1 Introduction to Static and Fatigue Design
2 Cuntze's Failure-Mode-Concept-based Strength Criteria
3 Cuntze's Fatigue Life Prediction Concept
4 Generation and Novel Interpretation of UD Haigh Diagrams
5 Steps of the Fatigue Life Prediction Method Proposed

Notes:

- ***Cuntze's Failure-Mode-Concept-based Static Strength Criteria*** **are applicable to isotropic, transversely-isotropic UD and orthotropic woven materials**
- as model parameters used are measurable: the strengths R_m and friction values μ
- the interaction exponent m is estimated due to mapping experience
- an artificial english denotation for the self-explaining German term Werkstoffanstrengung was created with QinetiQ during the WWFE reading material stressing effort *Eff*

Basic Features of Cuntze‘s Failure-Mode-Concept (FMC), 1995

<u>plus</u> the fact, that transversely-isotropic UD materials exhibit a *‘5-fold‘* **material symmetry characteristic** = 5 Strengths, 5 Failure Modes, 5 E, etc.

- **Each failure mode represents 1 independent failure mechanism**

  and thereby 1 piece of the complete *failure surface*

- **Each failure mechanism is governed by 1 basic strength** (is observed!)

- **Each failure *mode* can be represented by 1 failure *condition.***

  Therefore, equivalent stresses can be computed for each mode !!

  This is of advantage when deriving S-N curves and Haigh diagrams

  with minimum test effort.

Understanding the terms **Material Stressing Effort and Equivalent Stress**

Helpful “To turn the right screw“ in design is the delivery of

equivalent stresses and of **material stressing efforts *Eff***

***mode* material stressing effort * (in German “Werkstoffanstrengung”)**

The relationship is

$$Eff^{\text{mode}} = \sigma_{eq}^{\text{mode}} / R^{\text{mode}}$$

anology to ‘Mises’

$$Eff^{\text{fracture mode}} = \sigma_{eq}^{\text{fracture mode}} / R_m$$

$$Eff^{\text{Mises}} = \sigma_{eq}^{\text{Mises}} / R_{po.2}$$

mode equivalent stress

mode associated average strength* *(in test data mapping bar over $\bar{R}$)

- **material stressing effort** ***Eff = artificial technical term , created together with QinetiQ, UK, during the World-Wide-Failure-Exercises***

Cuntze's **Set** of **Modal 3D UD Strength Failure Conditions** ('criteria')

Cuntze = 'simple Mises ' amongst the UD criteria

Invariants, replaced by their stress formulations !!

[Cun04, Cun11]

strains from FEA

FF1 $Eff^{\|\sigma} = \breve{\sigma}_1 / \bar{R}^t_{\|} = \sigma^{\|\sigma}_{eq} / \bar{R}^t_{\|}$, $\breve{\sigma}_1 \cong \varepsilon^t_1 \cdot E_{\|}$ *

FF2 $Eff^{\|\tau} = -\breve{\sigma}_1 / \bar{R}^c_{\|} = +\sigma^{\|\tau}_{eq} / \bar{R}^c_{\|}$, $\breve{\sigma}_1 \cong \varepsilon^c_1 \cdot E_{\|}$

2 filament modes

IFF1 $\boxed{Eff^{\perp\sigma}} = [(\sigma_2 + \sigma_3) + \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^{\;2}}\,]/2\bar{R}^t_{\perp} = \boxed{\sigma^{\perp\sigma}_{eq} / \bar{R}^t_{\perp}}$

IFF2 $Eff^{\perp\tau} = [(\frac{\mu_{\perp\perp}}{1-\mu_{\perp\perp}}) \cdot (\sigma_2 + \sigma_3) + \frac{1}{1-\mu_{\perp\perp}} \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^{\;2}}\,] / \bar{R}^c_{\perp} = +\sigma^{\perp\tau}_{eq} / \bar{R}^c_{\perp}$

3 matrix modes

IFF3 $Eff^{\perp\|} = \{[\mu_{\perp\|} \cdot I_{23-5} + (\sqrt{\mu_{\perp\|}^{\;2} \cdot I_{23-5}^{\;2} + 4 \cdot \bar{R}_{\perp\|}^{\;2} \cdot (\tau_{31}^2 + \tau_{21}^2)^2}\,]/(2 \cdot \bar{R}_{\perp\|}^{\;3})\}^{0.5} = \sigma^{\perp\|}_{eq} / \bar{R}_{\perp\|}$

with $I_{23-5} = 2\sigma_2 \cdot \tau_{21}^2 + 2\sigma_3 \cdot \tau_{31}^2 + 4\tau_{23}\tau_{31}\tau_{21}$

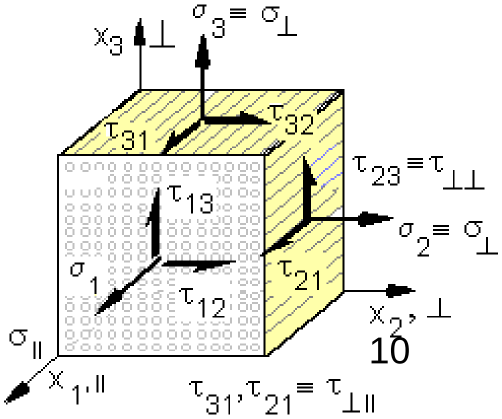
Interaction of modes:

$$Eff^m = (Eff^{\|\tau})^m + (Eff^{\|\sigma})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\|})^m = 1$$

with mode-interaction exponent $2.5 < m < 3$ from mapping tests data

Typical friction value data range: $0.05 < \mu_{\perp\|} < 0.3, \quad 0.05 < \mu_{\perp\perp} < 0.2$

see [Pet16] for measurement



Poisson effect * : bi-axial compression strains the filament without any σ_1

t:= tensile, c: = compression, || : = parallel to fibre, ⊥ := transversal to fibre

Modal treatment *requires* an **Interaction of the Single SFCs**

In the FMC:I

Interaction of adjacent Failure Modes in the mode transition zones

= by a ‘*series failure system*’ model that considers an

‘Accumulation’ of interacting mode– associated failure danger portions Eff^{mode}

$$Eff = \sqrt[m]{(Eff^{\text{mode 1}})^m + (Eff^{\text{mode 2}})^m + \ ....}$$ ***= 1 = 100% , if failure***

with mode-interaction exponent ***m ,*** *from mapping experience*

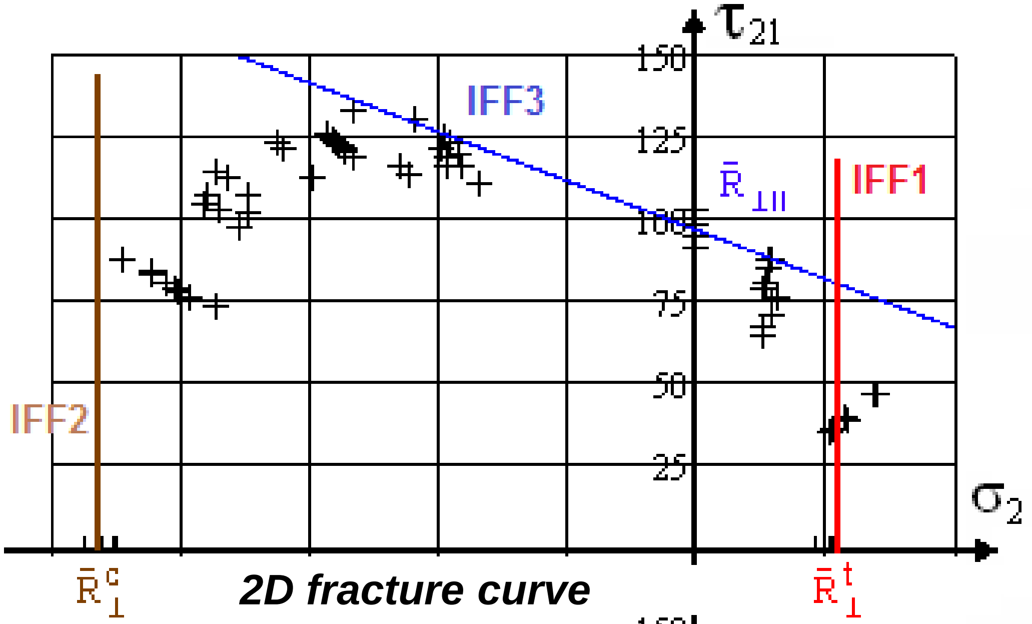
It is assumed engineering-like: m takes the same value for all

mode transition zones captured by the interaction formula above

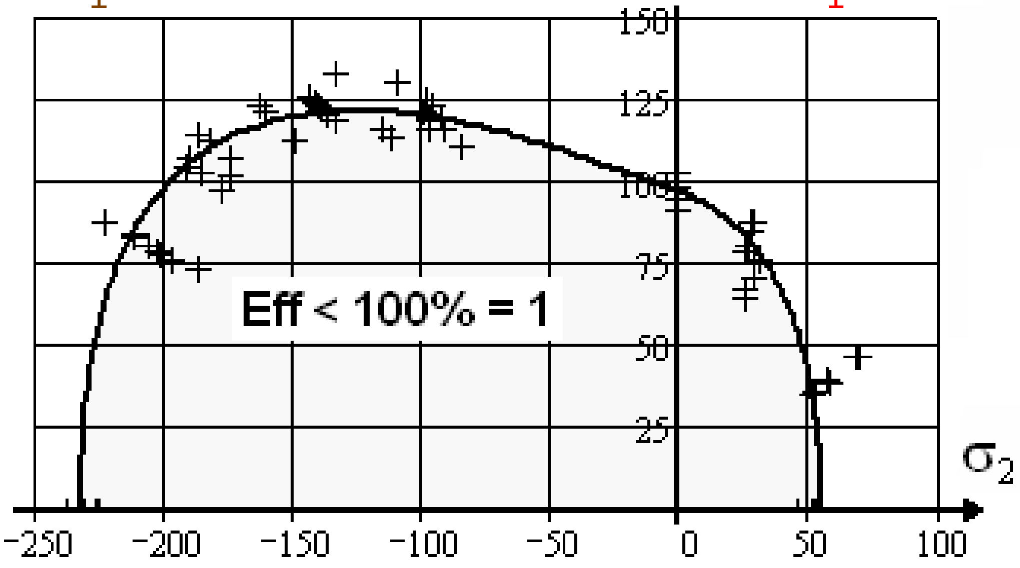
Visualization of Interaction: example UD Failure Modes

$\breve{\sigma}_1 = 0$

$\tau_{21}(\sigma_2)$ or $\{\sigma\}=(0,\sigma_2,0,0,0,\tau_{21})^T$

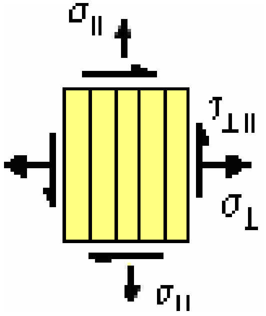


2D fracture curve



Mapping of course of IFF test data in a pure mode domain by the *single Mode Failure Conditions.*

3 IFF pure modes = straight lines !.

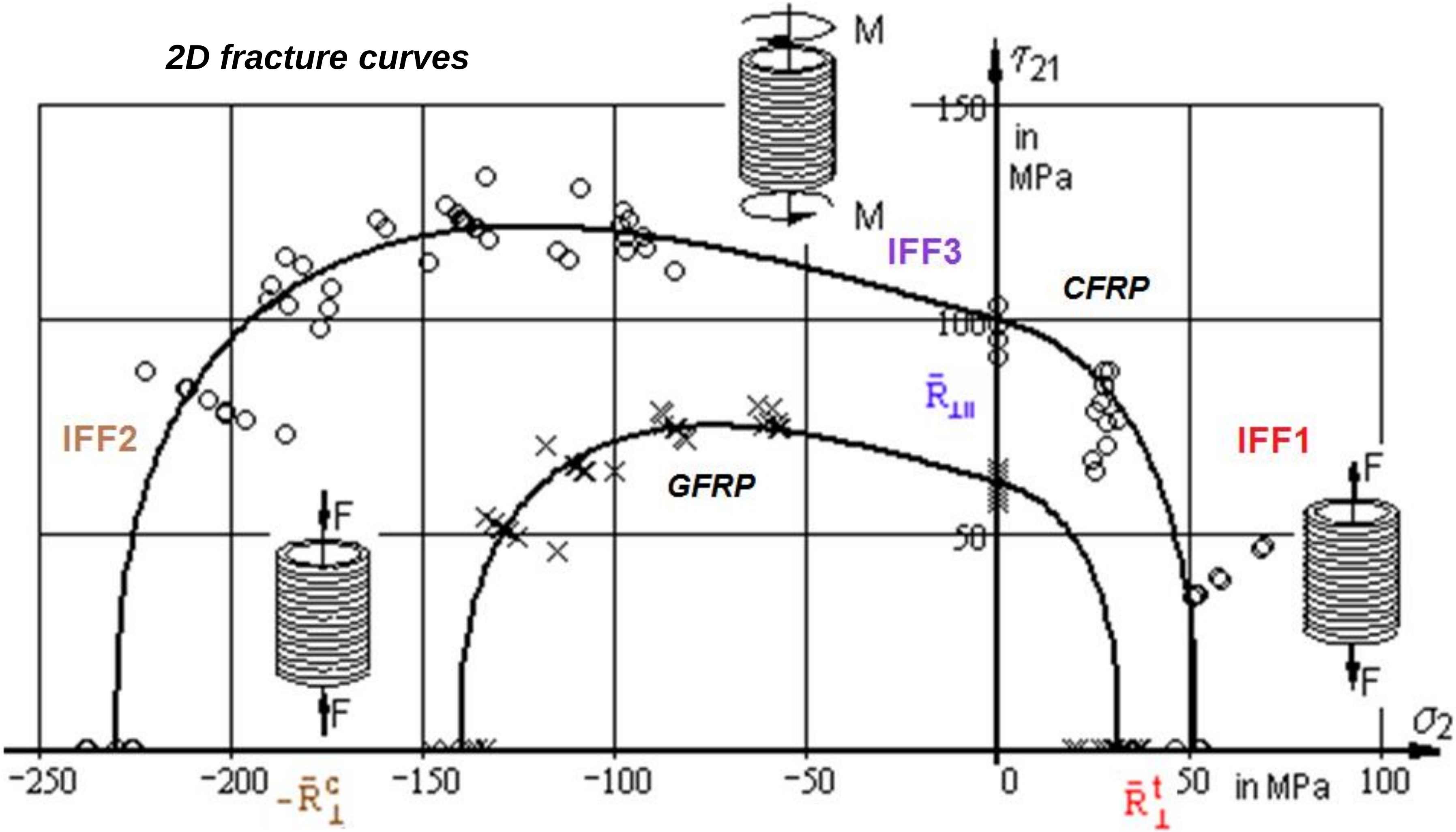


Mapping of course of test data by *the Interaction Model*

$$(Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\|})^m = 1$$

IFF 1-2-3 Cross–section of the Fracture Failure Surface (body)

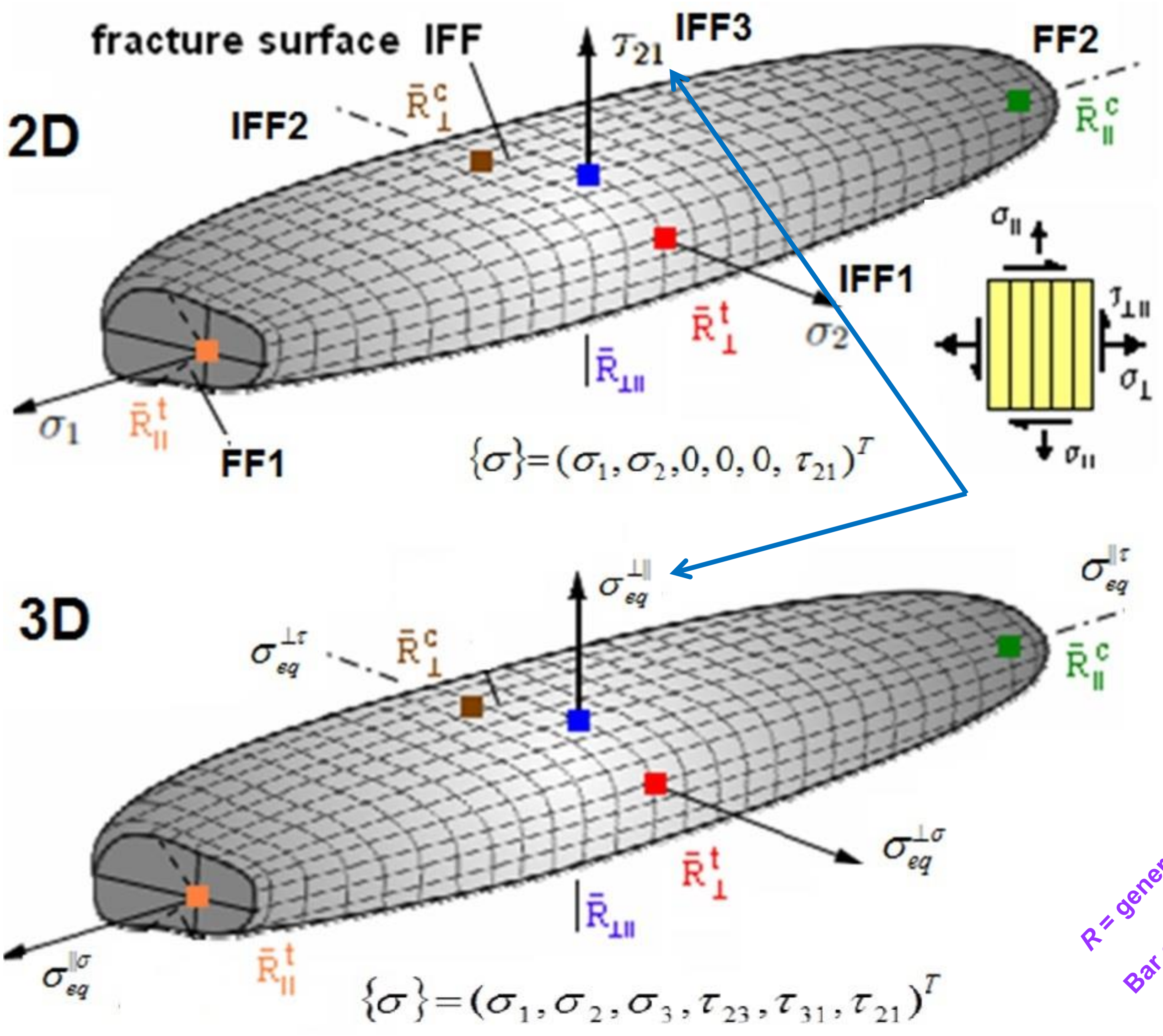
2D fracture curves



* Above tested were so-called isolated test specimens.

* For the presented fatigue approach embedded laminas are to consider!

2D ⟹ 3D **Fracture surface** by replacing the stress by the equival. stress



under fatigue damaging the fracture body is shrinking

R = general strength and statistically reduced
‘strength design allowable
Bar over R means average strength, applied
when mapping .

1 Introduction to Static and Fatigue Design
2 Cuntze’s Failure-Mode-Concept-based Strength Criteria
3 Cuntze’s Fatigue Lifetime Prediction (estimation) Concept
4 Generation and Interpretation of UD Haigh Diagrams
5 Steps of the Fatigue Lifetime Prediction Method Proposed

> ***„Fatigue is the black art,***
>
> ***to produce financial black holes“***

[J. Draper]

Therefore, in order to reduce very costly cyclic laminate test programs

the German Academic Research Group BeNa (proof of service strength),

founded by the author in 2010,

aims at :

A failure mode-based Lifetime Prediction Method,
lamina-oriented on the embedded lamina
in order to capture in-situ effects
and using <u>failure mode-based S-N curves</u>.

.

Some Notions

Stress Life Fatigue ← approaches → **Strain Life fatigue** (***ductile behaviour***)

Fatigue limit, endurance limit, and fatigue strength:
expressions, used to describe a cyclic property of a material

Behaviour:

- brittle : max stress (Oberspannung), σ_{max} is responsible for damaging
- ductile: amplitude stress σ_a *is* responsible for damaging (slip)

S-N curve (Wöhlerkurve): $R = \sigma_{min} / \sigma_{max}$

σ_a *and* σ_{max} (if brittle) are used

stress-life fatigue curve of a material, in terms of fracture cycles *N*, for a distinct applied stress $S \equiv \sigma(N)$. *(Note: Renders the weakening of a repeatedly loaded material)*

Haigh Diagram:

stress amplitude $\sigma_a(\sigma_m, R)$ is used

Mean stress σ_m influence f_M *of isotr. materials: prediction on basis of 2 test points* $(\sigma a_{R=-1}, 0)$, $\sigma a_{R=0}$, $\sigma a_{R=0}$),

$f_M = \sigma a_{R=-1} / \sigma a_{R=0}$, *represents the slope*

State of the Art : Cyclic Strength Analysis of UD-ply composed Laminates

- **No Lifetime Prediction Method available, applicable to any Laminate**
- **Procedures base – as with metals – on stress amplitudes and mean stress correction f_M**
- **Procedures base on specific laminates and therefore cannot be generally applied**
- **Up to now: Engineering Approach**

  ***Static Design Limit Strain* of $\varepsilon < 0.3\%$, practically means** negligible matrix-microcracking.

  Design experience proved: No fatigue danger given

- **Future : *Design Limit Strain* shall be increased** (EU-project: MAAXIMUS)

  Beyond $\varepsilon \approx 0.5\%$ *first filament breaks , diffuse matrix-microcracking changes to a discrete localized* one.

Often, *fiber-dominated laminates* are used in high performance stress applications.

UD-material should be better exploited !

Fatigue Damaging Drivers of Ductile and Brittle behaving Materials

- **Ductile Material Behavior** (example: isotropic metal materials)

  1 damaging mechanism *acts = "slip band shear yielding"*

  drives damaging under cyclic tensile, compressive, shear and torsional stresses:

  Therefore, this single mechanism can be described by one single strength formulation:

  the Mises Yield failure condition!

- **Brittle Behaving Material Behavior :** isotropic Materials

  2 damaging driving mechanisms act = *Normal Fracture failure mode (NF), Shear Fracture failure (SF)*

- **Brittle Behaving UD Material Behavior :** transversely-isotropic UD Materials

  5 damaging driving fracture failure mechanisms act ≡ 5 Fracture failure modes

Damaging Determination in Brittle behaving cyclically loaded Composites

Assumption:

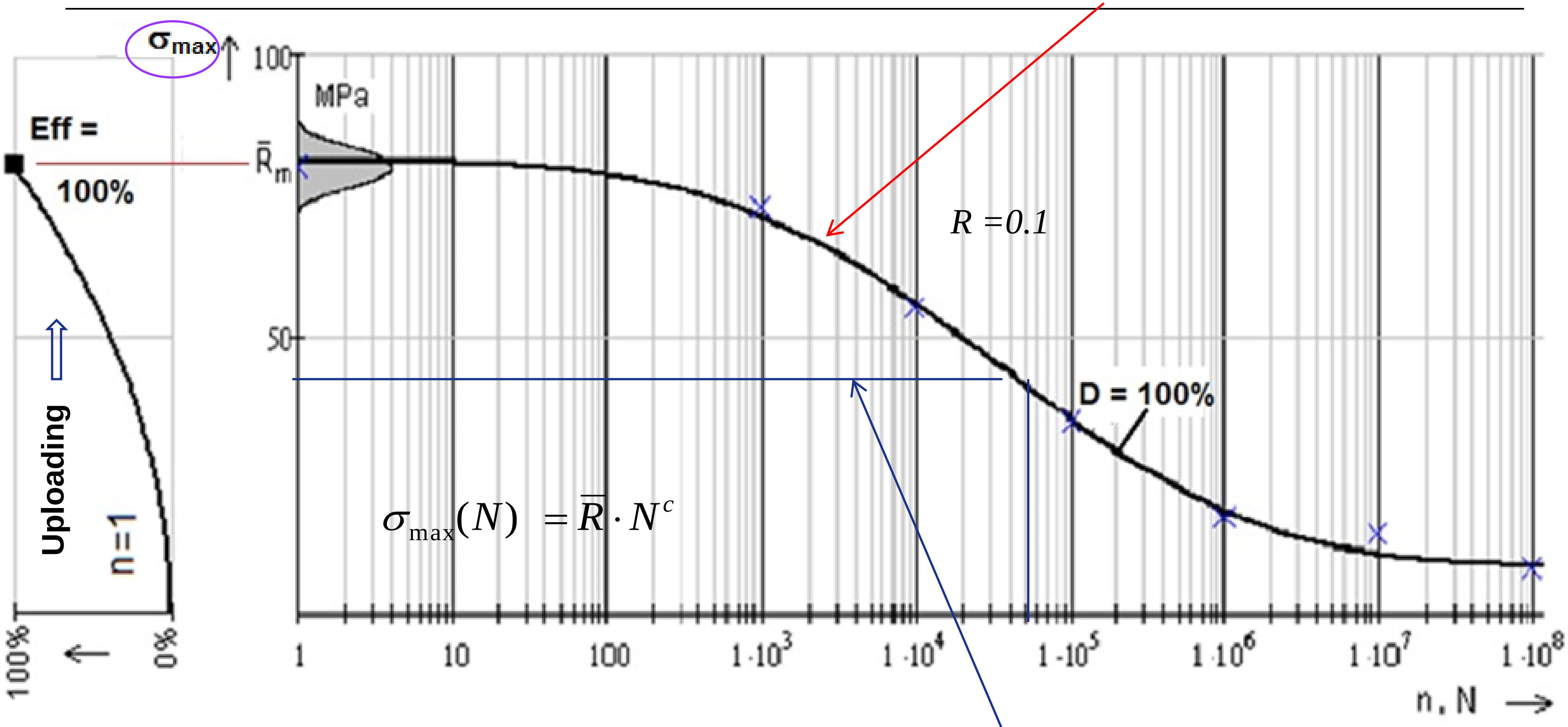
“If the failure mechanism (mode) cyclically remains the same

as in the static case t*hen*

- the fatigue damaging driving failure parameters are the same and

- the applicability of static SFCs is allowed for quantifying damaging portions !“

Cyclic development of damaging, average S-N-curve, brittle material



Static

R_m basic strength

Cyclic

residual strength $\sigma(R, N)$

Analogous limits of the material capacities :

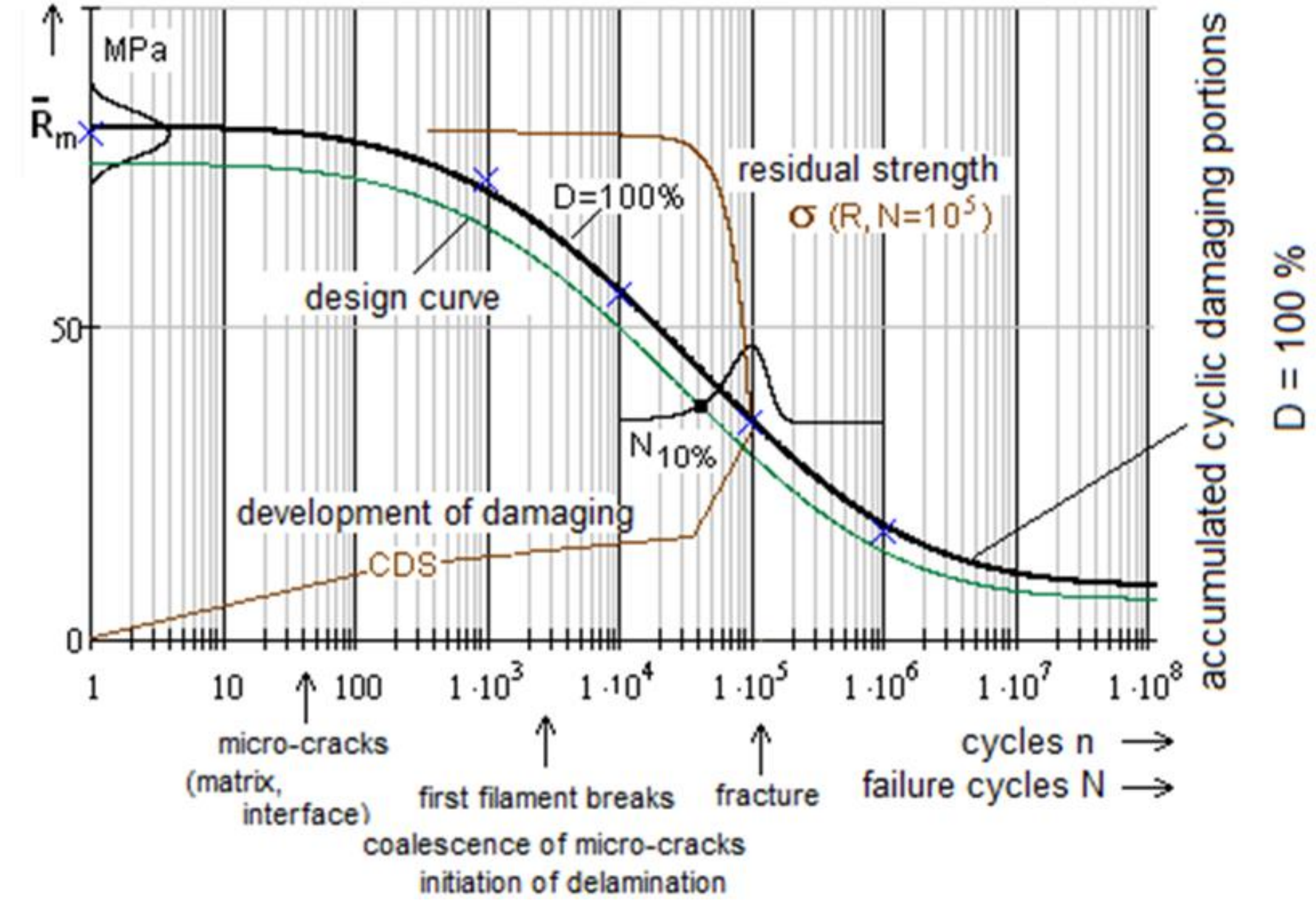
- Static : material stressing effort Eff (N=1) = 100 %
- Cyclic : material damaging sum D (N) = 100 %

= *sum of damaging portions*

S-N-mapping of brittle materials: use of σmax is advantageous compared to the amplitude σa !

Lin-Log S-N Curve: Average Curve (mapping) and Design Curve (verification)

$\sigma_{max} \Rightarrow \sigma_{eq,\,max}$ if more stresses act together within the envisaged failure mode (like Mises)



FF:= fibre failure. IFF:= Inter Fibre Failure, CDS:= characteristic damage state at the end of diffuse damaging

Steps in Fatigue Life Estimation

1 Input

Operational Loading: **Load time curves** (modeling rain flow, ..)

Time domain: Cycle-by-cycle or block-by-block (less computation effort)

Frequency dom.: Load spectra (loss of Reihenfolge)

Safety Concept: Design safety factor Life $j_{Life} = 3 – 10$, or

an Inspection interval, or an Replacement time approach

2 Transfer of operational loadings into stresses by using structural analysis

3 Domains of Fatigue Analysis

LCF: high stressing,

HCF: intermediate stressing 10.000 < n < 1.000.000, *rotor tube*

VHCF: low stressing and strains (SPP1466) > 10.000.000 *centrifuges, wind rotor blades*

4 Provision of Haigh Diagrams which involve all necessary S-N curves with generation of ‘Constant Fatigue Life (CFL) curves‘

3 Haigh Diagrams are provided : FF1-FF2, IFF1-IFF2, IFF3

1 Introduction to Static and Fatigue Design

2 Cuntze’s Failure-Mode-Concept-based Strength Criteria

3 Cuntze’s Fatigue Life Prediction Concept

4 Generation and Novel Interpretation of UD **Haigh Diagrams**

5 Steps of the Fatigue Life Prediction Method Proposed

AIM Cuntze: for brittle isotropic and UD-materials applicable

Automatic Establishment of the curved Constant Fatigue Life Curves (CFL) in Haigh Diagrams – for each single mode - on basis of

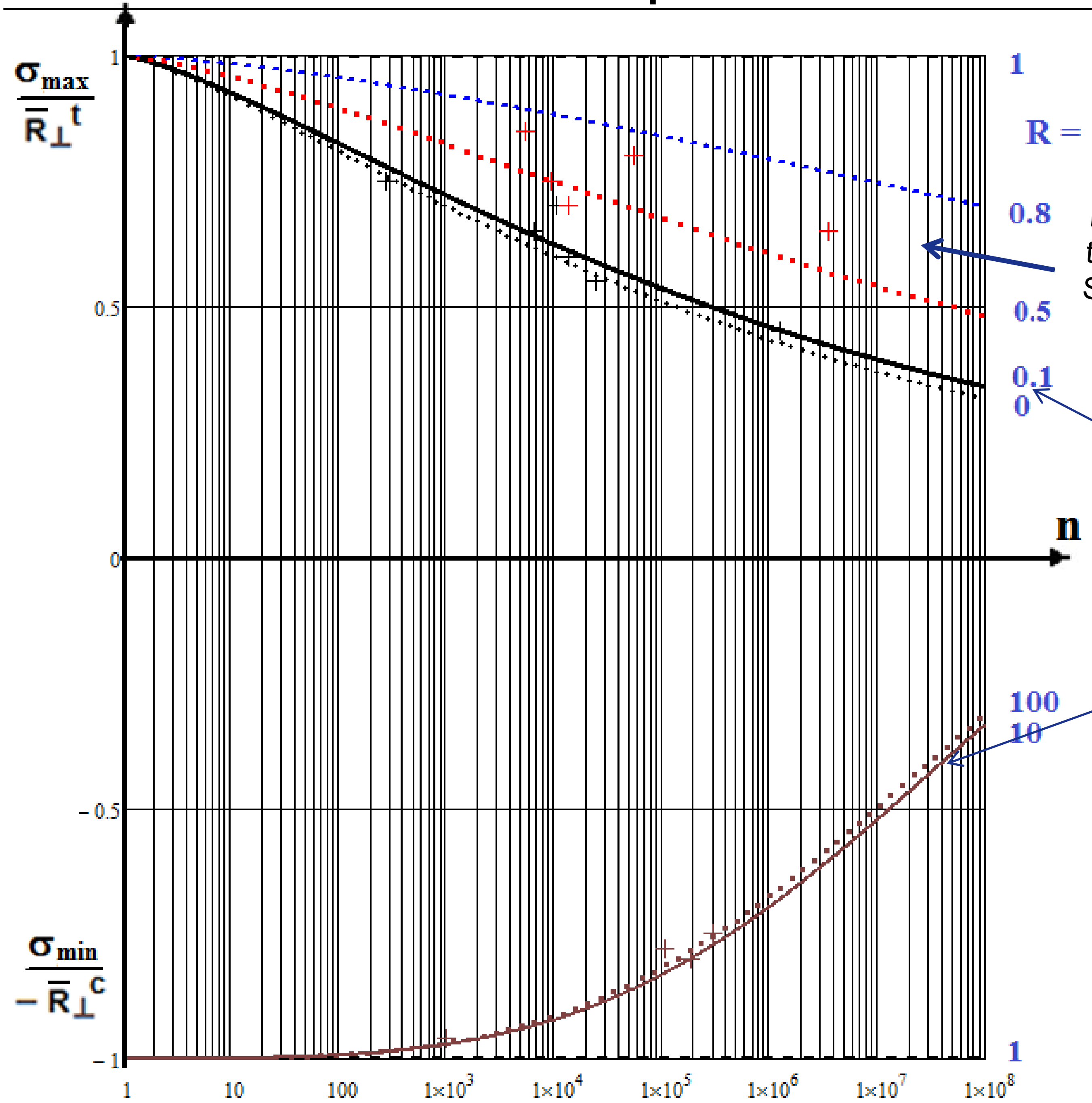
- a measured mode-decisive S-N curve as master curve, non-linear over the full range, plus
- a model [M. Kawai] to predict other necessary S-N curves on basis of above **Master S-N curve**.

In practice, usually, Haigh Diagrams are partly composed by straight lines. Is this reasonable especially for UD materials ?

Too many straight lines? Or curved lines ? [C. Hahne]

Original Kawai was failure-mode-linked further developed by Cuntze

How look Kawai model-predicted ‘Mode S-N curves ?



Example IFF1, IFF2‘

In the intermediate R-regime, the Kawai model-predicted S-N curves are conservative

on basis of the two given ‘Master mode S-N curves‘ 0.1, 10 ?

Kawai's Modified Fatigue Strength Ratio Ψ mode-linked applied to predict further necessary S-N curves on basis of one measured **Master S-N curve** $R = 0.1$ (NF), 10 (SF)

Searched model

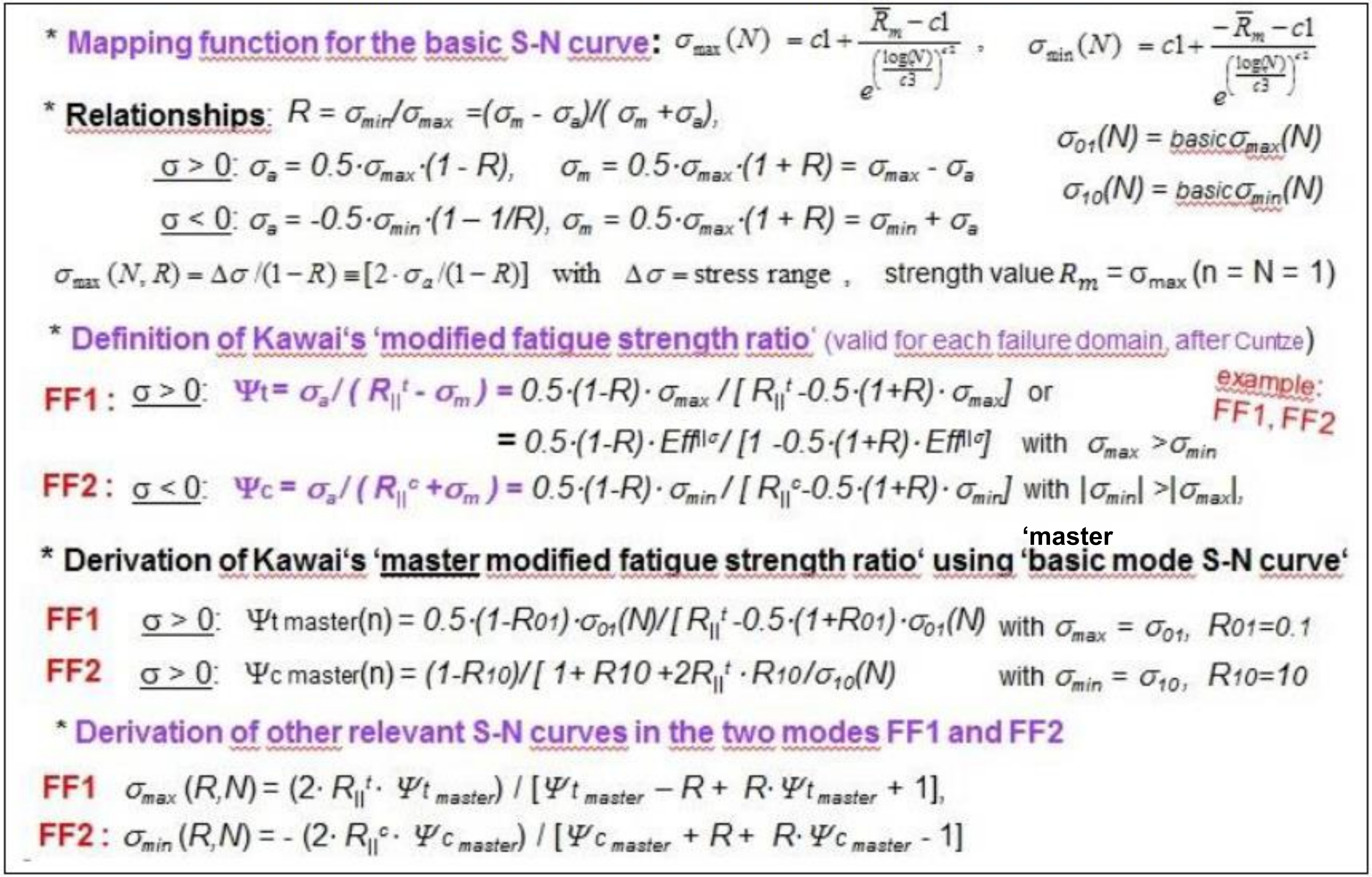


Table : Formulas to map the basic S-N curve and Kawai's Master Ψ-model

* **Mapping function for the basic S-N curve:** $\sigma_{\max}(N) = c1 + \frac{\overline{R}_m - c1}{e^{\left(\frac{\log(N)}{c3}\right)^{c2}}}$, $\sigma_{\min}(N) = c1 + \frac{-\overline{R}_m - c1}{e^{\left(\frac{\log(N)}{c3}\right)^{c2}}}$

* **Relationships**: $R = \sigma_{min}/\sigma_{max} = (\sigma_m - \sigma_a)/(\sigma_m + \sigma_a)$,

$\underline{\sigma > 0}$: $\sigma_a = 0.5 \cdot \sigma_{max} \cdot (1 - R)$, $\sigma_m = 0.5 \cdot \sigma_{max} \cdot (1 + R) = \sigma_{max} - \sigma_a$

$\underline{\sigma < 0}$: $\sigma_a = -0.5 \cdot \sigma_{min} \cdot (1 - 1/R)$, $\sigma_m = 0.5 \cdot \sigma_{max} \cdot (1 + R) = \sigma_{min} + \sigma_a$

$\sigma_{01}(N) = basic\,\sigma_{max}(N)$

$\sigma_{10}(N) = basic\,\sigma_{min}(N)$

$\sigma_{\max}(N, R) = \Delta\sigma/(1-R) \equiv [2 \cdot \sigma_a/(1-R)]$ with $\Delta\sigma$ = stress range , strength value $R_m = \sigma_{max}$ (n = N = 1)

* **Definition of Kawai's 'modified fatigue strength ratio'** (valid for each failure domain, after Cuntze)

FF1 : $\underline{\sigma > 0}$: $\Psi t = \sigma_a / (R_{||}^t - \sigma_m) = 0.5 \cdot (1-R) \cdot \sigma_{max} / [R_{||}^t - 0.5 \cdot (1+R) \cdot \sigma_{max}]$ or

$= 0.5 \cdot (1-R) \cdot Eff^{||\sigma} / [1 - 0.5 \cdot (1+R) \cdot Eff^{||\sigma}]$ with $\sigma_{max} > \sigma_{min}$

example: FF1, FF2

FF2 : $\underline{\sigma < 0}$: $\Psi c = \sigma_a / (R_{||}^c + \sigma_m) = 0.5 \cdot (1-R) \cdot \sigma_{min} / [R_{||}^c - 0.5 \cdot (1+R) \cdot \sigma_{min}]$ with $|\sigma_{min}| > |\sigma_{max}|$,

'master

* **Derivation of Kawai's 'master modified fatigue strength ratio' using 'basic mode S-N curve'**

FF1 $\underline{\sigma > 0}$: $\Psi t\,\text{master}(n) = 0.5 \cdot (1-R_{01}) \cdot \sigma_{01}(N) / [R_{||}^t - 0.5 \cdot (1+R_{01}) \cdot \sigma_{01}(N)$ with $\sigma_{max} = \sigma_{01}$, $R_{01} = 0.1$

FF2 $\underline{\sigma > 0}$: $\Psi c\,\text{master}(n) = (1-R_{10}) / [1 + R10 + 2R_{||}^t \cdot R_{10} / \sigma_{10}(N)$ with $\sigma_{min} = \sigma_{10}$, $R_{10} = 10$

* **Derivation of other relevant S-N curves in the two modes FF1 and FF2**

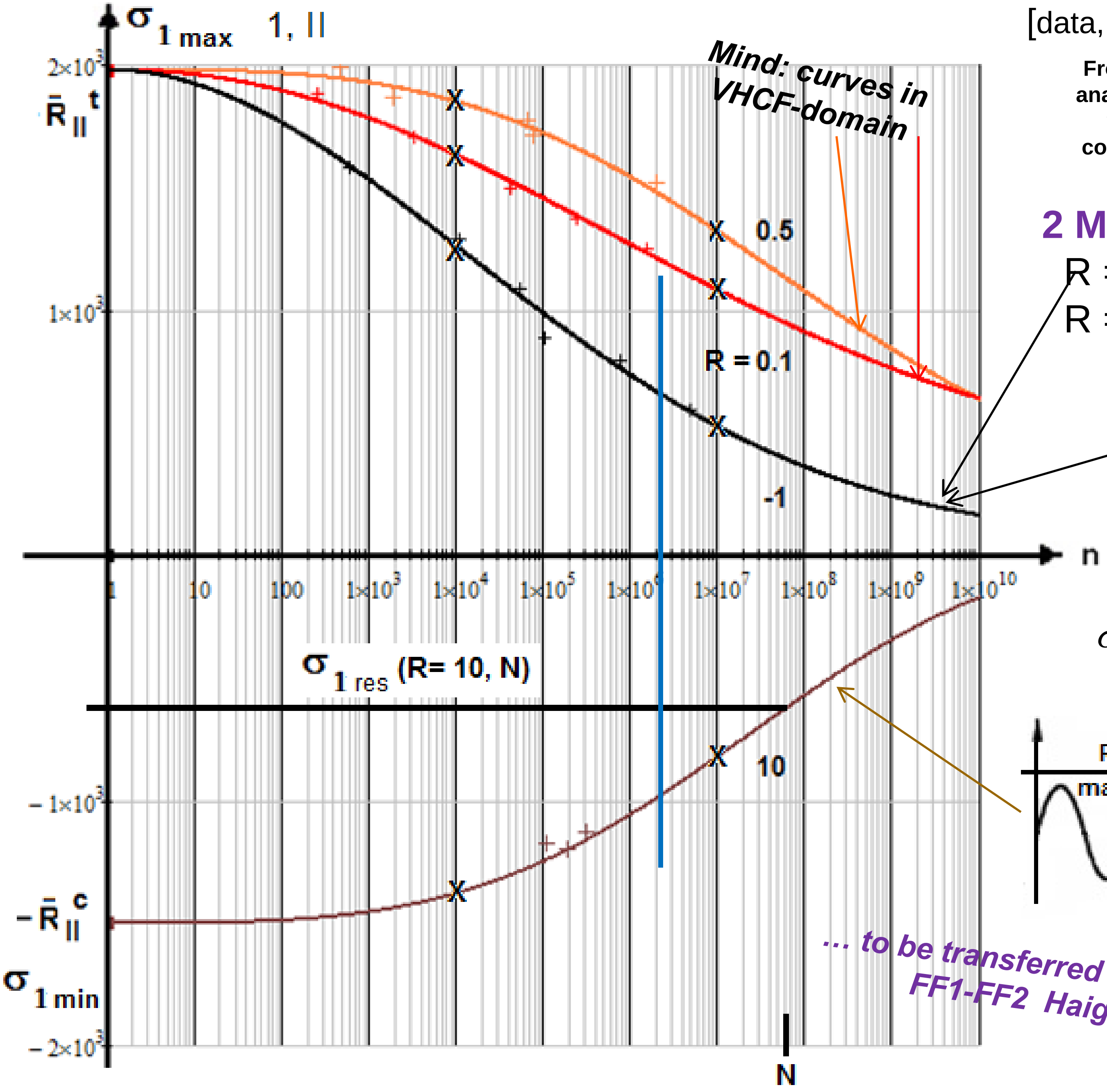
FF1 $\sigma_{max}(R,N) = (2 \cdot R_{||}^t \cdot \Psi t_{master}) / [\Psi t_{master} - R + R \cdot \Psi t_{master} + 1]$,

FF2 : $\sigma_{min}(R,N) = -(2 \cdot R_{||}^c \cdot \Psi c_{master}) / [\Psi c_{master} + R + R \cdot \Psi c_{master} - 1]$

S-N curve modelled : linearly, non-linearly in semi-log, log-log diagrams

Example: Individually lin-log mapped FF1-FF2-linked S-N curves

[data, courtesy Kawai-Suda]



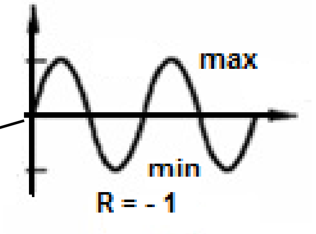
Mind: curves in VHCF-domain

From the mapped test data curve analytically determined are anchor points X for the prediction of constant fatigue life (CFL) curves

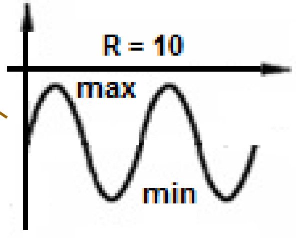
2 Master S-N-Curves:

R = 0.1 tension (FF1)

R = 10 compression



$$\sigma_{\max}(N) = c1 + \frac{\bar{R}_m - c1}{e^{\left(\frac{\log(N)}{c3}\right)^{c2}}}$$

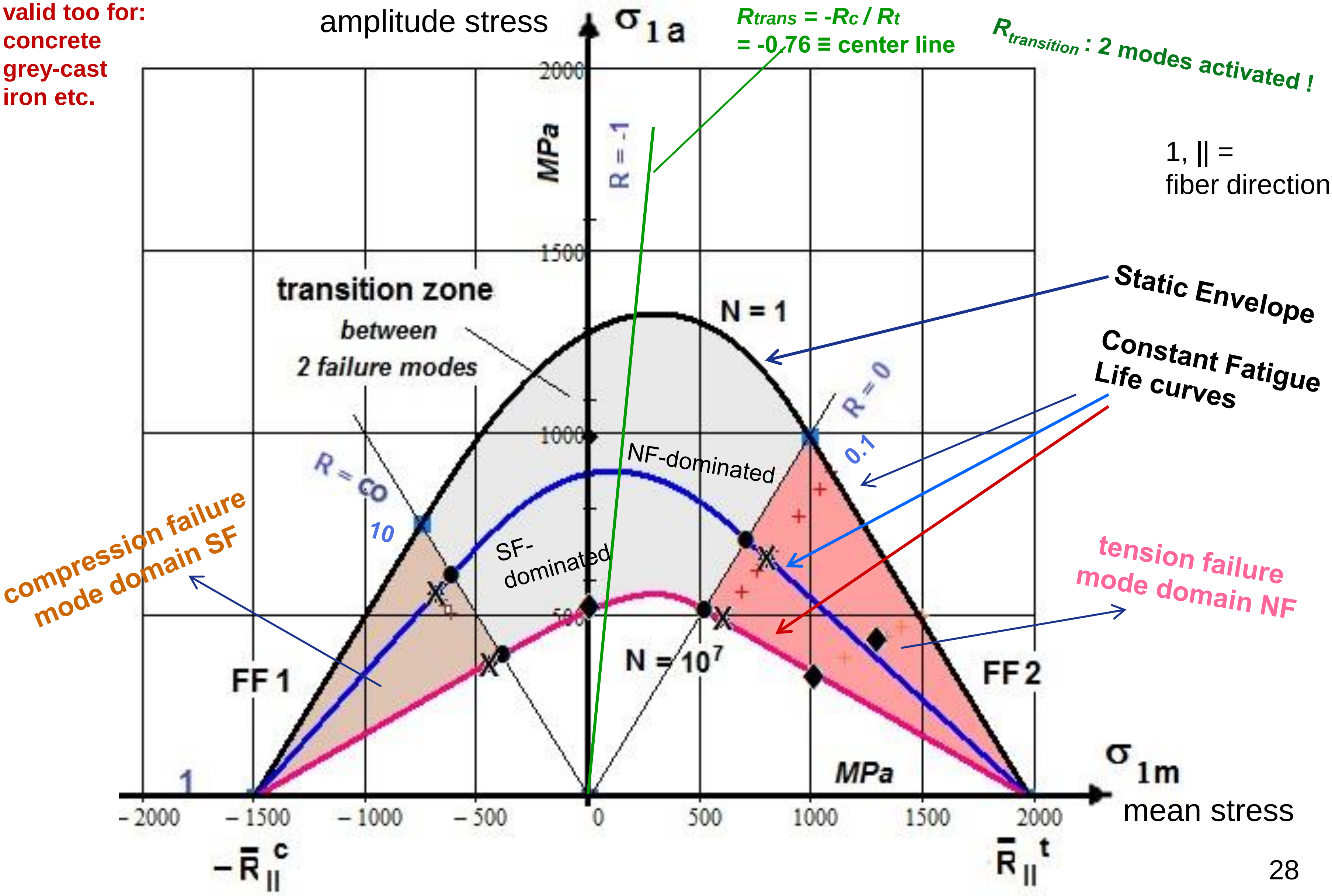


… to be transferred into the FF1-FF2 Haigh Diagram

Rigorous Interpretation of the Haigh diagram: example FF1-FF2 UD

displaying failure mode domains and transition zone, test data [Hah14]

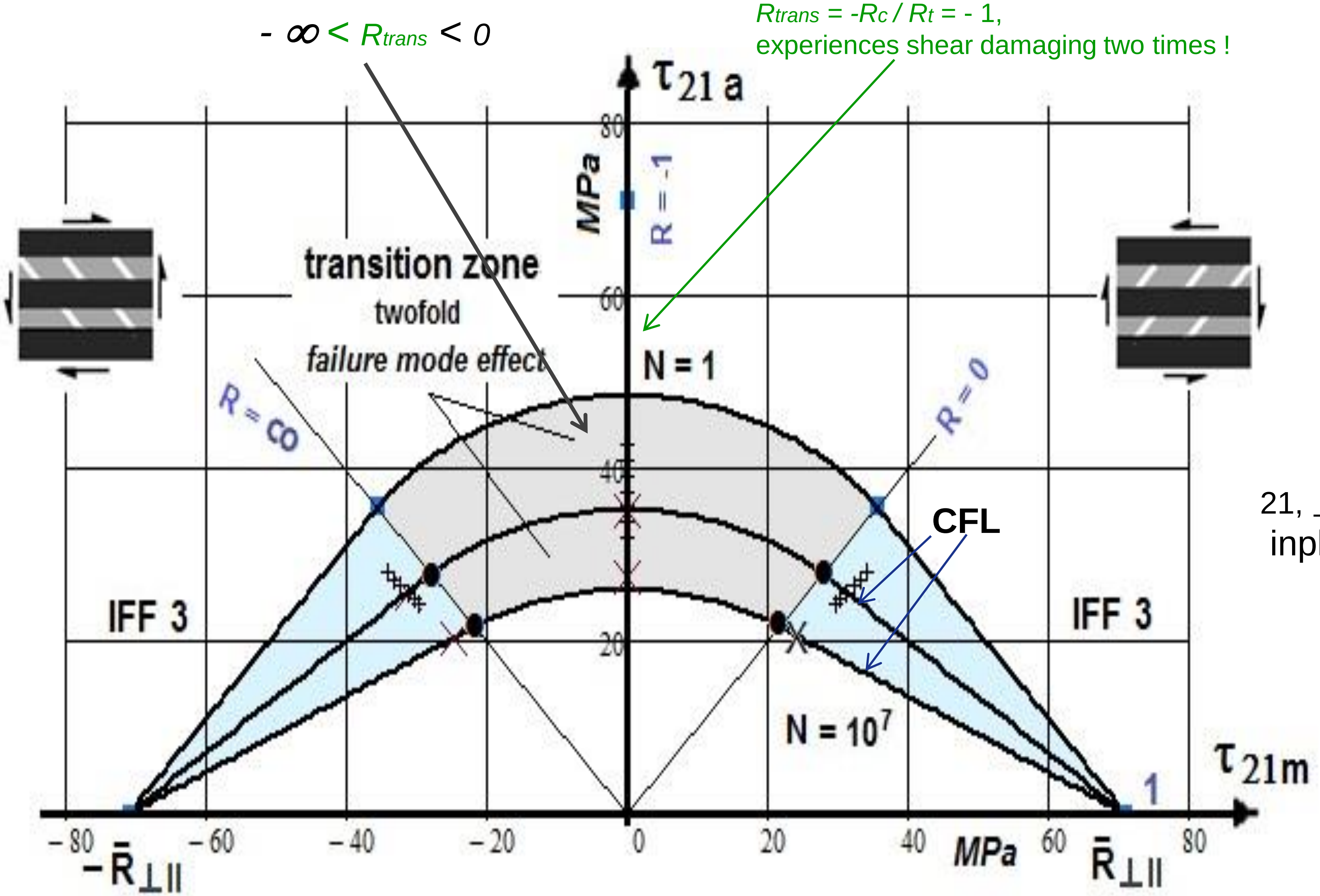
valid too for:
concrete
grey-cast
iron etc.



NF = Normal Fracture, **SF** = Shear Fracture, ***N*** **= fracture cycle number, CFL = Constant Fatigue Life**

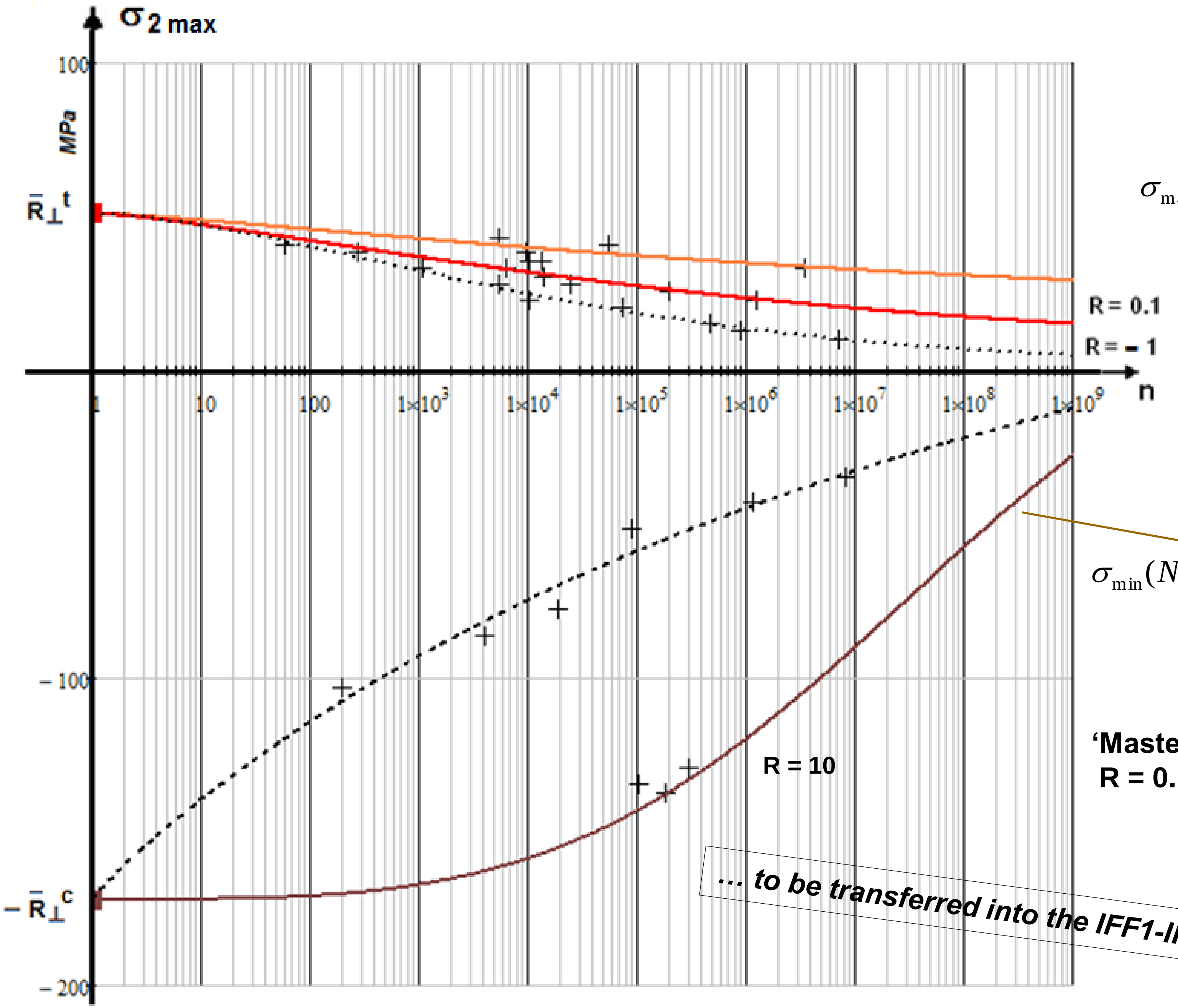
IFF3 UD Haigh diagram,

Display of a two-fold mode effect (a:= amplitude, m:= mean, N := number of fracture cycles, R := strength and R := $\sigma_{min}/\sigma_{max}$). Test data CF/EP, courtesy [Hahne14]



21, ⊥|| =
inplane shear

Mapped lin-log IFF1-IFF2-linked S-N curves *[data, courtesy C. Hahne]*



2, ⊥ = across
fiber direction

$$\sigma_{\max}(N) \;= c1 + \frac{\bar{R}_m - c1}{e^{\left(\frac{\log(N)}{c3}\right)^{c2}}}$$

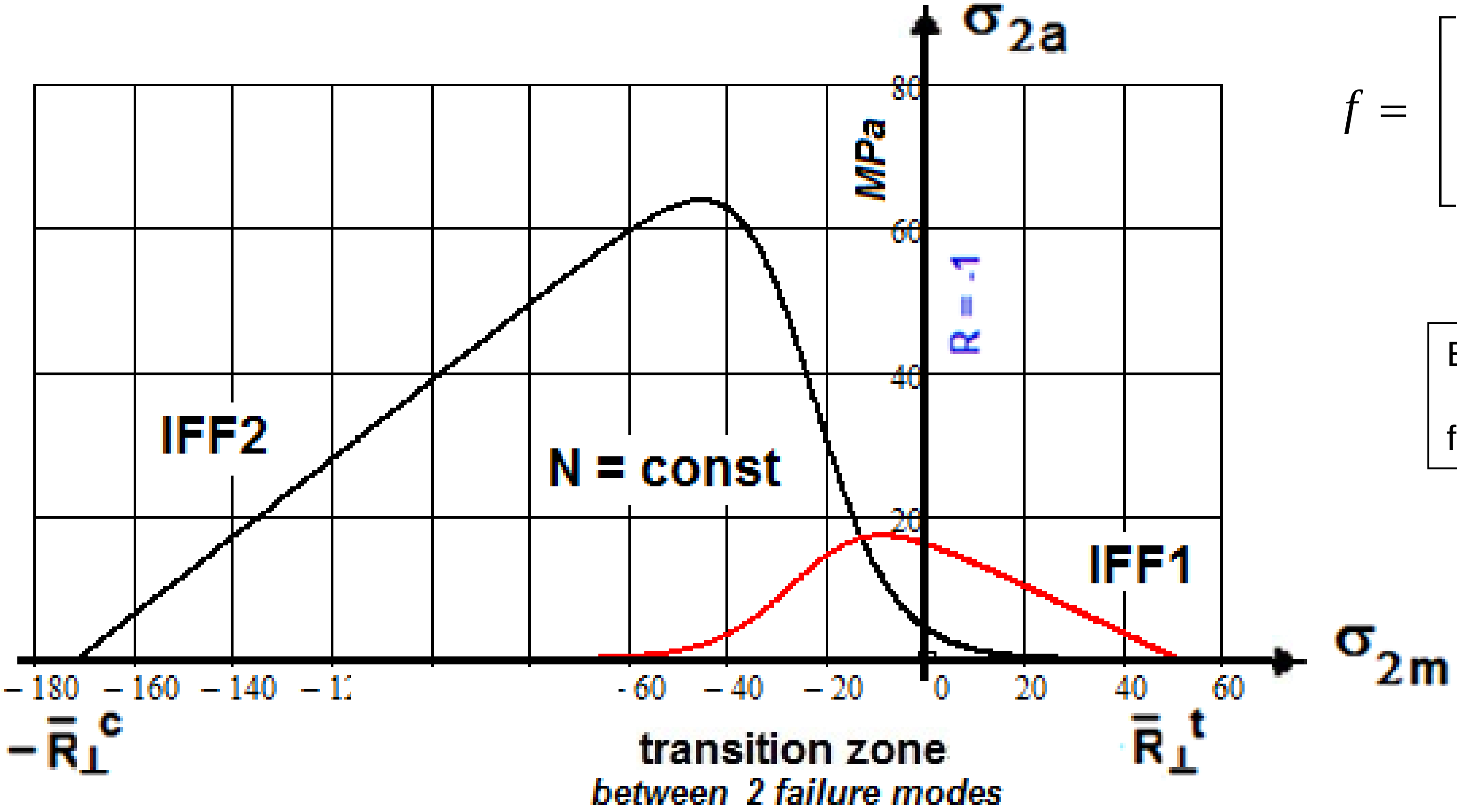
$$\sigma_{\min}(N) \;= c1 + \frac{-\bar{R}_m - c1}{e^{\left(\frac{\log(N)}{c3}\right)^{c2}}}$$

‘Master mode S-N curves‘ :
R = 0.1 and R = 10

… to be transferred into the IFF1-IFFF2 Haigh Diagram

How to obtain CFL curves in the Transition Domain ?

- *There is no problem to establish the Haigh diagrams FF and IFF3 due to the strength values being of similar size in each case: The static interaction formula was sufficient.*
- *For a Haigh Diagram for really brittle materials, **when R_{trans} is very different to -1**, a new solution procedure had to be used*. Chosen was an exponentially decaying function, that practiscally ends whwre the other mode reigns



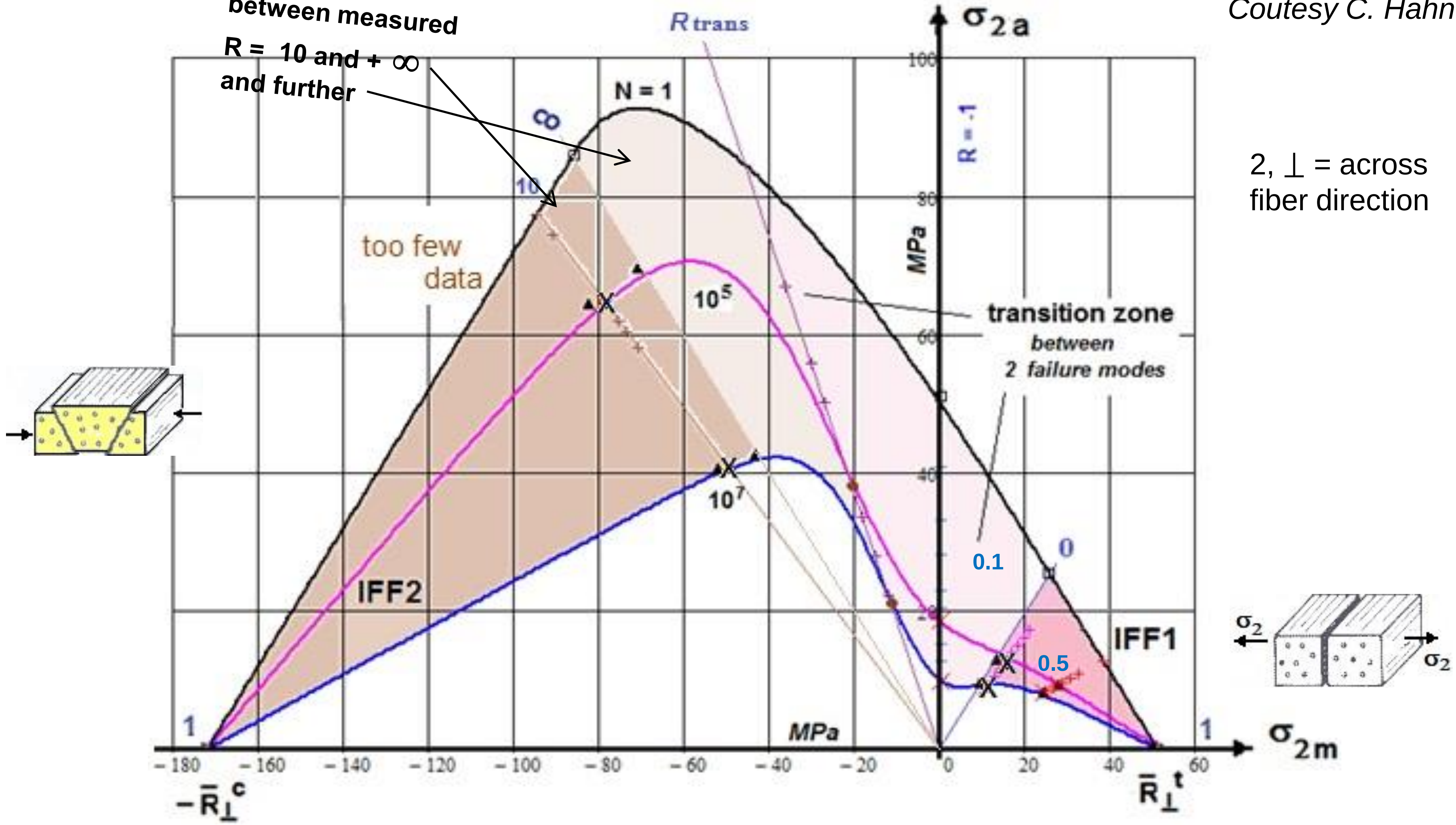
$$f = \left[\frac{1}{1 + e^{\frac{c_1 + \sigma_m}{c_2}}} \right] .$$

Example:
2, ⊥ = across fiber direction

→ failure domain-linked constant fatigue life (CFL) curves: ***σ_a (σ_m, R, N=constant)***

IFF1- IFF2 UD Haigh diagram

displaying the failure mode domains, transition zone



test-free area between measured R = 10 and + ∞ and further

$R_{trans} = -R_c / R_t = -3.4$

(Test data: Coutesy C. Hahne)

2, ⊥ = across fiber direction

- *Curve in the* IFF1 *domain looks non-linear !*
- *Check points from Ψ-prediction lie higher than points from S-N test data evaluation*

(The computed S-N curve X-points are anchor (checking) points for the to be predicted CFL curves)

1 Introduction to Static and Fatigue Design

2 Cuntze’s Failure-Mode-Concept-based Strength Criteria

3 Cuntze’s Fatigue Life Prediction Estimation Concept

4 Generation and Novel Interpretation of UD Haigh Diagrams

5 Steps of the Full Fatigue Life Prediction Method Proposed

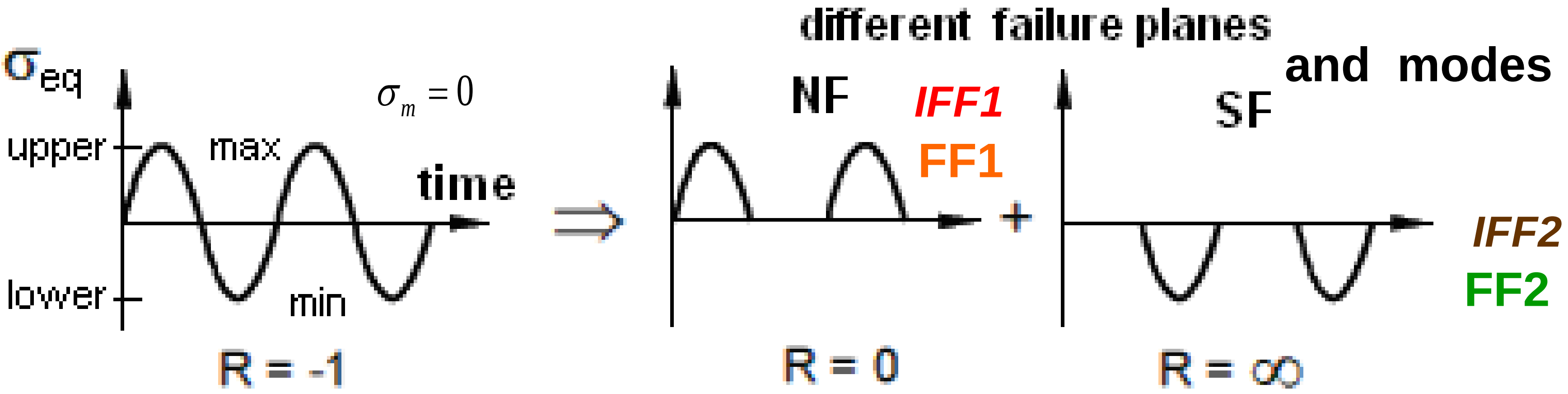
> *Keep in mind !*
>
> All is difficult prior to becoming simple!
>
> *[Moslik Saadi]*

Novel ? idea: Failure mode-wise modelling of Loading Cycles for high-performance 'fiber-dominated designed', UD laminas-composed laminates

---

For simply displaying the approach *it is chosen* : a *loading R = -1*

Separation due to the activated inherent different failure modes



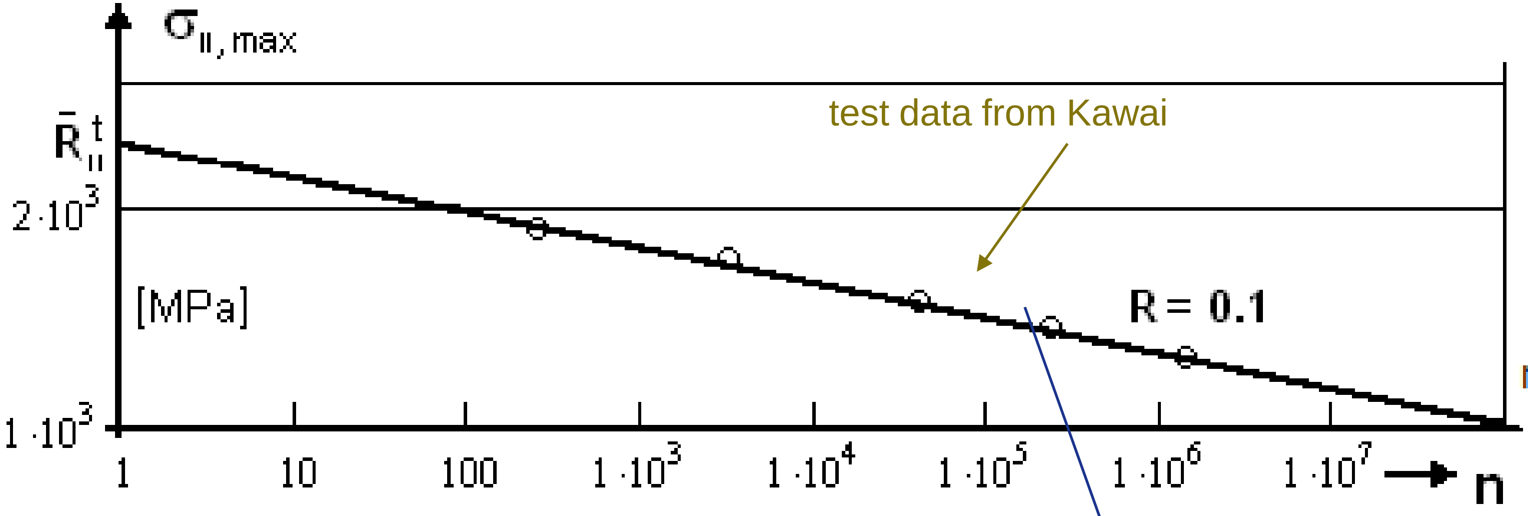
NF := Normal Fracture, SF := Shear Fracture

Step 1 : Failure mode-linked apportionment of cyclic loading (novel)

A specific **rain-fall** procedure must be applied

Mapping of S-N data and mode-representative ‘basic‘ *S-N curve*

Example:* FF1 *failure mode



applied as
master S-N curve
measured

Step 2 : S-N curve can be mapped, e.g. by a straight * line in a log-log graph

$$\sigma^{Master}_{\parallel,\ \max}(n) \approx \overline{R}^{\,t}_{\parallel} \cdot n^{c_{Master}} \; *$$

Measured curve used

as mode-representative ***Basic S-N curve*** **for FF1**

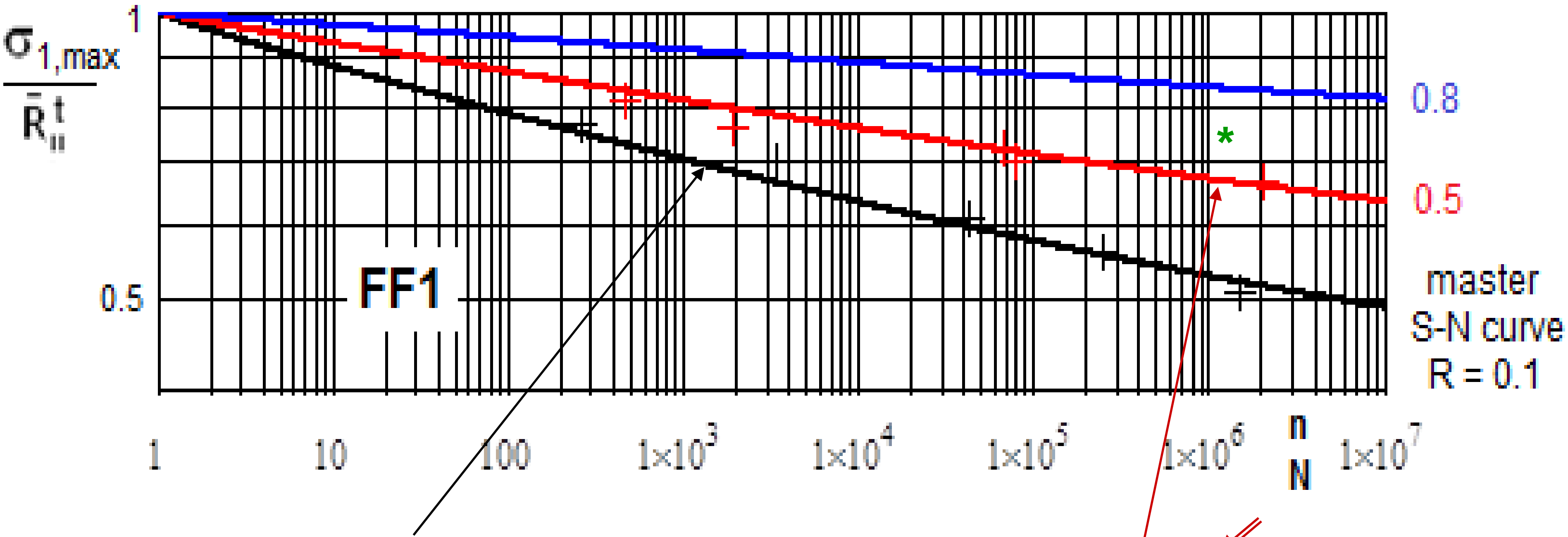
FF1 strength

In the general case of variable loading: Several S-N-curves are needed !

Master

* *more complicated S-N models are also applied !*

Prediction of needed other FF1 S-N curves from
Basic Mode S-N curve and Kawai model (Ψ Curve)



Given : $\sigma^{Basic}_{\|,\ max}(n) \approx \bar{R}^t_{\|} \cdot n^{c_{Basic}}$

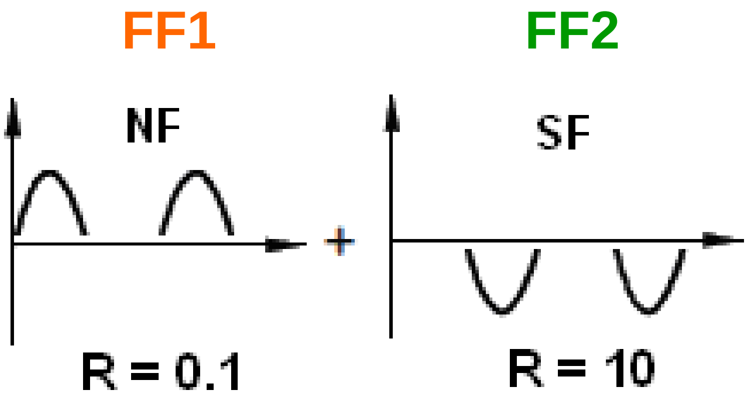
Searched : $\sigma^{pred}_{\|,\max} = \bar{R}^t_{\|} \cdot n^{c_{pred}}$

Slope of R = 0.5 ?

Step 3: Application of Kawai's 'Modified Fatigue Strength Ratio'.

*** improvable in the intermediate R-range**

Application of Miner-‘Rule‘ , for the *simple* loading *example* *R* = -1



$$D\ (FF1, FF2) = NF : (n_1 / N_1 + n_2 / N_2 + n_3 / N_3) + SF : (n_4 / N_4)$$

$$\Rightarrow D = D\ (FF1, FF2) + D\ (IFF1, IFF2, IFF3)\ \ \le\ \ D_{feasible}$$

from test experience

Step 4: Determination of Damaging Portions by Static Strength Criteria

Step 5: Mode-wise Accumulation of Damaging Portions (novel)

What was the main Objective of this Investigation ?
on basis of the ***'rigorous failure -mode thinking'***

For high performance composite parts:

Fatigue pre-dimensioning procedure for

'well-designed', UD laminas-composed laminates just by

- single lamina-dedicated mode-representative Master S-N curves, derived from *sub-laminate* test specimens, which capture the embedded ply (in-situ) effects,
- further necessary Kawai-model-predicted S-N curves
- automatically derivable CFL curves of the Haigh diagrams.

> Everything in the world is terminated by **chance** and **fatigue**.
>
> *Heinrich Heine*

Thank You !

Literatur

download possible from ***carbon~connected.de/Group/CCeV.Fachinformationen/Mitglieder***

[Ban16] Bansemir H.: *Certification Aspects*. Extended Abstract EC16, Augsburg 21-23. September 2016, conference publication

[Cun04] Cuntze R.: *The Predictive Capability of Failure Mode Concept-based Strength Criteria for Multidirectional Laminates*. WWFE-I, Part B, Comp. Science and Technology 64 (2004), 487-516

[Cun05] Cuntze R: *Is a costly Re-design really justified if slightly negative margins are encountered?* Konstruktion, März 2005, 77-82 and April 2005, 93-98 (reliability treatment of the strength problem)

[Cun09] Cuntze R.: *Lifetime Prediction for Structural Components made from Composite Materials – industrial view and one idea*. NAFEMS World Congress 2009, Conference publication.

[Cun12] HSB02000-01 *Essential topics in the determination of a reliable reserve factor*. 20 pages

[Cun13] Cuntze R.: *Comparison between Experimental and Theoretical Results using Cuntze's 'Failure Mode Concept' model for Composites under Triaxial Loadings - Part B of the WWFE-II*. Journal of Composite Materials, Vol.47 (2013), 893-924

[Cun13] Cuntze R.: *Tackling Uncertainties in Design – uncertain design parameters, safety concept, modelling and analysis.* Verbundwerkstoffe, GDM, 18. Symposium, Chemnitz 30.3. – 1.4. 2011

[Cun14] Cuntze R.: *The World-Wide-Failure-Exercises-I and –II for UD-materials – valuable attempts to validate failure theories on basis of more or less applicable test data*. SSMET 2014, Braunschweig, April 1 – 4, 2014, conference handbook

[Cun15] Cuntze, R: *Reliable Strength Design Veriftcation - fundamentals, requirements and some hints*. 3rd. Int. Conf. on Buckling and Postbuckling Behaviour of Composite Laminated Shell Structures, DESICOS 2015, Braunschweig, March 26 ~27, Extended Abstract, Conf. Handbook, 8 pages

[Cun16a] Cuntze R.*: Introduction to the Workshop – from Design Dimensioning via Design Verification to Product Certification*, Extended Abstract, Experience Composites (EC) 16, Symposium Handbook

[Cun16b]: *Classical Laminate Theory (CLT) for laminates composed of unidirectional (UD) laminae, analysis flow chart, and related topics*. Reworked HSB 37103-01, Draft, 2016, 58 pages

[Gai16] Gaier C., Dannbauer H., Maier J. and Pinter G.: *Eine Software-basierte Methode zur Betriebsfestig-keitsanalyse von Strukturbauteilen aus CFK*. CCeV-Austria, AG Engineering, Meeting St. Martin im Innkreis, Sept.8. Magna-Powertrain, Engineering Center Steyr

[Hah15] Hahne C.: *Zur Festigkeitsbewertung von Strukturbauteilen aus Kohlenstofffaser- Kunststoff-Verbunden unter PKW-Betriebslasten*. Shaker Verlag, Dissertation 2015, TU-Darmstadt, Schriftenreihe Konstruktiver Leichtbau mit Faser-Kunststoff-Verbunden, Herausgeber Prof. Dr.-Ing Helmut Schürmann

[HSB] German Aeronautical Technical Handbook *'Handbuch für Strukturberechnung*', issued in English by Industrie-Ausschuss-Struktur-Berechnungsunterlagen. TIB Hannover

[Kad13]Kaddour A. and Hinton M.: *Maturity of 3D failure criteria for fibre-reinforced composites: Comparison between theories and experiments*. Part B of WWFE-II, J. Compos. Mater. 47 (6-7) (2013) 925–966.

[Kaw04] Kawai M.: *A phenomenological model for off-axis fatigue behaviour of uni-directional polymer matrix composites under different stress ratios*. Composites Part A 35 (2004), 955-963

[Koc16] Koch I., Horst P. and Gude M.: *Fatigue of Composites – The state of the art*. EC16, Extended Abstract EC16, Augsburg 21-23. September 2016, conference publication

[Pet15] Petersen E., Cuntze R. and Huehne C.: *Experimental Determination of Material Parameters in Cuntze's Failure-Mode-Concept -based UD Strength Failure Conditions.* Submitted to Composite Science and Technology 134, (2016), 12-25

[Puc02] Puck A. and Schürmann H.: *Failure Analysis of FRP Laminates by Means of Physically based Phenomenological Models.* Composites Science and Technology 62 (2002), 1633-1662

[Rac87] Rackwitz R. and Cuntze R..: *System Reliability Aspects in Composite Structures*. Engin. Optim., Vol. 11, 1987, 69-76

[Roh14] Rohwer K.: *Predicting Fiber Composite Damage and Failure*. Journal of Composite Materials, published online 26 Sept. 2014 (online version of this article can be found at: http://jcm.sagepub.com/content/early/2014/09/26/0021998314553885)

[Sch06] Schürmann H.: *Konstruieren mit Faser-Kunststoff-Verbunden*. Springer-Verlag 2005

[Schu??] Schulte K.:

[Sho06] Shokrieh M.M. and Tahery-Behroz F.: *A unified fatigue model based on energy method.* Composite Structures 75 (2006), 444-450

[VDI2014] VDI 2014: German Guideline, Sheet 3 *Development of Fibre-Reinforced Plastic Components, Analysis*. Beuth Verlag, 2006. (in German and English, author was convenor)

[Wei11] Weinert A. and Gergely P.: *Fatigue Strength Surface: basis for structural analysis under dynamic loads*. CEAS Aeronautical Journal 2011, Vol.2, Issue 1, 243-252

Some Definitions needed for Modeling *What is ?*

Material : homogenized (smeared) model of the envisaged complex material which might be a material combination

Failure : structural part does not fulfil its functional requirements such as FF = fiber failure, IFF = inter-fiber-failure (matrix failure), leakage, deformation limit (tube widening, delamination size limit, ..) ⇨ **= a <u>project-defined</u> ‘defect‘**

Fatigue : process, that degrades material properties

Fatigue Life Stages (1) accumulation of damaging until initiation of a critical damage size (classical fatigue life prediction domain), (2) damage growth until onset of final fracture (domain of damage tolerance concepts), (3) separation (not of interest)

Damaging (not also damage, as used in English literature) **:** process wherein the results, the damaging portions, finally accumulate to a damage size such as a macro-scopic delamination. Accumulation tool usually used is *Palmgren-Miner‘s Damaging Accumulation Rule* (= model)

Damage : sum of the accumulated damaging or an impact failure, that is judged to be critical. Then, *Damage Tolerance Analysis* is used to predict damage growth under further cyclic loading or static failure under Design Ultimate Load

Haigh Diagram : involves all S-N curves required for fatigue life prediction.

Solution procedure , IFF1-IFF2 Haigh Diagram

Static strength failure $Eff = [(Eff^{NF})^m + (Eff^{SF})^m]^{m^{-1}} = 100\%$

$$\left(\frac{-(\sigma_{2m} - \sigma_{2a}) + |\sigma_{2m} - \sigma_{2a}|)}{2 \cdot \overline{R}_\perp^{\,c}}\right)^m + \left(\frac{\sigma_{2m} + \sigma_{2a} + |\sigma_{2m} + \sigma_{2a}|)}{2 \cdot \overline{R}_\perp^{\,t}}\right)^m = 1$$

The used static procedure still works for N = 1 with the interaction formula above delivering the CFL curve for N = 1 cycle, activating both NF + SF .

For higher N the interaction formula is engineering-like simplified. *It reads*:

$$\sigma_a(\sigma_m) = \left[\left(\frac{\sigma_{aSF}}{1 + e^{\frac{c_{1SF} + \sigma_m}{c_{2SF}}}}\right)^m + \left(\frac{\sigma_{aNF}}{1 + e^{\frac{c_{1NF} + \sigma_m}{c_{2NF}}}}\right)^m\right]^{1/m}$$

Thereby, an exponential decay function of the SF mode CFL curve for SF from R = ∞ down to zero at the end of the NF CFL curve at R = 0 is applied:

$$f = \left[\frac{1}{1 + e^{\frac{c_1 + \sigma_m}{c_2}}}\right] .$$