’ Some Proof ‘ of a 2nd <u>Basic</u> Stress Intensity Factor $K_{IIcr}{}^{c}$ beside $K_{Icr}{}^{t}$

4.2.1 General about the 2 basic fracture toughness quantities of isotropic materials

<u>Presumption</u>: An ideally homogeneous isotropic material in front of the cracktip. The following investigation is only for the ideal structural mechanics building of importance. In practice, there are usually no ideal homogeneous conditions at the cracktip.

Fracture Mechanics FM is the field of mechanics concerned with the study of the propagation of cracks. These cracks might have been there from the beginning or are formed under loading. Final fracture occurs when cracks propagate up to a limit defined by a critical stress intensity factor SIF or fracture toughness K_{cr}, respectively. The critical SIF K_{Ic} (later necessarily indexed $K_{Icr}{}^{t}$) is found in a plane strain condition, and is accepted as the defining (basic) property in Linear Elastic Fracture Mechanics, considering tension.

Comparing strength mechanics and fracture mechanics a question of the author is: “*Are there any links between them?” :*

- * Normal fracture <u>NF</u> acts perpendicular to the mathematically highest stress (‘most positive’) σ_I. If a centrally cracked test specimen is loaded up to a certain level, the crack theoretically well grows in the given fracture ‘plane’. $\bar{R}^{t}$ and $\bar{K}^{t}_{I\,cr}$ correspond, *<u>Fig.4-7</u>*.
- *Shear Fracture <u>SF</u> occurs under a compressive stress, that causes a critical combination of the Mohr stresses σ_n, τ_n , leading to a fracture plane angle $\Theta_{fp}{}^{c}$. If a cracked test specimen is loaded up to a certain level the question arises: Does the crack keep its original plane? "Is there a crack plane-linked ‘transition’ from 'Without crack' (strength mechanics) to 'With crack’ (fracture mechanics) also in the compression domain $I_1 < 0$ given?”

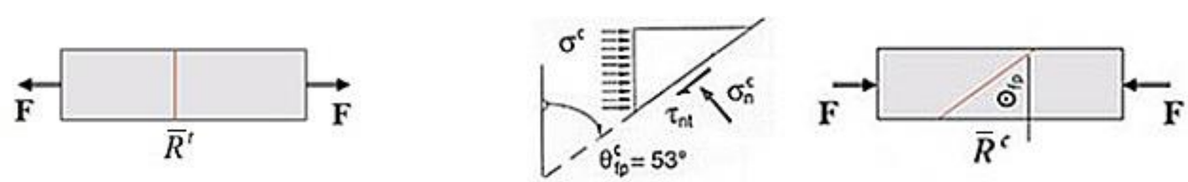


Fig.4-7: Fracture angles of brittle material under tension and compression; (left) NF with tensile strength, (right) SF with compressive strength

Note: In order to cope with the generally in structural engineering used indexing, one has to keep ^{c} for compression and ^{t} for tension and set critical $_{cr}$ for all fracture-mechanical quantities instead of the suffix $_{c}$.

A first response is: From material symmetry information one could conclude that the number of fracture toughness quantities or crack resistances, which are equivalent to the (basic) critical SIFs, is the same as the number of strength fracture resistances, namely R^{t} and R^{c}. The number of the (basic) critical SIFs may be also two, namely $K_{Icr}{}^{t} \equiv K_{Ic}$ and $K_{IIcr}{}^{c}$.

<u>Focusing tension</u>: According to the multi-dimensional stress state present cracks in materials usually do not propagate along their original crack plane but under so-called ‘mixed mode loading’ they twist on curved paths in which the specific singularity situation at the crack tip is decisive to finally achieve a Fracture Mechanics Mode I state.

The decomposition of a loading state into the three basic deformation modes, the fracture mechanics Modes-I, -II, -III, was introduced by Irwin and the different deformations, not the loadings, he indicated by arrows, see *Table 4-3*. These 3 deformation states are usually linked in literature, however, to crack driving loadings and then further to stresses: Mode I – Opening mode (*a tensile stress normal to the plane of the crack),* Mode II – Sliding mode (*a shear stress acting parallel to the plane of the crack and perpendicular to the crack front*), and Mode III – Tearing mode (*out-of-plane shear loading*).

Structural engineers, who apply FM tools for predicting lifetime by damage tolerance means, are used to think in stresses. Therefore they claim - considering the usual interpretation that the arrows are loadings - "*The fracture mechanics modes FM II and III are not in local equilibrium*". Bouquet's faces clearly depict this in *Table 4-3*. Of course, the consequence of being not in equilibrium is a turning of the original crack-plane into a direction normal to the principal *tensile* stress σ_I. And this happens to be. The crack runs out of the original ligament plane.

Focusing compression: There is another engineering discipline, namely geo-engineering with rock fracture mechanics, that is pretty decoupled from the tension domain in mechanical engineering, but where FM plays a big role. The cracks to be faced here under compression loads are many meters long and more. Here, a SIF K_{IIcr}^{c} is the focus but usually prevented by the secondary wing-cracking accompanied by splitting! Therefore, the situation to detect it and to measure it is complicated.
At the crack tip a local perturbation caused by for instance a stiff or a too large grain can change the local stress singularity situation by not generating a desired 'fine grained, homogeneous micro-structure'. Then the modelling-desired ideal homogenization state is violated and splitting of the brittle test specimen will occur.

Nevertheless, there is the author's postulate, employing crack path stability:

Only an angle-stable, self-similar crack growth plane-associated SIF is a 'basic' FM property.

Note

(1) *FM Mode I delivers a real ('basic') fracture resistance property generated under a tensile stress. Both the Modes II K_{IIc}, and III K_{IIIc} do not show a stable crack plane situation but are nevertheless essential FM model parameters to capture 'mixed mode loading' for performing a multi-axial assessment of the far-field stress state.*

(2) *With the Mode-II compressive fracture toughness K_{IIcr}^{c} it is like with strength. One says compressive failure, but actually shear (stress) failure is meant, compressive stress is 'only' the descriptive term. Therefore the (shear) index II is to take.*

Literature seems to support the author who assumes that there are two basic critical SIFs, only. His more detailed definition of such a basic SIF is: *The direction of the crack progress remains in the distinct plane if the stress situation remains the same and the ideal singularity situation at the crack tip is not changed by for instance a large grain.* In other words, the crack increases in its original plane, if the stress state remains in the crack case as in the formerly non-cracked strength case. This should be theoretically valid in the compression domain, too.

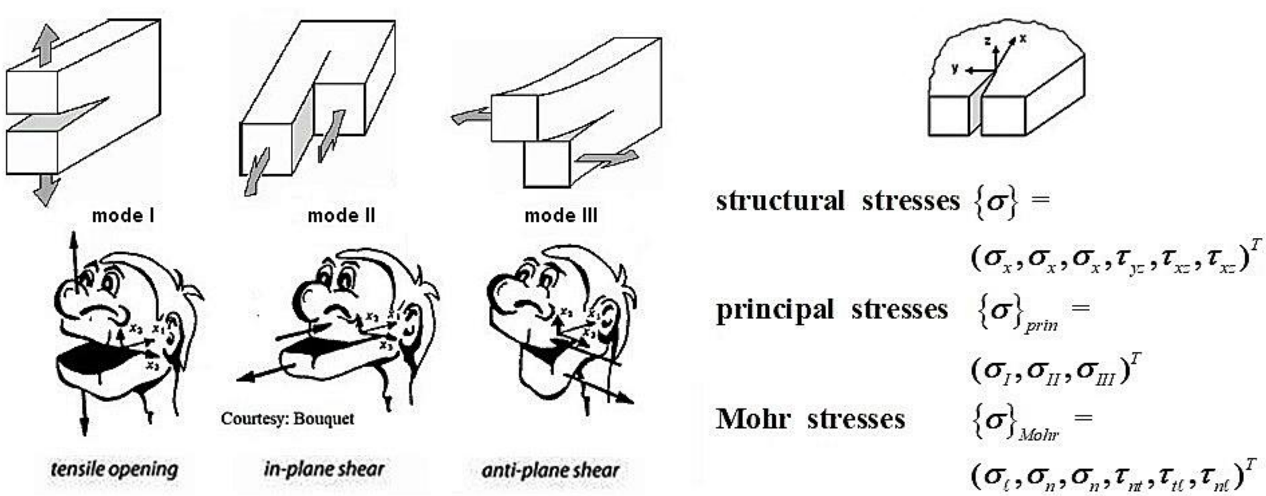
- Tension domain: One knows from $K_{Icr}{}^{t}$ (tension), that – viewing the transversal angle - it corresponds to R^{t}.
- Compression domain: The not generally known second basic SIF $K_{IIcr}{}^{c}$ seems to exist under ideal conditions. It corresponds to shear fracture SF happening under compressive stress R^{c} and leading to the angle $\Theta_{fp}{}^{c}$. The crack surfaces are closed for $K_{IIcr}{}^{c}$, friction sliding occurs.

The author's postulate "$K_{IIcr}{}^{c}$ exists" is firstly supported by an experiment with cracked test specimens under compression and secondly by a still available $K_{IIcr}{}^{c}$ formula in *[Pha03]* substantiated by the maximum value of the material stressing effort *Eff* for $\alpha = 90 - \Theta_{fp}{}^{c}$.

4.2.2 Some experimental proofing of $K_{IIcr}{}^{c}$

(1) A first proof of the author's postulate could be: There exists a minimum value of the compressive loading at a certain fracture angle. This means that the $K_{IIcr}{}^{c}$ becomes a minimum, too. Liu et al performed in [*Liu14*] tests using a cement mortar material. They describe the test investigations as
"*The specimens were square plates of 180mm×180mm×50mm, with three collinear artificial and penetrated cracks, which measure 20 mm in length. The ratio of cement, sand, and water was 1 : 1 : 0.35 by weight. The cracks were made by using a 0.1 mm film, placed during casting. Curing period was 28 days. Under controlled temperature 130°C for 2hrs, the films can be easily pulled out. The crack length and their interval distance are the same and equal 1.0 cm. The test specimens were loaded by a tri-axial loading device: The vertical loading is the major principal stress σ₁ and the two horizontal confining stresses are kept as constants during the process of vertical loading*".

Table 4-2: Fracture mechanics modes, stress states



One of the horizontal stresses is denoted σ3, and the other one σ2, as shown in Fig.4-8. In order to avoid the effect of the friction between the specimen and the loading device, the specimen surfaces were smeared with oil before testing".

→ The significant result of this test series is: A minimum value is located at about α = Θfpc ≈ 45°. That fits relatively well. Of course there is some difference between three collinear cracks and a single crack. The validity of the results, to use them as a proof, would have been improved if a single crack angle α = 50° had been tested, too.

Table 4-2 shall build up a feeling about the stress states and the generated fracture angles.

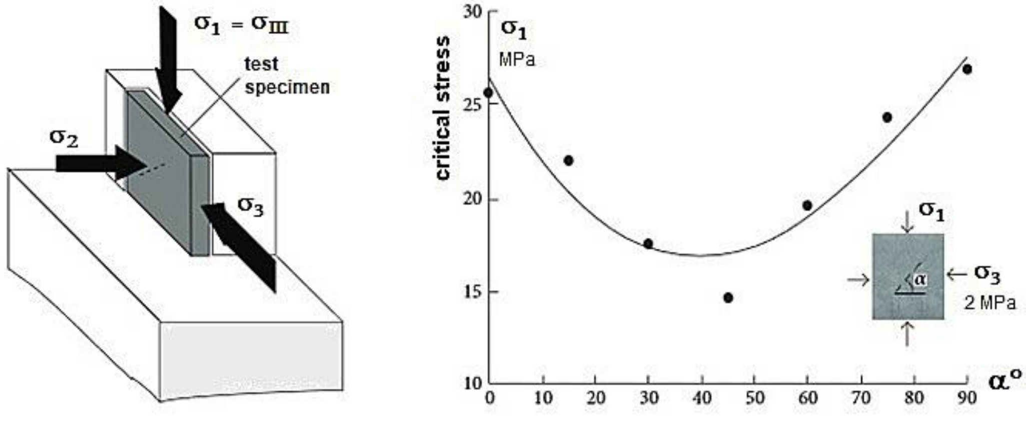


Fig.4-8: Scheme of the test set-up and of the test points obtained for cement mortar [Liu14], σ_1 represents the mathematical stress σ_{III} (largest compressive stress value). Here, literature defines $\Theta_{fp}^{\ c} = \alpha$

Table 4-2 shall build up a feeling about the stress states and the generated fracture angles.

Table 4-2: Considerations about stress state, possible fracture angle plane and fracture toughness
(α = inclination angle, angle $\Theta_{fp}^{\ c}$ is measured in compression test, differently defined)

Tension: 3D-stress state $\{\sigma\} = (\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{zx}, \tau_{xy})^T \equiv (\sigma_I, \sigma_{II}, \sigma_{III})^T$

with mathematical stresses $\sigma_I >$ (more positive) $\sigma_{II} > \sigma_{III}$

$\{\sigma\} = (0, \sigma_y, 0, 0, 0, 0)^T \equiv (\sigma_I, 0, 0)^T \Rightarrow$ $K_{Ic} = K_{Icr}^t$ (stable crack angle

$\Theta_{fp} = 0°$ $\leftrightarrow$ strength: $(\sigma_n^t, 0)^T$ with $\Theta_{fp} = 0°$ (fracture angle, NF)

$\{\sigma\} = (0, 0, 0, 0, 0, \tau_{xy})^T \equiv (\sigma_I, 0, \sigma_{III})^T \Rightarrow$ $K_{IIcr} \leftarrow$ (no stable crack angle)

$\{\sigma\} = (0, 0, 0, \tau_{yz}, 0, 0)^T \equiv (\sigma_I, 0, \sigma_{III})^T \Rightarrow$ $K_{IIIcr} \leftarrow$ (no stable crack angle)

if fully ductile, dense: $\rightarrow \Theta_{fp} = 45°$ inclined yield plane

Full farfield stress states: Superposition possible, if linear problem is applicable,

Mixed Fracture Mode condition: altering angle Θ_{fp} turns into the transversal plane of principal tensile stress

$\{\sigma\} = (0, \sigma_y, 0\ \tau_{yz}, 0, \tau_{xy})^T \equiv (\sigma_I, \sigma_{II}, \sigma_{III})^T \leftrightarrow 3D: (\sigma_n^t, \tau_{nt}, \tau_{n\ell})^T, \Theta_{fp}$(x,y,z)

The fracture mechanics modes II and III cause a turning of the original crackplane.

Compression: 3D-stress state $\{\sigma\} = (\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{zx}, \tau_{xy})^T \equiv (\sigma_I, \sigma_{II}, \sigma_{III})^T$

with mathematical stresses $\sigma_I > \sigma_{II} > \sigma_{III}$

$\{\sigma\} = (\sigma_x, 0, 0, 0, 0, 0)^T \equiv (0, 0, \sigma_{III})^T \Rightarrow$ K_{IIcr}^c (stable crack angle is assumed)

$\Theta_{fp}^{\ c}$ ideal crack tip situation presumed $\leftrightarrow$ strength: $(\sigma_{ncr}^t, \tau_{ntcr})^T$ with $\Theta_{fp}^{\ c}$ (SF fracture angle)

$\alpha \neq \Theta_{fp}^{\ c}$ $\leftrightarrow$ strength: $(\sigma_n^t, \tau_{nt})^T$ (no stable crack angle, wing cracks created)

Full farfield stress states: Superposition possible, if linear problem is given.

Different 3D stress states lead to different Mohr stress states which generate different failure planes Θ_{fp}

$\{\sigma\} = (\sigma_x, 0, 0\ \tau_{yz}, 0, \tau_{xy})^T \equiv (\sigma_I, \sigma_{II}, \sigma_{III})^T \leftrightarrow 3D: (\sigma_n^c, \tau_{nt}, \tau_{n\ell})^T, \Theta_{fp}$(x,y,z)

$\{\sigma\} = (\sigma_x, \sigma_y, 0\ , 0, 0, 0)^T \equiv (\sigma_I, 0, \sigma_{III})^T \leftrightarrow 3D: (\sigma_n^c, 0, \tau_{n\ell})^T, \Theta_{fp}$(x,y,z)

$\{\sigma\} = (\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{zx}, \tau_{xy})^T \equiv (\sigma_I, \sigma_{II}, \sigma_{III})^T \leftrightarrow 3D: (\sigma_n^c, \tau_{nt}, \tau_{n\ell})^T, \Theta_{fp}$(.x,y,z).

(2) Formulation and course of the SIF $K_{IIcr}{}^{c}$: The author believes – as a second proof for the existence of the fracture toughness $K_{IIcr}{}^{c}$ - that a formula is still available. P.H. Melville published in [*Mel77*] *(information from [Pha03], in literature for the author not available)*

$$K_{IIcr}^{c} = \sigma \cdot \sqrt{\pi \cdot a} \cdot \sin(\alpha) \cdot [\cos(\alpha) - \tan(\varphi) \cdot \sin(\alpha)] \quad \text{with}$$

σ = far field stress, a = half crack size, α = flaw (crack) angle and $\tan(\varphi) \cong \mu$.

The SIF depends on the size of the friction value μ. It is the highest, if $\Theta_{fp}{}^{c} = 90 - \alpha$ *(as defined here)* Fig.4-9.
The number on the curves in the right figure marks the maximum value of each 'friction' curve. Exemplarily assuming the usual linear Mohr-Coulomb $tan(\varphi) = \mu = 0.2$ means that $\Theta_{fp}{}^{c} = 50°$ A check of the special case "ductile" with $\mu = 0$ works as the angle α then correctly becomes the frictionless shear sliding angle or yield angle 45°.

Finding: → For a brittle material with its associated friction the SIF $K_{II}{}^{c}$ becomes highest when $\alpha = 90 - \Theta_{fp}{}^{c}$ and that it will lead to further crack growth in this plane.

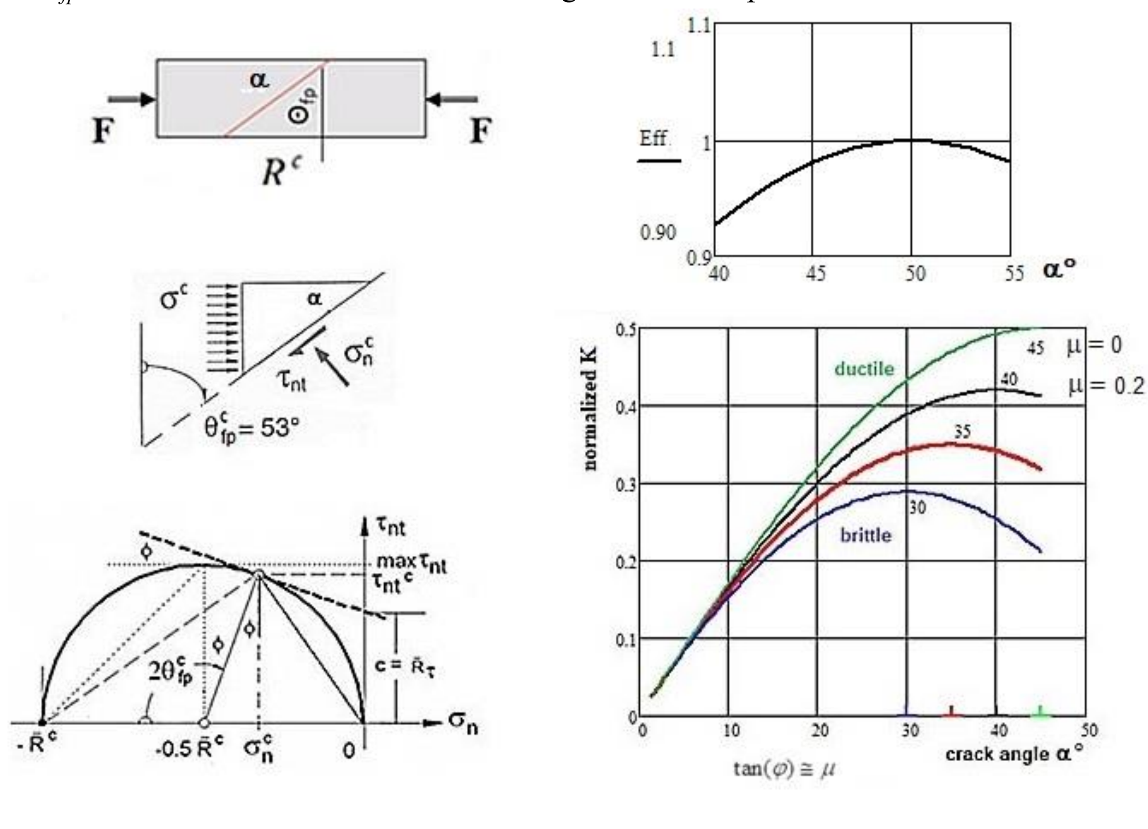


Fig.4-9: (left) the different angles in strength, Mohr-Coulomb; (right up) dependence of the material stressing effort Eff on the inclination angle α, (right down) $K_{IIcr}{}^{c}$ versus inclination crack α considering the friction value μ (here $\Theta_{fp}{}^{c}$ = 90° - α° is valid in literature [Mel77])

This means – in the case of isotropic materials - when linking Strength Mechanics and LEFM to investigate the crack growth angle:

* Domain $I_1 > 0$ (tension, classical fracture mechanics in mechanical engineering):
The maximum hoop stress in front of the crack-tip rules - after Erdogan-Sih - the growth direction of the crack. This practically means that a SFC for NF is employed when

investigating the turning of the crack in front of the crack-tip under multi-axial far field stress states. K_{Icr}^{t} (= K_{Ic}) rules.

* Domain $I_1 < 0$ (compression, civil engineering, rock mechanics):

Could it not be that under compression also a SFC for SF can be employed? This SFC considers the energy at fracture failure. At which fracture angle becomes the SF-SFC a minimum? This can be performed by using the material stressing effort *Eff* in combination with a minimum necessary energy amount $\mathcal{G} = K^2 \cdot (1-\nu^2) / E$.

One can pose the questions:

→ At which angle does *Eff* have a maximum? Applying linear Mohr-Coulomb the material stressing effort follows $Eff = \tau_n / (R^{\tau} - \mu \cdot \sigma_n)$ with R^{τ} the cohesive strength

→ At which angle takes the stress intensity K a maximum?

This was elaborated in the various pictures in *Fig.4-9*, above right side, with the response:

If the inclination angle corresponds to the fracture angle Θ_{fp}^{c} , then a critical state is generated.

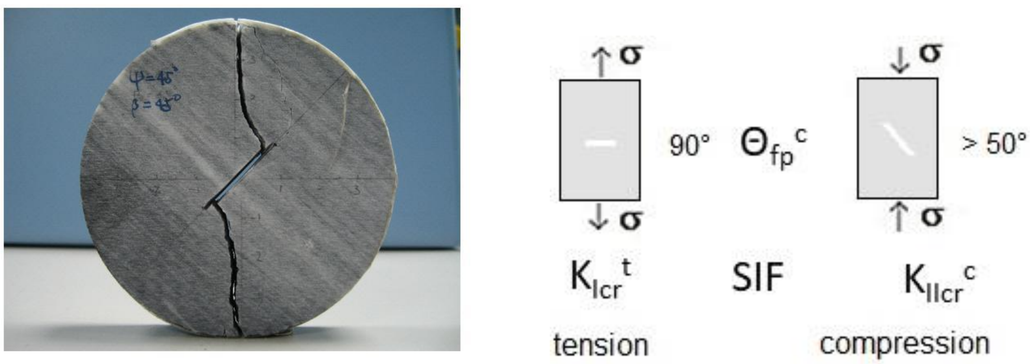


Fig.4-10, Brazilian Disk: Crack growth reality under vertical compression due to large grains ahead of the crack tip. [Sorry, originator of the picture could not be found in a search]. (right) comparison of Cracking angles of the two 'basic' SIFs, where the fracture plane theoretically will not change

<u>*LL*</u>:

A crack angle α, inclined the same as a compression-induced fracture shear angle Θ_{fp} of the formerly intact isotropic material is linked to minimum energy and to a maximum SIF. Both these values are critical quantities for further crack growth of the solid material element. This shear fracture angle should be inserted into a flaw-free Brazilian disk above and then compression tested and checked, whether the original crack angle remains stable.