COMPOSITES UNITED

Strength capacity of bi-axially pressed UD strands at turning points
of rotor blade loops and of hangers of network arch bridges.

Festigkeit bi-axial gedrückter UD-Stränge an Umlenkstellen von Rotorblattschlaufen und Hängern von Netzwerkbrücken.

- Derived on basis of Material Symmetry Facts and Cuntze's Failure Mode Concept FMC –

For aircraft engineering and for civil engineering

- Introduction with some Examples (intentionally from building industry)
- Design Idea of the UD Tape Strand Hangers
- Cuntze's Failure-Mode-Concept, Application to UD materials
- Design Verification focussing its Notions: Reserve Factor *RF*, *Eff (Werkstoffanstrengung)*, Failure Index $|F|$
- Discussion of Multi-axial Compressive Stress States in the Loops, *Eff* ↔ Strength Capacity
- Conclusions
- *Novel UD-model to Map the Influence of Porosity in the associated σ_2-σ_3-plane (formula, figure)*

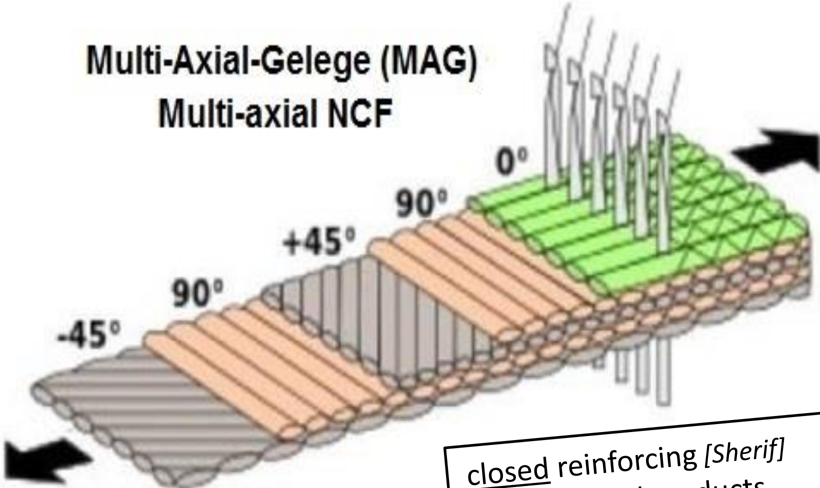
Numerical details of many slides serve the later reader for understanding but will be not presented!

Non-funded investigation.

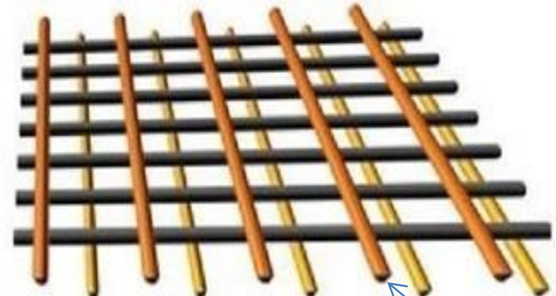
Prof. Dr.-Ing. habil. Ralf Cuntze VDI, retired from industry, engineer and hobby material modeler
Markt Indersdorf, Ralf_Cuntze@t-online.de, 0049 8136 7754

Fiber-reinforcing Products used in Engineering

Multi-Axial-Gelege (MAG)
Multi-axial NCF

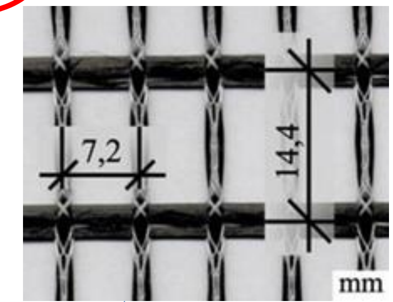


pultrusion
Stab-Gitter
bar-grid



strands
heavy tow

textile
fiber grid mats



stiff when placing,
plate armouring

Q-mat and R-mat
deliverable

closed reinforcing *[Sherif]*
semi-finished products

polymer
matrix

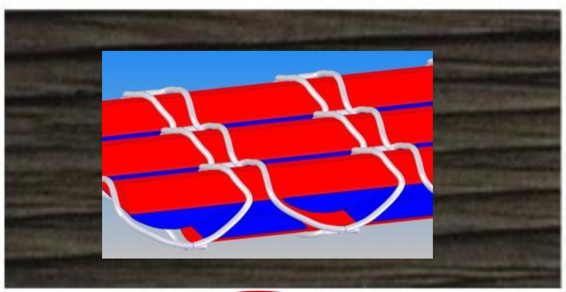
concrete
matrix

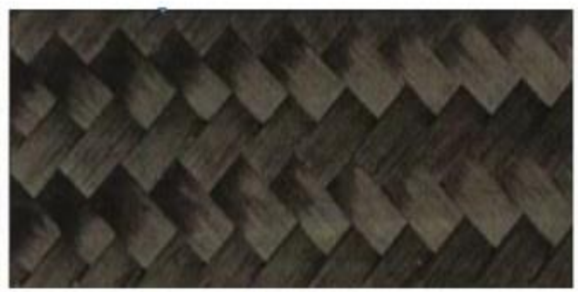
open reinforcing *[Sherif]*
semi-finished products

UD-Gelege
Non-Crimp Fabrics (NCF)

Gewebe
woven fabrics
Warp= Kette, Fill (Weft) = Schuss

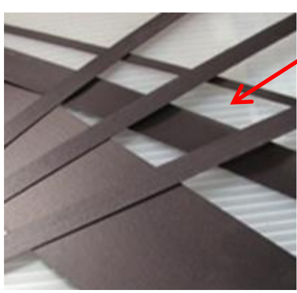
pre-preg
roll





flexible when placing,
beam reinforcement

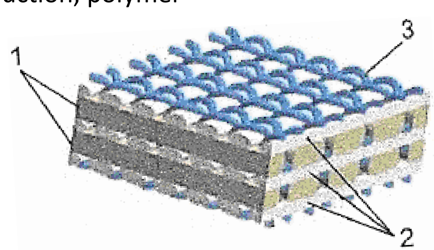
shotcrete



Tape, UD-lamina,
Lamella

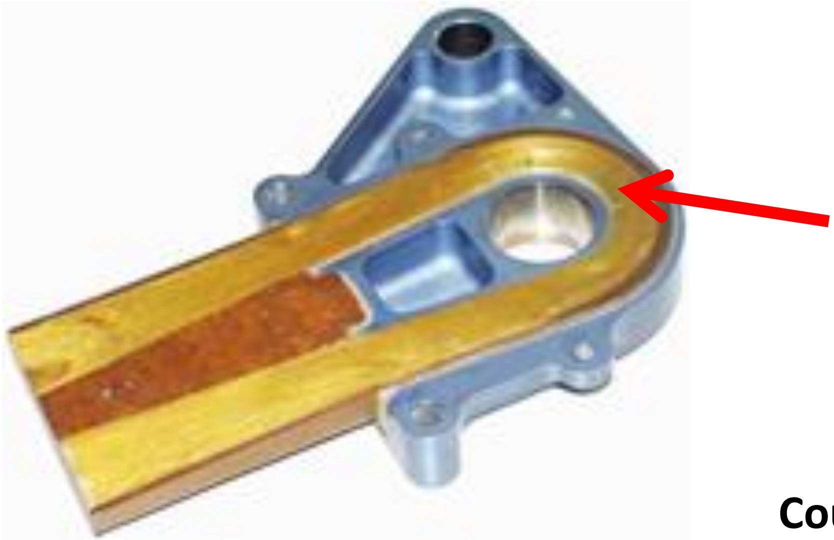
Lamelle ≡ *Gelege-Streifen,*
schmaler Gelege-Streifen ≡ strip, tape
breites Gelege-Stück ≡ sheet
Gelege = extremes Gitter, kein Faserabstand)

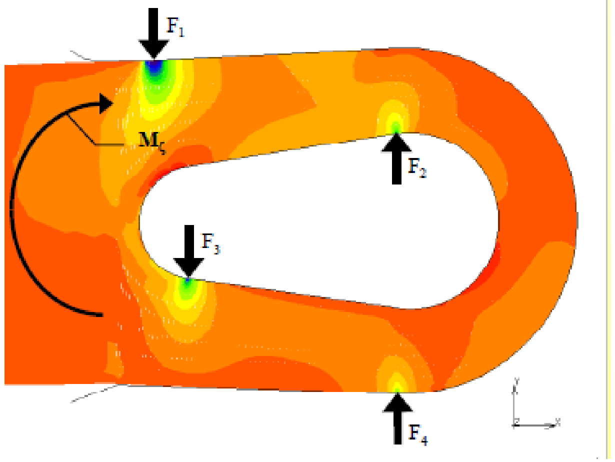
not in construction, polymer
matrix



(a lamella may consist of one UD-layer or from an angle-ply layer in case of shear reinforcement)

Various Rotor Blades





Courtesy: Eurocopter, Hauptrotorblätter
Rupert Pfaller

SAB 105mm CFK Heckrotorblätter

CFK-Zugglieder aus Strangschlaufen

seit 2003
CFK-Strangschlaufen für Krane und Baumaschinen

EMPA, Prof. Urs Meier

Special Automated Fabrication Process (turning points dificulty)

**Roving-
Placing Machine**

Special automated additive manufacturing process with endless fiber strands stored on the fixed 'truss' nodes. Lightweight 'fiber pavilion' made of 60 CFRP/GRP components

[***Knippers, Koslowski, ITKE Stuttgart; Menges, CD***]

BUGA-Heilbronn

Art-work and Load-carrying structure
(Kunstwerk und Tragwerk)

Montage des 1,4 t leichten CFK-Brückensegments von BaltiCo (*© BaltiCo*)
Dr. Dirk Büchler

Sassnitz auf Rügen besitzt nun eine 25 m lange Brücke, hergestellt mittels CFK-Strang-Ablegetechnologie der BaltiCo GmbH.

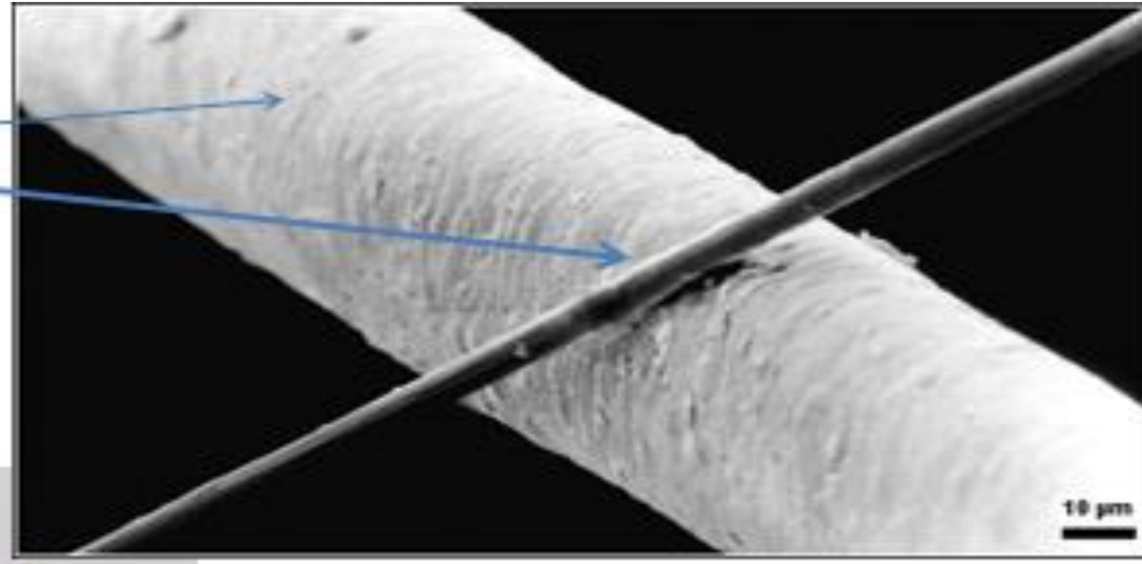
Im Juli 2020 wurde eine Fahrrad- und Fußgängerbrücke aus kohlenstofffaserverstärktem Kunststoff (CFK) der Firma BaltiCo in Sassnitz montiert, was aufgrund der gerade einmal 1,4 Tonnen Gewicht eines einzelnen Segments leichter als üblich gelang und innerhalb eines einzigen Tages erfolgte.

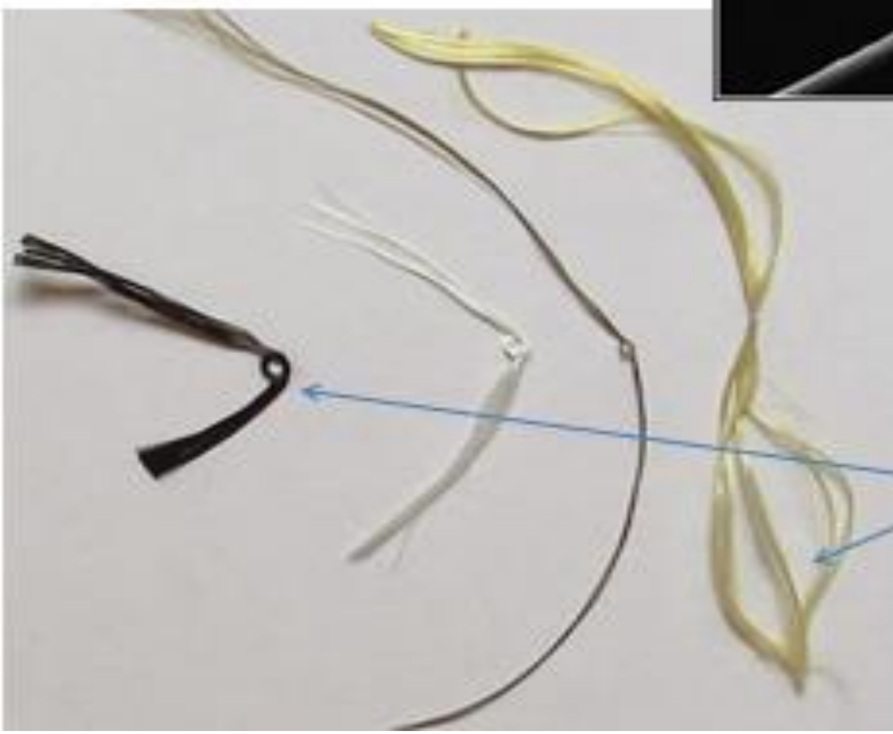
Technische Potentiale

- Materialeinsatz senken
- Carbon Footprint senken
- Nachhaltiger Bauen
- Filigraner Bauen
- Neue Designfreiheiten nutzen
- Lebensdauer erhöhen
- Funktionen integrieren
- Kosten reduzieren

Dr. Dirk Büchler, BaltiCo,
CU Bau Vorstandsmitglied

menschliches Haar
und Carbonfaser

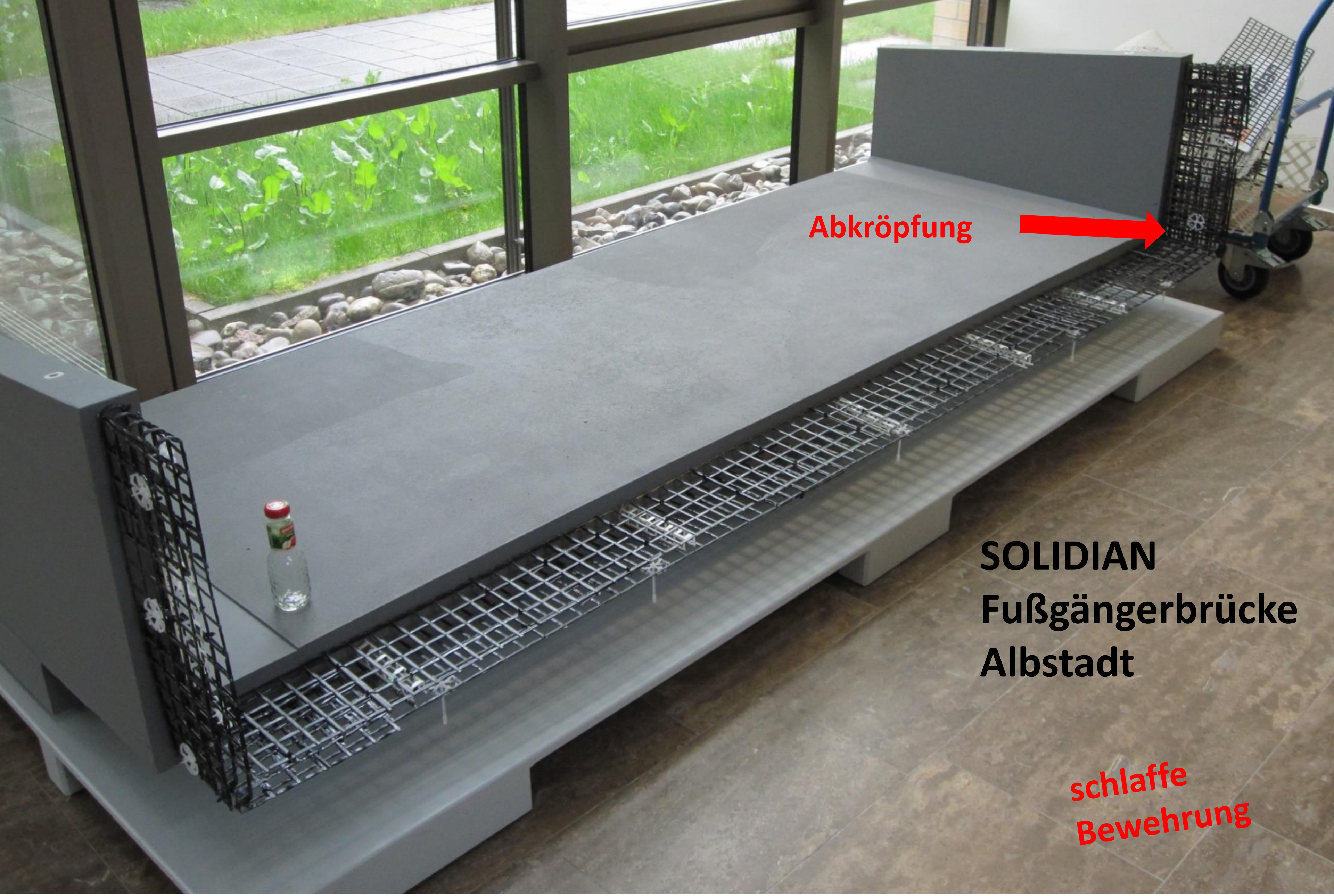




Fasern:
Carbon, Glas, Basalt,
Aramid

PROBLEMS:

(left, down) knot-test to visualize brittleness grade and fiber bending radius limits of fibers,
(right) a comparison of a carbon-fiber (Ø 7 µm) versus a hair (necessary for radius of armoring cages



Wäre zu dimensionieren nach: D96 DAfStb UA Nichtmetallische $_{nm}$ Bewehrung:
DAfStb-Richtlinie "Betonbauteile mit nichtmetallischer Bewehrung"

CFK-Hänger der Stuttgarter Stadtbahn-Brücke

The Stuttgart Stadtbahn bridge, installed over the A8 motorway in May 2020 in Germany, is the world's first network arch bridge that hangs entirely on tension elements made of carbon fiber-reinforced plastic (CFRP).
The 72 hangers are produced with Teijin (Wuppertal, Germany) carbon fiber, Tenax, by Carbo-Link AG (Fehraltorf, Switzerland).

[Dr. Winistörfer, Carbo-Link,
Prof. Urs Meier, EMPA]

Design challenge: Fiber Stress Concentration around the Loop

1. High secondary stresses (stress concentrations) occur around the bolt area when loading laminated strand loops. The more rigid the fibre and the “thicker” the loop is, the higher the secondary stresses are that will be generated.

Non-laminated strand loops solve this problem. Such a strand loop is made of a very thin UD tape of only 0.12 mm thickness. This tape is continuously laid in the desired number of layers around corresponding deflection bodies (bolts, pins) by an ‘endless tape lay-up device’.. There is no full bond between the individual layers, just friction bond. The last layer is laminated with the penultimate layer on a length of about 10 cm and anchored with it. *[Winistörfer, A.; Mottram, T.: Finite Element Analysis of Non-Laminated Composite Pin-Loaded Straps for Civil Engineering, Journal of Composite Materials, Vol. 35, 2001. Carbo-Link and EMPA Dübendorf, Prof. Urs Meier*].

When loading the ‘**friction bond**’ tape strand loop, a relatively even stress distribution is achieved due to relative shifts possible between the adjacent tape layers. The load transmission between the layers is carried out by friction. Better exploitation of the fiber strength over thickness!

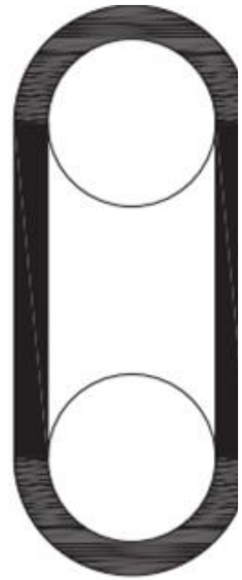
2. Lateral supports increase the load-bearing capacity of strand loops made of carbon fibre. This indicates that in dimensioning of loops it is necessary to apply a 3D-strength analysis (VDI 2014 Part 3, editor: R. Cuntze).

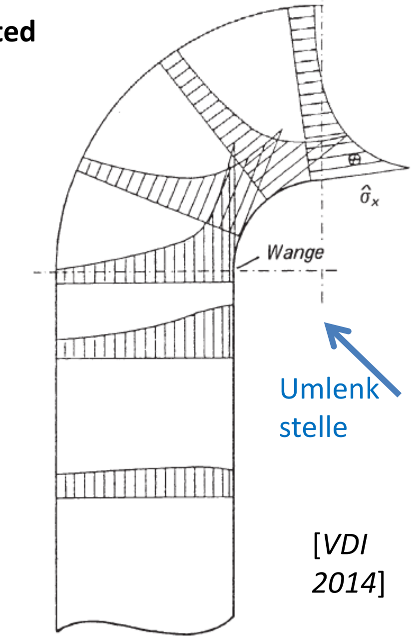
3. Full bond solutions suffer from tribologic wear (‘fretting’) over operational time which impacts the fatigue life. Protection is to foresee (Teflon, not topic here).

Production experience: **full bond**

When putting layer upon layer the ‘winding tension force’ will compress the former layer and reduce the efficiency of the former layer. This situation is improved by an optimal choice of the ‘winding tension force’ in the process together with intermediate consolidation.

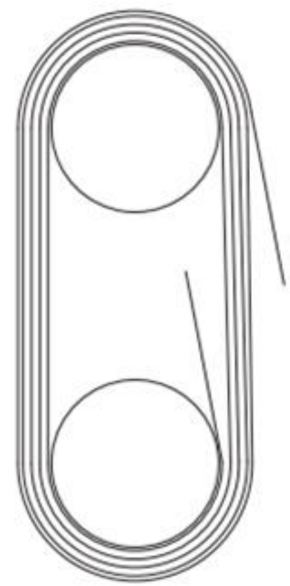
full bond, laminated





[*VDI 2014*]

$$\sigma_x = \sigma_{||}$$

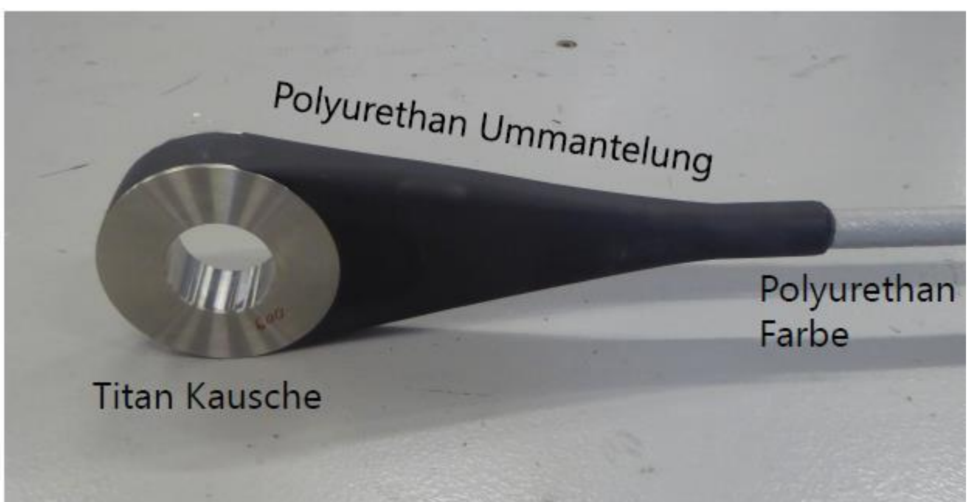


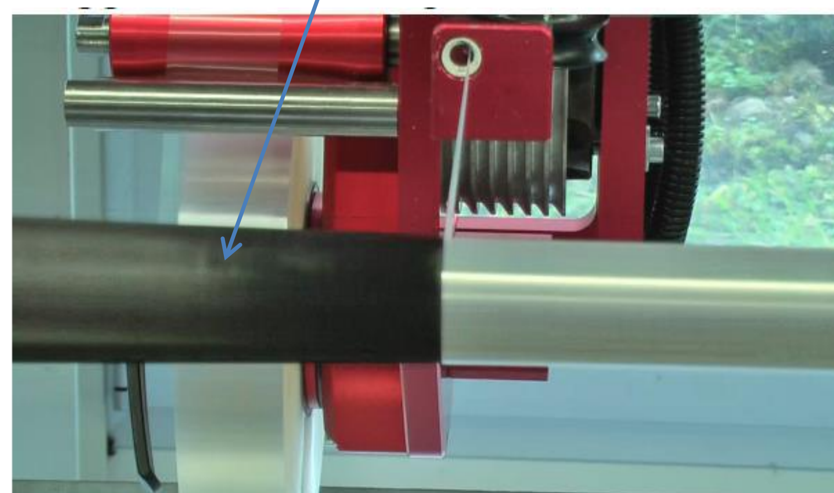
friction bond, not commonly cured
biegeschlaffe Konstruktion der Strangschlaufe
[*Carbo-Link*]

Fabrication of CFRP-Hangers *(pin-loaded tendons):* **Tensile rods with Tape Loop ends**

Lay-Up: Prepreg Bänder
Tenax®-J IMS60 E13 24K /EP

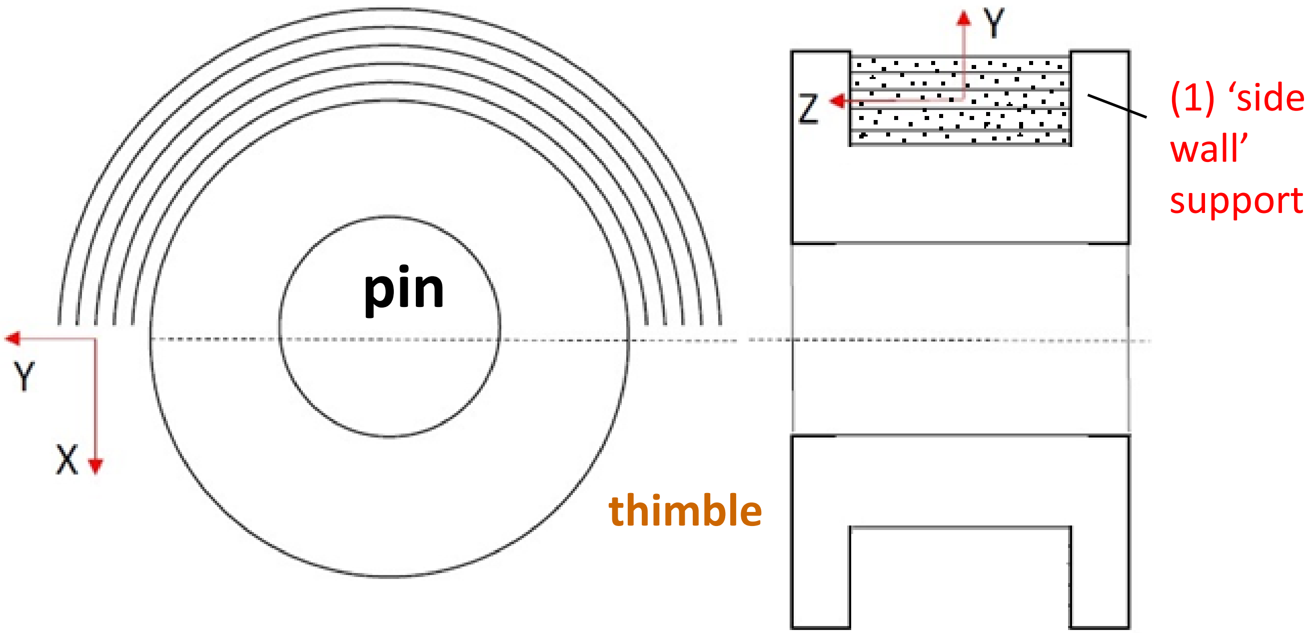
- **Built up by a narrow 'endless tape lay-up device'.**
- **Round tensile rod region is generated by wrapping a polyester yarn around.**
- **Dyneema sliding layer between thimble and pin.**
- **UD-strand is constrained in lateral direction in the thimble by the side supports.**

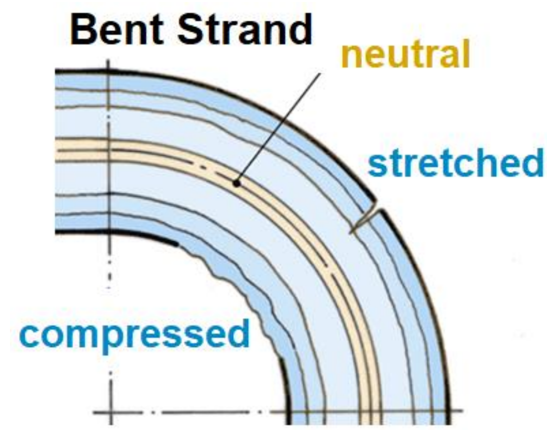




Bending of Filaments and of UD Strands and Tape Loops

friction-bond tape laying



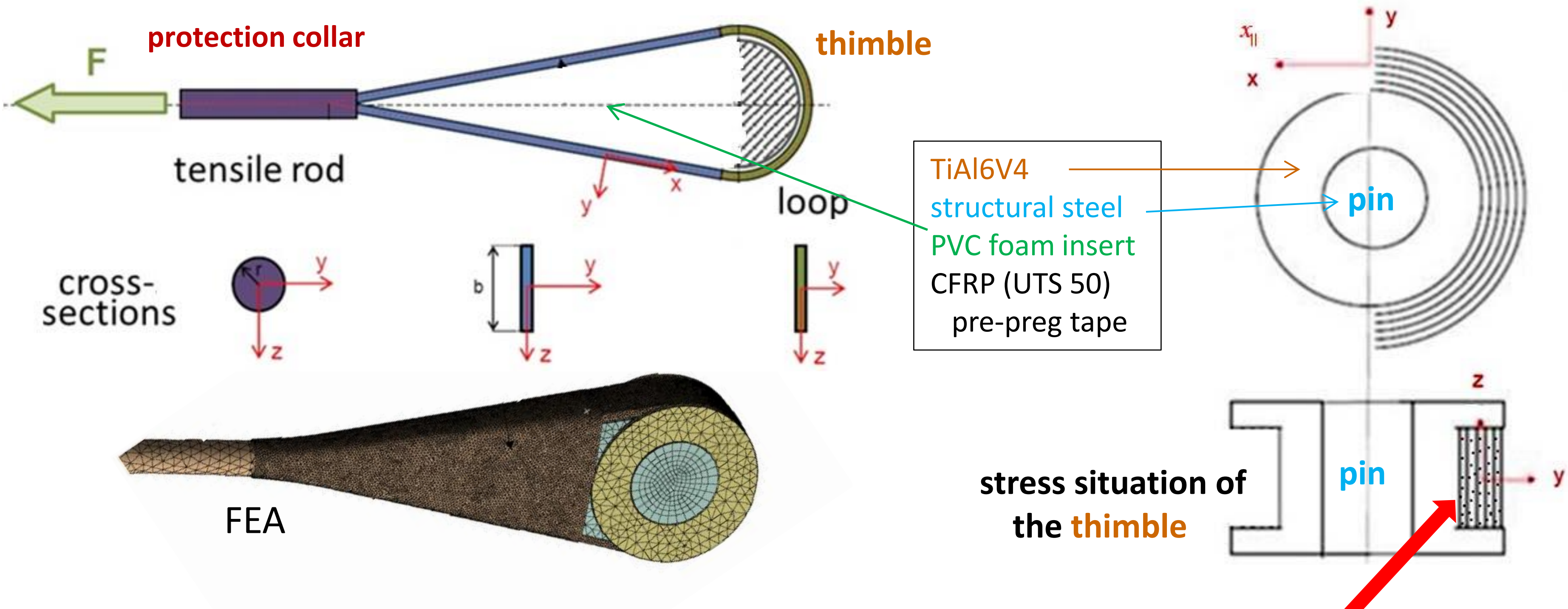


bent pre-preg tape.

(2)Crooked fibers do not carry load !

The turning locations are the challenging sites →

Structural Modelling, geometry, loading *[propriety sbp consultants, Stuttgart]*



$$\sigma_2^c \leftarrow \sigma_z^c, \;\; \sigma_2^c \leftarrow \sigma_y^c$$

Dimensioning Load Cases:
Axial loading, Pin rotation, Hygro-thermal effect, Friction.

The stresses in a loop can be roughly calculated if they are taken as the same as those in a thick-walled tube. This will also make it immediately clear that, in addition to the tangential stresses, radial stresses also dominate, being at their maximum at the inner edge , precisely, where the tangential stresses are also the largest!
These can be easily calculated from the transferred force *F* and the geometrical data for

Tenax® CARBON FIBERS

Definition	Grade		Filament Count	Tensile Strength MPa	Tensile Modulus GPa	Tensile Elongation %	Diameter µm	Density g/cm³	Electrical Resistivity ohm. cm
High Tensile	UTS50	12K	12000	4900	240	2	6.9	1.8	1.6×10^{-3}
		24K	24000	5000	240	2.1	6.9	1.79	1.8×10^{-3}

sbp: schlaich, bergermann & partner

3D-Design is mandatory and a Stress Assessment Tool

In the development of UD structural components
the application of 3D-validated Strength-Failure-Conditions SFCs ('criteria') $F = 1$
is one essential pre-condition for achieving in design verification
the required fidelity for structural product certification and production.

What is to provide?
3D-Strength Failure Conditions SFC to verify that Onset-of-Fracture does not occur.

That menas for the creation of the SFCs :
Using a **'Global'** Failure Function formulation **(Drucker-Prager, Tsai)** with ***F* = 1** or
using a **'Modal'** Failure Function formulation **(Mises, Cuntze)** with
the so-called **Material Stressing Effort (Werkstoffanstrengung) *Eff* = 100%.**

Eff = 100% is better understood by the engineer
and is valid in non-linear analysis where
Eff remains 100% during degradation that follows 'Onset-of-Fracture' !

'Modal' versus **'Global'** SFCs:
Modal means that only a test data set of one failure mode domain is mapped whereas global (Drucker-Prager isotropic, Tsai UD) means that mapping is performed over several mode domains, shear fracture mode SF with normal fracture mode NF.
The bottle-neck of global SFCs respectively 'Single Failure Surface Descriptions ' is, that any change in one of the 'forcibly married' modes requires a new global mapping which changes the failure curve in the physically not met mode, too.
Modal SFCs like Mises an Cuntze's SFCs generate real equivalent stresses (see Annex)

Experience with Material-Symmetry → *a physically sound Basis for the Generation of SFCs*

1 If a material element can be homogenized to an ideal crystal (= frictionless), then material symmetry demands for the Isotropic Material are:

- 2 *elastic 'constants'*, 2 *strengths*, 2 *fracture toughness values*, 2 *'basic' invariants* I_1, J_2 and 2 *strength failure modes*, for *yielding* two (NY, SY) and for *fracture* two (NF, SF) (→ for isotropic materials may be recognized a 'generic number' of 2. One needs just 2 invariants for formulate SFCs. This is valid as long as a one-fold acting failure mode is to describe by the distinct SFC and not a multi-fold one, such as for $\sigma_{II}^t = \sigma_{III}^t$, $\sigma_{II}^c = \sigma_{III}^c$)
- 1 *so-called physical parameter (such as the coefficient of thermal expansion CTE, and the coefficient of moisture expansion CME, friction* μ *, etc.)*

and for Transversely-isotropic UD-material :

- a generic number of 5 is witnessed for strengths etc and 2 for physical parameters.

2. *A real solid material model is represented by a description of the ideal crystal (frictionless) + a description of its friction behavior.* → Mohr-Coulomb requires for the real crystal another *physical parameter the inherent material friction value* μ*, namely is* 1 *for isotropic materials and* 2 *for UD materials*

3 Fracture morphology gives finally evidence

Each strength corresponds to a distinct *strength failure mode* and to a distinct *strength fracture type*, to Normal Fracture (NF) or Shear Fracture (SF).

author → **Material Symmetry-dedicated Derivation of FMC-based 'Modal' SFCs** →

UD-SFCs for 'Onset of fracture failure', mode interaction, and value domains

FF1 $\quad Eff^{\|\sigma} = \breve{\sigma}_1 / \bar{R}^t_{\|} = \sigma^{\|\sigma}_{eq} / \bar{R}^t_{\|}\,,$ $\quad \breve{\sigma}_1^{\,*} \cong \varepsilon^t_1 \cdot E_{\|}$ filament strains from FEA

FF2 $\quad Eff^{\|\tau} = -\breve{\sigma}_1 / \bar{R}^c_{\|} = +\sigma^{\|\tau}_{eq} / \bar{R}^c_{\|}\,,$ $\quad \breve{\sigma}_1 \cong \varepsilon^c_1 \cdot E_{\|}$ 2 filament modes

IFF1 $\quad Eff^{\perp\sigma} = [(\sigma_2 + \sigma_3) + \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^{\;2}}\,] / 2\bar{R}^t_{\perp} = \sigma^{\perp\sigma}_{eq} / \bar{R}^t_{\perp}$ Puck's mode A

IFF2 $\quad Eff^{\perp\tau} = [(\frac{\mu_{\perp\perp}}{1-\mu_{\perp\perp}}) \cdot (\sigma_2 + \sigma_3) + \frac{1}{1-\mu_{\perp\perp}} \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^{\;2}}\,] / \bar{R}^c_{\perp} = +\sigma^{\perp\tau}_{eq} / \bar{R}^c_{\perp}$ 3 'matrix' modes

$a_{\perp\perp}$

IFF3 $\quad Eff^{\perp\|} = \{[2\mu_{\perp\|} \cdot I_{23-5} + (\sqrt{(2\mu_{\perp\|})^2 \cdot I_{23-5}^{\;2} + 4 \cdot \bar{R}_{\perp\|}^{\;2} \cdot (\tau_{31}^2 + \tau_{21}^2)^2}\,] / (2 \cdot \bar{R}_{\perp\|}^{\;3})\}^{0.5} = \sigma^{\perp\|}_{eq} / \bar{R}_{\perp\|}$

with $\quad I_{23-5} = 2\sigma_2 \cdot \tau_{21}^2 + 2\sigma_3 \cdot \tau_{31}^2 + 4\tau_{23}\tau_{31}\tau_{21}$ [Cun04, Cun11]

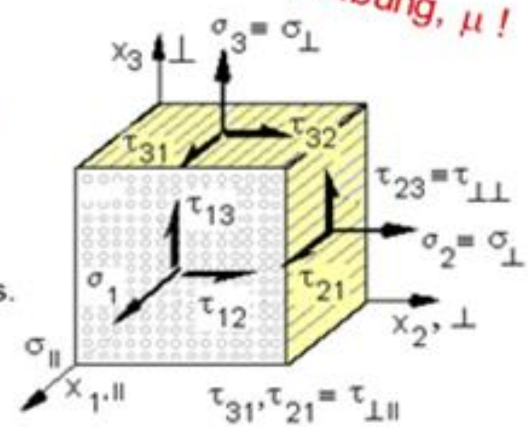
Modes-Interaction $\quad Eff^{m} = (Eff^{\|\tau})^m + (Eff^{\|\sigma})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\|})^m$

with influence IFF on FF : $\quad = 1 = 100\%$ is 'onset of failure'

Reibung, μ !

with mode-interaction exponent $\quad 2.5 < m < 3$ from mapping test data

Typical friction value data range: $\quad 0.05 < \mu_{\perp\|} < 0.3, \quad 0.05 < \mu_{\perp\perp} < 0.2$



Eff := material stressing effort (Werkstoffanstrengung), *R* := UD strength, σ_{eq} := equivalent stress.
Eff := artificial word, fixed with QinetiQ in 2011, to have an equivalent English term.
Poisson effect considered*: bi-axial compression strains a filament without any σ_1
t := tensile, c: = compression, || : = parallel to fibre, $\perp$:= transversal to fibre

With the SFCs above the author became winner of the World-Wide-Failure-Exercise-I and was top-placed in WWFE-II.
The derivation of the friction value from the original friction parameter $a_{\perp\perp}$ ($\mu_{\perp\perp}$) is presented on an attached slide.
The Effs are derived on basis of the 'Proportional Loading (stressing) Concept'.

‘Modal ‘ SFCs require the Interaction of the single Modes *

$$Eff \;=\; \sqrt[m]{(Eff^{\text{mode 1}})^m + (Eff^{\text{mode 2}})^m + \ldots} = 1 = 100\% \,, \qquad if \quad \text{Onset-of-Failure}$$

Eff = 100% represents the mathematical description of the surface of the (fracture) failure body

The interaction of adjacent failure modes is modelled with the ‘series failure system”.
That permits to formulate the total material stressing effort from all activated failure modes as ‘accumulation’ of *Eff*s or *in other words* as the sum of all the *failure danger proportions*.
m is mode interaction exponent.

Similarity of problem : ***commonly activated failure modes increase failure probability***

* In stability, the interaction of modes is ‘managed’ by design: *Placing all other modes far away of the global mode !*

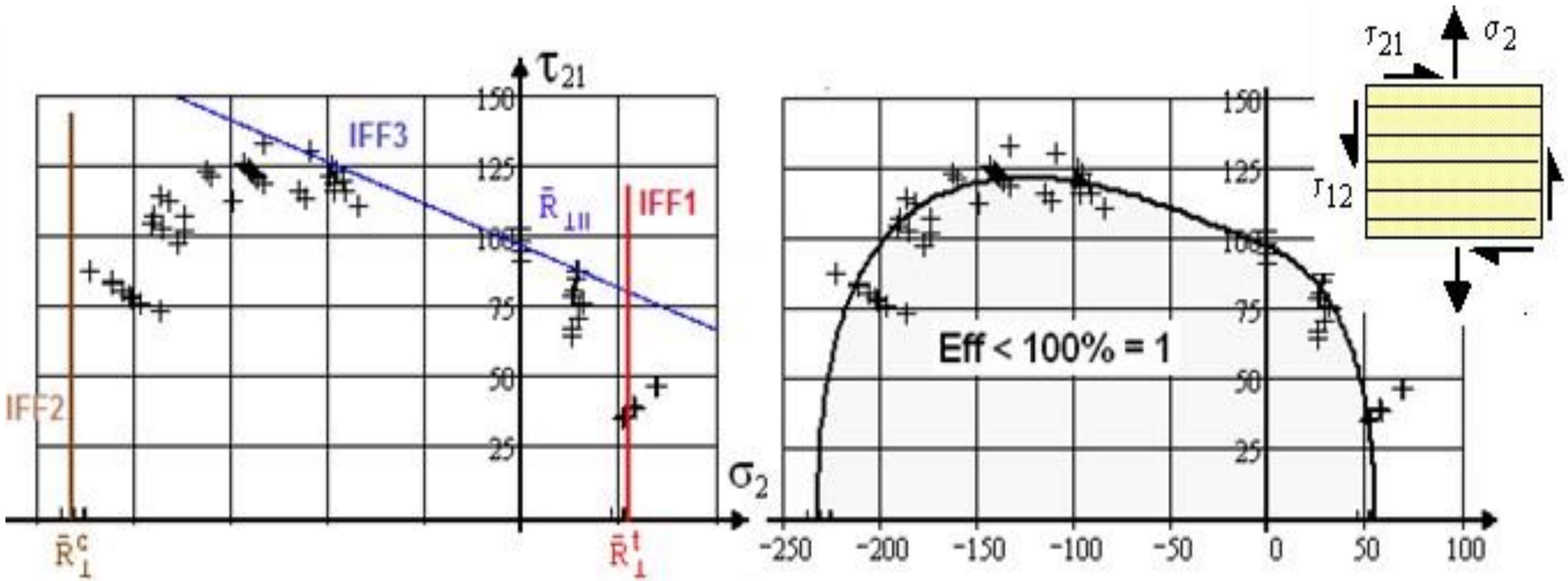
* In strength, all is more coupled, however, *the size of the equivalent stress of a UD mode can be used as design driver*

* In bonding, separation of the design critical adhesive failure from cohesive substrate failure.

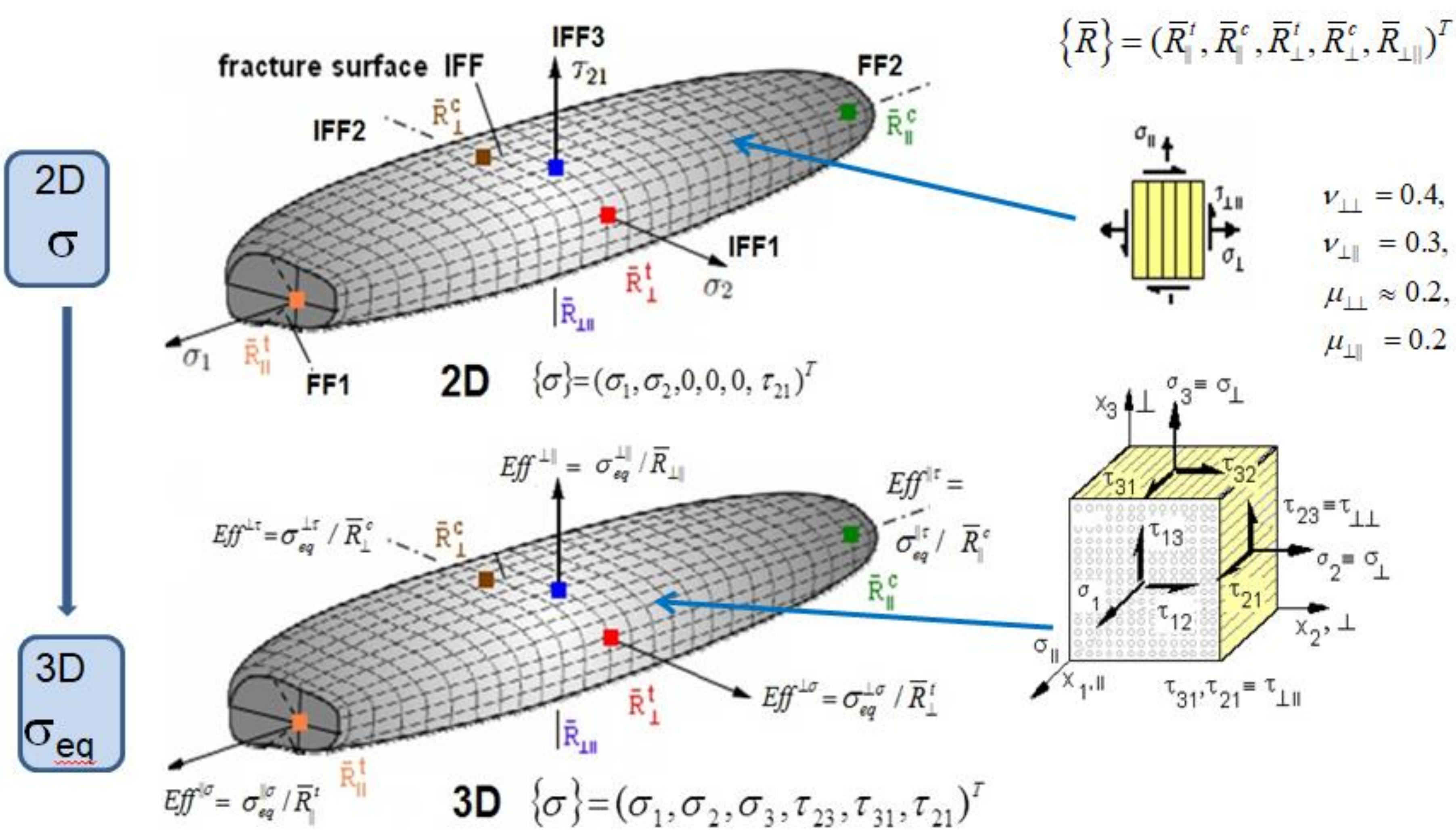
From mapping experience in the transition zone of modes:
The interaction exponent *m* for fracture failing brittle materials ($R^c/R^t > \approx 3$) the value is about $m = 2.6$.
A smaller *m* is conservative, on the ‘safe side’.

2D-Interaction, demonstrated in the IFF cross section, CFRP failure curve

2D stress state within the lamina, m=2.7



Fracture body of the UD material (lamina, lamella, sheet, tape) for 2D and 3D stress states



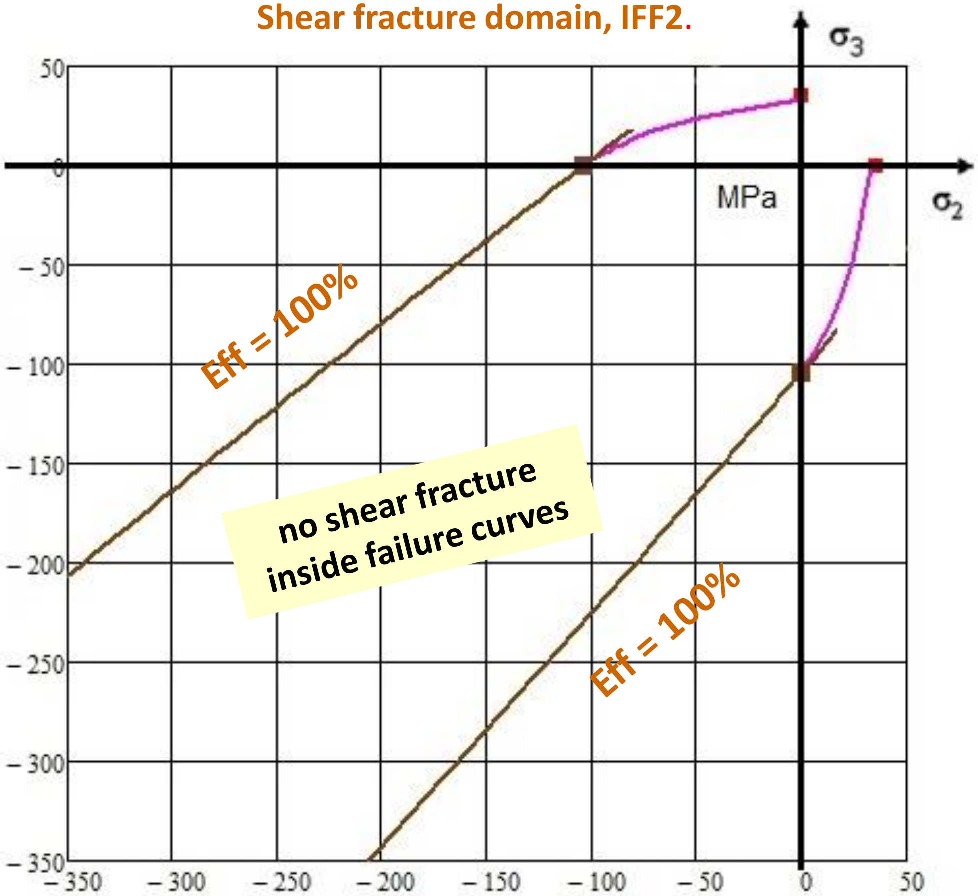
The use of the entity Eff excellently supports **'understanding the multi-axial strength capacity of materials'.**

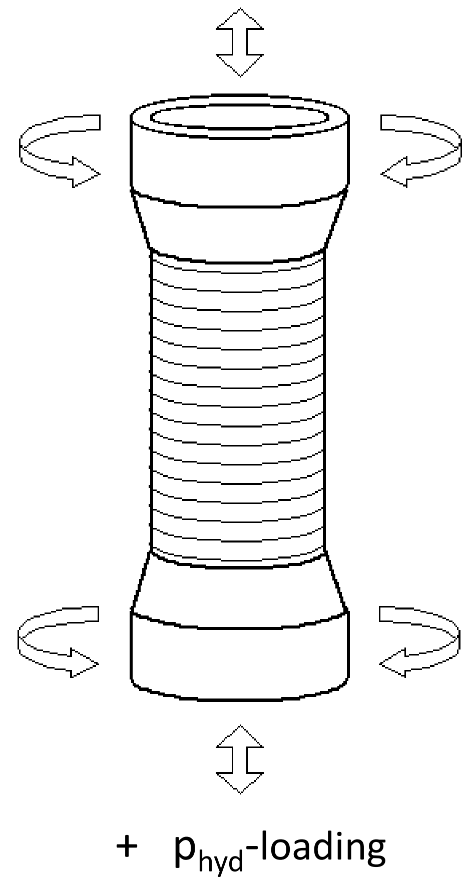
Independently of the multi-axial stress state: Eff (Werkstoffanstrengung) can never exceed 100%!

The measured UD fracture stress curves in the quasi-isotropic σ2-σ3-plane. Scheme of the 90°-wound tension/compression-torsion test specimen

Quasi-isotropic plane, where the 3D-stress state in the **turning point** takes place.

Shear fracture domain, IFF2.





Mind, however: Due to the Poisson effect, bi-axial compression leads to a tensile straining which usually is easily captured by an additional amount of tensile stress in the axially oriented fibers.

Demonstration of Static Strength in Design Dimensioning → Design Verification

The Design Strength of a structural part is demonstrated if

- no relevant strength failure (= limit state $\mathcal{G} = F - 1 = 0$ of a failure mode) is met
- all dimensioning load cases consider

  Each distinct load case with its various Failure Modes

  by a positive Margin of Safety $MoS > 0$ or a Reserve Factor $RF > 1$.

RF := 'Load View' of the Design Verifier

A further increase of the loading is allowed

if $RF > 1$ and similar if $Eff < 100\% = 1$.

Reserve Factor (is load-defined) : as ***RF = failure load / applied Design Load***

Non-linear analysis finishes if Eff = 100% which means RF = 1

$$RF = \frac{\text{(non-linearly) Predicted Failure Load}}{j \cdot \text{Design Limit Load}} > 1.$$

with $j :=$ design factor of safety, $DUL = j_{ult} \cdot$ Design Limit Load

Material Reserve Factor : ***f_{Res} = strength / applied stress ≡ Puck's Stretch Factor f_S***

essential for linking →

$$f_{Res} = \frac{1}{Eff} = \frac{\text{Strength Design Allowable } R}{\text{Stress at } j \cdot \text{Design Limit Load}} > 1.$$

if linear analysis: $f_{Res} = RF = 1 / Eff$, $Eff \equiv Puck's\ f_E$

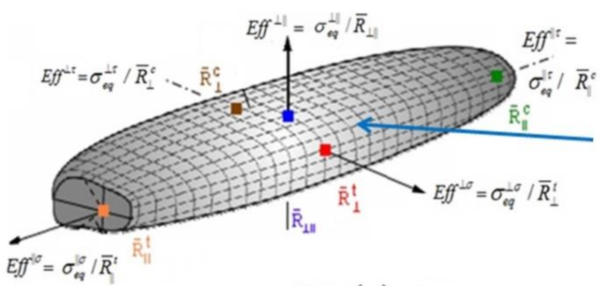
'Proportional loading (stressing) concept'

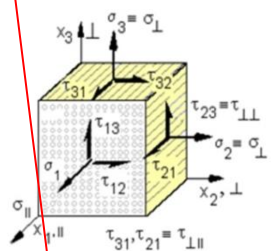
f_{Res} in construction $= (R_k / \gamma_M) / (\sigma \cdot \gamma_S) = R_k \cdot \gamma_S / (\sigma \cdot \gamma_M)$

with γ_M, γ_S the partial safety factors applied for material and stress

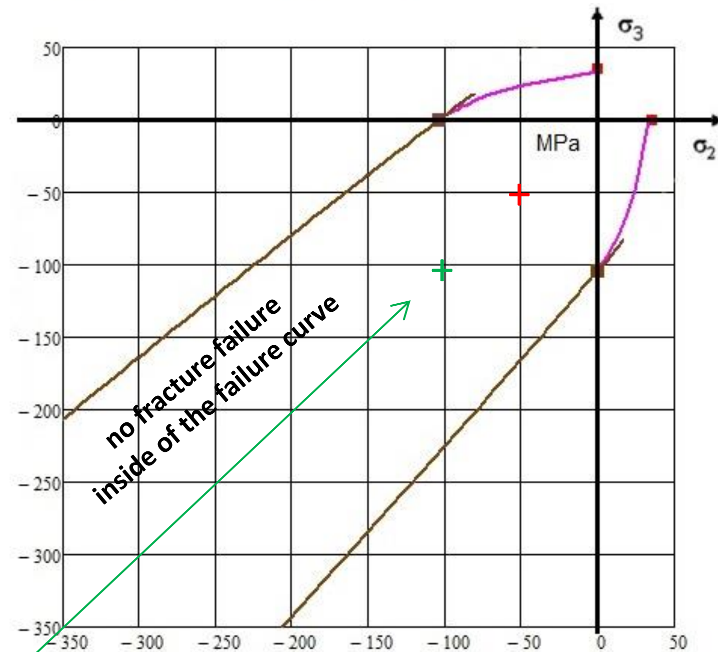
Discussion of Lateral Stress States in General and in the UD-hanger Strand Loop. Visualisation of the *Eff* values of 3 different Compression Stress States. *IFF 2-mode domain*

IFF2 $$Eff^{\perp\tau} = [(\frac{\mu_{\perp\perp}}{1-\mu_{\perp\perp}})\cdot(\sigma_2+\sigma_3) + \frac{1}{1-\mu_{\perp\perp}}\sqrt{(\sigma_2-\sigma_3)^2+4\tau_{23}^2}\,] \,/\, \bar{R}_{\perp}^{c} = +\sigma_{eq}^{\perp\tau} \,/\, \bar{R}_{\perp}^{c}$$





3D $\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$



3 stress states and its Eff values

Stress States			Eff in %
	σ_2	σ_3	
1D	0	$-R_{\perp}^{c}$	100
2D	$-0.5\,R_{\perp}^{c}$	$-R_{\perp}^{c}$	24 +
σ^{cc}, σ^{cc}	$-R_{\perp}^{c}$	$-R_{\perp}^{c}$	**-52** +
			$a_{\perp\perp} = 0.26$, $b_{\perp\perp} = a_{\perp\perp} + 1$
			proportional stressing

Achtung:
Es steigt keine Festigkeit sondern die Schubbruchgefahr Eff wird kleiner !!

- Multiaxial compression lowers the *Eff* of IFF2.
- In the case of 'dense' UD materials bi-axial compression causes no fracture failure, formally indicated by a **negative *Eff***, which physically means *Eff* = 0.
- bi-axial compression generates a tensile stress because the constraining fibers withstand the axial straining.

Finally the application: **Assessment of a Chosen Stress Data Set <u>in the Loop Groove</u>**

3D Stress state: General, modes FF1, FF2, IFF1, IFF2, IFF3

$$\{\sigma\} = (\sigma_1,\ \sigma_2,\ \sigma_3, \tau_{23},\ \tau_{31},\ \tau_{21})^T = (\sigma_1,\ \sigma_2^{\,pr}, \sigma_3^{\,pr},\ 0,\ \tau_{31}^{\,pr},\ \tau_{21}^{\,pr})^T = (\sigma_{||},\ \sigma_{\perp 2},\ \sigma_{\perp 3}, \tau_{\perp\perp},\ \tau_{\perp||3},\ \tau_{\perp||2})^T$$

$$\{\sigma_{eq}^{mode}\} = (\sigma_{eq}^{||\sigma},\ \sigma_{eq}^{||\tau},\ \sigma_{eq}^{\perp\sigma},\ \sigma_{eq}^{\perp\tau},\ \sigma_{eq}^{||\perp})^T \text{ with } \{\bar{R}\} = (\bar{R}_{||}^t, \bar{R}_{||}^c, \bar{R}_{\perp}^t, \bar{R}_{\perp}^c, \bar{R}_{\perp||})^T \quad \text{when mapping}$$

Acting 3D Stress state, caused by 'bias' load situation: *inner layer, higher tangential stress*

stress dedications: $\sigma_x = \sigma_1 = \sigma_{||}$ (see VDI figure, slide 10),

$\sigma_y = \sigma_{radial} = \sigma_{\perp 3}$ (from zero to compressive maximum in the groove, where $\sigma_{||}$ is also highest).

$\{\sigma\} = (\sigma_1,\ \sigma_2,\ \sigma_3, \tau_{23},\ \tau_{31},\ \tau_{21}) = (1700, -80, -100, 10, 20, 40)$ *in MPa*. *Chosen*

a full 3D assessment is required in Design Verification and not i.e. two 2D-plane assessments, only

Other Input: Strength Design Allowables, curve parameters etc. *Chosen example, in MPa*

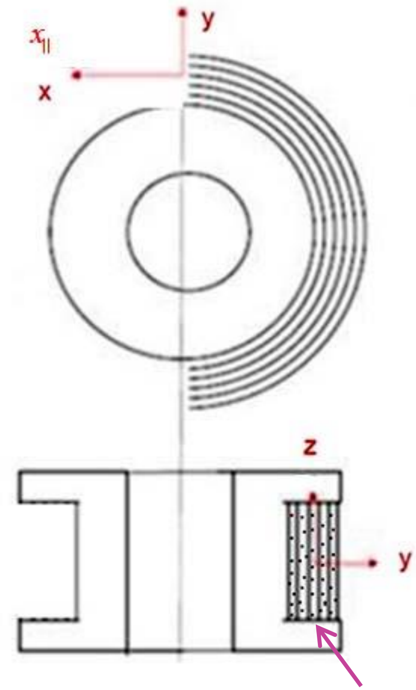
$$\{\bar{R}\} = (2560,\ 1590,\ 73,\ 185,\ 90)^T \rightarrow \{R\} = (R_{||}^t, R_{||}^c, R_{\perp}^t, R_{\perp}^c, R_{\perp||})^T = (2000,\ 1300,\ 50,\ 140,\ 60)^T$$

$$\mu_{\perp||} = 0.21,\ \mu_{\perp\perp} = 0.206,\ a_{\perp\perp} = \mu_{\perp\perp} / (1 - \mu_{\perp\perp}) = 0.26,\ \nu_{\perp||} = 0.32,\ m = 2.6,$$

$$\{E\} = (165000,\ 165000,\ 8400,\ 8400,\ 5600)^T \text{ MPa.}$$

$$Eff^{\,m} = (\mathrm{Eff}^{||\sigma})^m + (\mathrm{Eff}^{||\tau})^m + (\mathrm{Eff}^{\perp\sigma})^m + (\mathrm{Eff}^{\perp\tau})^m + (\mathrm{Eff}^{\perp||})^m \rightarrow Eff = \left(()^m + ()^m + ()^m + ()^m + ()^m\right)^{m^{-1}}$$

$$Eff_{IFF} = \left((\mathrm{Eff}^{\perp\sigma})^m + (\mathrm{Eff}^{\perp\tau})^m + (\mathrm{Eff}^{\perp||})^m\right)^{m^{-1}}.$$



caused by side walls of the groove:

$$\varepsilon_{||} = -\nu_{\perp||} \cdot (\sigma_2^c + \sigma_3^c) / E_{||}$$
$$= -\nu_{||\perp} \cdot (\sigma_2^c + \sigma_3^c) / E_{\perp}.$$
$$\sigma_{1,bi-axial} = E_{||} \cdot \varepsilon_{||}.$$

Results **without** compressive constraining by stiff side 'walls': Interaction heavily acts, similar to stability !

$$Eff^{||\sigma} = 0.74,\ Eff^{||\tau} = 0,\ Eff^{\perp\sigma} = 0,\ Eff^{\perp\tau} = 0.94,\ Eff^{\perp||} = 0.30,\ Eff^{IFF} = 0.95,\ Eff = 1.12$$

Results **with** compressive constraining by stiff side 'walls': Mandatory design !

$$Eff^{||\sigma} = 0.75,\ Eff^{||\tau} = 0,\ Eff^{\perp\sigma} = 0,\ Eff^{\perp\tau} = 0.08,\ Eff^{\perp||} = 0.30,\ Eff^{IFF} = 0.31,\ Eff = 0.78.$$

Interesting Information on 3D compressive stress states: *Ultra-High-Performance-Concrete UHPC [IfM Dresden]*
Impact of hydrostatic compression on strength capacity.

1D- and 3D-test results → *Eff* remains 100% for $(-\bar{R}^c = -160,\ 0,\ 0)$ and $(-224-6,\ -6,\ -6)$.

Lessons Learned: *Eff* = 100% : The (technical) strength is not increased. *Eff* is 100% for both the two failure states

Standard question and FAQ: What about the porosity in the Strand??

Cuntze's **Novel** Model to map the influence of porosity on the strength capacity of the bi-axially compressed UD-Strand

* Failure Function for a dense UD material

$$F^{SF} = [a_{\perp\perp} \cdot I_2 \; + \; b_{\perp\perp} \cdot \sqrt{I_4}] \; / \; \bar{R}_{\perp}^{c} = 1$$

$$= \; [a_{\perp\perp} \cdot (\sigma_2 + \sigma_3) \; + \; b_{\perp\perp} \cdot \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}{}^2}] \; / \; \bar{R}_{\perp}^{c} = 1$$

with $a_{\perp\perp} = b_{\perp\perp} - 1$ after inserting $\bar{R}_{\perp}{}^{c} = 104$ MPa

* Failure Function for a porous UD material (index po, *author's simple approach*)

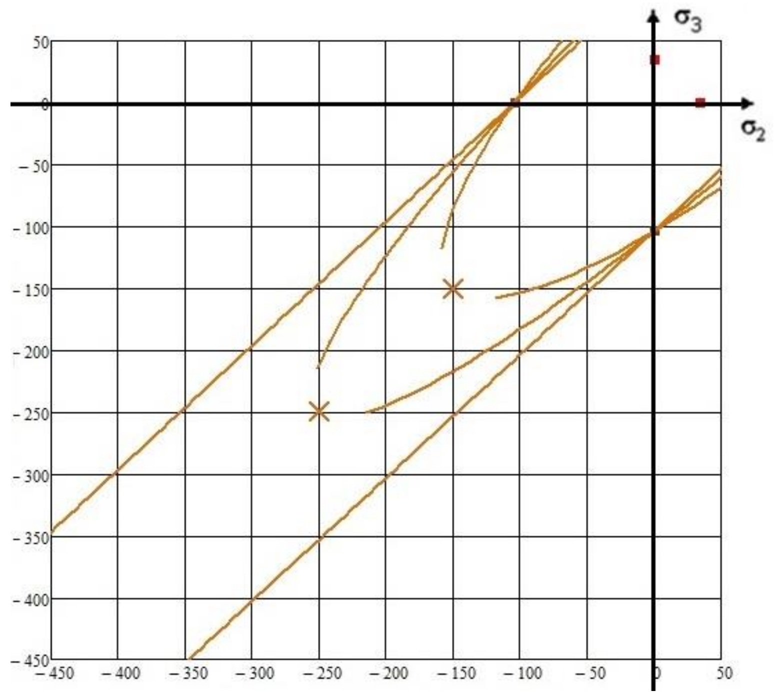
$$F^{SF}_{porosity} = \sqrt{a_{\perp\perp po}{}^2 \cdot I_2{}^2 + \; b_{\perp\perp po}{}^2 \cdot I_4} - \; a_{\perp\perp po} \cdot I_2 \;] \; / \; 2\bar{R}_{\perp}^{c} = 1$$

$$= \sqrt{a_{\perp\perp po}{}^2 \cdot (\sigma_2 + \sigma_3)^2 + \; b_{\perp\perp po}{}^2 \cdot \sigma_2 - \sigma_3)^2 + 4\tau_{23}{}^2} - \; a_{\perp\perp po} \cdot (\sigma_2 + \sigma_3) \;] \; / \; 2\bar{R}_{\perp}^{c} = 1$$

The two curve parameters are determined here from insertion of the

compressive strength point $(\sigma_2 = -\bar{R}_{\perp}^{c}, \; 0)$ and

bi-axial compressive fracture stress point $(\sigma_2 = -\bar{R}_{\perp}^{cc}, \; \sigma_3 = -\bar{R}_{\perp}^{cc})$.



Fracture failure curves of UD material regarding two different porosity grades.

$a_{\perp\perp po}$ for 0, 0.10, 0.22 with $b_{\perp\perp po}$ for 4.0, 3.5, 2.9 .

- Ideal dense materials possess no porosity.
- A fully porous material may be defined by $R_{\perp}^{cc} \cong R_{\perp}^{c}$.
  This case can be modelled like foam materials in the quasi-isotropic domain [*Cun16a*].

Conclusions

regarding the 3D-stress state in the ‘groove’ of the thimble

1. A SFC has to map 3D stress states. It can be validated, principally, by 3D-test data sets only. If just 2D test data is available, then the necessary 3D mapping quality is not fully proven.
2. Each failure stress state belongs to *Eff* = 100% and represents one point on the surface of the failure body. This is valid for 1D- (these are the strength values), for 2D- and for 3D-stress states
3. In the case of a multiaxial compressive stress state the strength does not increase but the risk to fracture may become smaller, indicated by *Eff* which becomes lower than *Eff* ($R_\perp^c$) = 100 % !
4. No ‘usual side walls’: A 1D-compressive radial stress state in the groove of the thimble would lead to IFF2-caused wedge failure of the laterally compression-loaded UD loop strand
5. With side walls: The radial compression generates a positively acting 3D-compressive stress state. Hence, side walls of the groove are mandatory despite of the fact that the fibers experience a little higher tensile stress.
6. Hooked fibers or tapes practically do not carry the desired tensile loading. Fibers or tapes loosing some stretch, caused by the winding or tape-laying process, carry much less tensile loading and are not efficiently used.
7. Thin tapes (layers) help to exploit the capacity of the fibers.
8. Reducing the bond between the layers increases the strength capacity of the loop *[CarboLink idea]*.

→ ***Design ‘makes’ structural Strength,*** **q.e.d.**

Condition versus criterion: $F = 1$ versus $F < = > 1$

NOTIONS

Damage (Beschädigung): physical harm, which captures in English as well micro-damage (Schädigung) as macro-damage (Schaden)

Eff: material stressing effort *Eff* = f (*Eff* ^{modes}) representing as interaction equation - *captures the damaging portions of all activated modes* - the mathematical equation of the surface of the fracture (failure) body

Equivalent stress σ_{eq}: (a) equivalent to the stress state, as performed in σ_{eq}^{Mises}, and (b) comparable to the value of the strength *R* which dominates one single failure mode or failure type. *Eff* , equivalent stress and strength *R* are positive-defined.

Failure: state of inability of an item to perform a required function in its limit state. A structural part does not fulfil its functional requirements such as the failure modes Onset-of-Yielding, brittle fracture (NF, SF, Crushing Fracture CrF), Fiber-Failure FF, Inter-Fiber-Failure IFF (matrix failure), leakage, deformation limit (tube widening), delamination size limit, frequency bound, or heat flow etc. A failure is a project-defined 'defect'. For each failure mode a Limit State with *F* = Limit State Function or Failure Function is to formulate.

Failure Mode: Failure mode is a commonly used generic term for the types of failures, is a name for a potential way a system may fail (in design verification usually a project- associated failure)

Failure Surface and Failure Body: the surface of the failure body is the shape defined by *F* = 1 or *Eff* = 100% = 1

Failure Type (isotropic): NF, SF, CrF, Normal Yielding NY, Shear Yielding SY

Flaw versus micro-crack: a micro-crack is a sharp flaw (Ungänze), grade of singularity is decisive

Fracture: separation of a whole into parts

Fracture (failure) body: Surface of the tips of all fracture (failure stress) vectors. Fracture is the failure of brittle materials

Material: 'homogenized' (macro-)model of the envisaged complex solid or heterogeneous material combination which principally may be a metal, a lamina or further a laminate stack analyzed with effective properties. Homogenizing (smearing) simplifies modelling

Material behavior: *brittle* behavior could be characterized with the complete loss of tensile strength capacity at first fracture, R^t. Quasi-brittle behavior shows - after reaching R^t - a slight strain hardening followed by a gradual decay of tensile strength capacity during a strain softening domain. *Ductile* behavior is accompanied by a gradual increase of tensile stress (strain hardening), and after reaching R^t a strain softening domain follows

Material Stressing Effort (*Werkstoffanstrengung, nicht Werkstoffausnutzung, das Verschnitt etc berücksichtigt*): definition as $Eff = \sigma_{eq} / R$; max*Eff* =100% is reached at $F = 1 = 100\%$. Just for 100% *F* corresponds to *Eff*

'Modal' versus 'Global' SFCs: Modal means that only a test data set of one failure mode domain is mapped whereas global (Drucker-Prager isotropic, Tsai UD) means that mapping is performed over several mode domains. The bottle-neck of global SFCs respectively 'Single Failure Surface Descriptions' is, that any change in one of the 'forcibly married' modes requires a new global mapping which changes the failure curve in the physically not met mode, too. The advantage of 'Modal' SFCs is to obtain – analogously to Mises - physically plausible equivalent stresses for each failure mode

Multi-fold stress state: example isotropic material: $\sigma_I = \sigma_{II}$, $\sigma_I = \sigma_{II} = \sigma_{III} \rightarrow \sigma_{hyd}$; 3-fold)

Proportional loading: procedure, how the material stressing effort *Eff* is derived from the failure function *F*. In the case of a non-homogeneous function *F* the associated values are only equal for the failure state *F* = 1, respectively for *Eff* = 100%

Reserve factor: ratio of a 'resistance value' and a so-called 'action value'. *RF* > 1 permits a further increase of loading. This is terminated by *Eff* = 100% 'material stressing effort' (Werkstoffanstrengung) in the last critical Hot Spot, when no more stress redistribution in the structural component is possible

Strength: in engineering linked to a uni-axial fracture stress. (1) Characteristic strength: in mechanical engineering the typical average strength, in civil engineering a reduced (5% fractile) average strength value. (2) Design strength: a statistically reduced average strengt. $\overline{R}$ = bar over the *R* which means *average* strength and which is to apply when mapping, like here. *R* = general strength and also the statistically reduced 'strength design allowable.

Stress (not stress component!): component of the stress tensor defined as force divided by the area of cross-section.

Validation: result of a successful qualification of a model (i.e. material model)

(design) Verification: fulfillment of a set of design requirement data