Correlations between and Interrelationships of the Fracture Behaviours of Isotropic Materials, Unidirectional Composites, and Woven Fabrics

- depicted on basis of Cuntze's Failure Mode Concept

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Summary:

Phenomenological, invariant-based strength (failure) conditions are applied for a variety of materials which might possess a dense or a porous consistency. These materials are brittle and ductile isotropic materials, brittle unidirectional laminae and brittle woven fabrics. The failure conditions are based on the author's Failure Mode Concept (FMC).

Essential topics of the paper are:
- 'Global fitting' versus 'failure mode fitting',
- a brief derivation of the FMC,
- the presentation of the FMC-based strength failure conditions for the material families above
- the visualization of the derived conditions
- the outlining of correlations and interrelationships of the materials.

Conclusions drawn from the investigations:
- The FMC is an effective concept that very *strictly* utilizes a failure mode thinking as well as the application of *material symmetry-related invariants.* It has proven to be a helpful tool in simply fitting the course of multi-axial material strength test data, and it finally can capture several failure modes in one equation avoiding the shortcomings of the usual 'global fitting' strength conditions.
- Very different but similar behaving materials can be basically treated with the same strength condition. This makes to think about a procedure which -in case of a new material- more simply and less costly should enable us for an engineering assessment of multi-axial states of stress. And, this can be based on a less big test campaign plus the available information from results of a similar behaving material.
- More representative multi-axial test data should be available. They are necessary to really make a three-dimensional validation of the various failure conditions of all structural materials possible, even of some standard ones.

Keywords:

Failure criteria, correlations, isotropic brittle and ductile materials, UD laminae, woven fabrics

1 Introduction

Design Verification demands for reliable reserve factors and these –besides a reliable structural analysis- for reliable strength failure conditions. Such a condition is the mathematical formulation of a failure curve or a failure surface, respectively. In aerospace, the static design verification has to be performed on Flight Load level which corresponds to *onset of (global) yielding* as well as on Design Ultimate Load (*DUL*) level which corresponds to *onset of fracture.* The first requires a yielding failure condition in case of isotropic ductile materials or a similar condition in case of composites, based on a similar deterioration for a composite as yielding for an isotropic material. The latter requires fracture conditions as strength failure conditions which usually means fibre failure in case of composites.

Material (strength) failure of non-cracked structural parts is addressed only, and not stability failure or damage tolerance as well as physical and material nonlinearities in the analysis (see for UD material, [Cun04]). The contents of this paper reflects a presentation at the CDCM06 in Stuttgart.

In general, failure conditions shall assess a multi-axial stress state in the critical material point by utilizing the uniaxial strength values R and an equivalent stress σ_{eq} which represents a distinct actual stress state. Further, they shall allow for inserting stresses of the utilized different coordinate systems (COS) into usually stress-formulated failure conditions, and optimally -if possible- into invariant-based ones.

Such failure conditions have to be generated for combinations of dense & porous, ductile & brittle behaving materials. These can be isotropic materials, transversely-isotropic (UD:= unidirectional) materials and rhombically-anisotropic materials (woven fabrics) and in future for structural textiles, stitched or braided or knitted. The structural build-up of the latter will require a quasi-ductile treatment for these basically entirely brittle behaving 'material'.

Most often, failure conditions map a course of multi-axial test data by one global equation (i.e. [Tsa71]) not taking care whether the data belong to one or more failure mechanisms or failure modes. Therefore, extrapolations out of the mapped or fitted domain may lead to erroneous multi-axial strengths. Further, if a correction change in the domain of one failure mode has to be made due to improved test results it may affect the failure surface or failure curve domain of another independent failure mode. This is a mathematical consequence but not a physically correct one [Har93, VDI97].

Driven from the shortcomings of such a 'global fitting' the author since 1994 looked for a 'failure mode-related fitting'. The procedure 'How to determine such *mode failure conditions*?' he termed the Failure Mode Concept FMC [Cun98]. The FMC is a concept that very strictly uses a failure mode thinking (more than others prior or later to the author, such as v. Mises with the HMH hypothesis [Mis1913], Hashin [Has80], and Christensen [Chr97]. Nevertheless, the FMC shall capture several failure modes in one equation without the short-comings of the classical global failure conditions.

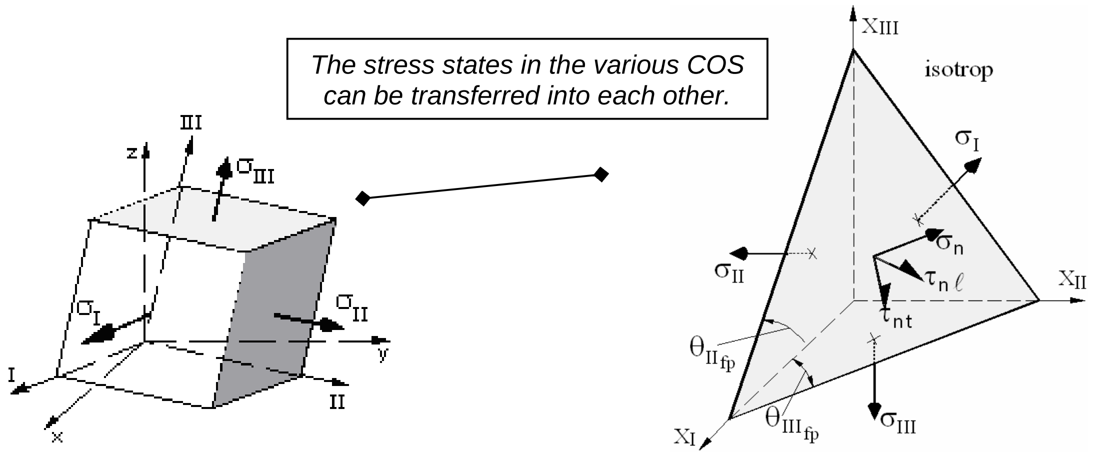
The concept is also based –as far as the material homogenization permits to do it- on material symmetry-related invariants, which have proven to be a helpful tool in simply fitting multi-axial strength test data. The application of invariants in the generation of strength failure conditions has benefits due to the fact that material symmetry [Cun98a], together with the findings of Beltrami [Bel1885], support the choice of an invariant to be utilized in a distinct failure condition. An invariant is a combination of stresses –powered or not powered- the value of which does not change when altering the coordinate system. Invariants are optimum for the formulation of advantageous scalar strength failure conditions.

Existing links in the mechanical behaviour show up: Different structural materials can possess similar material behaviour or can belong to the same class of material symmetry. For instance: a brittle

porous concrete in the compression domain can be basically described by the same failure condition like a very ductile behaving light-weight steel in the high tension domain when pores are generated.
This has the consequence: The same strength failure function *F* can be used for different materials and more information is available for pre-dimensioning and modelling from past experimental results of a similarly behaving material. Therefore the message is: Use these benefits!
Special ***aim*** of the paper shall be ***a global view of the material links/correlations/ interrelationships and not a detailed information on the derived failure conditions.***

2 Stress States & Invariants

There are various kinds of stresses which may be inserted into a strength failure condition. In the case of isotropic materials we face: principal stresses, structural component stresses, and Mohr's fracture plane stresses, [Moh1900, Pau61]. These stresses can be transferred into each other. ***Fig. 1*** outlines all these stresses and the associated invariants for **isotropic materials**.



Principal stresses

$$\{\sigma\} = (\sigma_I,\ \sigma_{II},\ \sigma_{III})^T$$

$$I_1 = (\sigma_I + \sigma_{II} + \sigma_{III})^T = 3\sigma_{oct} \equiv f(\sigma),$$

$$6J_2 = (\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2 = 4(\tau_{III}^2 + \tau_{II}^2 + \tau_I^2) = 9\tau_{oct}^2 \equiv f(\tau)$$

Structural component stresses

$$\{\sigma\} = (\sigma_x,\ \sigma_y,\ \sigma_z,\ \tau_{yz},\ \tau_{xz},\ \tau_{xy})^T$$

'isotropic invariants'!

$$I_1 = (\sigma_x + \sigma_x + \sigma_z)^T$$

$$6J_2 = (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + (\sigma_x - \sigma_y)^2 + 6(\tau_{yz}^2 + \tau_{zx}^2 + \tau_{xy}^2)$$

Mohr's fracture plane stresses

$$\{\sigma\} = (\sigma_\lambda,\ \sigma_n,\ \sigma_t,\ \tau_{nt},\ \tau_{t\lambda},\ \tau_{\lambda n})^T$$

$$I_1 = (\sigma_\lambda + \sigma_n + \sigma_t)^T$$

$$6J_2 = (\sigma_n - \sigma_t)^2 + (\sigma_t - \sigma_\lambda)^2 + (\sigma_\lambda - \sigma_n)^2 + 6(\tau_{nt}^2 + \tau_{t\lambda}^2 + \tau_{n\lambda}^2)$$

$$27J_3 = (2\sigma_I - \sigma_{II} - \sigma_{III})(2\sigma_{II} - \sigma_I - \sigma_{III})(2\sigma_{III} - \sigma_I - \sigma_{II}),$$

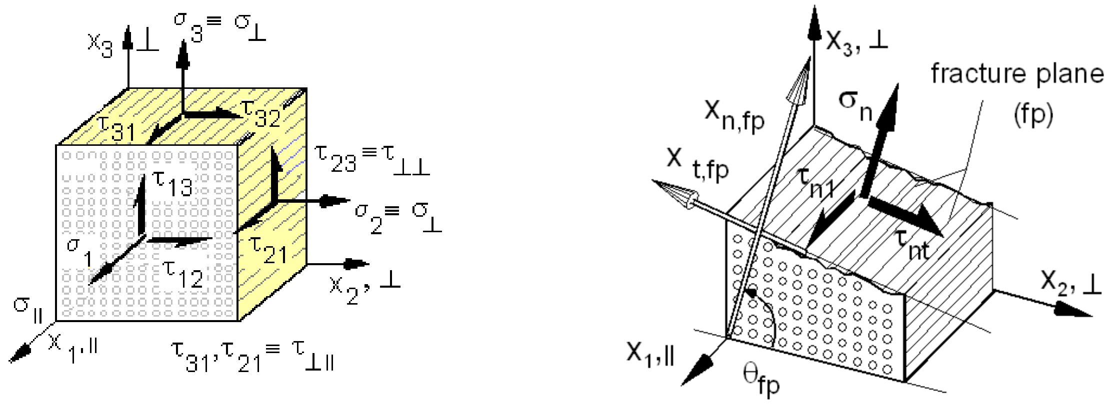
$$I_\sigma = 4J_2 - I_1^2/3, \quad \sigma_{mean} = I_1/3$$

Fig. 1*: Isotropic Material, 3D stress states & invariants*

Formulations of strength failure conditions may use, such as recently performed by Hashin/Puck for unidirectional laminae, Mohr's postulate: "*Fracture is determined by the stresses in the fracture plane!*" Most often this has an formulation advantage but additionally the determination of the angle of the inclined fracture plane is needed.

In case of **transversely-isotropic material** (UD composite) the associated stresses and invariants are depicted in ***Fig. 2***. It is to be seen that three kinds of stresses are applied: lamina COS–based stresses, Mohr stresses, and –as there is a quasi-isotropic plane existing- somehow quasi-principal stresses.

In case of **orthotropic material** (rhombically-anisotropic, ***Fig. 3)***, just a formulation in fabrics lamina stresses makes sense. And, one has to deal with more, however simpler invariants.



$\{\sigma\}^{quasi-isotropic\ \ plane}_{principal} = (\sigma_1, \sigma_2^p, \sigma_3^p, 0, \tau_{31}^p, \tau_{21}^p)^T$

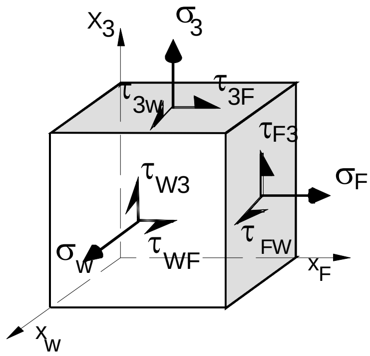
$\{\sigma\}_{la\,min\,a} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$

$\{\sigma\}_{Mohr} = (\sigma_\lambda,\ \sigma_n,\ \sigma_t,\ \tau_{nt},\ \tau_{t\lambda},\ \tau_{\lambda n})^T$

UD invariants *[Boe85]*

$I_1 = \sigma_1\ , \quad I_2 = \sigma_2^p + \sigma_3^p$	$I_1 = \sigma_1, \quad I_2 = \sigma_2 + \sigma_3$	$I_1 = \sigma_\lambda, \quad I_2 = \sigma_n + \sigma_t$
$I_3 = \tau_{31}^{p\ 2} + \tau_{21}^{p\ 2}$	$I_3 = \tau_{31}^2 + \tau_{21}^2$	$I_3 = \tau_{t\lambda}^2 + \tau_{n\lambda}^2$
$I_4 = (\sigma_2^p - \sigma_3^p)^2 + 0$	$I_4 = (\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2$	$I_4 = (\sigma_n - \sigma_t)^2 + 4\tau_{nt}^2$
$I_5 = (\sigma_2^p - \sigma_3^p)(\tau_{31}^{p\ 2} - \tau_{21}^{p\ 2}) + 0$	$I_5 = (\sigma_2 - \sigma_3)(\tau_{31}^2 - \tau_{21}^2) + -4\tau_{23}\tau_{31}\tau_{21}$	$I_5 = (\sigma_n - \sigma_t)(\tau_{t\lambda}^2 - \tau_{n\lambda}^2) - 4\tau_{nt}\tau_{t\lambda}\tau_{n\lambda}$

Fig. 2*:Transversely-isotropic material, 3D stress states & invariants*



Woven fabrics material element. *Warp (W), Fill(F).*

Stress vector:

$$\{\sigma\}_{la\,min\,a} = (\sigma_W, \sigma_F, \sigma_3, \tau_{3F}, \tau_{3W}, \tau_{FW})^T$$

Fabrics invariants *[Boehler]*:

$$I_1 = \sigma_W\ ,\ I_2 = \sigma_F\ ,\ I_3 = \sigma_3$$

$$I_4 = \tau_{3F}\ ,\ I_5 = \tau_{3W}\ ,\ I_6 = \tau_{FW}$$

Fig. 3*: Orthotropic (rhombically-anisotropic) material, 3D stress state & invariants, [Boe95]*

3 Observed Strength Failure Modes and Strengths

Of high interest for the establishment of material strength conditions is the number of strength failure modes and the number of strengths, observed in the fracture tests, [Mas94].

For the various **isotropic materials** several typical failure modes can be differentiated between: a) b*rittle behaviour/dense consistency, b) brittle behaviour/ porous consistency, and c) ductile behaviour/dense consistency.*

Specifics are:

a) Two failure modes: Normal Fracture (NF) under tension and Shear Fracture (SF) under compression are recognized. NF, due to the poor deformation prior to fracture and the smooth fracture surface (fractography reveals), is also termed *cleavage fracture.* SF exhibits shear deformation prior to fracture, a knowledge, which will be helpful for the choice of invariants with the formulation of the strength condition. Two strength have to be measured.

b) Two failure modes are found: NF and Crushing Fracture (CrF). The latter shows a volumetric deformation prior to fracture which will be helpful for the choice of invariants, too. Remarkable for the result of a compression test is that there is a full decomposition of the texture, a hill of fragments (crumbs) remains. Two strength have to be measured.

c) Prior to fracture just one failure mode is to be identified: SF under tension. Shear deformation is observed prior to fracture (at maximum load) and then diffuse and later, local necking + void growth (means a volumetric change) prior to rupture in the so-called 'Gurson domain', [Cun02]. This SF is also termed t*earing fracture* and shows dimples under tension. One strength, the *load-controlled* value R_m^t*, is* to be measured. The corresponding compressive strength is neither existing nor necessary for design, because deformation-limiting design requirements will not permit to go that far. However, if the vicinity of a highly strained location will take over the load locally and it happens that $\sigma_{eq} > R_m^t$. Then the *deformation-controlled multiaxial* strength at rupture may be considered in design [Cun02]. If the Finite Element Analysis output delivers true stresses then the true strength can and should be applied to be consistent.

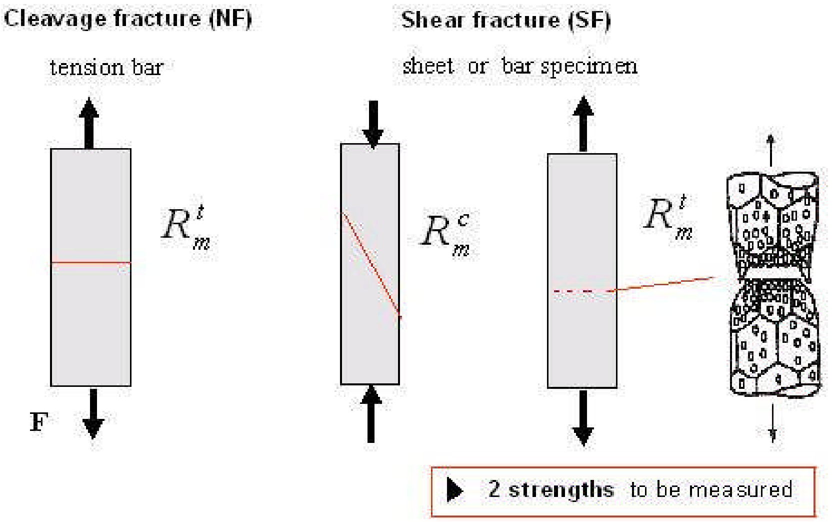


Fig. 4: *Fracture failure modes and strengths of isotropic materials*

For **transversely-isotropic material** (brittle Uni-Directional lamina) fractography of test specimens reveals (***Fig. 5***) that 5 fracture modes exist in a UD lamina: 2 FF (Fibre Failure) + 3 IFF (Inter Fibre Failure). From basic knowledge and test experience is known: common practice is measuring 5 strengths for an accurate design.

Of highest importance for failure are the FF. However learned from component tests, Puck's wedge failure mode *IFF3* might be hazardous like an *FF*. It depends on the stack and the entire loading.

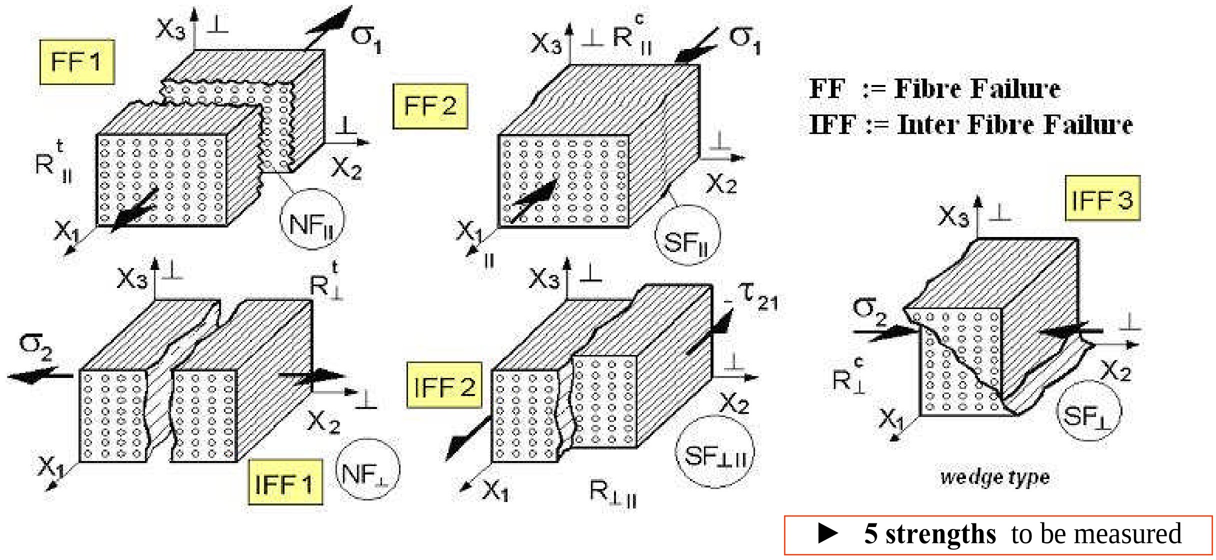


Fig. 5: *Fracture failure modes of UD materials*

For **rhombically-anisotropic or orthotropic material** (woven fabrics, ***Fig. 6***) from testing and theory is learned: There are more strength failure modes than for UD material (causes more mode interaction domains), but simpler invariants are existing and the hope that a simpler formulation of single strength conditions is possible, too. Fractography however exhibits no clear failure modes. In this material case multiple cracking is always observed.

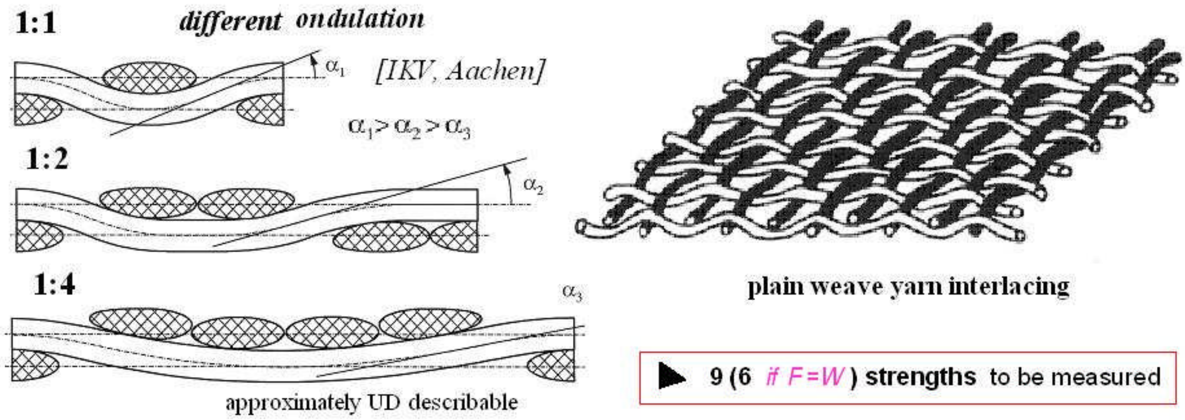


Fig. 6: *Fracture failure of woven fabrics material*

It is caused under tension, compression, bending, shear. No clear mode-related strengths are observed. Therefore, they have to be defined according to the orthotropic material symmetry [Cun98]. From the left part of the figure one can conclude: Modelling depends on fabrics type.

Due to the various fibre pre-forms: from roving, tape, weave, and braided (2D, 3D), knitted, stitched, or mixed as in a pre-form hybrid the needed variety of failure conditions to be developed in case of the 'higher' structural textiles becomes obvious.

4 Attempt for a Systematization

The design process is faced with numerous different materials, different material behaviour, various failure modes and a lot of more or less validated strength failure conditions. The question arises: Is there a possibility in this complex design business to find a procedure to figure out failure conditions which are simple enough, however, describe physics of each failure mechanism sufficiently well? Is a systematization helpful? ***Fig. 7*** displays a scheme of (material) strength failure conditions for isotropic material and directly compares it with the actually used brittle UD material.

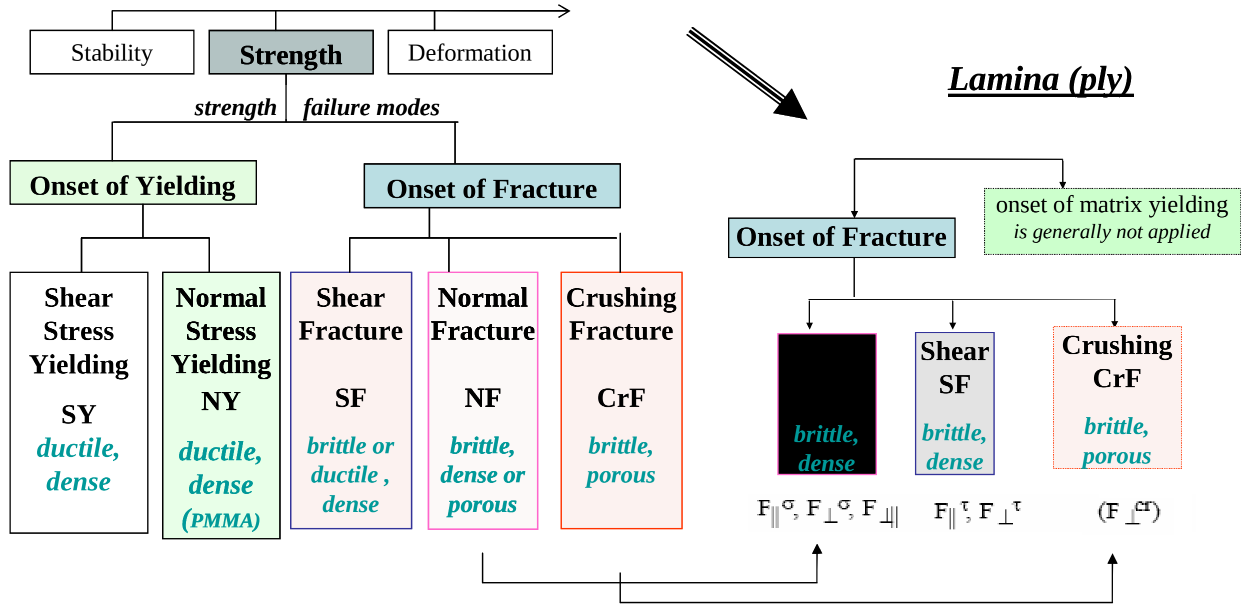


Fig. 7: *Scheme of Strength Failure Conditions for isotropic material and brittle UD*

What may be learned from the isotropic part of the figure and the study of the associated failure conditions: The same mathematical form of a failure condition (interaction of stresses within one mode) is valid from *onset of yielding* to *onset of fracture,* if the physical mechanism remains, e.g. shear yielding in case of ductile steels.

In general, the growing yield body (SY) is confined by the fracture surface (SF or NF). The figure also emphasizes that one kind of failure which is normally not addressed, Normal Yielding (NY), seems to exist (see last figure) for the chain-based texture of PMMA (plexiglass).

The arrows denote the coincidences between brittle UD laminae and brittle isotropic materials. Delamination failure of laminates –built up from the UD material- is not addressed here.

Regarding *Fig. 7* the establishment of strength failure conditions needs to be further structured.

Fig. 8 gives an overlook on homogenizing a material on different structural levels or scales. In case

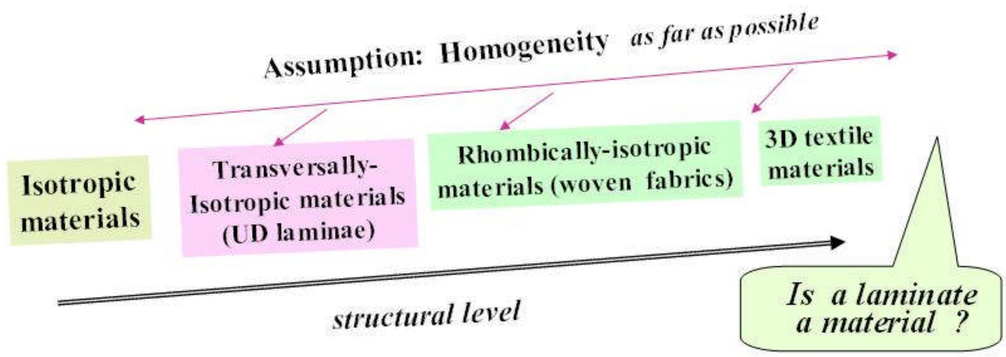


Fig. 8: *Material homogenizing + modelling, material symmetry aspects*

of elasticity modelling homogenization or 'smearing' is applicable for the pre-assessment of elasticity properties whereas in case of strength modelling the smearing process may not be so effective due to

the fact that, e.g. for UD, the micromechanical fibre strength σ_{1f} determines fracture and not the macro-mechanical tensile stress σ_1, utilized in the lamina model.
Material symmetry shows that the number of strengths is identical to the number of elasticity properties! Using material symmetry requires that homogeneity is a valid assessment for the *task-determined* model. The application of material symmetry provides with the minimum number of strength and elasticity properties to be measured which is of high benefit.

Structural analysis, design dimensioning and design verification strongly depend on material behaviour and consistency. In ***Fig.9*** the author proposes a classification of various homogenized

	allocation to *Design Verification levels DYL, DUL* wrt. material behaviour	
Failure type / *Consistency*	**brittle, semi-brittle** DUL	**(quasi-) ductile** DYL
dense	fibre re-inforced plastics , mat, woven fabrics, grey cast iron, matrix material, amorphous glass C90-1,	Glare, ARALL, metal alloys braided textiles
porous	foam, fibre re-inforced ceramics	sponge

failure: fracture failure functional/usability limit

Fig. 9*: Proposed classification of homogenized materials*

materials considering the so-called *failure type* which is linked to brittle and ductile behaviour, and considering the *material's consistency*. Practically, design driving in the two static design verifications cases (on Design Ultimate Load and Design Yield Load level, corresponds to flight load) is for the fracture–related DUL a brittle behaviour describing condition and for DYL a ductile behaviour describing one. Structural composites usually display brittle behaviour.

5 Short Derivation of the Failure Mode Concept (FMC)

Cuntze tries to formulate easy-to-handle homogeneous invariant-based failure conditions with stress terms of the lowest possible order. The conditions in mind shall be 'engineering-like' and shall not make a search of the fracture plane necessary.
Requirements for the development of those failure conditions are:

- simply formulated + numerically robust
- physically-based, and
- practically just need the (few) information on the strengths available at pre-dimensioning. Further parameters shall be assessable
- condition shall be a mathematically homogeneous function.

Possibilities of formulating a failure condition are given by applying:

- stresses (strains have the disadvantage of neglecting residual stresses) or
- invariants (FMC employs *stress* invariants).

There are two different formulations possible: a 'global formulation' and a 'mode-wise formulation'. The associated equations read, [Cun98a, VDI97].:

1 global failure condition : $F(\{\sigma\},\{R\}) = 1$ (usual formulation) ;

Several mode failure conditions : $F(\{\sigma\}, R^{mode}) = 1$ (used in the FMC).

A failure condition is the mathematical formulation, $F = 1$, of the failure surface which in case of the global formulation may include several failure modes including all stresses and strengths (F >=< 1 is termed failure criterion). In contrast, the FMC includes all mode-active stresses but just the mode-governing strength.
From application of global conditions the lesson had to be learned: A change, necessary in one failure mode domain, may have an impact on other physically not related failure mode domains, but in general <u>not</u> on the safe side [Har93]

Experience on isotropic and UD material shows, see also [Chr98]:

- Each of the observed fracture failure modes is linked to one strength
- Material symmetry says: ***Number of strengths = number of elasticity properties***
- Application of invariants for composites is also possible.

  As an example, UD material can serve: $R_{\|}^{t}, R_{\|}^{c}, R_{\perp\|}, R_{\perp}^{t}, R_{\perp}^{c}$ *and* $E_{\|}, E_{\perp}, G_{\|\perp}, \nu_{\perp\|}, \nu_{\perp\perp}$.

Due to the experience above the **FMC postulates** in its 'phenomenological engineering approach': **Number of failure modes = number of strengths,** too**!** This means for isotropic material 2 modes and for the transversely-isotropic UD material 5.

Reasons for choosing invariants when generating failure conditions are presented by Beltrami [Bel1885]. He assumes: "*At 'onset of yielding' the material possesses a distinct strain energy density W ". This is* composed of two portions:

$$W = \left[\frac{1-2\nu}{6E} \cdot I_1^{\,2} + \frac{2+2\nu}{6E} \cdot J_2 \right]$$

the *dilatational energy* (I_1^2) and the *distortional energy* (J_2 ≡ *Mises*), wherein I_1^2 describes the *volume* change of the cubic material element and J_2 its change of the *shape*. These changes can be witnessed by the fracture morphology of test specimens.

In order to formulate a relatively simple failure condition one chooses as invariant a term that respects whether the cubic material element will experience a volume change in the considered mode or a shape change. The same is valid for UD material.
So, from Beltrami, Mises (HMH), and Mohr/Coulomb (friction) may be derived "Each invariant term in the *failure function* ***F*** may be dedicated to one physical mechanism in the solid = cubic material element" :

- volume change	: I_1^2	*(dilatational energy)*	I_1^2, I_2^2
- shape change	: J_2 (Mises)	*(distortional energy)*	I_3 , I_4
- friction	: I_1 (Mohr-Coulomb)	*(friction energy)*	I_2.
	isotropic invariants		UD invariants

<u>Note</u>: Above two I_1 are different for isotropic and UD material.

The idea behind the FMC was: A possibility exists to more generally formulate failure conditions failure mode-wise (e.g. shear yielding of Mises, Puck 1991 for UD failures) and stress invariant-based (J_2 etc.). The latter has still been performed for the isotropic materials by Mises and later numerous other authors such as in [Gol66], and for UD Boehler [Boe85], Hashin [Has 80], Christensen [Chr97], Jeltsch-Fricker [Jel96] etc., and fabrics [Boe95, Mec 98].

So, the question arose "*What is new with the FMC?"* This is:

- the *strict* failure mode thinking
- the individual interaction of a failure mode with the other modes by having no impact on another pure mode domain
- an a-priori reduction of the possibilities to formulate failure conditions.

Concluding on the previous context the following detail aspects can be listed:

- 1 failure mode represents 1 independent failure mechanism
- 1 failure condition represents 1 failure mechanism (interaction of stresses).
- 1 failure mechanism is governed by 1 basic strength
- Interaction of modes by a probabilistic theory-based 'rounding-off' approach' formulated as a series failure system model

$$(Eff)^{m} = (Eff^{mode\ 1})^{m} + (Eff^{mode\ 2})^{m} + \ldots + .. = 1\ .$$

with the *stress effort Eff = portion of load-carrying capacity of the material* which corresponds to $\sigma_{eq}^{mode} / R^{mode}$, and with the (Weibull-related) interaction coefficient m.

For an UD example, the interaction of the 3 IFF shall be visualized. All IFF failure modes are interacted together with the FF in one single (global) failure equation

$$Eff^{\,m} = (Eff_{||}^{\tau})^{m} + (Eff_{||}^{\sigma})^{m} + (Eff_{\perp}^{\sigma})^{m} + (Eff_{\perp||})^{m} + (Eff_{\perp}^{\tau})^{m} = 1.$$

Herein, the stress efforts of the 3 pure IFF modes (they form straight lines in ***Fig.10***) read:

$$Eff_{\perp}^{\sigma} = \frac{\sigma_2}{\bar{R}_{\perp}^{t}}, \qquad Eff_{\perp||} = \frac{|\tau_{21}|}{\bar{R}_{\perp||} - \mu_{\perp||} \cdot \sigma_2}, \qquad Eff_{\perp}^{\tau} = \frac{-\sigma_2}{\bar{R}_{\perp}^{c}}.$$

For usually applied UD materials the value of *m* is *2.5 – 3*. Experience shows, approximately the same value may be taken for all interaction zones. ***Fig. 10*** is based on a hoop wound GFRP tube, E-glass/LY556/HT976. It depicts the straight pure mode curves and the interaction curve (σ_2, τ_{21}).

For information: Puck's *action plane IFF approach* uses a modified Mohr/Coulomb theory to physically interact the three brittle IFF. The unknown IFF fracture angle is determined when the so-called *action plane of maximum stress effort* is 'found'.

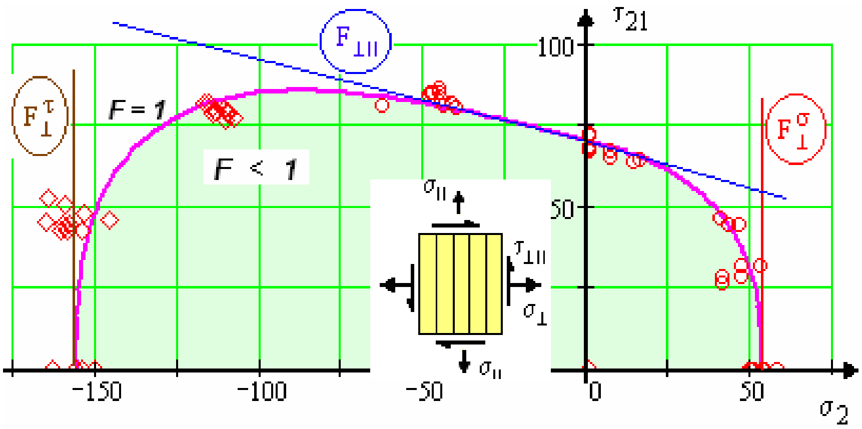


Fig. 10: *Visualization of the interaction procedure*

6 Visualizations of some Derived Failure Conditions

For a variety of different behaving materials failure conditions have been derived and applied to available own and multi-axial strength test data from literature. The results are visualized in the ***Figs. 11*** through ***18***.

At first, the fracture failure curve for a **grey cast iron** is presented in ***Fig.11a***. The data are well mapped by the given pure mode conditions and the interaction equation. The 2D curve is substantiated by two 3D figures, ***Figs.11b***, in a Lode coordinates diagram which demonstrate the applicability of the conditions. Viewing the data, the difference between the so-called tensile and the compressive meridian can be neglected and a rotational symmetric fracture body assumed.

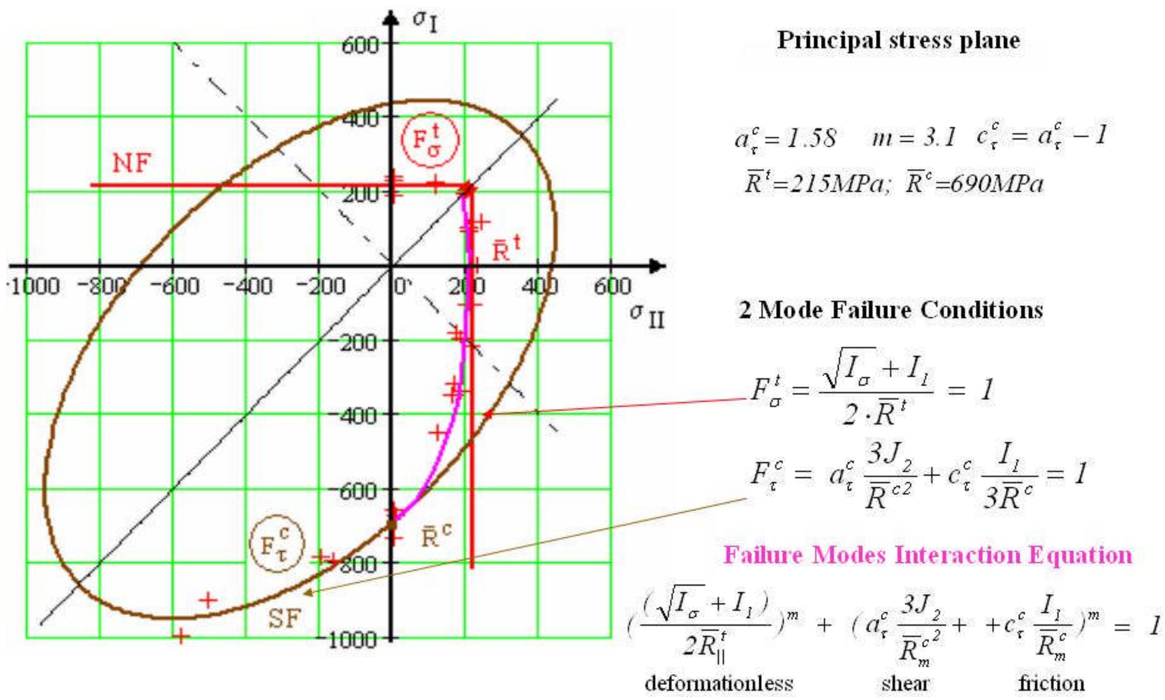


Fig. 11a: *2D failure curves of grey cast iron (brittle, dense, microflaw-rich), [Coffin]*

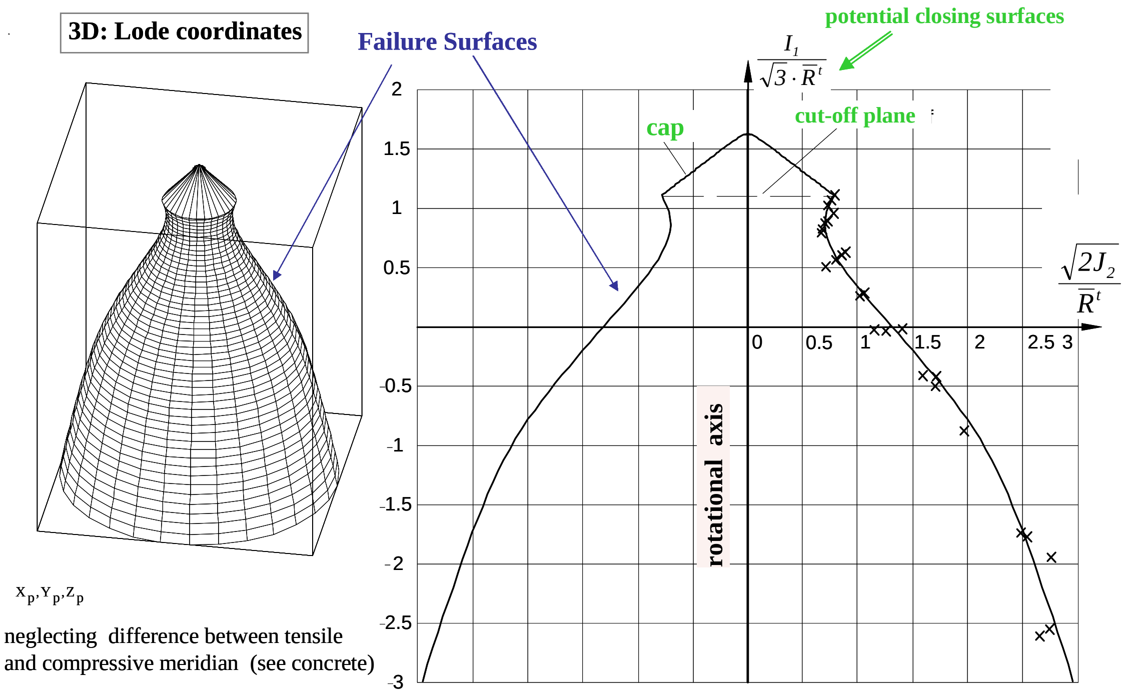


Fig. 11b: *3D failure surfaces of grey cast iron, [test data: Coffin]*

In case of **concrete**, the situation is more complex when viewing the test data. It shows a big bandwidth. The reason for this bandwidth is not only the test scatter but the stress-state dependent failure probability causing non-coaxiality in the octahedral plane. The difference between the so-called tensile (extension) meridian and the com-pressive meridian is to be considered; the 120° symmetry comes to act, ***Fig.12a***.

The invariant J_3 is applied [Boer89] to simply describe the non-coaxiality, which the author is believing to be the result of a joint failure probability, [Awa 78], Rac87], due to the double action of the failure mechanism or failure mode, respectively.

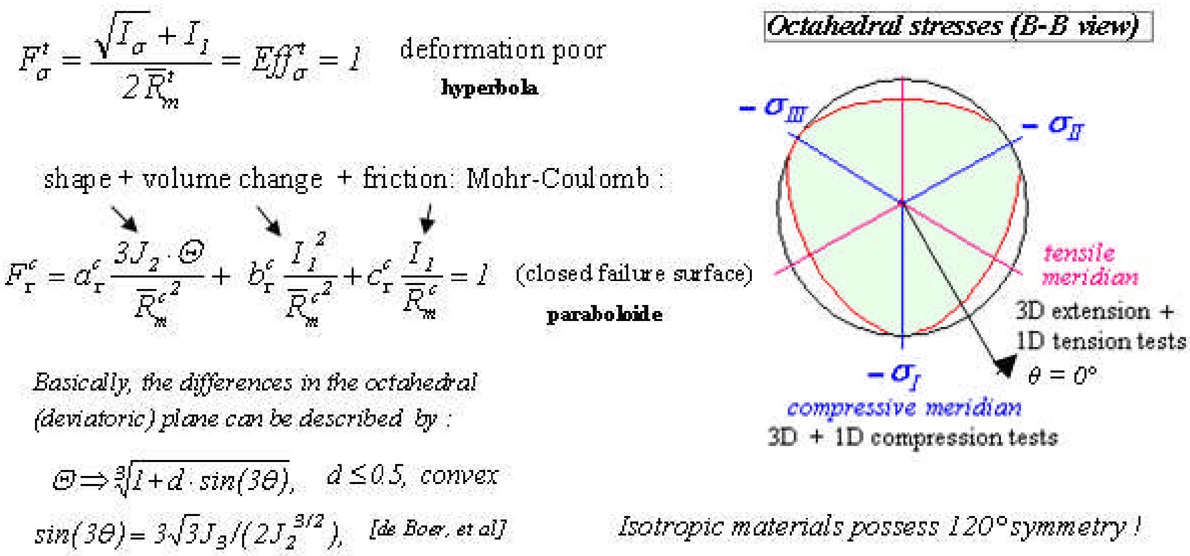


Fig. 12a*: 3D failure condition of Concrete (brittle, microflaw-rich)*

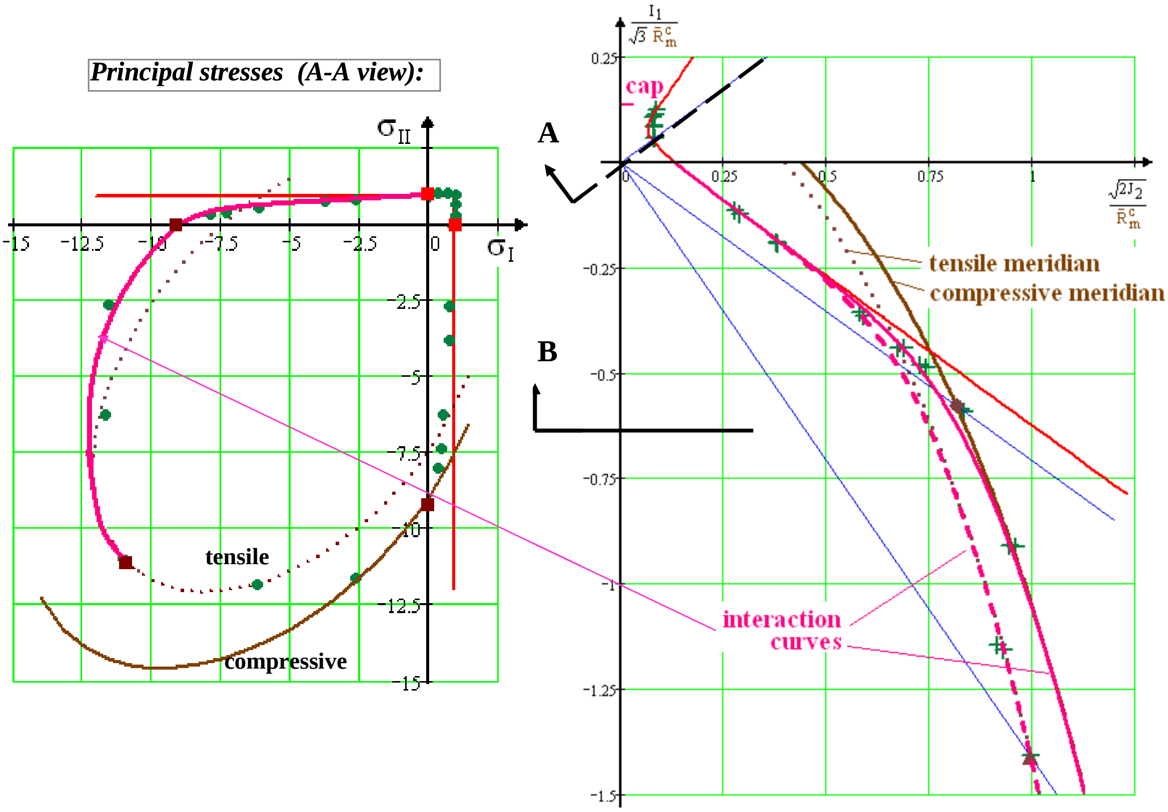


Fig.12b *: 3D and 2D failure surfaces/failure curves of Concrete regarding the meridians*

Fig.12b displays in the 2D figure the failure curves derived when taking the tensile meridian equation

[Kup70] and the compressive meridian equation, only, and the interaction curve which maps the transition from the determined compressive meridian in the compressive strength regime to the tensile meridian determined regime around the bi-axial strength. The 3D figure outlines the non-co-axiality around the rotational axis.

For brittle, porous **monolithic ceramics, *Fig. 13*** depicts the highly porosity-dependent failure curve. Learned*: the same failure condition as for porous concrete can be applied.*

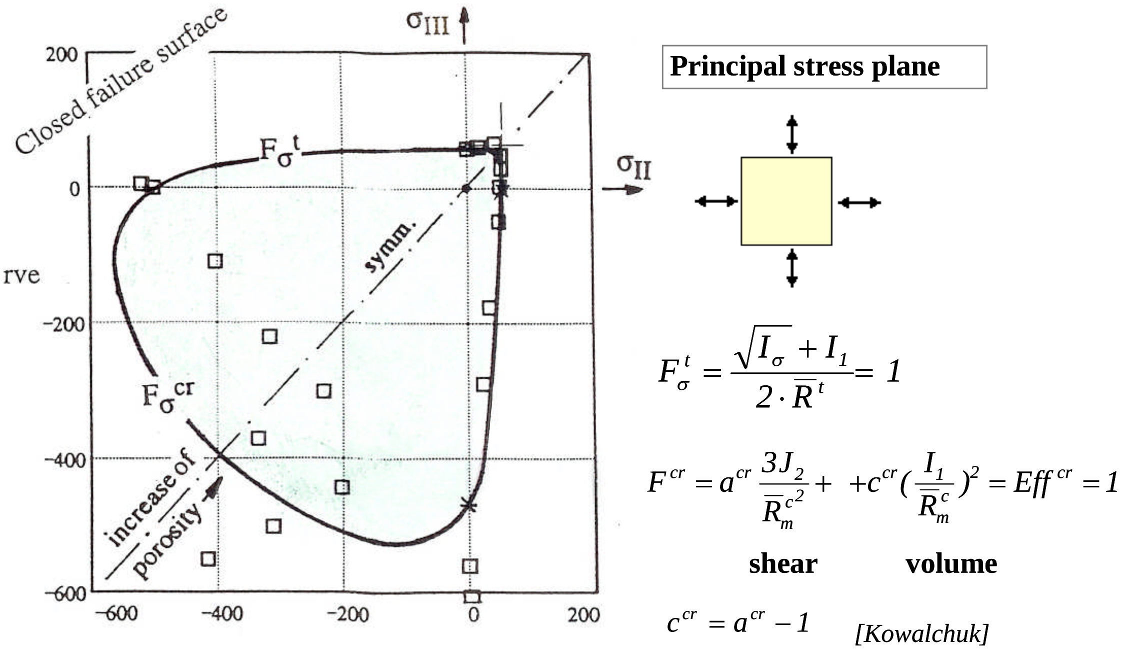


Fig. 13*: 2D failure curves of monolithic ceramics (brittle, porous, microflaw-rich; [Kow83]*

From [Thi97] the following set of strength data has been provided for **C/C fibre-reinforced ceramics.** This means for a brittle porous ceramics lamina based on a UD tape fibre constituent.

Invariants, applied in 3D case are: for friction (I_3, I_2), for shear (I_4), and friction (I_2). This reduces for a plane stressing to the interaction equation including the three IFF failure modes, indicated in ***Fig. 14.*** For the equations, see Fig. 17.

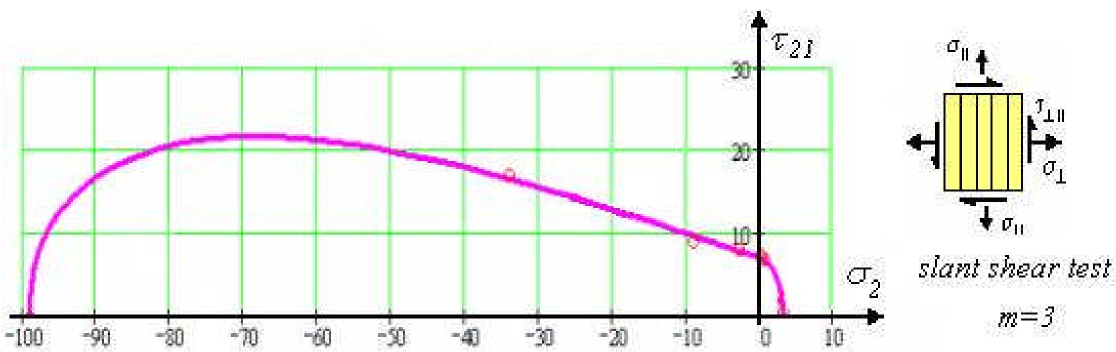


Fig.14*: In-plane shear-transversal normal stress failure curve of a UD-based C/C, [Thi77]*

For the brittle, dense **glass C 90-1** a 2D failure curve in the principal stress plane is displayed and a 3D failure surface in the Lode diagram. The interaction of the test data is good.

Learned*: the same failure condition as for grey cast iron can be applied.*

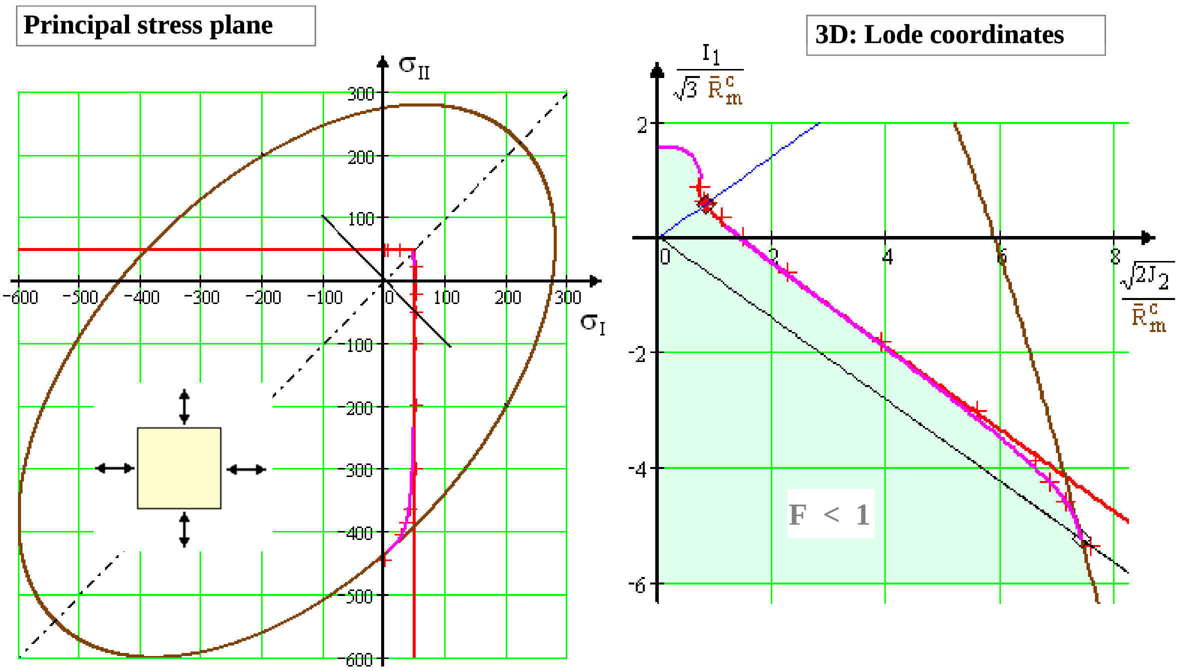


***Fig. 15**: 3D and 2D failure surfaces/failure curves of glass C 90-1 (brittle, dense; [Kow83])*

For **UD lamina fibre reinforced plastics** the in-plane stresses-caused fracture is visualized in ***Fig.16***. Learned: *Same failure condition as with UD-CMC.*

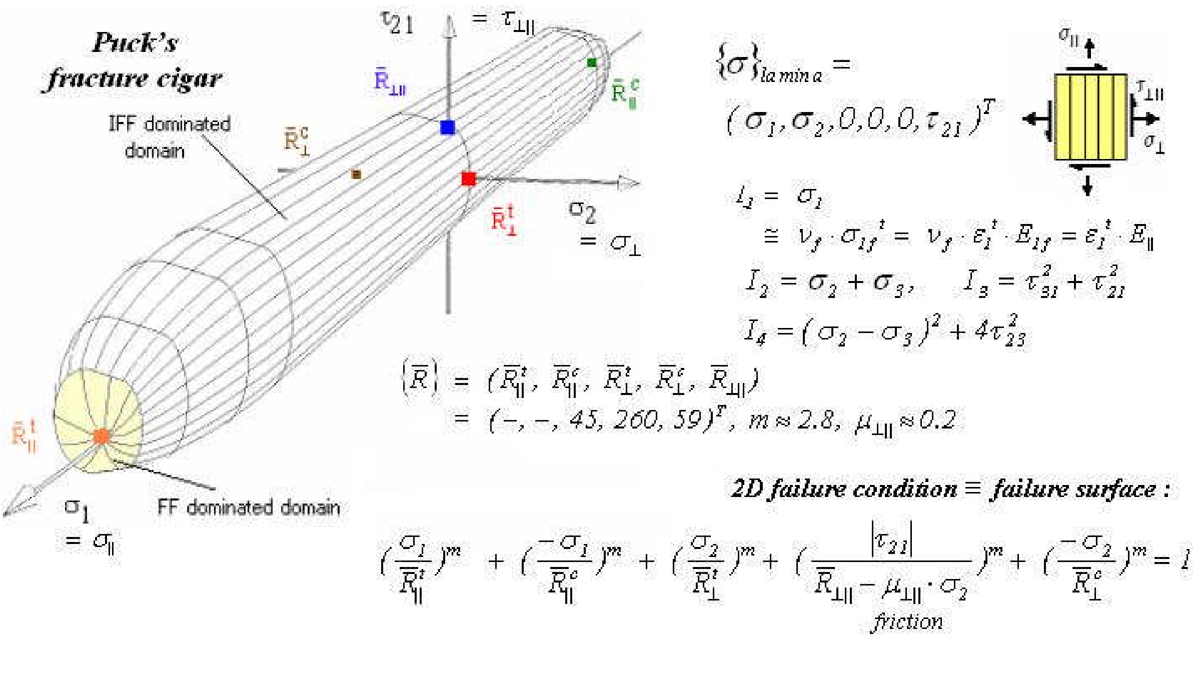


***Fig. 16**: 2D failure surface of FRP UD lamina (brittle, dense, microflaw-rich)*

For 2D visualizations of other UD materials the reader be referred to [Cun03, 06].

Remark: The invariant-based UD failure conditions have been 2D-validaded (real 3D test data are world-wide missing) by 14 test cases of the World-Wide Failure Exercise-I(1993-2003, [Hin02 and 04]). Winner of the contest were the FMC-conditions [Cun04a,b] and Puck's action-plane conditions [Puc02], both, non-funded elaborations. Later, the author further simplified his UD FMC failure conditions [Cun06].

A WWFE-II was recently initiated to really validate the available UD failure conditions by tri-axial strength data.

For two other **carbon fibre-reinforced fabrics ceramics, *Fig. 17***, failure curves are presented. The interaction mapping worked. Here it is to be noted: for woven fabrics, test information for a real validation is not yet available.

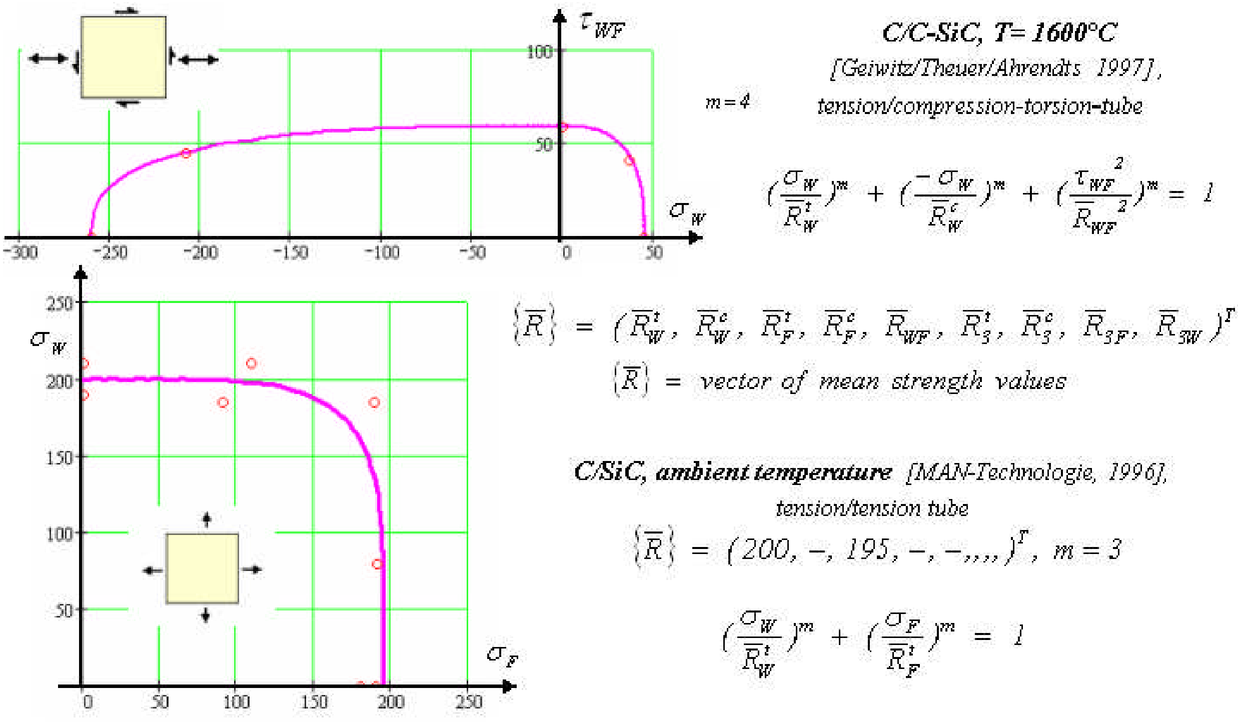


***Fig. 17**: 2D failure curve of C/SIC (brittle, porous fabric; [Gei97)*

Finally, the mapping of a course of multi-axial **PMMA** tensile strength values shall attract attention because this type of yielding might be described as Normal Yielding (NY). ***Fig.18*** displays a non-convex failure curve.

Learned: *Preliminary mapping investigations indicate that* F_σ^t *combined with the Θ-function from concrete which varies with the stress state and by utilizing* $I_1^{\,2}$ *seem to have the capability to solve this complicated mapping problem.*

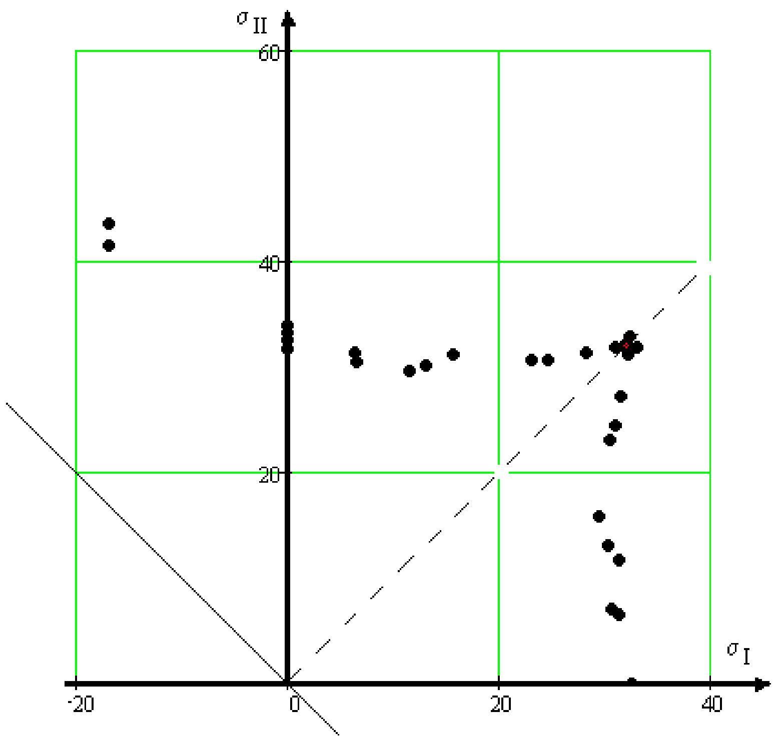


***Fig.18**:*
Multi-axial strength data of PMMA (plexiglass)

7 Conclusions

From Failure Mode Concept applications can be concluded the FMC is an efficient concept, that improves and simplifies design verification. It is applicable to brittle/ductile, dense/porous, isotropic/ anisotropic materials, if clear failure modes can be identified, and if the homogenized material element experiences a volume change or a shape change or material-internal friction.

Many material (behaviour) cross-links have been outlined, e.g. a compressed brittle porous concrete can be described like a highly tensioned ductile porous metal in the so-called Gurson domain.

The concept delivers a global formulation of 'individually' combined independent failure modes, without the well-known short-comings of global failure conditions which mathematically combine independent failure modes.

Outlook: Even in smooth stress regions a strength condition can be only a necessary condition which may be not sufficient for the prediction of 'onset of fracture'. This is valid for instance for the in-situ lateral strength of an embedded lamina, see [Fla82, Leg02]. And also, when applying test data from (*isolated)* lamina tensile coupons to an *embedded* lamina in a laminate analysis, one has to consider that tensile coupon tests deliver test results of *weakest link type*. An embedded or even an only one surface-sided constraint lamina, however, possesses a *redundant* behaviour.

In case of discontinuities such as notches with steep stress decays only a *toughness + characteristic length-based energy balance condition* may form a sufficient fracture condition. Attempts to link 'onset of fracture/cracking' prediction methods for structural components are actually undergone.

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