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Strength Failure Conditions of the Various Structural Materials: Is there some Common Basis existing?

Ralf G. Cuntze[1]

Abstract: The paper deals with the application of phenomenological, invariant-based strength conditions (fracture failure) and their interrelationships. The conditions have been generated and are just applied here for a variety of materials. These might possess a dense or a porous consistency, and belong to brittle and ductile behaving isotropic materials, brittle unidirectional laminae and brittle woven fabrics. The derivation of the conditions was based on the author's so-called Failure Mode Concept (FMC) which basically builds up on the hypotheses of Beltrami and Mohr-Coulomb.

Essential topics of the paper are: 'global fitting' versus 'failure mode fitting', a short derivation of the FMC, the presentation of the FMC-based strength failure conditions for the material families mentioned above, and the visualization of a variety of conditions. Various links or interrelationships between the materials are outlined.

Conclusions which may be drawn from the laborious investigations: 1. The application of Beltrami's assumption together with the consideration of material friction forms a common basis in the determination of strength conditions. 2. The FMC is an efficient concept because it very *strictly* utilizes a 'thinking in failure modes' as well as an application of *material symmetry-related invariants*. It has proven to be a helpful tool in simply fitting the course of multi-axial strength test data, and it finally can capture several failure modes in one equation avoiding the shortcomings of the usual 'global fitting' strength conditions. 3. Different but similar behaving materials can be basically treated with the same strength condition.

Keyword: Failure criteria, visualization, brittle and ductile isotropic materials, brittle UD and woven fabrics laminae, material behaviour links

1 Introduction

Design Verification (see Fig. A1) demands for reliable reserve factors and these – besides a reliable structural analysis – for reliable strength conditions. Such a condition is the mathematical formulation of a failure curve or of a failure surface. In aerospace, the static design verification has to be performed for *onset of (global) yielding* on flight load level and for *onset of fracture* (cracking) on Design Ultimate Load (*DUL*) level. The former usually requires a yield condition and the latter requires one or more fracture conditions as strength failure conditions.

In general, such failure conditions shall assess a multi-axial stress state which acts in the critical material point by utilizing just one uniaxial strength R and the equivalent stress σ_{eq} representing the multi-axial stress state above. Further, they shall allow for inserting stresses from the utilized various coordinate systems (COS) into stress-formulated failure conditions, and optimally into invariant-based ones, if possible.

Failure conditions have to be generated for dense & porous, ductile & brittle behaving materials. These can be isotropic materials, transversally-isotropic (UD := unidirectional) materials and rhombically-anisotropic materials (woven fabrics) and in future for even 'higher structural textiles', stitched or braided or knitted. The structural build-up of the latter may require in future a quasi-ductile treatment for these entirely brittle behaving textile 'materials'.

[1] Prof. Dr.-Ing. habil. Ralf G. Cuntze, formerly MAN Technologie AG, Augsburg, Germany. D-85229 Markt Indersdorf, Tel. & Fax: 0049 8136 7754, E-mail: Ralf_Cuntze@t-online.de

Most often, failure conditions map a course of multi-axial test data by one global equation (i.e. Tsai and Wu (1971)) not taking care whether the data belong to one or more failure mechanisms or failure modes. Therefore, extrapolations out of the mapping domain may lead to erroneous results. Further, if a correction change in the domain of one failure mode has to be made it may affect the failure surface or the curve domain of another independent failure mode. This is a mathematical consequence but not a physically correct one [Hart-Smith (1993), Cuntze et al (1997)].

Driven from the shortcomings of such a 'global fitting' the author looked since 1995 for a 'failure mode-related fitting'. The procedure 'How to determine such *mode failure conditions?*' he termed the Failure Mode Concept FMC [Cuntze (1999)]. This FMC is a concept that very strictly uses the failure mode thinking (more than other authors, such as Christensen (1997) and Hashin (1980). Mises with the HMH hypothesis, 1913, faced a single failure mode, the yielding, only).

The FMC is also based – as far as the material homogenization permits to do it – on material symmetry-related invariants, which have proven to be a helpful tool in simply fitting multi-axial strength test data. The application of invariants in the generation of strength failure conditions has benefits due to the fact that material symmetry [Cuntze (1999)], together with the findings of Beltrami (1885), support the choice of an invariant to be utilized in a distinct failure condition. An invariant is a combination of stresses – powered or not powered – the value of which does not change when altering the coordinate system. Invariants are optimal for the formulation of the advantageous *scalar* strength failure conditions. Idea is that the FMC enables to simply capture several failure modes in one equation without the short-comings of classical global conditions.

Existing links in the mechanical behaviour show up: fully different structural materials can possess similar material behaviour and may belong to the same class of material symmetry. For instance: a brittle porous concrete in the compression domain can be basically described by the same failure condition like a very ductile behaving light-weight steel in the high tension domain when pores (void nucleation) have been generated.

This has the consequence: The same failure function F can be used for different materials and more information is available for pre-dimensioning and modelling from past experimental results of a similarly behaving material. Therefore the message is: Use these benefits by thinking about a procedure which -in case of a new material- more simply and less costly will enable the designer for an engineering assessment of multi-axial stress states, early in the product development, on basis of a less big test campaign plus the available information from the similar behaving material.

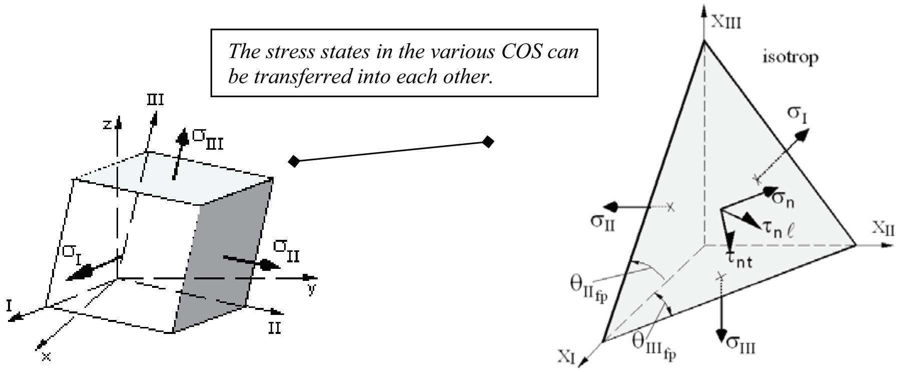
Special aim of the paper – in the given page frame – shall be *a global view of the material links/coincidences/interrelationships* and *not a detailed information on the presented failure conditions*. The strength failure of non-cracked structural parts is addressed only, and not stability failure or damage tolerance (see the figure in the Annex) or physical and material nonlinearities in the analysis (for this aspect, see for e.g. for UD material [Cuntze and Freund (2004), Cuntze (2004)]).

2 Stress States & Invariants

There are various kinds of stresses which may be inserted into a strength failure condition. In the case of isotropic materials one faces: principal stresses, structural component stresses, and Mohr's fracture plane stresses, [9, 10]. These kinds of stresses can be transferred into each other. Fig. 1 outlines all these stresses and the associated invariants for **isotropic materials**.

Formulations of strength failure conditions may follow, such as performed by Hashin/Puck for unidirectional laminae, Mohr's postulate: "*Fracture is determined by the stresses in the fracture plane!*" This has a formulation advantage but makes the determination of the angle of the inclined fracture plane necessary. The failure condition is not scalar any more.

In the case of **transversely-isotropic material**

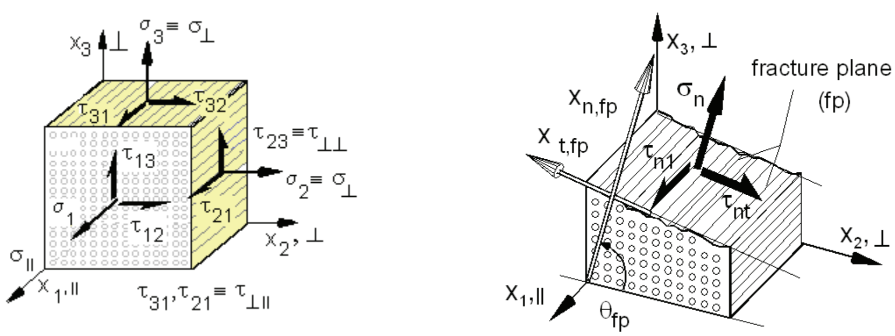


Principal stresses	Structural component stresses	Mohr's fracture plane stresses
$\{\sigma\} = (\sigma_I,\ \sigma_{II},\ \sigma_{III})^T$	$\{\sigma\} = (\sigma_x,\ \sigma_y,\ \sigma_z,\ \tau_{yz},\ \tau_{xz},\ \tau_{xy})^T$	$\{\sigma\} = (\sigma_\ell,\ \sigma_n,\ \sigma_t,\ \tau_{nt},\ \tau_{t\ell},\ \tau_{\ell n})^T$
	'isotropic invariants'!	
$I_1 = (\sigma_I + \sigma_{II} + \sigma_{III})^T = 3\sigma_{oct} \equiv f(\sigma)$,	$I_1 = (\sigma_x + \sigma_x + \sigma_z)^T$	$I_1 = (\sigma_\ell + \sigma_n + \sigma_t)^T$
$6J_2 = (\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2 = 4(\tau_{III}^2 + \tau_{II}^2 + \tau_I^2) = 9\tau_{oct}^2 \equiv f(\tau)$	$6J_2 = (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + (\sigma_x - \sigma_y)^2 + 6(\tau_{yz}^2 + \tau_{zx}^2 + \tau_{xy}^2)$	$6J_2 = (\sigma_n - \sigma_t)^2 + (\sigma_t - \sigma_\ell)^2 + (\sigma_\ell - \sigma_n)^2 + 6(\tau_{nt}^2 + \tau_{t\ell}^2 + \tau_{n\ell}^2)$

$$27J_3 = (2\sigma_I - \sigma_{II} - \sigma_{III})(2\sigma_{II} - \sigma_I - \sigma_{III})(2\sigma_{III} - \sigma_I - \sigma_{II}),$$

$$I_\sigma = 4J_2 - I_1^2/3, \qquad \sigma_{mean} = I_1/3$$

Figure 1: Isotropic Material, 3D stress states & invariants



$\{\sigma\}^{quasi-isotropic\ \ plane}_{principal} = (\sigma_1, \sigma_2^p, \sigma_3^p, 0, \tau_{31}^p, \tau_{21}^p)^T$	$\{\sigma\}_{lamina} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$	$\{\sigma\}_{Mohr} = (\sigma_\ell,\ \sigma_n,\ \sigma_t,\ \tau_{nt},\ \tau_{t\ell},\ \tau_{\ell n})^T$

UD invariants *[10a]*

$I_1 = \sigma_1, \quad I_2 = \sigma_2^p + \sigma_3^p$	$I_1 = \sigma_1, \quad I_2 = \sigma_2 + \sigma_3$	$I_1 = \sigma_\ell, \quad I_2 = \sigma_n + \sigma_t$
$I_3 = \tau_{31}^{p\ 2} + \tau_{21}^{p\ 2}$	$I_3 = \tau_{31}^2 + \tau_{21}^2$	$I_3 = \tau_{t\ell}^2 + \tau_{n\ell}^2$
$I_4 = (\sigma_2^p - \sigma_3^p)^2 + 0$	$I_4 = (\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2$	$I_4 = (\sigma_n - \sigma_t)^2 + 4\tau_{nt}^2$
$I_5 = (\sigma_2^p - \sigma_3^p)(\tau_{31}^{p\ 2} - \tau_{21}^{p\ 2}) + 0$	$I_5 = (\sigma_2 - \sigma_3)(\tau_{31}^2 - \tau_{21}^2) + -4\tau_{23}\tau_{31}\tau_{21}$	$I_5 = (\sigma_n - \sigma_t)(\tau_{t\ell}^2 - \tau_{n\ell}^2) - 4\tau_{nt}\tau_{t\ell}\tau_{n\ell}$

Figure 2: Transversely-isotropic material, 3D stress states & invariants

(UD composite) the associated stresses and invariants are depicted in Fig. 2. It is to be seen that three kinds of stresses are applied: lamina COS-based stresses, Mohr stresses, and – as there is a quasi-isotropic plane existing – quasi-principal stresses.

In the case of **orthotropic material** (rhombically-anisotropic, Fig. 3), just a formulation in fabric's lamina stresses makes sense. Now, one has to deal with more, however simpler invariants.

3 Observed Strength Failure Modes and Strengths

Of high interest for the establishment of material strength conditions is the number of strength failure modes and the number of strengths, observed in the fracture tests, Masters (1994).

For the various **isotropic materials** several failure modes can be differentiated which are typical for: *a) brittle behaviour/dense consistency, b) brittle behaviour/ porous consistency, and c) ductile behaviour/dense consistency.* Their features are:

a) Two failure modes: Normal Fracture (NF) under tension and Shear Fracture (SF) under compression are recognized. NF is also termed, due to the poor deformation prior to fracture and the smooth fracture surface (fractography reveals it), *cleavage fracture*. SF exhibits shear deformation prior to fracture, a knowledge, which is helpful for the choice of invariants when formulating the strength condition. *Two strength have to be measured*.

b) Two failure modes are found: NF and Crushing Fracture (CrF). The latter shows a volumetric deformation prior to fracture which will be helpful for the choice of invariants, too. Remarkable for the compression test is here that there is a full decomposition of the texture, a hill of fragments (crumbs) remains. *Two strength have to be measured*.

c) Just one failure mode can be identified: SF under tension. Shear deformation is observed prior to fracture (at maximum load) and then occurs diffuse and later local necking + void growth (means a volumetric change) prior to rupture in the so-called 'Gurson domain', Gurson (1997). This SF is also termed t*earing fracture* and shows dimples under tension. One strength, the *load-controlled* value R_m^t, is to be measured. The corresponding compressive strength is neither existing nor necessary for design, because deformation-limiting functional design requirements will not permit to go that far. However, if the vicinity of a highly strained location will take over the load locally and if it happens that $\sigma_{eq} > R_m^t$, then the *deformation-controlled multiaxial* strength at rupture ('Gurson domain') may be considered in design [Cuntze (2002)]. Then true stresses and true strengths should be applied.

For the brittle uni-directional lamina, which can be modelled as a **transversely-isotropic material**, the fractography of test specimens reveals (Fig. 5) that 5 Fracture modes exist in a UD lamina: 2 FF (Fibre Failure) + 3 IFF (Inter Fibre Failure). From basic knowledge and test experience is known: It's still common practice to measure 5 strengths for accurate designing.

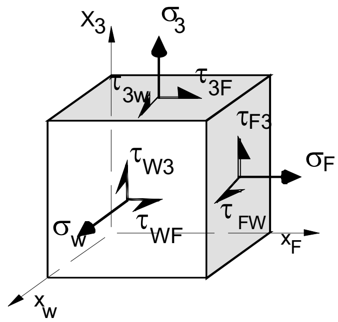
Of highest importance for failure are the FF, however, learned from component tests, Puck's wedge failure mode IFF3 might be hazardous like an FF. Its criticality depends on the stack and the entire loading.

For **rhombically-anisotropic or orthotropic material** (woven fabrics, Fig. 6) from testing and theory is learned: there are more strength failure modes existing than for a UD material. This causes more mode interaction domains, however the designer is just faced with simple invariants and can hope that a simpler formulation of the individual strength conditions is possible.

Unfortunately for most of the textiles, fractography will not exhibit clear failure modes. In these materials always multiple cracking is caused under tension, compression, bending, or shear. No clear mode-related strengths are observed. Therefore, they have to be defined according to the number orthotropic material symmetry is fixating, see [Cuntze (1999)].

From the left part of the Fig. 6 can be concluded: Modelling depends on fabrics type.

Due to the various fibre pre-forms: from roving, tape, weave, and braided (2D, 3D), knitted,



Woven fabrics material element. *Warp (W), Fill(F).*

Stress vector:

$$\{\sigma\}_{lamina} = (\sigma_W, \sigma_F, \sigma_3, \tau_{3F}, \tau_{3W}, \tau_{FW})^T$$

Fabrics invariants *[Boehler 1995, 14]*:

$$I_1 = \sigma_W, \; I_2 = \sigma_F, \; I_3 = \sigma_3$$

$$I_4 = \tau_{3F}, \; I_5 = \tau_{3W}, \; I_6 = \tau_{FW}$$

Figure 3: Orthotropic (rhombically-anisotropic) material, 3D stress state & invariants, Boehler (1995)

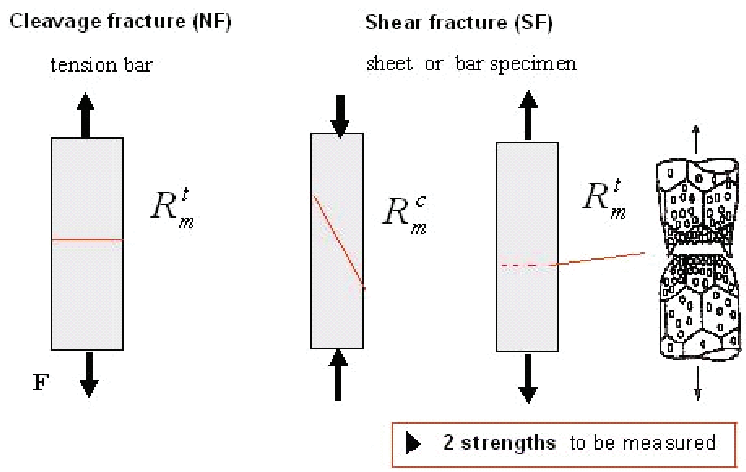


Figure 4: Fracture failure modes and strengths of isotropic materials

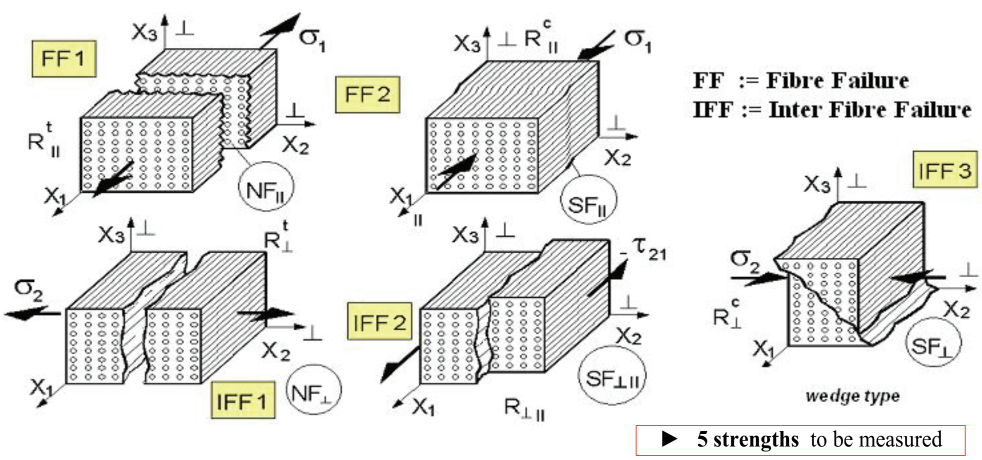


Figure 5: Fracture failure modes of UD materials, Cuntze et al (1997)

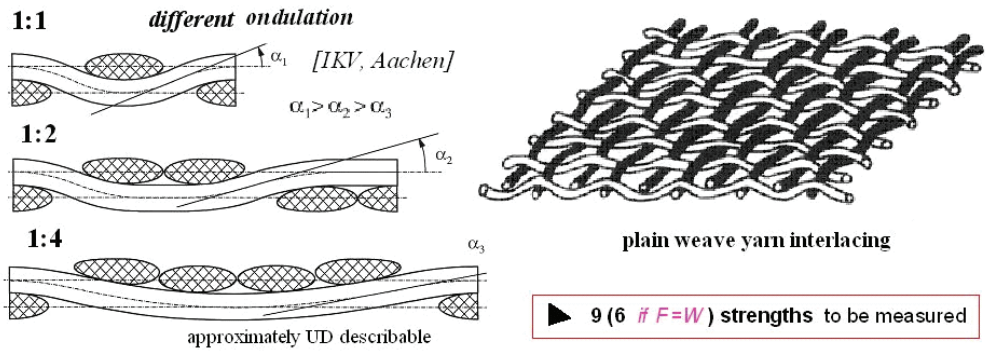


Figure 6: Fracture failure of woven fabrics material

stitched, or mixed as in a pre-form hybrid it becomes obvious that a large variety of failure conditions has to be developed in case of the 'higher level structural textiles'.

4 Attempt for a Systematization

The design process is faced with numerous different materials, different material behaviour, various failure modes and a lot of more or less validated strength failure conditions. So, the questions arise: "Is there a possibility in this complex design business to find a procedure to figure out failure conditions which are simple, however, describe physics of each failure mechanism sufficiently well?" And therefore "Might be a systematization helpful?"

Fig. 7 displays a scheme of (material) strength failure modes for isotropic materials and directly compares them with the *brittle* UD materials which are only used as structural materials, presently.

What may be learned from the 'isotropic part' of the figure and from studying the associated failure conditions? The same mathematical form of a failure condition (means interaction of stresses within one mode) is valid from *onset of yielding* to *onset of fracture*, if the physical mechanism remains, such as with shear yielding in case of ductile steels. In general, the growing yield body (SY) is confined by the fracture surface (SF or NF). The figure emphasizes that one failure mode, Normal Yielding (NY), should also exist according to the proposed system. PMMA (plexiglass) with its chain-based texture really shows NY!

The arrows denote the coincidences between brittle UD laminae and brittle isotropic materials. Delamination failure of laminates – built up from the UD laminae as building blocks – is not addressed here.

Regarding Fig. 7 the establishment of strength failure conditions needs to be structured as it will be proposed in the next chapter.

Fig. 8 gives an overlook on homogenizing a material on different structural levels or scales. In the case of elasticity modelling a homogenization or 'smearing' is applicable for pre-assessment of elasticity properties whereas in case of strength modelling the smearing process may not be so effective due to the fact that, e.g. for UD, the micromechanical fibre strength σ_{1f} determines fracture and not the macro-mechanical tensile stress σ_1, utilized in the lamina model.

Material symmetry shows that the number of strengths is identical to the number of elasticity properties! Using material symmetry in material modelling requires that homogeneity is a prerequisite. However, the application of material symmetry beneficially fixes the number of properties to be measured to a minimum one. In this context should be mentioned: The choice of the material model is always dependent on the efficiency a structural task can be solved and the quality required.

Structural analysis and design verification strongly depend on the behaviour and the consistency of the material. In Fig. 9 the author

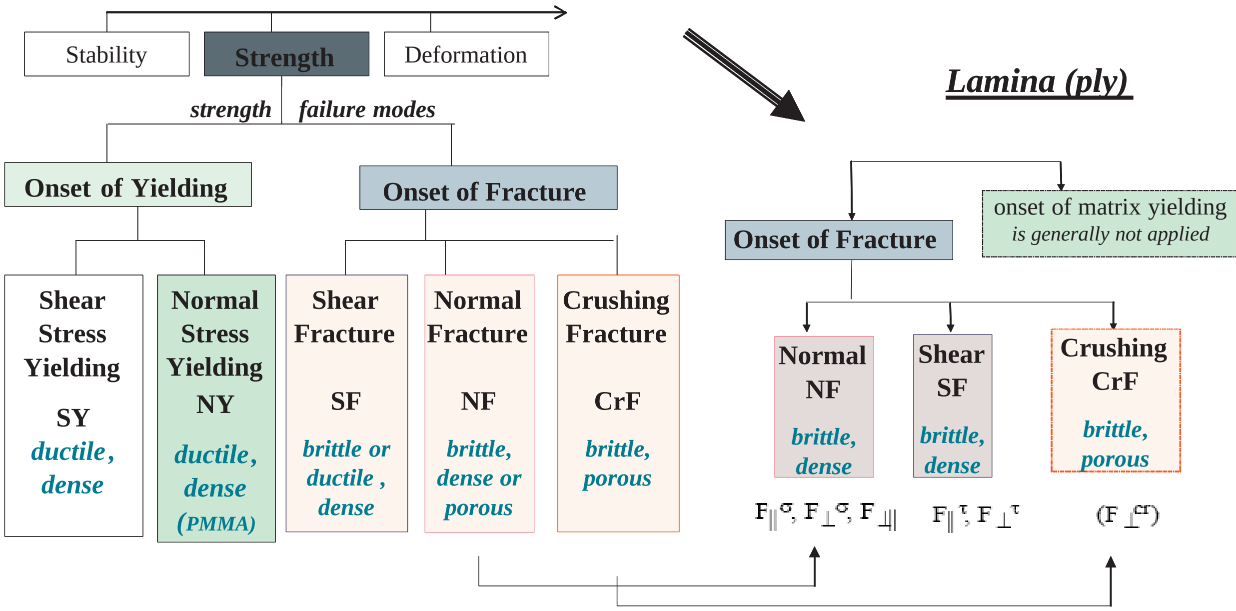


Figure 7: Scheme of Strength Failure Conditions for isotropic material and brittle UD

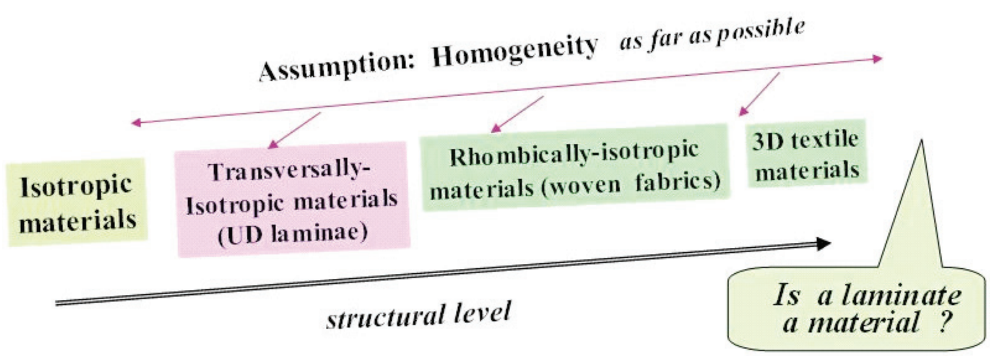


Figure 8: Material homogenizing + modelling and material symmetry aspects

	allocation to *Design Verification levels DYL, DUL* wrt. material behaviour	
Failure type / *Consistency*	**brittle, semi-brittle** DUL	**(quasi-) ductile** DYL
dense	fibre re-inforced plastics, mat, woven fabrics, grey cast iron, matrix material, amorphous glass C90-1,	Glare, ARALL, metal alloys braided textiles
porous	foam, fibre re-inforced ceramics	sponge

failure: fracture failure functional/usability limit

Figure 9: Proposed classification of homogenized materials

proposes a coarse classification of the various homogenized materials considering the so-called *failure type* which is linked to brittle and ductile behaviour, and further considering the *material's consistency*. Practically, design driving on the two static design verification levels is for the fracture-related design ultimate load (DUL) a brittle behaviour describing strength condition and for design yield load (DYL) or pressure a ductile behaviour describing one.

Structural composites usually display brittle behaviour.

5 Short Derivation of the Failure Mode Concept (FMC)

Applying the FMC, Cuntze tries to formulate easy-to-handle homogeneous invariant-based failure conditions with stress terms of the lowest possible order. The conditions in mind shall be 'engineering-like' and shall not make a search of the fracture plane necessary which would be necessary when using a typical Mohr-Coulomb formulation.

So, the essential requirements for the development of the failure conditions are:

- physically-based,
- simply formulated + numerically robust and
- practically just need the information on the strengths available at pre- dimensioning; further model parameters shall be assessable on the safe side
- condition shall be a mathematically homogeneous function.

There are two different formulations possible: a 'global formulation' and a 'mode-wise formulation'. The associated equations read, [Cuntze et al (1997), Cuntze (1999)].:

One global failure condition:
$F(\{\sigma\},\{R\}) = 1$ (usual formulation) ,

Several mode failure conditions:
$F(\{\sigma\},R^{\text{mode}}) = 1$ (used in the FMC).

A failure condition is the mathematical formulation, $F = 1$, of a failure curve or surface. In the case of a global formulation the condition has to capture several failure modes including all stresses and strengths ($F >=< 1$ is termed failure criterion). In contrast, the FMC includes the mode active stresses and just the mode-governing strength.

From application of global conditions the lesson had to be learned: A change, necessary in one failure mode domain, may have an impact on other physically not related failure mode domains, but in general <u>not</u> on the safe side (see e.g. Hart-Smith (1993)).

Possibilities of formulating a failure condition are given by applying:

- stresses (strains have the disadvantage of neglecting residual stresses) or
- invariants (FMC employs *stress* invariants).

Experience on isotropic and UD material shows, see also Christensen (1998):

- Each of the observed fracture failure modes is linked to one strength
- Material symmetry says:
  Number of strengths = number of elasticity properties
  Example UD material:
  $R^t_{||}, R^c_{||}, R_{\perp||}, R^t_{\perp}, R^c_{\perp}$ and $E_{||}, E_{\perp}, G_{||\perp}, \nu_{\perp||}, \nu_{\perp\perp}$.
- Application of invariants for composites is also possible.

Due to the experience above the **FMC postulates** in its 'phenomenological engineering approach': **Number of failure modes** equals **number of strengths!** This means for isotropic material 2 and for transversely-isotropic UD material 5 properties.

> *<u>Mind</u>: In general, failure conditions include yielding and fracture failure modes. A fracture failure surface which may consist of several parts confines the growing yield surface. A yield surface is usually describing just one mode, the shear yielding SY (for PMMA there are 2 yield modes however: SY + NY), whereas the fracture surface usually*

describes several independent fracture modes. A yielding failure mode and a fracture failure mode can be described by the same procedure but the equations are usually different except if the material's yielding behaviour will remain till fracture.

Reasons for choosing invariants when generating failure conditions are presented by Beltrami (1885). He assumes: *"At 'onset of yielding' the material possesses a distinct strain energy density W"*. This is composed of two portions: the *dilatational energy* (I_1^2) and the *distortional energy* ($J_2 \equiv$ Mises) in

$$W \cdot 6E = (1-\nu) \cdot I_1^2 + (2+2\nu) \cdot J_2,$$

wherein E is the Young's modulus, I_1^2 describes the *volume* change of the cubic material element and J_2 its change of the *shape*. These changes can be witnessed by the fracture morphology.

In order to formulate a relatively simple scalar failure condition one chooses as invariant a term that respects whether the cubic material element will experience a volume change in the considered mode or a shape change. The same is valid for UD material. In the case of brittle behaving materials one energy term is to be added, the friction energy, which is linked to a Mohr-Coulomb behaviour.

So, from Beltrami, Mises (HMH), and Mohr/Coulomb (friction) may be derived "Each invariant term in the *failure function* ***F*** may be dedicated to one physical mechanism in the solid = cubic material element", see Table 1:

The idea behind the FMC was: A possibility exists to very generally formulate failure conditions failure mode-wise (e.g. shear yielding of Mises or later Puck for inter fibre fracture failures of UD materials) and stress invariant-based (J_2 etc.). The latter has still been performed for the isotropic materials by Huber-Mises-Hencky and later numerous other authors, and for UD material Boehler (1995), Hashin (1980), Christensen (1997), Jeltsch-Fricker (1996) etc., and for fabrics [Boehler (1995), Meckbach (1998)].

So, the question arose *What is new with the FMC?* This is:

- the strict thinking in failure modes
- the individual interaction of a failure mode with the other modes by having no impact on another pure failure mode domain
- an a-priori reduction of the possibilities to formulate failure conditions.

Concluding on the previous context the following detail aspects can be listed:

- 1 failure mode represents 1 independent failure mechanism
- 1 failure condition represents 1 failure mechanism (interaction of stresses)
- 1 failure mechanism is governed by 1 strength.

What is finally missing is the interaction of failure modes. This shall be performed here by a probabilistic theory-based 'rounding-off' approach' formulated as a *series failure system* model

$$(Eff)^m = (Eff^{\,\text{mode}1})^m + (Eff^{\,\text{mode}2})^m + . + . + . = 1$$

with the so-called (global) *stress effort Eff*, representing the actual *portion of the 'load'-carrying capacity of the material*, and with the (Weibull-related) interaction coefficient m. The *mode stress efforts* are the contributions of each participating failure mode. Each failure mode is characterized by one strength and therefore an equivalent stress is given for each mode according to

$$Eff^{mode} = \sigma_{eq}^{mode} / R^{mode}.$$

For an example, namely the UD material, the interaction of the 3 IFF shall be visualized. All three IFF failure modes are interacted together with the FF in one single (global) failure equation

$$Eff^m = (Eff_{||}^{\tau})^m + (Eff_{||}^{\sigma})^m + (Eff_{\perp}^{\sigma})^m + (Eff_{\perp||})^m + (Eff_{\perp}^{\tau})^m = 1.$$

Table 1: Interrelationships of the invariants with physical mechanisms and energy

- volume change :	I_1^2	*(dilatational energy)*	I_1^2, I_2^2
- shape change :	J_2 (HMH, 'Mises')	*(distortional energy)*	I_3 , I_4
- friction :	I_1 (Mohr-Coulomb)	*(friction energy)*	I_2.
	isotropic invariants		UD invariants

Herein, the stress efforts of the 3 pure IFF modes (form straight lines in Fig. 10) read:

$$Eff_{\perp||} = \frac{|\tau_{21}|}{\overline{R}_{\perp||} - \mu_{\perp||} \cdot \sigma_2}, \quad Eff_{\perp}^{\sigma} = \frac{\sigma_2}{\overline{R}_{\perp}^{t}},$$

$$Eff_{\perp}^{\tau} = \frac{-\sigma_2}{\overline{R}_{\perp}^{c}}.$$

For usually applied UD materials the value of m is $2.5-3$. Approximately the same value may be taken for all interaction zones. Fig. 10 is based on a hoop wound GFRP tube, E-glass/LY556/HT976. It depicts the straight pure mode curves and the interaction curve (σ_2, τ_{21}).

6 Visualizations of some Derived Failure Conditions

The failure conditions addressed here are most often termed strength criteria (the term condition is more accurate due to the fact that $F = 1$ is applied). They employ the strength properties required by the material symmetry associated with the chosen (homogenized) material model (isotropic, transversely isotropic, rhombically anisotropic). This material model is an *ideal* one and is treated as a crystal. However, to formulate a strength condition for a *real* material missing parameters are to be determined. These are the internal friction properties of the brittle materials for which this work is necessary to do, only.

For a variety of differently behaving materials failure conditions have been derived and applied to available own and multi-axial strength test data from literature. The results are visualized in the Figs. 11 through 18:

* At first, the fracture failure curve for a **grey cast iron** (data Coffin) is presented in Fig.11a. The data are well mapped by the given pure mode conditions and the interaction equation. The 2D curve is substantiated by two 3D figures, Figs.11b, in a Lode coordinates diagram which demonstrate the applicability of the conditions. Viewing the scatter of the data, the difference between the so-called tensile and the compressive meridian (compare the next material, concrete) can be neglected and a rotationally symmetric fracture body assumed. *Learned from application to an epoxy matrix: the same failure condition can be taken.*

* In case of **concrete**, the situation is more complex. The test data show a big bandwidth. The reason for this bandwidth is not only the test scatter but the stress-state dependent failure probability causing non-coaxiality in the octahedral plane. The difference between the tensile (extension) meridian and the compressive meridian is to be considered. Now the isotropy-inherent '120° material (crystal) symmetry' comes to act, Fig.12a.

Usually, the invariant J_3 is applied [de Boer and Dresenkamp (1989)] to describe this non-coaxiality. This has been done without any physical explanation. The author however is believing (based on own calculations) that the '120° pop-in' is the result of a joint failure probability, due to a doubly activated failure mode, see also [Awaji and Sato (1978), Rackwitz and Cuntze (1987)]. The application of J_3 looks a little simpler.

In the failure condition F_{τ}^{c} in Fig. 12 the volume change term I_1^2 becomes active when the concrete is porous and not dense. Then the friction term I_1 will vanish.

* For brittle, porous **monolithic ceramics**, Fig. 13 depicts the highly porosity-dependent failure curve. *Learned: the same failure condition as for porous concrete can be applied.*

* For the brittle, dense **glass C 90-1** a 2D failure curve in the principal stress plane is displayed and a 3D failure surface in the Lode diagram. The interaction of the test data is good. *Learned: the*

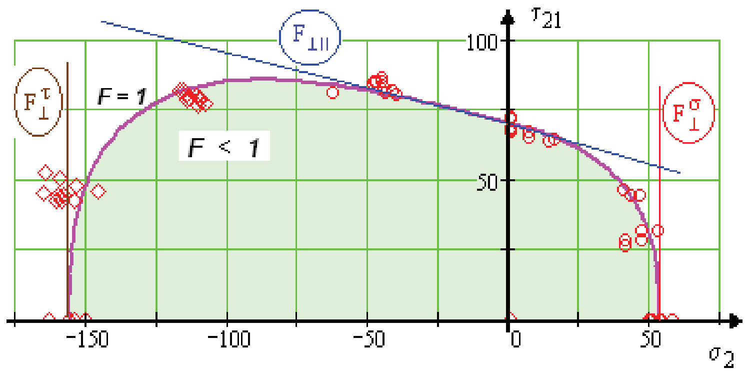


Figure 10: Visualization of the interaction procedure (2D formulation with simple $Eff_{\perp||}$ equation [Cuntze (2007)])

same failure condition as for grey cast iron can be applied.

* From Thielicke (1997) the following set of strength data has been provided for **C/C fibre-reinforced ceramics**. It is a brittle porous ceramics lamina based on a UD tape. Invariants, applied in 3D case were: for friction (I_3, I_2), for shear (I_4), and friction (I_2). This reduces for a plane stressing to the interaction equation including the three IFF failure modes, indicated with the respective failure condition in Fig. 15.

* For **UD lamina fibre reinforced plastics** the in-plane stresses-caused fracture is visualized in Fig. 16. *Learned: Same failure condition as with UD-CMC.*

For 2D visualizations of other UD materials the reader be referred to [Cuntze, R.G. and Freund (2004), Cuntze (2004, 2006)]. As delamination conditions are not a topic of this paper the reader may be referred to [Cuntze (2007)].

<u>*Remark*</u>: The invariant-based UD failure conditions – as all others – have been 2D-validaded (sufficient 3D test data are world-wide missing) by 14 test cases of the World-Wide Failure Exercise (1993-2003, Hinton et al (2002), Hinton et al (2004)). Winner of the contest were the FMC-conditions [Cuntze, R.G. and Freund (2004), Cuntze (2004)] and Puck's action-plane conditions [Puck, A. and Schürmann (2002)], both, non-funded elaborations. Later, the author further simplified his UD FMC failure conditions [Cuntze (2006, 2007)].

Recently, a WWFE-II [Kaddour and Hinton, M.J.] has been started to predict again in a Part A a set of fracture curves and stress-strain curves for 12 test cases consisting of UD laminae and various UD laminae-composed laminates subjected to different stress states in the multi-axial compression domain. This exercise aims for a 3D validation. A Part B, planned for 2008, will contain the comparison with the test curves.

* For two other **carbon fibre-reinforced fabrics ceramics**, Fig. 17, failure curves are presented. The utilized test data have been published in Geiwitz et al (1997) and Cuntze (1998).

The interaction mapping worked for this specific stress combination. Here it is to be noted: for woven fabrics, test information for a real validation is not yet available.

* As a last example it should be mentioned wrt *one* failure mode (e.g. yielding) that the application of the invariants for failure conditions of adhesives in a bonded joint is principally the same as for soil and rock material. In these cases however the interaction of *several* failure modes is in practice globally executed.

7 Conclusions

Material relationships:

* Many material (behaviour) links have been outlined, e.g. a compressed brittle porous concrete can be described like a tensioned ductile porous metal in the Gurson domain

▶ Drawing conclusions from the lessons learned

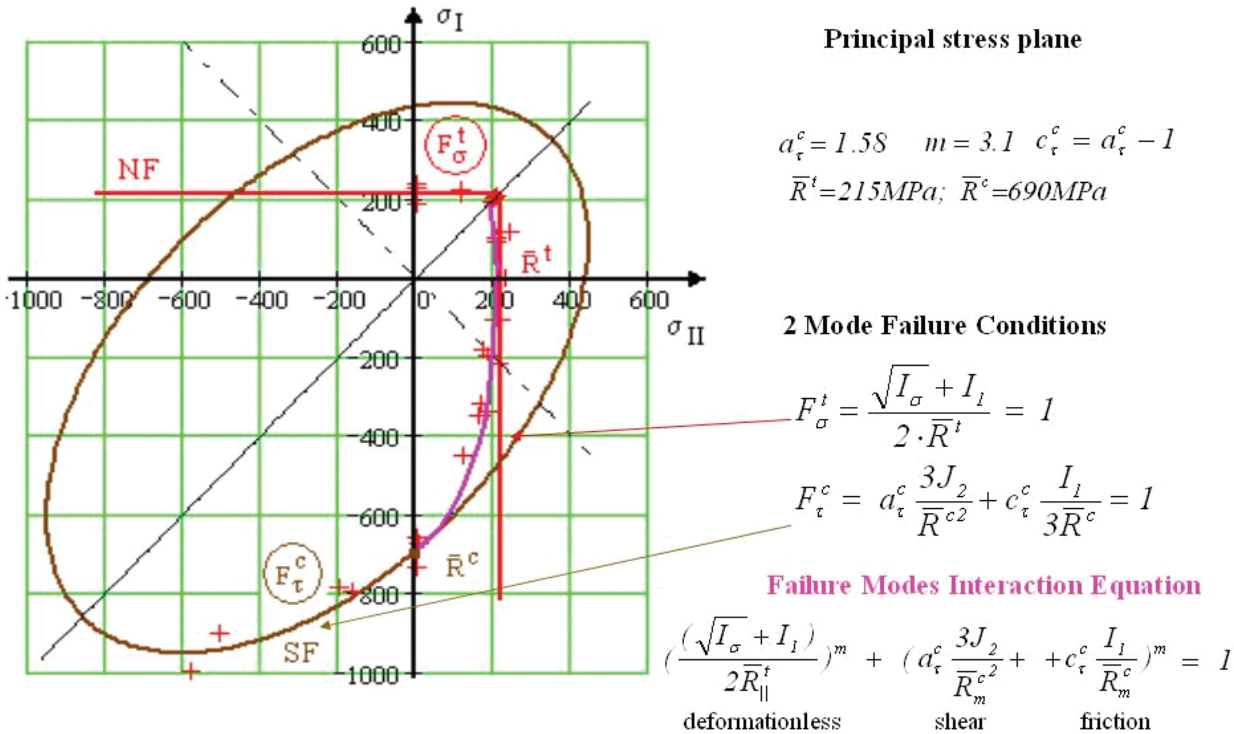


Figure 11a: 2D failure curves of grey cast iron (brittle, dense, microflaw-rich)

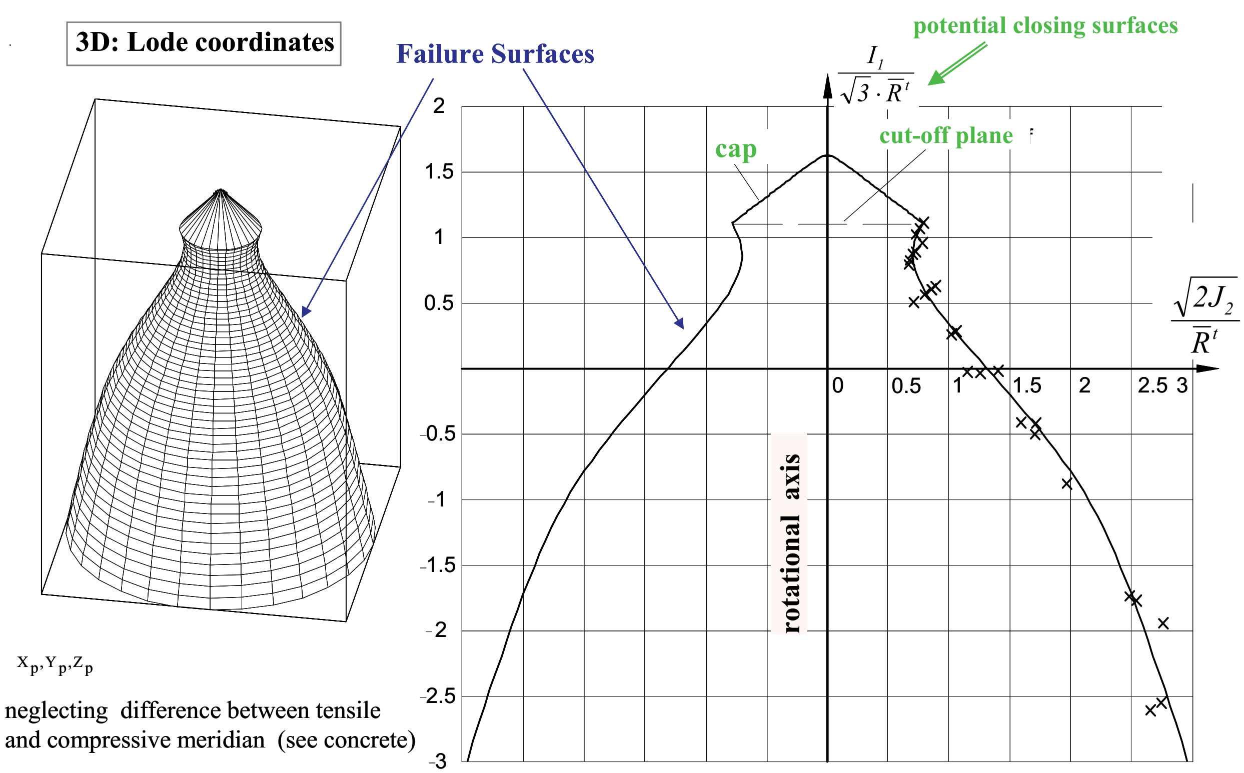


Figure 11b: 3D failure surfaces of grey cast iron

$$F_{\sigma}^{t} = \frac{\sqrt{I_{\sigma}} + I_1}{2\bar{R}_m^t} = Eff_{\sigma}^{t} = 1$$

deformation poor
hyperbola

shape + volume change + friction: Mohr-Coulomb :

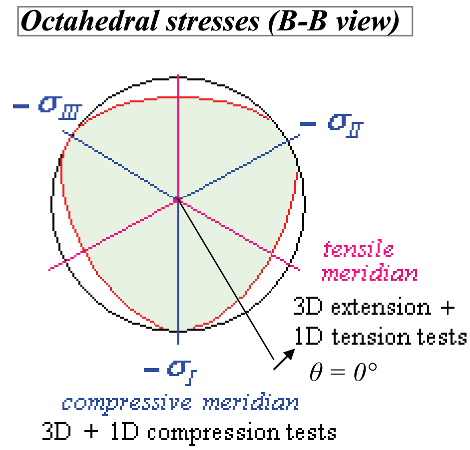
$$F_{\tau}^{c} = a_{\tau}^{c}\frac{3J_2 \cdot \Theta}{\bar{R}_m^{c\,2}} + b_{\tau}^{c}\frac{I_1^{\,2}}{\bar{R}_m^{c\,2}} + c_{\tau}^{c}\frac{I_1}{\bar{R}_m^{c}} = 1$$

(closed failure surface)
paraboloide

Basically, the differences in the octahedral (deviatoric) plane can be described by :

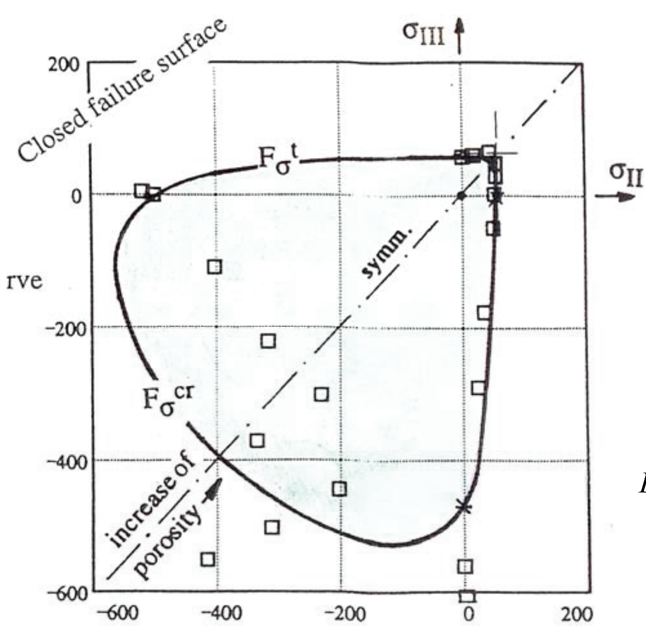
$$\Theta \Rightarrow \sqrt[3]{1 - d \cdot sin(3\theta)}, \quad d \le 0.5, \ convex$$

$$sin(3\theta) = 3\sqrt{3}J_3/(2J_2^{\,3/2}), \quad \text{[de Boer, et al]}$$

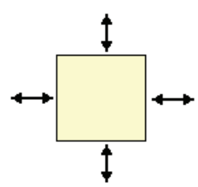


Isotropic materials possess 120° symmetry !

Figure 12: 3D and 2D failure surfaces/failure curves of concrete (brittle, microflaw-rich)



Principal stress plane



$$F_{\sigma}^{t} = \frac{\sqrt{I_{\sigma}} + I_1}{2 \cdot \bar{R}^t} = 1$$

$$F^{cr} = a^{cr}\frac{3J_2}{\bar{R}_m^{c\,2}} + \ + c^{cr}\left(\frac{I_1}{\bar{R}_m^c}\right)^2 = Eff^{cr} = 1$$

shear **volume**

$c^{cr} = a^{cr} - 1$ *[Kowalchuk]*

Figure 13: 2D failure curves of monolithic ceramics (brittle, porous, microflaw-rich; Kowaltschuk and Giginjak (1983)

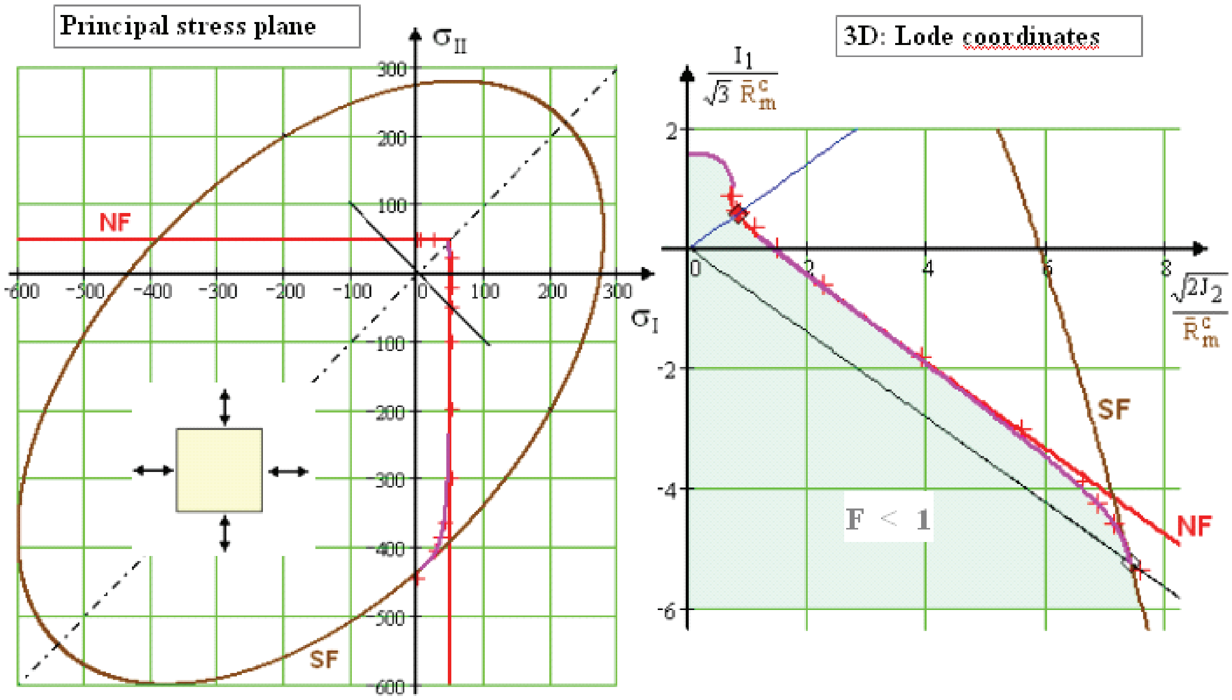


Figure 14: 3D and 2D failure surfaces/failure curves of glass C 90-1 (brittle, dense; Kowaltschuk and Giginjak (1983))

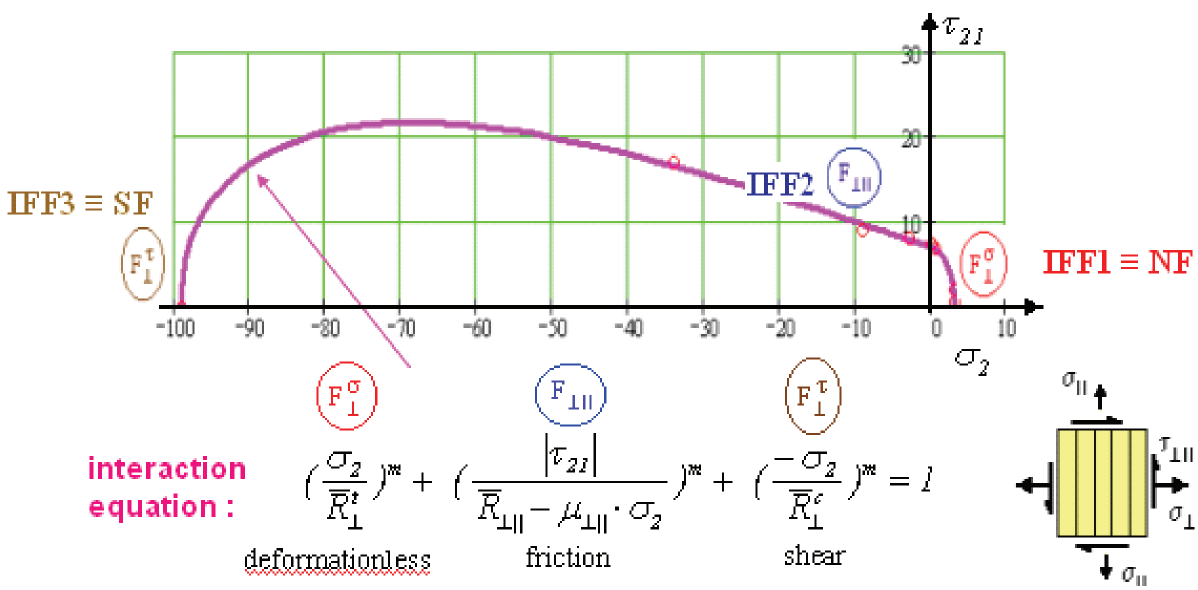


Figure 15: In-plane shear-transversal normal stress failure curve of a UD-based C/C, [Thielicke (1997)] $\{\overline{R}\} = (\overline{R}^t_{||}, \overline{R}^c_{||}, \overline{R}^t_{\perp}, \overline{R}^c_{\perp}, \overline{R}_{\perp||}) = (-,-,3,99,7)^T, m = 2.3, \mu_{\perp||} = 0.3.$

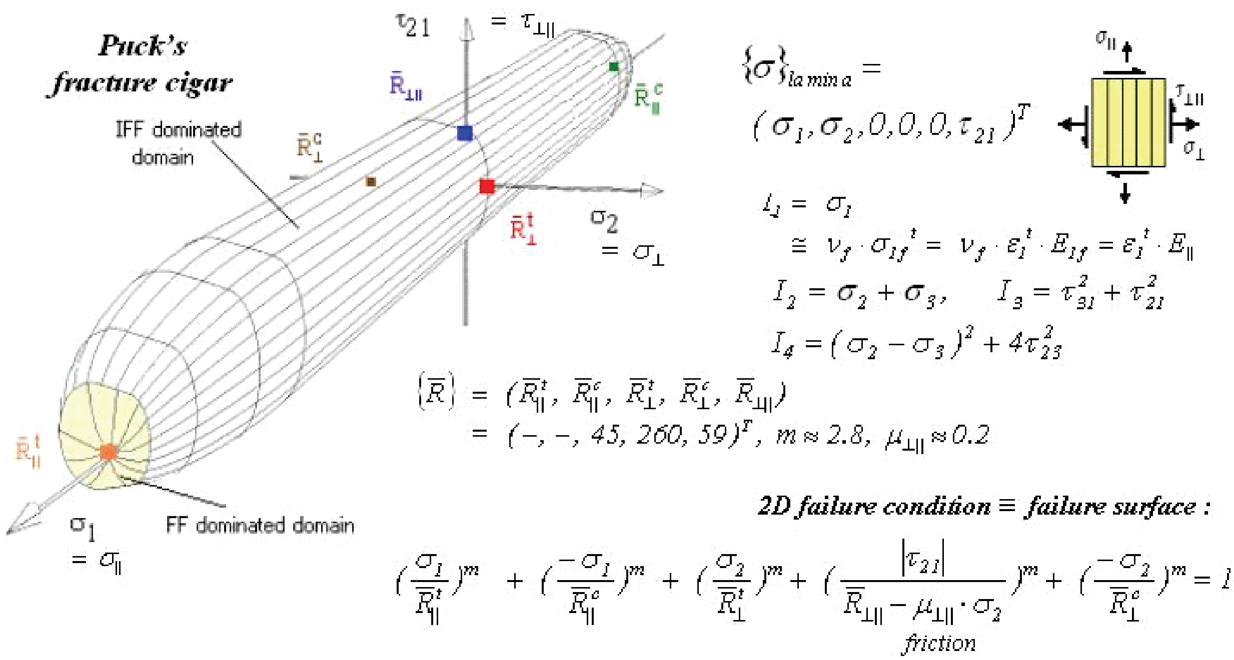


Figure 16: 2D failure surface of FRP UD lamina (brittle, dense, microflaw-rich, VDI 2014)

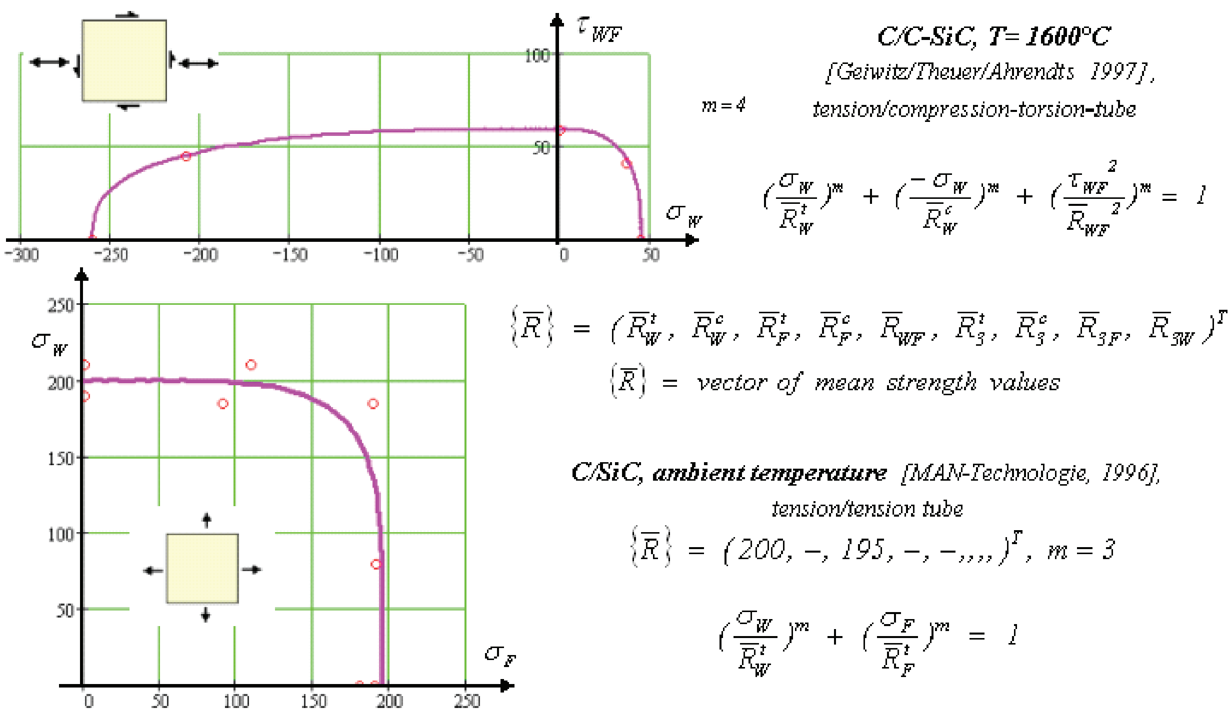


Figure 17: 2D failure curve of C/SIC (brittle, porous fabric, [Geiwitz et al (1997) and Cuntze (1998)])

on the various failure conditions and skipping the more sophisticated parts in them a *Common Basis is* obvious: It's more or less *Beltrami's hypothesis + the consideration of friction all the conditions are based on!*

FMC validation:

* From Failure Mode Concept applications can be concluded the FMC is an efficient concept, that improves and simplifies design verification. It is simply applicable to brittle/ductile, dense/porous, isotropic/anisotropic materials, if clear failure modes can be identified, and if the homogenized material element experiences a volume change or a shape change or material internal friction.

* It delivers a global formulation of 'individually' combined independent failure modes, without the well-known short-comings of global failure conditions which mathematically combine independent failure modes.

Design hints and remarks:

* Even in smooth stress regions a strength condition can be only a necessary condition which may be not sufficient for the prediction of 'onset of fracture', i.e. for the in-situ lateral strength in an embedded lamina see Flaggs and Kural (1982), Leguillon (2002).

* When applying test data from (*isolated lamina*) tensile coupons to an *embedded* lamina in a laminate, one has to consider that tensile coupon tests deliver test results of *weakest link type*. An embedded or even an only one-sided constraint lamina, however, possesses *redundant* behaviour,

* In case of discontinuities such as notches with steep stress decays only a *toughness + characteristic length-based energy balance condition* may form a sufficient fracture condition, [VDI 2014 (2006)]. Attempts to link 'onset of fracture/cracking' prediction methods for structural components are actually undergone, [Leguillon (2002)]

-More representative multi-axial test data should be available. They are necessary to really make a three-dimensional validation of the various failure conditions of the presently used structural materials possible, even of some standard ones.

Acknowledgement: The author thanks all persons who have discussed with him aspects of this non-funded research work and thereby contributed to an improvement of its contents.

References

Awaji, H. and Sato, S. (1978): A Statistical Theory for the Fracture of Brittle Solids under Multiaxial Stresses. *Intern. Journal of Fracture*, 14, R13-16.

Beltrami, E. (1885): Sulle Condizioni di Resistenza dei Corpi Elastici. *Rend. ist. d. sci. lett.,Cl. mat. nat.* 18, 705-714.

Boehler, J.P. (1985): Failure Criteria for Glass-Fiber Reinforced Composites under Confining Pressure. *J. Struct. Mechanics*, 13, 371

Boehler, J.P. (1995): *Personal note to the author on Fabric Invariants*.

Christensen, R.M. (1997): Stress based Yield/ Failure Criteria for Fiber Composites. *Int. J. Solids Structures*, 34, no. 5, 529-543.

Christensen, R.M. (1998): The Numbers of Elastic Properties and Failure Parameters for Fiber Composites. *Transactions of the ASME*, Vol. 120, 110-113.

Cuntze, R.G. (1998): Strength Prediction for Multi-axially Loaded CMC-Materials. *3rd European Workshop on thermal Protection Systems*. ESA-ESTEC: Noordwijk, WP P141.

Cuntze, R.G. (1999): *The Failure Mode Concept - A new comprehensive 3D-strength Analysis Concept for Any Brittle and Ductile behaving Material*. Europ. Conf. on Spacecraft Structures, Materials and Mechan. Testing. ESA-CNES-DGLR-DLR; Braunschweig, Nov. 1998, *ESA SP-428*, 269-287.

Cuntze, R.G. (2004): The Predictive Capability of Failure Mode Concept-based Strength Criteria for Multidirectional Laminates. Elsevier, Part B., *Composites Science and Techn.*, 64, 487-516 (WWFE-I).

Cuntze, R.G. (2006): Efficient 3D and 3D Failure Conditions for UD Laminae and their Application within the Verification of the Laminate Design. Elsevier, *Composites Science and Technol-*

ogy , 66, 1081-1096.

Cuntze, R.G. (2007): Prediction of Static and In-plane Fracture Failure of UD Laminae composed of Fibre-reinforced Plastics. 4th *NAFEMS Nordic Seminar on Material's Modelling. Sandvika*, March 2007, Conference CD.

Cuntze, R.G., Deska, R., Szelinski, B., Jeltsch-Fricker, R., Meckbach, S., Huybrechts, D., Kopp, J., Kroll, L., Gollwitzer, S., and Rackwitz, R. (1997): *Neue Bruchkriterien und Festigkeitsnachweise für unidirektionalen Faserkunststoffverbund unter mehrachsiger Beanspruchung –Modellbildung und Experimente–.* VDI-Fortschrittbericht, Reihe 5, Nr. 506, 250 pages. (*New fracture criteria (Puck's criteria) and Strength 'Proof of Design' for Uni-directional FRPs subjected to Multiaxial States of Stress –Model development and experiments–.* In German).

Cuntze, R.G. and Freund, A. (2004): The Predictive Capability of Failure Mode Concept-based Strength Criteria for Multidirectional Laminates. Elsevier, Part A. *Composites Science and Technology*, 64, 343-377 (WWFE-I).

Cuntze, R.. and Memhard, D. (2002): Evaluation of the Tension Rod Test for Ductile Material Behaviour -Key item for the establishment of a procedure to assess strain-controlled 'hot spots'. *European Conf. on Spacecraft Structures, Materials and Mechanical Testing*, ESA-CNES-DGLR-DLR, Toulouse.

de Boer, R. and Dresenkamp, H.T. (1989): Constitutive Equations for concrete in failure state. *J. Eng. Mech*, 115 (8), 1591-1608.

Flaggs, D.L. and Kural, M.H. (1982): Experimental Determination of the In Situ Transverse Lamina Strength in Graphite Epoxy Laminates. *J. Comp. Mat.*, Vol. 16, S. 103-116.

Geiwitz, W., Theuer, A. und Ahrendts, F.J. (1997): *Experimentelle Bestimmung eines Versagenskriteriums für faserverstärkte Keramik.* Konferenz, Verbundwerkstoffe und Werkstoffverbunde. Kaiserslautern, Konferenzhandbuch.

Gurson, A.L. (1977): Continuum Theory of Ductile Rupture by Void Nucleation and Growth. Part 1:Yield criteria and flow rules for porous ductile media. *J.Eng. Mater. Techn.*, 99, 2-15.

Hart-Smith, L.J. (1993): An Inherent Fallacy in Composite Interaction Failure Curves. *Designers Corner, Composites*, 24, 523-524

Hashin, Z. (1980): Failure Criteria for UD Fiber Composites. *J. of Appl. Mech.*, 47, 329-334.

Hinton, M. J., Kaddour, A. S. and Soden, P. D. (2002): A Comparison of the Predictive Capabilities of Current Failure Theories for Composite Laminated, Judged against Experimental Evidence. *Composite Science and Technology*, 62, 1725-1797; (WWFE-I).

Hinton, M.J., Soden P.D., Kaddour A.S. (2004): Failure criteria in fibre reinforced polymer composites: The World-Wide Failure Exercise. Elsevier (ISBN: 0-08-044475-X), 700 pages; (WWFE-I).

Jeltsch-Fricker, R. (1996): Bruchbedingungen vom Mohrschen Typ für transversal-isotrope Werkstoffe am Beispiel der Faser-Kunststoff-Verbunde. ZAMM 76, 505-520.

Kaddour, A.S. and Hinton, M.J.: *Instruction to Contributors of the Second World-Wide Failure Exercise (*WWFE-II), Part A.

Kowaltschuk, B.I. and Giginjak, F.F. (1983): in Russian, Kiew, Naikowa Dumka.

Leguillon, D. (2002): Strength or Toughness? –A criterion for crack onset at a notch. *Europ. J. of Mechanics A/Solids*, 21, 61 – 72.

Masters, J. (1994): Fractography of Modern Engineering Materials. *Composites and Metals*. 2nd volume. ASTM STP1203.

Meckbach, S. (1998): Invariants of Cloth-reinforced Fibre Reinforced Plastics. *Kasseler Schriften zur angewandten Mathematik*, Nr. 1/1998 (in German).

Mohr, O. (1900): Welche Umstände bedingen die Elastizitätsgrenze und den Bruch eines Materials? *Civilingenieur*, XXXXIV, 1524-1530, 1572-1577.

Paul, B. (1961): A modification of the Coulomb-Mohr Theory of Fracture. *Journal of Appl. Mechanics*, pp.259-268.

Puck, A. and Schürmann, H. (2002): Failure Analysis of FRP Laminates by Means of Physically based Phenomenological Models. *Composites Science and Technology*, 62, 1633-1662.

Rackwitz, R. and Cuntze, R.G. (1987): System Reliability Aspects in Composite Structures. *Eng.' Opt.*, Vol. 11, pp. 69-76.

Thielicke, B. (1997): *Die Ermittlung der interlaminaren Scherfestigkeit von kohlenstofffaserverstärkten Kohlenstoffen mit dem Druckscherversuch im Temperaturbereich zwischen Raumtemperatur und 2000˚C*. Dissertation, Uni Karlsruhe.

Tsai, S.W. and Wu, E.M. (1971): A General Theory of Strength for An-isotropic Materials. *Journal Comp. Materials*, 5, 58-80.

VDI 2014 (2006): German Guideline, Sheet 3 *'Development of Fibre-Reinforced-Plastics components, - Analysis'*.(German/English issue, Beuth Verlag, (author was convenor).

Thank you !

R. Cuntze

ANNEX

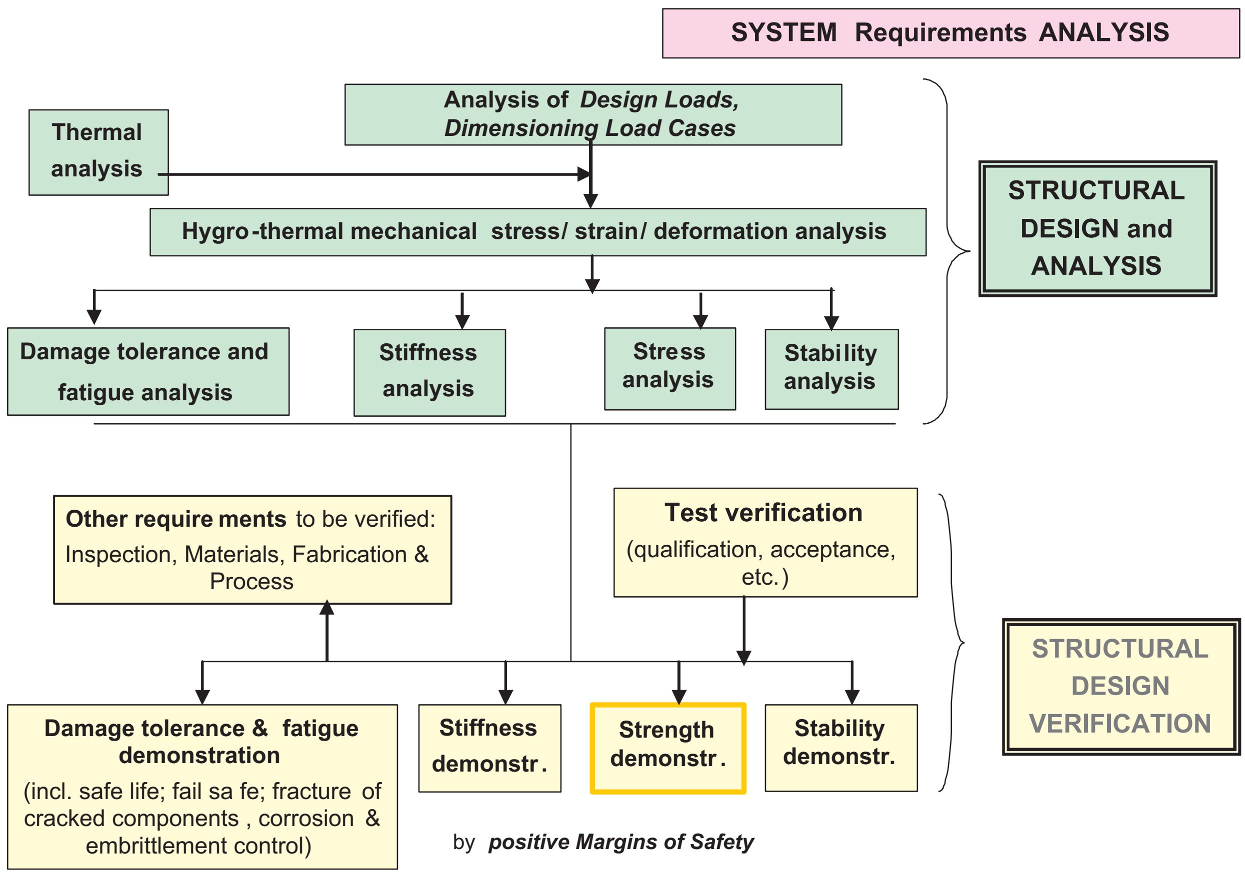


Figure A1: Flow diagram *Design Analyses* and *Design Verifications*