Keywords: Reserve factor, margin of safety, load definitions, influences, non-linear

Summary

The present HSB sheet depicts a survey of influences that might determine the goodness of the computed load-defined reserve factor RF. Examples for some different stress states and materials are provided.

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Contents

1 General

In design verification and justification, respectively, structural integrity must be guaranteed. To accomplish this in design, design load cases are generated, failure modes are identified, failure conditions (criteria) are established, and Factors of Safety FoS are applied.

Some sort of a measure that a failure will be not happen is a reliable Reserve Factor RF, where a reserve factor is the ratio of a resistance value and an action value. Determination of such a reliable reserve factor is performed considering all influencing items. Therefore, information on essential influencing items on the reserve factor is collected in this sheet in order to enable one to understand the base and to assess the quality of the obtained reserve factor values. Influences from numerical analysis and applied program codes are not addressed here.
However, it should be mentioned that the structural model (i.e. whether the structure is linearly or non-linearly analysed) is chosen by the designing and analysing engineers. If a linear model is chosen, it may be fully sufficient for design verification despite of the fact that the structure may respond locally non-linearly.

As there is a discrepancy between the various definitions and terms of loads and of safety concepts used for different types of air and space vehicles, and as there are some differences in the determination of the reserve factor itself, these topics are briefly presented, too. Further, the terms of 'allowable stress' and '(strength) design allowable' are elaborated and visualized for future unambiguous use.
The term 'design' (already used by NASA decades ago) has the meaning that the quantity is applied by the designing engineers. This involves the technical terms design loads and strength design allowables.

Central objective of structural design verification is the demonstration of a 'non-negative' (in spacecraft termed 'positive') Margin of Safety, $MoS = RF - 1$, for each failure mode and each single dimensioning load case. Such a dimensioning load case corresponds to a maximum load case occurring during the life of the structure.

2 List of Abbreviations and Symbols

Abbreviation	Description
DBL	design buckling load
F	failure function
FF	fiber failure (tensile failure and compression failure or kinking))
FMC	failure mode concept
FRP	fiber reinforced plastics
IFF	inter-fiber-failure (matrix tensile failure including interface failure, matrix compressive failure (wedge failure), and in-plane shear failure)
NF, SF	normal fracture, shear fracture
R	resistance (e.g. material strength, structural stability)
S	action
SY	shear yielding
UD	uni-directional

Symbol	**Unit**	**Description**
f_{RF}	–	material reserve factor obtained with linear analysis models
j	–	value of the factor of safety
ll	$^{a)}$	limit load (basic design load in aircraft, in German: sichere Last)
m	–	mode-interaction exponent of strength failure modes
p_f	–	probability of failure
DLL	$^{a)}$	design limit load (basic design load in spacecraft)
DUL	$^{a)}$	design ultimate load (generally used in aerospace)
DYL	$^{a)}$	design yield load
E	MPa	Young's modulus
Eff	–	material stressing effort (in German: Werkstoff-Anstrengung)
F	N	force (single load)
FoS	–	factor of safety for design dimensioning (also used SF, FS)
K_{PM}	–	uncertainty factor in spacecraft to capture uncertainty of load derivation
MoS	–	margin of safety in design verification $MoS = RF - 1$ (also used MS)
RF	–	reserve factor in structural design verification
$\Re$	–	structural reliability ($\Re = 1 - p_f$)
σ	MPa	stress
σ_{eq}	MPa	equivalent stress associated to a failure mode (e.g. v. Mises shear yielding)

$^{a)}$same unit as property under consideration

Index	**Description**
$_{0.2}$	denotes onset of yielding (marks a deformation restriction in design)
$_{bru}$	bearing ultimate
$_{cr}$, $_{fr}$	critical, fracture
$_{R}$	residual
$_{ult}$	ultimate (usually marks a fracture level in design)
$_{t}$, ^{t}, ^{c}	torsion, tension, compression
$^{\sigma}$, $^{\tau}$	failure driving internal stress
$_{I, II, III}$	indexes of principal stresses
$_{\|\|}$, $_{\perp}$	fiber parallel (longitudinal), lateral (transversal)

3 Analyses

3.1 Introduction

A reserve factor RF is the ratio of a resistance value and an action value. Hence, - according to this definition - it is referred to loading and not to stresses.
In the case of structures the reserve factor may be derived at stress level (this means the level of the usually 'homogenized' material), at section-load level or at load level. If a linear analysis is adequate - and as long as pre-stresses and residual stresses are not present - the stress is proportional to load and the stress-based derived reserve factor f_{RF} is equivalent to the load-based reserve factor RF.

Structures experience a variety of loading conditions (also called 'actions'), depending on the particular role and function. Sources of loadings stem from the operations take-off, cruising and landing of an aircraft, from the weight of the structure itself on earth, from manufacturing, testing, assembling, and from transportation. Considered are thereby environmental effects such as gust and turbulence, temperature, moisture and aging.
A so-called 'resistance' (term in the respective standards, Ref. [1]) such as a strength of materials, stiffness and geometry of the structural part must be adequate to resist all imposed loading without unacceptable distortion or failure. This characteristic shall be retained under the influence of all relevant environmental conditions (such as temperature, humidity, vacuum, radiation, atomic oxygen, debris, lightning, impact) whilst being optimized to be mass and cost-efficient.
When generating reserve factors, the application of the 'right' failure condition and it's correct application are of high influence.

3.2 Definitions and terms around the reserve factor

For more detailed information refer to HSB 00100-01 , Ref. [6].

Action S: external stress or loading applied to the structural part or used in design
<u>Examples:</u> Loads, fluxes, forced deformation, pressure, hygro-thermal loading, static loads, quasi-static loading from vibration placed in equilibrium, transient and impact loading

Basic Design Load (novel term, defined in *Tab. 1* in order to obtain a common basis in aerospace):
Maximum load expected in service when a structure is used according to the design mission.
Design loads are based in
- Aircraft design: on *limit load* ll (in German: sichere Last)
- Spacecraft design: on *Design Limit Load* DLL which includes the uncertainty of the load derivation introduced by the load engineer

Design Allowable (for resistance only): statistically-based minimum value
<u>Examples:</u> At load level (e.g. buckling resistance or joint strength), or at flux level, or at stress level (material A-basis or B-basis strength design allowables).
<u>Note 1:</u> In the case of an unknown distribution, BOEING takes the lowest of 299 strength test points as A-value and the lowest of 29 test points as B-value.

Note 2: Especially in composite design, a *limit design strain* is also used; its value is project-specific.

Design Load: factorized basic design load (for instance $DYL,\ DUL$) used by the designing engineer

Design Value: value of a property used in design that is assumed to consider its uncertainty adapted to a specific case (the design allowable is adapted to a specific design case)

Design Verification: demonstration that the design fulfills the requirements

Dimensioning Load Case: a physically possible, driving design (maximum) load case which is of a certain probability of occurrence

Equivalent Stress: stress value combining effects of those stresses which are active in a failure mode.
Examples: the v. Mises equivalent stress $\sigma_{eq}^{\text{Mises}}$ for the shear yielding failure mode in case of ductile material behaviour and the maximum principal stress in case of a brittle tensile fracture failure mode $\sigma_{eq}^{NF} = \sigma_I$

Factor of Safety (FoS) or Safety Factor: deterministic factor (based on long experience) which converts the given basic operational loading (real loading) into a higher design loading (fictitious, both, in aircraft and spacecraft engineering) in order to introduce safety.
Example: for spacecraft: $DUL = j_{ult} \cdot DLL$; for aircraft: $j_{ult} \cdot ll$
Note: In aerospace the FoS are load-increasing factors.

Failure Condition: mathematical formulation of the failure surface $F = 1 = 100\%$

Failure mode: observable effect of the mechanism through which the failure occurs.
Examples: normal fracture and shear fracture, local buckling, leakage, given deformation limit, excessive wear, corrosion, initiation of yielding, delamination

Hygro-thermal-mechanical analysis: analysis which includes the response of the structure to humidity, temperature and mechanical loading

Load Factor: multiplication factor in flight of inertial weights of an aircraft (positive upward n_z)

Limit Load: maximum external load expected in service derived from 'system load scenario analyses on full aircraft models'

Margin of Safety ($MoS = RF - 1$) or Safety Margin: fraction by which the resistance exceeds the product *'design limit load* DLL times FoS' (example spacecraft)
Note 1: A non-negative (positive) margin is to be demonstrated in design verification for each failure mode.
Note 2: In contrast to the required design quantity FoS, the MoS may be a test-related value.

Material Allowable: statistically based resistance property of a material, of a joint, or of a structural part.
Note 1: It is not necessarily a strength design allowable R_m; it may be an average value $\overline{R}_m$

instead, if mapping of test data has been performed.
Note 2: At AIRBUS, it is a statistics-based value.

Material Design Allowable: At AIRBUS, it is a value used for certification.

Material Design Value: resistance value adapted to specific design processes

Material Reserve Factor f_{RF}: reserve factor obtained in linear analysis
Note: if load is proportional to stress, $f_{RF} = RF$

Material Stressing Effort Eff: (in German: Werkstoff-Anstrengung; an equivalent term is not available in English language) exertion, that a material experiences under a stress state, generated by external loading and residual stresses.
Note: To be applied in non-linear analysis where stress is not proportional to load anymore. It is an agreed term with QinetiQ in the UK, the organizers of the World-Wide-Failure-Exercise on composite failure conditions.

Mechanical Properties: properties of the material such as elastic, physical, or strength properties.
Note: When analysing the structural behaviour, usually average (typical) elastic and average physical properties (e.g. hygro-thermal ones) are used. Generally, a stress-strain curve is also such a physical property of the material. In strength analysis statistically-based ('reduced') strength properties are used to capture the scatter of the material strength.

Reserve factor RF: load-defined factor as ratio of a *resistance value* and an *action value*.
Note: Similar to the definition of the Factor of Safety FoS, the Reserve Factor RF is referred to loads.

Resistance: a material property or a structural property counteracting the applied loading
Examples: load resistance and buckling resistance (which are structural resistance properties at load level), strength (which is a material resistance property R at stress level), or sometimes given as a limiting strain

Safety Concept: deterministic or even probabilistic concept to capture the uncertainties in design
Note: These concepts consider the uncertainties of the design variables in a different manner to ensure a conservative reliable design.

Structural Integrity: characteristic of a structural element that enables it to withstand the load environment and the usage imposed during service

Structural Reliability $\Re$: ability of a structure to fulfill the operational requirements during a distinct lifetime with a distinct reliability.
Note: Structural Reliability considers the probability of combinations of the significant scattering (stochastic, random) design variables. Currently, in aerospace the two essential design variables load and strength are separately treated, but not really stochastically combined as in real structural reliability applications using a probabilistic concept.

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3.3 Derivation of basic design loads and of design loads

The loading values are provided by the load engineer, derived from system load scenario analyses of full aerospace models. These are analysed by the designing engineer in respect of potential load cases to be used as basic design loads.
Note: The term 'basic design load' is used in order to obtain a common logic for aircraft and spacecraft.

Table 1: Loading denotations in aerospace engineering

Design & Analysis	**Aircraft** (generally used terms)	**Spacecraft** (ECCS-E-30 Standard)
loads from system load scenario analyses of full aerospace structures (generated by load engineer based on pre-design structural models)	balanced global aircraft loadings (from certification and operational requirements)	Limit Load (LL)
basic design load	limit load (ll) (in German: sichere Last)	Design Limit Load $DLL = LL \cdot K_{PM}$ (K_{PM} consideres the uncertainty of the system analysis model and project goal uncertainties, usually $K_{PM} \approx 1.2$)
load assumed for design to a detrimental permanent deformation failure	design load equal to limit load ($ll \cdot j_{0.2}$ with $j_{0.2} = 1$, for civil aircraft and partially for military aircraft)	Design Yield Load $DYL = DLL \cdot j_{0.2}$ (usually $j_{0.2} \approx 1.1$)
load assumed for design to a fracture failure	ultimate load ($ll \cdot j_{ult}$) (often $j_{ult} = 1.5$, j_{ult} may be seen composed of a load uncertainty factor 1.25 and of a design uncertainty factor 1.2, refer to Ref. [14])	Design Ultimate Load $DUL = DLL \cdot j_{ult}$ (usually $j_{ult} \approx 1.25$)
load combinations are considered to derive Dimensioning Load Cases		

The generation of the basic design loads is an iterative process that follows the design development. For instance in spacecraft engineering, the design is based on *Limit Loads* (***Tab. 1***), derived from system load analysis of the full spacecraft structural model. This analysis is the so-called Launcher-Coupled Dynamic Analysis which delivers a so-called *Limit Load* (LL). It should be not confused with the aircraft-technical term *limit load* ll (in German: sichere Last). *Tab. 1* further depicts that in aircraft design a similar approach is used. The limit load ll in civil aircraft practically corresponds to the Design Yield Load level in spacecraft.

After consideration of all potential load cases an essential goal of the designer - for quick project decisions necessary - is the definition of a reasonable, minimum number of design driving load cases from the basic design loads.

More details considering loads, their denotations in aerospace and the derivation of the design driving load cases will be given in a separate HSB sheet (not yet issued).

3.4 Structural analysis (stress-strain field determination) and strength analysis

3.4.1 Stress and strain analysis

Structural analysis - considering the external boundary conditions and internal restraints - converts the environmental loadings into internal forces, fluxes, stresses and strains for all the load cases.
It is principally the execution of the stress-strain analysis.

General approach

- Use of adequate models to map the structural behaviour
- Consideration of all relevant load cases
- Use of adequate physical properties such as
  - average (typical) values of the stochastic design variables in order to end up with a structural behaviour which meets real behaviour best (which means with a 50% expectance value)
  - average stress-strain relationship (is to be seen as a physical quantity) in the applied models
- Consideration of environmental influences such as temperature, humidity, radiation. They may have a substantial impact on the coefficient of thermal expansion, Young's modulus and strength design allowables.
- Data input from a material handbook is to be carefully selected in order to match the real hardware (e.g. with respect to plate, sheet, extrusion, heat treatment).

Utilization of properties in input

- Stress and Strain Analysis (structural analysis):
  Average elasticity properties and nominal geometry (thickness, length) to represent average structural behaviour
  Note: In structures with multiple load paths, loads are distributed according to the stiffness properties of the individual load paths. Therefore, it is essential to use average stiffness values (material and geometrical properties) to end up with typical load distributions. Otherwise it

might happen that predicted load distributions are 'un-typical' with the result that stresses may be predicted too low and incorrect locations of critical points are assessed.

- Strength Demonstration (design verification):
  One-sided tolerance bands (for static and fatigue strength) and two-sided tolerance bands (for thickness, Young's modulus) are considered.

- Stiffness Demonstration:
  Due to stiffness requirements upper and lower tolerance limits

- Application of A-value and B-value Design Allowables (statistically based):
  - A-values: Application of the Safe Life Concept (single load path, where failure of a single element leads to the loss of structural integrity)
  - B-values: Application of the Damage Tolerance Concept (multiple load paths, redundancy).

3.4.2 Stability analysis

In stability analysis - even for 'simple' stability-endangered structural elements - seldom a test series is available to compute statistically-based stability design allowables.

Aircraft certification standards usually do not require statistics to be performed on stability tests. For example, in EASA CS 25.307 (b) is cited: "When static or dynamic tests are used to show compliance with the requirements of CS 25.305 (b) for flight structures, appropriate material correction factors must be applied to the test results, unless the structure or part thereof, being tested has features such that a number of elements contribute to the total strength of the structure and the failure of one element results in the redistribution of the load through alternate load paths." Thus, for all multiple load path structures a statistics-aimed testing is not required by CS 25.

In spacecraft, an improvement of the predictions of the traditional stability analysis result is aimed at by applying a statistically based estimation procedure for the failure load. This requires knowledge about the design variables such as geometrical tolerances, imperfections and scatter of properties. This knowledge is the input of - for instance a Monte-Carlo method - which predicts sensitivities and delivers a statistically based failure load, an improved value compared to the traditional analysis one.

3.4.3 Strength analysis

- Generally, strength analysis can be not performed in one shot together with stress-strain analysis (in fact, it is an iterative process).

- Strength analysis can be separately performed as a 'post-processing work' to account for:
  - scatter of the design variables (e.g. upper or lower Young's modulus in stiffness requirement cases, minimum local thickness values),
  - strength design allowables (statistical minimum values).

Essential in the strength analysis is the used *Failure Condition* $F = 1$. It's aim is to assess multiaxial stress states by just utilizing the basic uni-axial strength values, which are mandatory in design, anyway. Types of strength failure conditions are (see Ref. [10]):

- *One global* strength failure condition:

$$F(\{\sigma\},\{R\}) = 1$$

where F is the failure function (with $F = 1$ the failure condition, and $F >=< 1$ the failure criterion); $\{\sigma\}$ is the vector of stresses (not the stress vector) or stress state; $\{R\}$ is the vector of strength data (formally composing all the different material strengths).
The global strength failure condition uses interpolation functions (e.g. polynomials) when considering the usual occurrence of more than one failure mode.

- *Several mode* strength failure conditions:

$$F(\{\sigma\},\{R_{mode}\}) = 1$$

For example, mode strength failure conditions are used in Cuntze's FMC, Ref. [9].

Note: The application of a global condition may have a drawback: The underlying global fit of the full course of test data can mathematically combine independent physical failure modes. This may lead to erroneous reserve factor results when computing the multi-axial failure stress state, because a data change in one of the modes influences another independent mode.

Strength of a design is demonstrated, if 'No relevant strength failure condition (to be understood as the limit state of a failure mode) is met or exceeded for all dimensioning load cases', resulting in a non-negative (positive) Margin of Safety MoS.

If the design verification is performed with non-linear models (material non-linearity with or without geometrical non-linearity) then the determination of a RF or a MoS via the *material stressing effort* Eff (in German: Werkstoff-Anstrengung) is required. A material can maximally sustain 100%.

Fig. 1 illustrates the situation for a linear model, a non-linear model and a linear model including residual stress using an example for fiber reinforced plastic material. Stress due to the applied loading is denoted as $\{\sigma\}_L$. The linear model is represented by a straight line (i.e. stress is proportional to loading), the non-linear model by a curved line being close to the linear model for low stress values. The non-linear model includes micro-damaging effects due to diffuse matrix-cracking and some yielding of the matrix material. With increasing Inter-Fiber-Failure (IFF)-related damaging, loading is more and more transferred to the stiff fibers. The third model combines linear material response with residual stress $\{\sigma\}_R$ (again stress is not proportional to loading).

The design curve $F(\sigma_2, \tau_{21}, R^t_\perp, R^c_\perp, R_{\perp||}) = 1$ intersects with the stress coordinate axes at points defined by the strength design allowables $R^c_\perp,\ R^t_\perp$ and $R_{\perp||}$. 'Initial failure load' is reached where the lines representing the models intersect with the design line (i.e. at that load where the material stressing effort counts 100%). The reserve factor RF takes the value which is necessary to multiply the loads with in order to obtain the failure condition $F(\sigma_2, \tau_{21}, R^t_\perp, R^c_\perp, R_{\perp||}) = 1$.

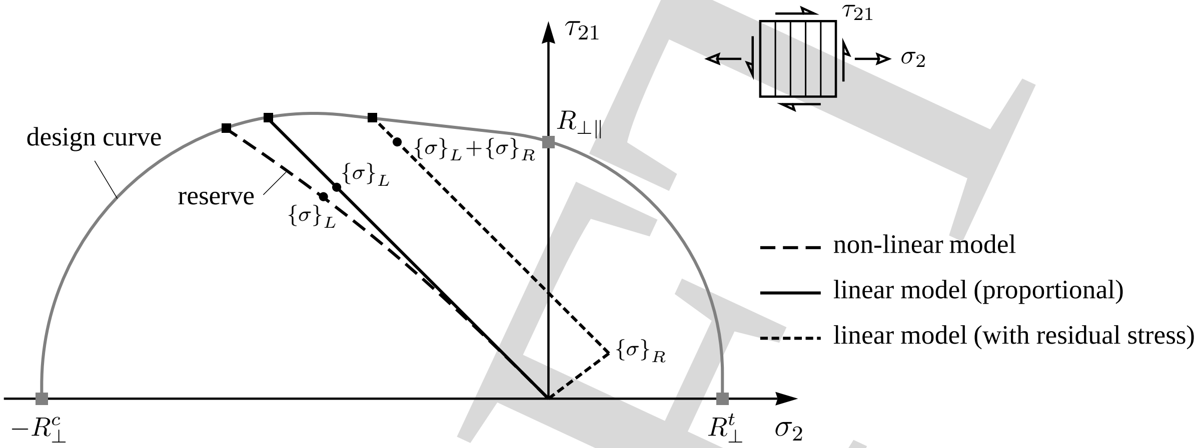

Figure 1: Illustration of different analysis models; UD lamina (ply) used as example

Features to be kept in mind for non-linear analysis are:

- Stressing is re-distributed to stiffer regimes (in case of composites to the fibers)
- In contrast to the linear model, an altered load path (according to local weakening) may be predicted by the non-linear model
- Large strains and large displacements may occur (which on principle includes yielding of metals and quasi-yielding (micro-cracking caused))
- Change of shape of the loaded structure is possible
- Release of residual stresses according to degradation growth in case of monotonic and cyclic loading and in hygro-thermal environment
- The assessment of the critical stress state is performed with the obtained Eff and the strength design allowable. Global yielding is not permitted.
  If a critical strain has to be assessed then a *limit design strain* needs to be defined.

3.4.4 Failure conditions for combined loading

Under combined loading, several contributors act together to generate failure. Formulas to predict the reserve factor RF are offered for three structural levels, see also Refs. [15, 7] (in this context, the reserve factor RF is sometimes called load multiplier λ in the literature):

$$\text{(a)}\quad \left(RF\cdot\frac{N}{N_{fr}}\right)^{c_N} + \left(RF\cdot\frac{M}{M_{fr}}\right)^{c_M} + \ldots = 1 \quad \text{forces, moments}$$

$$\text{(b)}\quad \left(RF\cdot\frac{n}{n_{fr}}\right)^{c_n} + \left(RF\cdot\frac{m}{m_{fr}}\right)^{c_m} + \ldots = 1 \quad \text{stress resultants and couples} \qquad (3\text{-}1)$$

$$\text{(c)}\quad \left(f_{RF}\cdot\frac{\sigma}{\sigma_{fr}}\right)^{c_\sigma} + \left(f_{RF}\cdot\frac{\tau}{\tau_{fr}}\right)^{c_\tau} + \ldots = 1 \quad \text{normal and shear stresses}$$

with the associated fracture values (subscript $_{fr}$) and the interaction exponents c_j; they should be obtained from tests or from handbooks or could be estimated by theory, at least. Above equations are valid for a fixed course of test data.
Each of the three equations describes a failure surface or a failure curve. Equations (a) and (b) belong to combinations of loads at load and flux level and equation (c) to combinations of stressings (a better choice is an adequate strength failure condition). The equations reflect linear and non-linear behaviour. They are used on the resistance side, both, for material failure (strength) and structural failure (stability)
Above equations are engineering compromises. The goodness of the computed reserve factor depends on the goodness of the recommended exponents. It must be checked, thereby, whether the exponents are based on a measured combined failure state and whether they are obtained from fitting failure data of a ductile, a brittle or an intermediately behaving material.

In general, the above equations can not be solved analytically for RF. Thus, numerical techniques are necessary.

Examples for further applications on combined loading are given in HSB 51200-01 (Ref. [7]).

3.5 Safety concepts in strength analysis

3.5.1 Classical safety concepts and (design) factor of safety j

A safety concept formerly used is the so-called 'Allowable stress' concept. In the last decades, it has been replaced by the 'Strength design allowable' concept, see Ref. [11]. The change from one to the other concept is accompanied with confusion caused by applying 'former' terms in the 'new' concept.
Under the prerequisite of linearity, above two concepts can be mathematically transformed into each other.

- 'Allowable stress' safety concept:
  The resistance quantity 'strength ' (A- or B-value) is reduced by the safety factor j, see ***Fig. 2***, which proves:

$$\text{allowable stress} = \frac{\text{strength design allowable}}{j} \tag{3-2}$$

  Note: At least since 1926 by M. Mayer (Ref. [11]), a safety concept on basis of allowable stresses is questioned.

- 'Strength design allowable' safety concept:
  The action quantity as the applied external loading is increased by the safety factor j.
  Note 1: This concept, presently used in aerospace, is still a deterministic concept.
  Note 2: It should be always checked, whether the use of a linear model results in an acceptable design.

  The Safety Factor is a load-defined FoS which enlarges a given loading to a computational design load
  - factor for design, when viewing the limit load ll (aircraft) as basic load level
  - factor for design, when viewing the Design Limit Load DLL (spacecraft design)

- on top of the FoS, so-called project factors and system margins may be used which depend on design policy
Note 1: The magnitude of a FoS j is based on proven processes and validated methods for analyses, tests and manufacturing. Inaccurate analyses are not covered by the FoS.
Note 2: Higher FoS values are applied for 'verification by analysis only' or in spacecraft if later on a higher reliability is required than was considered in the derivation of the basic design loads. A FoS value j is related to a specific design policy or a specific project.
Note 3: A safety factor will never be computed.

- 'Allowable stress' safety concept versus 'Strength design allowable' safety concept:
At present there is the problem that terms are often mixed up at the stress level. An 'allowable stress' is not anymore a term in the safety concept presently used in aerospace. This word should be avoided, because its un-reflected use may yield in-accurate RF values.

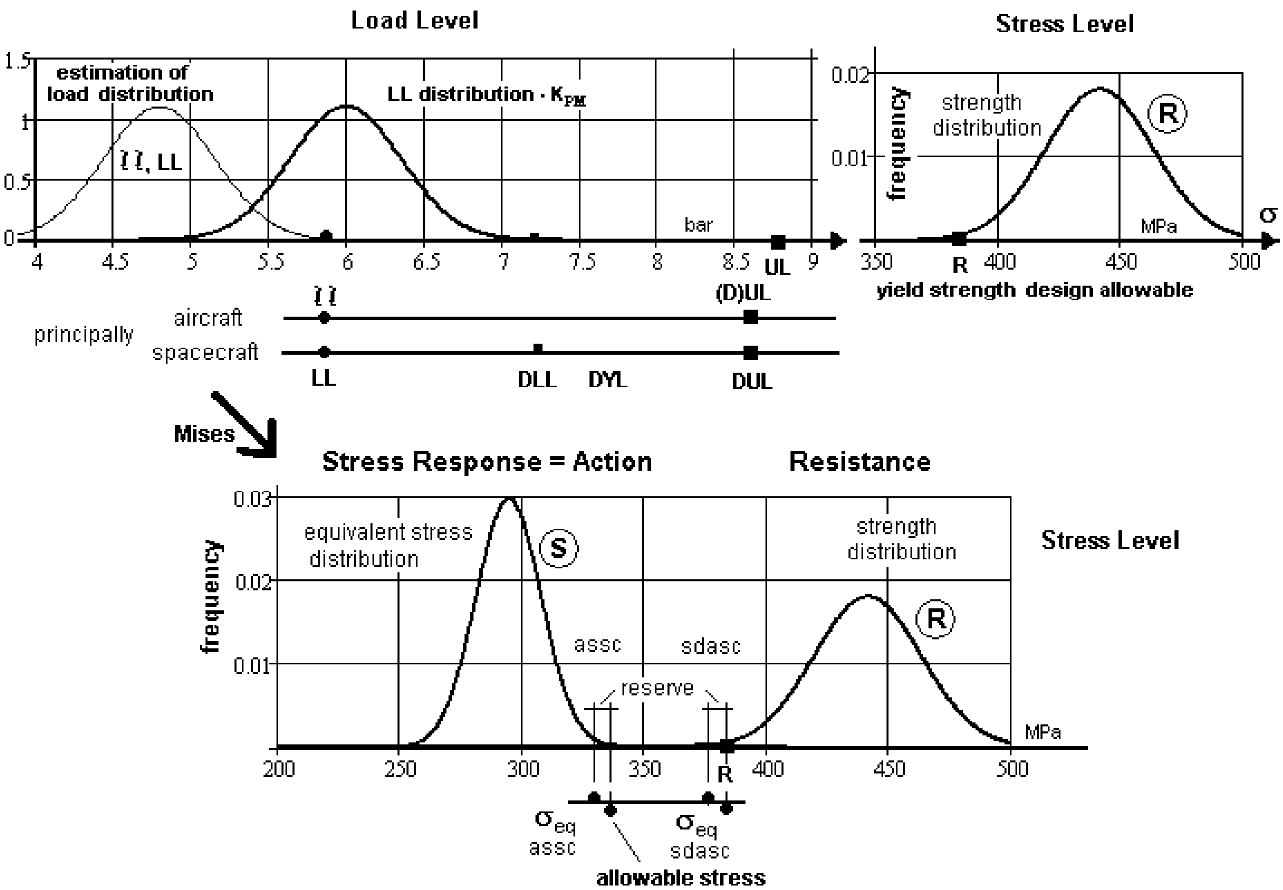

Figure 2: Example 'Pressure vessel under internal pressure': visualization of differences of (1) aerospace load terms and of (2) 'allowable stress' safety concept (assc) and 'strength design allowable' safety concept (sdasc)

Using an example, 'Pressure Vessel' in *Fig. 2*, the various terms are visualized. The comparison of the terms is performed at stress level. Therefore, the loads are transferred at first into stress quantities to obtain a strength associated level. This is achieved for the ductile behaving material used via the von Mises equivalent stress.

Remark: According to $ll < DLL$ in the upper left figure, it can be concluded that in many cases in aircraft design linear analyses may be sufficient.

AGARD cites in Ref. [14] "When other physical effects (thermal, aging, ...) occur in limit conditions, specific FoS must be applied successively and separately on each of these effects", if physically reasonable. The verification of such multi-physical effects should be demonstrated by a test in order to reduce the mass-driving effect of piling-up factors of safety.
The new safety concepts have been created in order to avoid a piling-up.

3.5.2 Outlook at partial safety factor concept and probabilistic safety concept

Partial safety factor concepts (where global FoS is split) and also probabilistic safety concepts are used in several disciplines such as civil engineering. These concepts capture the *combination* of all scattering design variables with their respective statistical distributions. It's application results in a more reliable stress response followed by a more reliable RF. Nevertheless, in case of the partial safety factor concept - as the simplest probabilistic safety concept - dedicated FoS are used for the design. The standards are not yet capable of dealing with feasible survival probabilities (as attempted with the ARIANE 5 launcher some twenty years ago).
For aircraft engineering it has been proposed to at first replace the global FoS concept, that integrally covers *all* uncertain design variables, by the most simple *partial* safety concept (so-called semi-probabilistic method, also termed Level I method) in order to put 'safety' there where the uncertainty is recognized (loads, calculation, test, manufacturing imperfections, etc.).
In this context up to now, in aerospace at most, just the two driving design variables (load and strength) are stochastically treated, but each separately. This is practically still performed at ESA. There, one (partial) factor of safety (e.g. K_{PM}) is used in the derivation of the DLL and the usual FoS is applied for design. Such a concept is beneficially e.g. for a more reliable determination of deformation in the case of statically indeterminate structures.
In the long range, a probabilistic verification will be the objective in aerospace to determine a structural reliability $\Re = 1 - p_f$. This would afford a re-work of the design policy including the establishment of 'reasonable' design limit values.

Probabilistic concepts require stochastic modeling which means statistics and an assumption of the distribution type (such as Normal, Log-Normal, Weibull, Extreme Value, or something else) of each essential uncertain design variable. This is basically the same as with the classical procedure where both these models are required on principle, too, the mechanical modeling and the stochastic modeling. Applying a probabilistic concept, in addition, a so-called logical modeling of the failure system is chosen (as a series failure system or a parallel failure system or a combination) considering the effect on the probability of commonly acting failure modes and obtaining a so-called 'joint failure probability'.
Even, if only the type of distribution can be assessed and used in analysis and not the tails of the distribution, more reliable information is generated with the probabilistic concept.

3.6 Computation of a strength reserve factor value (*RF*)

3.6.1 Case: Linear analysis

Strength:

”Stress is proportional to a loading “.

Consequently, it holds for the basic design load that

$$j_{ult} \cdot \sigma_{eq}(\text{basic design load}) = \sigma_{eq}(j_{ult} \cdot \text{basic design load}). \tag{3-3}$$

In the linear analysis case RF corresponds to a stretch factor f_{RF} of the applied vector of stresses to meet the failure curve (2D) or the surface of the failure body (3D). The failure point lies on the elongation *(Fig. 1, left)* of the applied stress state vector. This means, the failure point is known when the applied vector is known.

Effects of a multi-axial stress-state are accounted for by the equivalent stress, the associated material behaviour (brittle, ductile or intermediate in the addressed stress state domain) and the prevailing failure mode.

The following equations are applied in aerospace design for isotropic materials (metals, adhesives, most matrices, etc.)

$$RF_{yield} = \frac{R_{0.2}}{\sigma_{eq,yield}}, \quad RF_{ult} = \frac{R_m}{\sigma_{eq,ult}}; \text{ and } \quad RF_{br,yield} = \frac{R_{br,yield}}{\sigma_{eq,yield}}, \quad RF_{bru} = \frac{R_{bru}}{\sigma_{eq,bru}}. \tag{3-4}$$

Although in linear anlyses stresses and loads are proportional (i.e. result are the same for stress level and load level), formulations at load level should be preferred.

On principle, the equations are similar for composites.

Stability:

In stability verification - even for the statistically treatable stability-endangered structural elements - seldom a test series is available as basis to compute statistically-based stability design allowables. For a compression-loaded column (strut), this situation may be given, however. Then, it can be said that

$$RF_{stab} = \frac{N_{fr}}{N_{DBL}}. \tag{3-5}$$

3.6.2 Case: Non-linear analysis

“Stress is not proportional to a loading”.

Consequently, no stress formulation is permitted anymore according to Eq. (3-3) and the reserve factor is determined load-related. It holds

$$j_{ult} \cdot \sigma_{eq}(\text{basic design load}) \geq \sigma_{eq}(j_{ult} \cdot \text{basic design load}) \tag{3-6}$$

This can be interpreted such that the stress ’smoothes’ due to non-linear behaviour. Failure loading is iteratively, load step-wise computed and is that loading level where the growing vector of the stress state on its load path meets the failure surface or a failure curve (as shown in *Fig. 1*). This

failure surface is identical to the mathematical formulation $F = 1$ or $Eff = 100\%$.
A lower limit may be reached if numerical problems prohibit further increase of 'loading'. The uploading process is thereby checked by calculating Eff (necessary for design verification) on its way up to the maximum $Eff = 100\%$. Hence, the predicted failure loading reached in numerical analysis may be smaller than the true failure loading.

The reserve factor reads

$$RF_{ult} = \frac{\text{achieved failure load}}{DUL} . \qquad (3\text{-}7)$$

The procedure, how RF is computed in the case of non-linearity, will be dealt with in a specific HSB sheet (not yet issued).

Note: If the ratio $R_m/R_{p0.2} > j_{ult}/j_{p0.2}$, then a material-nonlinear analysis is not necessary for verification in spacecraft design because the DUL drives the design.

4 Strength Examples for the Determination of a *RF*

Some examples for the determination of a reserve factor for strength problems are provided. They consider the usual 'Worst Case' loading. They do not regard effects from prior up- and un-loadings which should be covered by the applied residual strength.
Here, just computations on the stress level are demonstrated, because the different aspects can be made obvious on this level.

The choice of the strength failure condition must be adequate to the material behaviour in order to obtain an accurate RF value.

4.1 Material failure of a ductile behaving metallic material

Task:

Is *onset of shear yielding failure* SY achieved under limit load ll?

Given:

- Applied loading: shear stress $\tau_{xy}(ll) = 65\,\text{MPa}$ (linear analysis)
- Strength design allowable $R_{p0.2} = 125\,\text{MPa}$
- Necessary failure condition (limit state function): $\sigma_{eq}^{\text{Mises}} = R_{p0.2}$

Calculation:

To solve the task, the failure condition for the "von Mises yielding" mode is applied (formulas are given in HSB 51101-01 and HSB 51101-02, Refs. [8, 9])

- equivalent stress: $\sigma_{eq}^{Mises} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \cdot \sigma_y + 3 \cdot \tau_{xy}^2} = \tau_{xy} \cdot \sqrt{3}$ (since $\sigma_x = \sigma_y = 0$)

Results:

- $f_{RF} = R_{p0.2}/\sigma_{eq}^{\text{Mises}} = 125/(65 \cdot \sqrt{3}) = 1.11 > 1$.

Assessment: Onset of shear yielding is not yet achieved under this stressing. f_{RF} is larger than 1, the design is verified.

Note: In spacecraft, the designing engineer uses another basic loading for this work case, namely $DYL = j_{0.2} \cdot DLL$.

4.2 Material failure of a brittle behaving uni-directional (*UD*) material

Task:

Is *onset of inter-fiber-failure* achieved under DLL?
This question is essential for leakage of tank walls. Investigated is the critical lamina of the laminate wall.

Given:

UD laminas-composed laminate

- Applied loading: $\{\sigma\}_L\,(DLL) = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{13}, \tau_{12})^T = (525, -85,\ 0,\ 0,\ 0,\ 56)^T$ MPa (vector containing all stresses, it is not the stress vector)

- Residual stress state: $\{\sigma\}_R = (\sigma_1, \sigma_2, \sigma_3, 0, 0, 0)^T = (3,\ 10,\ 0,\ 0,\ 0)^T$ MPa

- Strengths:$\{R\} = \left(R_{||}^t, R_{||}^c, R_{\perp}^t, R_{\perp}^c, R_{\perp||}\right)^T = (1050,\ 725,\ 36,\ 150,\ 63)^T$ MPa

- Elasticity quantities: $\{E\} = \left(E_{||},\ E_{\perp}, G_{||\perp}, \nu_{\perp||}, \nu_{\perp\perp}\right)^T$, $E_{||} = 150000\,\text{MPa}$

- Internal friction coefficient of the UD material: $\mu_{\perp||} = 0.3$

- Failure condition: In total, 5 failure modes are required from material symmetry reasons for UD material. 3 of them are affected by the applied 2D stress state.

- Mode interaction exponent: $m = 2.6$

Calculation:

2D failure conditions from HSB 51301-03(Ref. [10])

Chosen analysis model: Linear analysis, $f_{RF}^{mode} \equiv RF^{mode}$

Determination of the material reserve factors RF^{mode}: the indexes σ and τ indicate that stress (tensile or shear), which is active in the respective failure mode. For each activated failure mode one mode reserve factor is computed:

- $RF_{||}^{\sigma} = R_{||}^t/\sigma_1 = 1050/(525 + 3) = 1.99,$
- $RF_{\perp}^{\tau} = R_{\perp}^c/\sigma_2 = -150/(-85 + 10) = 2.0,$
- $RF_{\perp||} = (R_{\perp||} - \mu_{\perp||} \cdot \sigma_2)/\tau_{21} = (63 - 0.3 \cdot (-85 + 10))/56 = 1.53.$

It is possible - even for transversely-isotropic material - to formulate an equivalent stress for each single failure mode (2D formulation):

$$\sigma_{eq}^{||\sigma} = \sigma_1 \equiv \varepsilon_1 \cdot E_{||}, \qquad \sigma_{eq}^{\perp\tau} = \sigma_2, \qquad \sigma_{eq}^{\perp||} = \tau_{21} \cdot R_{\perp||}/(R_{\perp||} - \mu_{\perp||} \cdot \sigma_2)\,.$$

The full reserve factor RF is the result of the interacting mode reserve factors RF^{modes}

- $(1/RF_{DLL})^m = (1/RF_{||}^{\sigma})^m + (1/RF_{\perp}^{\tau})^m + (1/RF_{\perp||})^m, \quad RF_{DLL} = 1.15 > 1.$

Results:

As (material) reserve factor is achieved $RF_{DLL} = 1.15$. This corresponds to $Eff_{DLL} = 1/1.15 = 0.87$. The design is verified.
Driving mode is lateral compression, which is a wedge-shaped failure mode of the *lamina (ply)* responsible for onset of delamination in the *laminate*. This mode is the most critical IFF mode.
A coarse design information, that $\varepsilon_1 = \sigma_1/E_{||} = 528/150000 = 0.35\%$, fits to the limit design strain level often applied as design sizing concept.

Final remarks:

(1) Depending on the chosen strength failure condition, the value of the reserve factor may significantly deviate. Therefore, a physically adequate strength failure condition should be chosen. Bi-axial stress states will cause the largest difference (120° symmetry, see HSB 53101-02).

(2) The computation of multi-axial failure stresses of compressed materials can not be based on strength information, alone. According to Mohr-Coulomb, material friction properties μ have to be provided, too.

5 Application Hints

- Investigating a MoS value by probabilistic means (see Ref. [4]) points out that the MoS-value or the RF-value are not real quantitative measures for failure prediction. They are only agreed measures.
  Reducing the standard deviation by some percent results in a higher benefit than increasing the mean (average) value by the same value.
- Deterministic analysis: A set of design parameters is obtained with which the failure limit states are not reached by a not quantifiable distance represented by RF. No real measure is given with RF.
  Probabilistic analysis: A set of design parameters is obtained which is a set of coordinates of the so-called most probable failure point. This causes more work. However, the obtained failure probability ($p_f = 1 - \Re$) is a measure for above distance because it considers all possible combinations of the scattering (random) design variables.
- Load terms shall be used carefully in order to speak about the same load level.
- An 'allowable stress' in the former safety concept one could 'allow', but one should not use it according to the present application of a safety format that does not work with allowable stresses anymore (see Fig. 1). There is a risk to mix-up this term and numbers.
- The use of the single term 'allowable' instead of 'strength design allowable' (more general: resistance design allowable) is misleading because one would never allow this level.

- The term ’allowable strain’, one should also avoid according to the ’allowable stress’ reasons. One should use the term ’design strain limit value’ ε_{lim} (in Ref. [2] termed ’strain limit’). Its value depends on operational requirements and design policy.
For instance for UD-laminates ε_{lim} might be about 0.3%. This practically means ’approximately linear structural behaviour’. Some companies use this value for reasons to have a good chance for repair or for avoiding a fatigue or a damage tolerance demonstration.

- A RF is - according to its definition - a design load-factorizing quantity (How many percent loading can be imposed up to a distinct loading level?). As long as the stress is proportional to loading, it can be interpreted as a *material failure* in structural analysis.

- Practically, the determination of a reserve factor in case of *structural* failure (stability) can be seldom performed. Test series are generally missing and the application of a probabilistic procedure (Monte-Carlo method or others) on the basis of imperfection knowledge in the special application just improves the ’design-by-analysis’ situation. Besides this, design loading limits of instability-endangered structures can not be defined always, e.g. for compression-loaded panels.

- Global strength failure models usually capture several failure modes. But, it is scientifically not correct to employ polynomial (mathematical curve-fit) interaction failure models - used in the global failure condition models - whenever the physical mechanism of failure changes.

- Essential influences on stress level and thereby on RF are to be considered, especially for composites. These influences may be generated from manufacturing caused residual stresses, curing stresses from shrinking, visco-elastic behaviour, fiber waviness, hygro-thermal environment, etc. For UD-composites applications which are placed in the compressive domain the residual stress effectiveness is naturally smaller than for those in the tensile domain.

Keep in mind:
Excellent FEA alone does not guarantee reliable designs and reliable reserve factors. The choice of the ’right’ Dimensioning Load Cases from all the possible load cases may be the most responsible and important task of the designing engineer who has to decide in a short time in critical situations. An adequate application of the chosen ’Safety Concept’ is mandatory. Appropriate properties (the use of minimum values is not always on the safe side) should be taken according to the specific task to be solved. Failure theories - especially for composites - need carefully to be applied and should consider the actual material behaviour under stress state and temperature. The same material may behave ductile and brittle depending on stress state and environment. Therefore, one can not call a material a ductile or a brittle one.