Cracking at statically-loaded Notches ***using***
Fracture Mechanics (FM) and novel Finite Fracture Mechanics (FFM)
- Analyses of so-called 'Open Hole Panels' -

Summary

Full Design Verification requires the verification of Strength and of Damage Tolerance in the case of potentially cracked (macro-damaged) statically-loaded structural components under sudden overloading.

The Strength Analysis (SA) requires that the effective multi-axial stress state is not above the given Strength Design Allowable and the Damage Tolerance Analysis (DTA) the same for the so-called residual strength of the structural component containing a pre-crack.

Lying between Strength analysis and Fracture Mechanics (FM) analysis 'Onset-of-Cracking' (OoC) is experienced at stress concentration sites such as notches like open holes in a panel of a sufficiently brittle material. In this context, Leguillon's Hypothesis [1] says

"*A* (generating) *crack is* (becomes) *critical when and only when both the released energy and the local stress reach critical values along an assumed finite crack*".

This novel hypothesis, 'Neuber'-improving, shall be presented here. It captures the prediction of the instantaneous OoC. The name of the tool is Finite Fracture Mechanics (FFM), see *Fig.1*. It predicts for notched components that loading level where the Strength Failure Criterion (SFC) equals the FM criterion or it determines as a coupled (hybrid) stress-energy criterion the critical loading that causes the finite crack size Δa_c. Because FM is one part of the FFM as introduction and for better understanding at first the well-known FM analysis tool R-curve shall be presented.

smooth structure	**notched structure**	"transition domain"	**cracked structure**
no steep stress decay SFC	stress concentration Neuber method (up to now)	'onset-of cracking' *assumed crack, FFM* (novel replacement)	stress intensity *real pre-cracks, FM* *'no hole' and 'with hole'*

Fig. 1: Stress situations in a structural component

Fig.2 visualizes the task to be solved. For practical application the concept of a linear-elastic stress intensity factor K may be sufficient and is usually applied. Coordinates used are depicted.

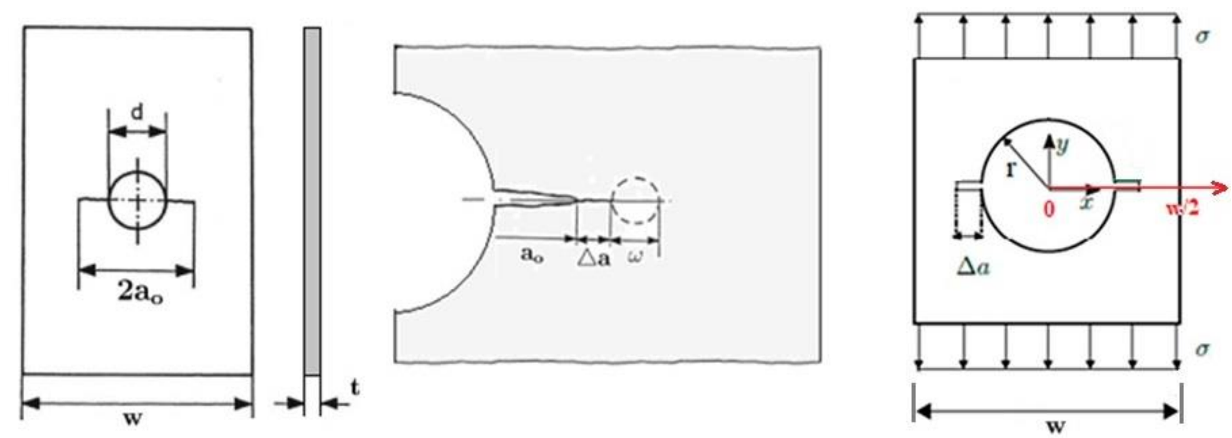

Fig. 1: Plate strip with a central open hole and an existing through crack of the size $a = a_0 - r$.

(left) characterization of an open hole panel with existing crack , w= plate width, t = plate thickness,
(center) crack growth details in the case of slight crack tip yielding ω *of not fully brittle materials,*
(right) σ *= remote tensile stress, leading to cracking for* $\sigma = \sigma_{\text{fail}}$, Δa *= assumed FFM crack ,* $d = 2r$.

Key Words: Residual strength, critical crack length, R-curve, Finite Fracture Mechanics, coupled criterion.

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1 General

There are three approaches available to perform Design Verification (DV) for occurring static stress situations: Strength Failure Criteria (SFC), Continuum (micro-)Damage Mechanics (CDM, *not yet DV-capable*) criteria and Fracture Mechanics (FM) criteria for cracked (macro-damaged) components. A novel approach is the hybrid tool Finite Fracture Mechanics (FFM) which captures the 'onset-of-cracking' (OoC) at stress concentration (SC) points and at higher stress singularities.

The FFM is a coupled (hybrid) criterion that fills a gap in FM by assuming an instantaneous formation of a crack of finite size [1, 2]. Intention is to initially show the classical application of FM, because FM provides one part tool of the FFM. *Fig.1* gave a survey on the situations faced.

Due to FFM, the Neuber method is now obsolete, but falls as a special case. What Neuber called "support length" is precisely the crack length supplied by the FFM, without the need for acceptance or experimental identification!

The provided analyses are restricted to the 2D-case, 3D-extension will be a future task.

A SFC is a necessary condition but might not be a sufficient condition for the prediction of ‘Onset-of-cracking’, seen here as onset of failure:

*This is known for the author for about 50 years from the so-called ‘thin layer effect’ of UD-layer-composed laminates: *Due to being strain-controlled, the material flaws in a thin lamina (*transversely-isotropic material*) cannot grow freely up to micro-crack size in the thickness direction, because the neighboring laminas act as micro-crack-stoppers*. Considering fracture mechanics, the strain energy release rate, responsible for the development of damage energy in the 90° plies - from flaws into micro-cracks and larger -, increases with increasing ply thickness. Therefore, the actual absolute thickness of a lamina in a laminate is a driving parameter for initiation or onset of micro-cracks, i.e. [*Fla82*].

*Further and generally more known in metallic applications is in the case of discontinuities of the here focused isotropic materials such as notch singularities with steep stress decays: only a *toughness + characteristic length-based energy balance condition* may form a sufficient set of two fracture conditions.

When applying SFCs usually ideal solids are viewed which are assumed to be free of essential micro-voids or micro-crack-like flaws, whereas applying Fracture Mechanics the solid is considered to contain macro-voids or macro-cracks, respectively.

Since about 20 years Finite Fracture Mechanics (FFM) tries to fill a gap between the continuum mechanical strength analysis and the classical FM analysis. FFM is an approach to offer a criterion to predict the crack onset in brittle isotropic and UD materials.
This is a bridge that had to be built from the strength failure to the fracture mechanics failure ground. Attempts to link SFC-described ‘onset of fracture’ prediction methods and FM prediction methods for structural components have been performed. Best known is the still cited Hypothesis of Leguillon, within he assumes cracks of finite length Δa. Thus using FFM one obtains one more unknown but also a further equation to solve together with the SFC the equation system.
This coupled criterion does not refer to microscopic mechanisms to predict crack-*nucleation*!

Considering FFM it is referred to the literature [1, 2, 3]

2 List of Symbols

Symbol	Unit	Description
a	mm	crack length
a_0	mm	initial crack length (open hole panel: crack a + hole radius r)
a_c	mm	*critical* crack length
a_e	mm	*effective crack length* $a_e = a_p + \omega/2$
a_p	mm	*physical crack length* $a_p = a_0 + \Delta a$
c_{ij}		abbreviating functions and abbreviations
$f(a)$		correction function of the stress intensity factor (SIF)
f_d		correction function concerning the hole diameter
f_w		correction function concerning the specimen width w
t	mm	panel specimen thickness
w	mm	width of panel, test specimen
Δa	mm	stable increase of a due to static loading
Δa_e	mm	effective crack elongation (R-curve abscissa) $\Delta a_e = a - a_0$
A	mm	parameter of the R-curve model
B	-	parameter of the R-curve model
E	MPa	Young's modulus (MPa = N/mm^2)
F	N	force
$\mathcal{G}_{Ic}$	$MPa \cdot m$	Cracking resistance: potential strain energy release rate at failure. Under plane strain conditions (most critical case) $\mathcal{G}_{Ic} = K_{Ic}^{\ 2} \cdot (1- \nu^2) / E$ $dW \sim \sigma_{applied}^{\ 2} \cdot mm / E \equiv MPa \cdot mm$, $\sqrt{m} = 31.6 \cdot \sqrt{mm}$
$K(\sigma,a)$	$MPa \cdot \sqrt{m}$	Cracking action: stress intensity factor, (SIF) $K = \sigma \cdot \sqrt{\pi \cdot a} \cdot f(a)$,
K_{as}	$MPa \cdot \sqrt{m}$	parameter of the R-Curve model (asymptotic value of R-curve)
K_b	$MPa \cdot \sqrt{m}$	parameter of the R-Curve model (value at beginning of R-curve)
K_{app}	$MPa \cdot \sqrt{m}$	apparent fracture toughness (general) = critical SIF (not the often used K_c)
K_p	$MPa \cdot \sqrt{m}$	physical value of the SIF K: $K_p = \sigma \cdot \sqrt{\pi \cdot a_p} \cdot \sqrt{\sec(\pi \cdot a_p / w)}$, sec =1/cos
K_e	$MPa \cdot \sqrt{m}$	Effective SIF: $K_e = \sigma \cdot \sqrt{\pi \cdot a_e} \cdot \sqrt{\sec(\pi \cdot a_e / w)}$, often termed K_R
K_{Ic}	$MPa \cdot \sqrt{m}$	Cracking resistance: critical SIF (fracture mechanics Mode I testing) at onset of unstable sharp crack propagation in the plane strain state = most brittle condition, otherwise called K_c; or = fracture toughness of uni-axially tensile-loaded, minimum ductile (*brittle*) material specimens = material resistance to crack propagation $K_{Ic} = \sigma \cdot \sqrt{\pi \cdot a_c} \cdot f = \sigma_c \cdot \sqrt{\pi \cdot a} \cdot f$
K_R	$MPa \cdot \sqrt{m}$	Cracking resistance, R-curve ordinate
R-curve	$MPa \cdot \sqrt{m}$	material *Resistance* to fracture curve in case of slow, stable crack propagation from a sharp notch, accompanied by growth of the plastic zone at the crack-tip (*unfortunately also the letter R was taken*)
R; R_{p02}	MPa	failure stress ≡ strength (*Resistance to stress action*); tensile yield str.
dW	MPa·mm	energy $dW \sim \int \sigma \cdot \varepsilon \cdot d\varepsilon = \int \sigma^2 / E \cdot d\varepsilon$
ν	-	Poisson's ratio
ω	mm	full plastic zone at the crack-tip
σ	MPa	Action: remote (far field) uniform tensile stress
σ_c	MPa	critical value of σ = residual strength

(In structural mechanics x is usually the length coordinate, but in fracture mechanics the net section direction)

3 Analysis using the Crack Growth Resistance curve = ‘R-curve’

3.1 *General on Fracture Mechanics quantities and R-curve Concept*

Basic assumption: Use of largest crack size that can be expected, following the ‘weakest link’ failure model and regarding quality assurance measurement limits.

In the Damage Tolerance procedure of cracked (macro-damaged) structural components two basic questions are posed in analysis:

1. What is the static strength if a crack is present (residual strength problem)?
2. How is the propagation behavior of the present crack (large crack growth problem)?

In order to perform this for isotropic materials some different quantities are used to predict the stress state at the crack tip caused by a far-field stress or remote stress, respectively.

*The stress intensity factor (SIF) K, applied to homogeneous linear elastic materials. Its measured size depends on test specimen width w, the crack size a, the location of the present crack and the material. It can be written as $K = \sigma \cdot \sqrt{\pi \cdot a_0} \cdot f(a/w)$, where the SIF K_I of the fracture mechanics mode I is applied here, (*Fig. 4*).

*The strain energy release rate $\mathcal{G}$, defined as the instantaneous loss of total potential energy Π per unit crack growth area (crack length Δa · plate thickness t) of the fresh surface S, by $\mathcal{G} = -\Pi / S$. In the case of brittle materials for its ‘basic’ Fracture Mode-I a relationship exists $\mathcal{G}_I = K_I^2 / E'$ with $E' = E/(1-\nu^2)$ for plane strain.

*The J-integral J, characterizing the singular stress field at the crack tip in nonlinear elastic-plastic materials where the size of the plastic zone is small compared to the crack length. It is one way of determining the strain energy release rate $\mathcal{G}$. For brittle materials J corresponds to $\mathcal{G}$.

Macrocrack extension occurs when the stress intensity factor (SIF) K attains a critical value. Thereby the *Action-linked* SIF is entirely dependent on the structure geometry and loading condition, whilst the *Resistance-linked R*-curve is basically a material property dependent on temperature, environment, and loading rate as well the geometric test specimen range, etc.

Crack-growth resistance curves, the so-called *R*-Curves, are used here to predict:

- the residual strength of the structure for a given crack position and crack length,
- the critical length of an initial crack under given loadings.

These curves are conveniently plotted with crack extension Δa instead of crack size a, because the shape of the R-curve does not vary with the crack size.

* For very brittle materials with its flat *R*-curves, there is no stable crack extension and the initial crack size a_0 is the same as the critical crack size a_c. Then a single value of toughness characterizes the material, the cracking resistance K_{Ic}.

* For ductile materials (*such as low strength steels*) with a rising *R*-curve there is no single value of toughness that characterizes the material. Reason is that the plastic zone ω at the crack tip increases with crack growth and length, hence the energy dissipated to overcome plastic deformation will increase. In materials with a rising *R*-curve, stable crack growth occurs and the critical crack size will be larger than the initial crack size. Mind: These *R*-curves (italic *R* letter) shall not be mixed up with the ‘R-curves’ in fatigue $R = \min\sigma / \max\sigma$.

Fracture mechanics regards small scale ductility (usually described by its diameter ω) at the crack tip and multi-axial loading, *Fig 2*.

In the case of a mixed-mode loading and opening of a crack, the energy release rate consists of the three parts $\mathcal{G}_I$, $\mathcal{G}_{II}$, $\mathcal{G}_{III}$ that correspond to the respective three fracture modes. The fracture-effective formulation then is $\mathcal{G} = \mathcal{G}_I + \mathcal{G}_{II} + \mathcal{G}_{III}$.
Crack extension occurs when above strain energy release rate $\mathcal{G}$ attains a critical value $\mathcal{G}_c$. In the case of fracture it becomes $\mathcal{G} \geq \mathcal{G}c$. $\mathcal{G}$ is directly related to the stress intensity factor *K*. It is associated in two-dimensional fracture mechanics with the loading modes (Mode-I, Mode-II, or Mode-III) the so-called Mixed-Mode Problem, applicable to cracks under plane stress, plane strain and anti-plane shear, see *Fig.4*. For the Fracture Mode-I, the energy release rate $\mathcal{G}$ is related to the Mode-I stress SIF K_I for a linearly-elastic material.

The two questions at the beginning of this sub-chapter can be answered using the analytical methods of fracture mechanics. For practical application the concept of the linear-elastic *K* is usually applied:
"*A structural component will fail in the case of static loading if the stress intensity factor (SIF) K of a brittle material reaches its critical value at K = K_c, termed fracture toughness, which depends on the material behavior*".
The determination of the K_c -values requires in the so-called *K*-concept the fulfilment of a geometric bound in order to achieve the real minimum K_{Ic}-value by a test specimen thickness of

$$t \ > \ 2.5 \cdot (K_{Ic} \,/\, R_{p0.2})^2 \ \rightarrow \ \sigma_c = K_{Ic} \,/\, (\sqrt{\pi \cdot a_0} \cdot f(a_0)\,.$$

Instead of the "Plain Strain Fracture Toughness" K_{Ic} (which is a material property but subject to certain minimum geometric requirements), an "Apparent Fracture Toughness" is inevitably to apply, adapted to the current geometric conditions.

A plot of strain energy release rate $\mathcal{G}$ versus crack extension Δa for a particular loading situation is termed driving force curve $\mathcal{G}(\Delta a)$. The driving force for crack propagation can be quantified by above characterizing parameters K, $\mathcal{G}$, or J. A plot of *R* versus crack extension Δa is a resistance curve, as still cited termed *R*-curve $R(\Delta a)$.

3.2 *Models for R-curve (resistance) and for Stress Intensity Factor (SIF)-curve*

3.2.1 Resistance: *R*-curve, ordinate K_R (*using a test data mapping function*)

For well mapping the test data course of the *R*-curve J. Broede proposed the mapping function

$$K_e(a) = K_{as} - (K_{as} - K_b) \cdot \frac{1-B}{\exp\left(\frac{\Delta a_e}{A}\right) - B} \quad \text{with inverse} \quad \Delta a_e = A \cdot \ln\left(B + (1-B) \cdot \frac{K_{as} - K_b}{K_{as} - K_e}\right)$$

$$\Rightarrow \quad \text{new } a_0 = a_0 + \Delta a_e = a_0 - A \cdot \ln\left(B + (1-B) \cdot \frac{K_{as} - K_b}{K_{as} - K_e}\right)$$

in [2] including the effective quantities K_e and Δa_e. The plot $K_e(\Delta a_e)$ is termed effective *R*-curve.

3.2.2 Action: *Stress Intensity Factor (SIF)-curve, K_{SIF}* (using a width correction function f_w)

With the so-called geometry correction functions *f* - *correcting the original infinite plate term* $\sqrt{\pi \cdot a}$ - concerning hole diameter (index d) and width (index w) of the centrally cracked panel ('plate strip') the SIF reads for the two cases:

Panel, version 'No hole' $_{nh}$:

$$K_{nh} = \sigma \cdot \sqrt{\pi \cdot a} \cdot f_w(a) \quad \text{with} \quad f_w(a) = \sqrt{\sec \frac{\pi \cdot a}{w}} \quad \text{capturing the panel width}$$

$$\boxed{K_{nh}(a) = \sigma \cdot \sqrt{\pi \cdot a} \cdot \sqrt{\sec \frac{\pi \cdot a}{w}} \quad , \qquad (\sec = 1/\cos).}$$

Panel, version 'With hole' $_{wh}$: (Tada delivered in [9] a hole considering correction function *f(a))*:

$$K = \sigma \cdot \sqrt{\pi \cdot a} \cdot f(a) \quad \text{with} \quad f(a) = f_d(a) \cdot f_w(a) \quad \text{in the case of an open hole panel}$$

$$f_d(a) = \sqrt{1 - \frac{r}{a}} \cdot (1 + 0.358 \cdot \frac{r}{a} + 1.425 \cdot \left(\frac{r}{a}\right)^2 - 1.578 \cdot \left(\frac{r}{a}\right)^3 + 2.156 \cdot \left(\frac{r}{a}\right)^4), \quad f_w(a) = \sqrt{\sec(\frac{\pi \cdot r}{w}) \cdot \sec(\frac{\pi \cdot a}{w})}.$$

$$\boxed{K_{wh}(a) = \sigma \cdot \sqrt{\pi \cdot a} \cdot \sqrt{1 - \frac{r}{a}} \cdot (1 + 0.358 \cdot \frac{r}{a} + 1.425 \cdot \left(\frac{r}{a}\right)^2 - 1.578 \cdot \left(\frac{r}{a}\right)^3 + 2.156 \cdot \left(\frac{r}{a}\right)^4 \cdot \sqrt{\sec \frac{\pi \cdot r}{w} \cdot \sec \frac{\pi \cdot a}{w}}.}$$

3.3 Conditions to Determine the Unknowns: *critical quantities σ_c, a_{ce}*

'*Crack growth will occur when dG/da > dR/da and G ≥ R*'.

This corresponds to '*The driving force curve is tangent with the R-curve*' as depicted in *Fig.3*. It can be interpreted as the critical condition when the energy available in the component for crack growth exceeds the maximum amount that the material can dissipate. In order to solve this task the following conditions must be met:

3.3.1 $K_{SIF}(\sigma_c, a_{ce}) = K_e(a_{ce} - a_0)$ with $\Delta a_e = a - a_0$. This means, that

firstly the coordinates of the touch point of SIF curve with R-curve are to determine.

$$\boxed{K_e(a) = \sigma \cdot \sqrt{\pi \cdot a} \cdot f(a) = K_{as} - (K_{as} - K_b) \cdot \frac{1 - B}{\exp\left(\frac{\Delta a_e}{A}\right) - B} \ .}$$

3.3.2 $dK_{SIF}(\sigma_c, a_{ce}) / da = dK_e(a_{ce} - a_0)/da$. This means, that

secondly, the two slopes of both the curves must become the same at the touch point, task which requires a differentiation (*Mathcad 15 code* symbolic *application*), delivering

$$\boxed{\frac{dK_e}{da} \Rightarrow \sigma \cdot \frac{d}{da}\left(\sqrt{\pi \cdot a} \cdot f(a)\right) = K_{as} + \frac{(B-1) \cdot (K_b - K_{as})}{B - \exp(\frac{\Delta a_e}{A})} \Leftarrow \frac{dK_R}{da} \ .}$$

For the SIF-curve holds for the two versions, $\mathrm{SIF_{nh}}$ no hole and $\mathrm{SIF_{wh}}$ with hole:

$$\text{'No hole'}: \quad \frac{dK_{\mathrm{SIFnh}}}{da} = \frac{d\left(\sigma\cdot\sqrt{\pi\cdot a}\cdot\sqrt{\cdot\sec\dfrac{\pi\cdot a}{w}}\right)}{da}$$

$$= \sigma\cdot\frac{\sqrt{\pi}/(2\sqrt{a}\cdot caw)+\pi^{1.5}\cdot\sqrt{a}\cdot saw/(w\cdot caw^2)}{2\cdot\sqrt{\sqrt{\pi\cdot a}/caw}}, \; saw=\sin\left(\frac{\pi\cdot a}{w}\right), caw=\cos\left(\frac{\pi\cdot a}{w}\right).$$

'With hole' :

$$\frac{dK_{\mathrm{SIFwh}}}{da} = \frac{d\left(\sigma\cdot\sqrt{\pi\cdot a}\cdot\sqrt{1-\dfrac{r}{a}}\cdot(1+0.358\cdot\dfrac{r}{a}+1.425\cdot\left(\dfrac{r}{a}\right)^2-1.578\cdot\left(\dfrac{r}{a}\right)^3+2.156\cdot\left(\dfrac{r}{a}\right)^4\bullet\sqrt{\sec\dfrac{\pi\cdot r}{w}\cdot\sec\dfrac{\pi\cdot a}{w}}\right)}{da}$$

$$= \sigma\cdot\sqrt{\pi\cdot a}\cdot\left[c3\cdot\sqrt{1-\frac{r}{a}}\cdot(c1)+\frac{c3\cdot\sqrt{1-\frac{r}{a}}\cdot(c2)}{2\cdot a}+\frac{r\cdot c3\cdot(c2)}{2\cdot a^2\cdot\sqrt{1-\frac{r}{a}}}+\frac{\sin\left(\frac{\pi\cdot a}{w}\right)\cdot\sqrt{1-\frac{r}{a}}\cdot(c2)}{\sqrt{a}\cdot 2\cdot w\cdot\cos\left(\frac{\pi\cdot a}{w}\right)^2\cdot\cos\left(\frac{\pi\cdot r}{w}\right)\cdot c3}\right]$$

and the abbreviation functions

$$c1=\frac{4.734\cdot r^3}{a^4}-\frac{8.624\cdot r^4}{a^5}-\frac{0.358\cdot r}{a^2}-\frac{2.85\cdot r^2}{a^3}, \quad c2=\frac{0.358\cdot r}{a}+\frac{2.156\cdot r^4}{a^4}+\frac{1.425\cdot r^2}{a^2}-\frac{1.578\cdot r^3}{a^3}+1, \quad c3=\sqrt{\frac{1}{\cos\left(\frac{\pi\cdot a}{w}\right)\cdot\cos\left(\frac{\pi\cdot r}{w}\right)}}$$

* In the HSB sheet 62232-3 *J. Broede* mapped the *R*-curve by an appropriate analytical model, model parameters were determined there and finally $\sigma_c = R_{res}$ was derived by iteratively increasing the crack size up to a_c. This provides the failure stress for the maximally sustainable loading of the pre-cracked component.

* In *Table 1*, bottom, Cuntze delivers a continuous implicit mathematical computation.

3.4 Solution of the equation set to predict the unknowns

The Mathcad computation delivers the searched quantities for the open hole panel. *Fig.3* provides the full data set. In the computation, the usually in $\mathrm{MPa}\cdot\sqrt{\mathrm{m}}$ given fracture toughness (= critical SIF) is taken, which however requires a final factorization of the obtained critical stress by $\sqrt{1000}$ to get into the MPa, mm system.

In *Fig.3,* for the envisaged panel, the *R*-curve is plotted together with two SIF-curves, one for an initially guessed reference stress of $\mathrm{sig_{wh}}$=15 (dashed) and one for the computed critical reference value $\mathrm{sig_c}$ =12.5 (**bold**).

*For the ‘no hole-panel’ the critical SIF reads $K_c = 180\,\mathrm{MPa}\cdot\sqrt{\mathrm{m}} = 180\cdot\sqrt{1000}\,\mathrm{MPa}\sqrt{\mathrm{mm}}$ and the results are: $a_{ce} = 55.4$ mm, $\sigma_c = 12.5\cdot\sqrt{1000} = 396$ MPa .

*For the ‘hole panel’, in order to check any influence of the hole the associated rising **SIF-curve** was plotted, too. The same tangent point is obtained for this SIF-curve.

The computation of the 'no hole-panel' delivers as critical stress = residual strength, the value σ_{res}= 396 MPa (Mathcad computation scheme in *Table 1*).

*Table 1: Determination of the touch point = instability tangent point (*w width effect, no hole*)*

Vorgabe $\quad sigw = 11 \quad a = 33$

$$sigw\cdot\sqrt{\pi\cdot a}\cdot\sqrt{Sek\left(\frac{\pi\cdot a}{w}\right)} = Kas - (Kas - Kb)\cdot\frac{1-B}{\exp\left(\frac{a-a0}{A}\right)-B} \qquad \textbf{(point)}$$

$$\frac{\pi\cdot sigw\cdot\left(\dfrac{\cos\left(\frac{\pi\cdot a}{w}\right)+\frac{\pi\cdot a}{w}\cdot\sin\left(\frac{\pi\cdot a}{w}\right)}{\cos\left(\frac{\pi\cdot a}{w}\right)^2}\right)}{2\cdot\sqrt{\dfrac{a\,\pi}{\cos\left(\frac{\pi\cdot a}{w}\right)}}} = \frac{e^{\frac{a-a0}{A}}\cdot(B-1)\cdot(Kb-Kas)}{A\cdot\left[B-e^{\frac{1}{A}\cdot(a-a0)}\right]^2} \qquad \textbf{(slope)}$$

$$Aa := Suchen(sigw, a) \qquad Aa = \begin{pmatrix}12.5\\55.4\end{pmatrix} \qquad sigw := Aa_0 \quad sigw = 12.5 \qquad awc := Aa_1 \quad \boxed{awc = 55.4}$$

$$\sigma wc := sigw\cdot\frac{MPa\cdot\sqrt{m}}{\sqrt{mm}} \qquad \sigma wc := sigw\cdot\sqrt{1000}$$

$$KSIFwc0 := sigwc\cdot\sqrt{\pi\cdot awc}\cdot\sqrt{Sek\left(\frac{\pi\cdot awc}{w}\right)} \qquad w = 300 \qquad \boxed{KSIFwc0 = 180} \qquad \boxed{\sigma wc = 396}$$

For information, however – no practical effect in *Fig.3* comparing the blue curve KSIFwh – the associated (point) condition with considering the hole is added below:

$$sig\cdot\sqrt{\pi\cdot a}\cdot\left(c3\cdot\sqrt{1-\frac{r}{a}}\cdot c1 + \frac{\sqrt{\pi}\cdot sig\cdot c3\cdot\sqrt{1-\frac{r}{a}}\cdot c2}{2\cdot\sqrt{a}} + \frac{\sqrt{\pi}\cdot sig\cdot r\,c3\cdot c2}{2\cdot a^{1.5}\cdot\sqrt{1-\frac{r}{a}}} + \frac{\pi^{1.5}\cdot\sqrt{a}\cdot sig\cdot\sin\left(\frac{\pi\cdot a}{w}\right)\cdot\sqrt{1-\frac{r}{a}}\cdot c2}{2\cdot w\cdot\cos\left(\frac{\pi\cdot a}{w}\right)^2\cdot\cos\left(\frac{\pi\cdot r}{w}\right)\cdot c3}\right)$$

$$= Kas + \frac{(B-1)\cdot(Kb-Kas)}{B-e^{\frac{1}{A}\cdot(a-a0)}}$$

with the to be inserted abbreviation functions c1, c2, c3

The computation of the critical crack length a_c at the end of static loading is determined by the application of the formula below and there inserting K_{ec} (see application later). As K-values are usually given in $MPa\cdot\sqrt{m}$ this is intentionally widely followed here!

$$\text{new } a_0 = a_0 + \Delta a_{ec} = a_0 - A\cdot\ln\left(B + (1-B)\cdot\frac{K_{as}-K_b}{K_{as}-K_{ec}}\right) .$$

<u>*Results:*</u> *The R-test curve (resistance, marked KR) captures all physical effects such as small scale yielding at the crack tip, marked by the letter ω! It is effective, therefore K_e. Therefore, in order to be compatible the SIF-curve (action, marked KSIF) has to incorporate this effect. It does not depend on a_0,w.*

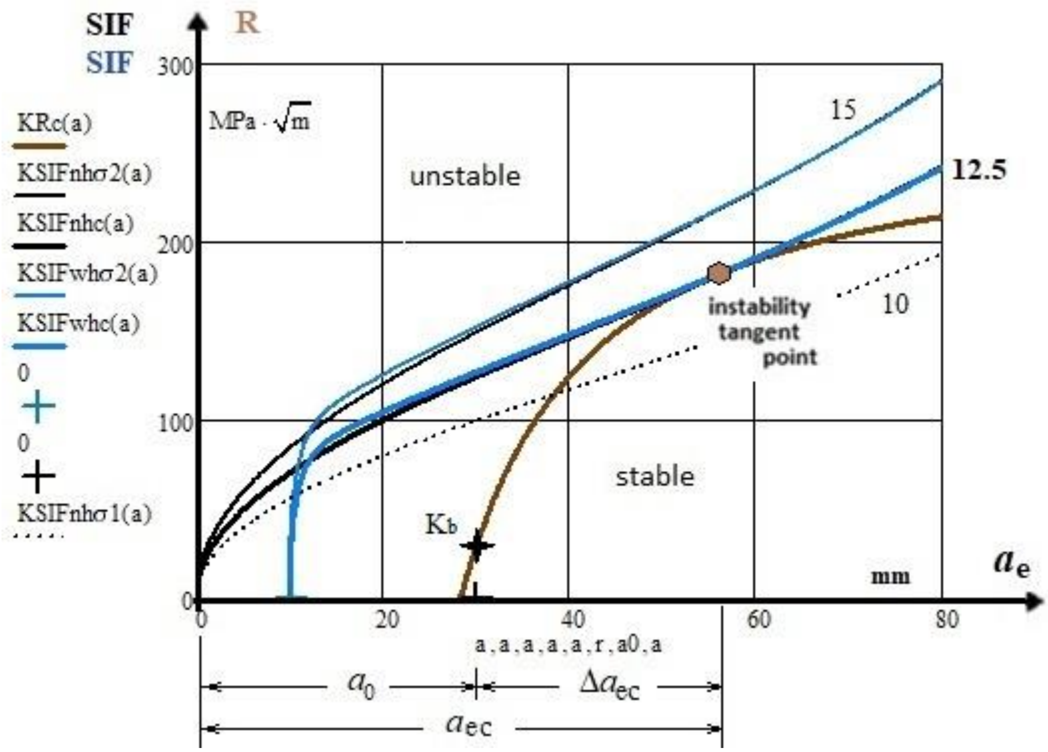


Fig.3: Wide panel example (HSB 62232-03) with w =300 mm, *t = 8*mm, *a_0 = 30* mm, *d =20* mm.
Aluminum Alloy 7475-T7351 in LT-direction: A =55.7 mm, *B = 0.75, K_{as} =246* $\text{MPa}\cdot\sqrt{\text{m}}$ *,*
K_b =29 $\text{MPa}\cdot\sqrt{\text{m}}$ *, R_{p02} =425* MPa *(B-value for t = 6...38* mm*) .*
Instability point: K_e = 180 $\text{MPa}\cdot\sqrt{\text{m}} = \text{MPa}\cdot\sqrt{\text{mm}}\cdot\sqrt{1000}$ *, a_{ce} = 55.4* mm.
SIF-curve: reference stresses in MPa, to factor by $\sqrt{1000} = 31.6$: $\sigma_2 = 15 > \sigma_c = 12.5 > \sigma_1 = 10$, *Table* 1 .
(For simplification the simple letter a was taken in the formulas instead of ae)

Note:

The R-curve does not run out from a_0. This is caused because just the test data domain has to be fitted best. In the HSB sheet this end is therefore not sketched. The model point K_b lies on the a_0-line.

► The test-based R-curve is essential for FFM to determine in future a more correct fracture toughness value K_{app} instead of the previous K_{Ic} for the usually FFM-treated very brittle material.

4 Analysis using Finite Fracture Mechanics (FFM)

4.1 General

To prove Structural Integrity several design verifications (DVs) must be performed for components having the following features: Smooth, notched (stress concentrations) and cracked (stress singularities), see *Fig.4, left*. Thereby, static and cyclic loadings must be taken into account focusing uni-axial and multi-axial stress states.

FFM-focus here is static loading under uni-axial stresses, which means Mode I-linked.

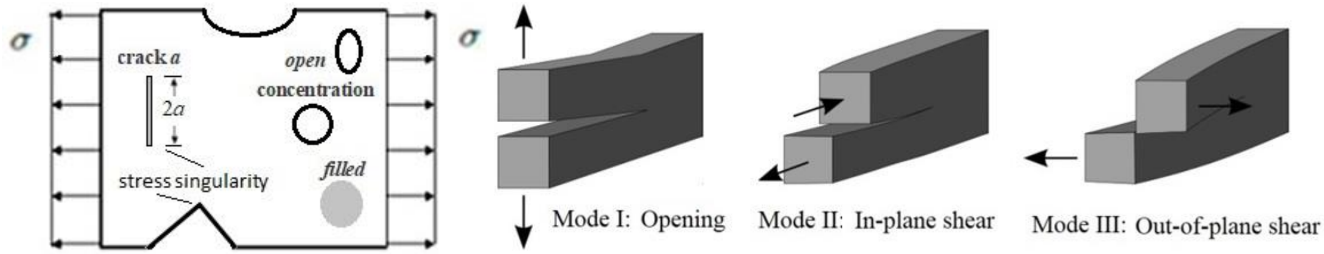


Fig. 4: (left) Stress concentrations and stress singularities under uni-axial stressing. (right) The 3 FM-modes, crack length a

The following levels are relevant when generating stress-related DV tools:

1. Stresses: *Strength Failure Conditions (SFC), as local design verifications* to predict onset-of-cracking (*several strength fracture failure modes and one yield mode,* practically just one for tension loading,)
2. Stress concentration: *Application of (local) stress concentration factors K_t* to predict onset-of-cracking (fracture) for the assessment of these internal discontinuity-caused probably locally infinite, singular stresses.
3. Stress intensity (singularity): (*non-local*) *Fracture mechanics methods using stress intensity factors $K \sim \sigma \cdot \sqrt{\pi \cdot a}$ (SIFs) and fracture toughness (*representing the resistance of brittle materials to the propagation of flaws under an activated stress, assuming: the longer the flaw, the lower the bearable fracture stress) *being a critical K_c which is needed for a crack to grow under monotonic loading. For the usually envisaged tension loading* (pressure-linked geo-mechanics is not the focus) *there are three fracture mechanics modes to consider as depicted in Fig.4 above.*

All design verifications are required <u>in parallel</u> in accordance with the applicable regulations.

Tackling above three structural cases, then it can be attributed:

1. Stresses: *In the strength* fracture *failure criterion (SFC) strength values R (isotropic: here $R^t \equiv R_m$) are to insert, which capture any flaws and micro-cracks in the material data set of the test specimen. All effects are considered.*
2. Stress concentrations: *Experience tells that the application of a SFC with the application of a factor K_t is not sufficient. Here, a non-local DV method is required, which combines a strength fracture criterion and fracture mechanics criterion. This is the focus of FFM.*
3. Stress intensity: The necessary *('large') crack size value is identified by Quality Assurance or fixed as the minimum measurable crack size. The crack situation at hand is to model and toughness values K_{Ic} are to insert. A large crack analysis does not need a coupled DV in order to predict onset-of- further cracking, because the SFC is fulfilled.*

<u>Note</u>:
There are stress-related and strain-related SFCs. Stress-related ones have the advantage, compared to strain-related ones that "*Residual stresses can be simply incorporated"*).

4.2 Introduction

Since about 20 years Finite Fracture Mechanics (FFM) intends to fill the gap between the continuum mechanical strength failure criteria (SFC) and the classical FM. FFM is an approach to offer a criterion to predict ‘Onset-of-Cracking’ in brittle isotropic and UD materials. This is a bridge that had to be built from strength failure to fracture mechanics failure.

Attempts to link SFC-described ‘Onset-of-Cracking (OoC, fracture)’ prediction methods and FM prediction methods for structural components have been performed. Best known is the still cited Hypothesis of Leguillon “*A crack is critical when and only when both the released energy and the local stress reach critical values along an assumed finite crack*”. Within the FFM, Leguillon assumes instantaneous cracks of finite length Δa. Thus, using FFM one obtains one more unknown but also one more equation to solve together with the SFC the equation system.

Of the basic two previous FFM concept variants, the integral concept used here has proven to be the best. In this case, the stress curve is averaged over the fictitious, critical crack length for the SFC, i.e. converted into a locally evenly distributed stress curve averaged over this length.

As long as this is done over a comparatively small area, this is fine, but if it is a very large crack depth, where the crack extends far into an area of the stress profile where the stress peak has already been significantly reduced, the stress value averaged in this way becomes quite small. The question then is whether this procedure can still lead to a valid SFC application. In the future therefore, it would make sense to limit the range over which the stress curve is averaged appropriately in such cases?

This coupled criterion does not refer to microscopic mechanisms to predict micro-crack nucleation.

Reasons to develop the FFM were some facts from studying ‘Onset-of-Cracking’:

- <u>Isotropic material</u>

  The minor failure behavior of absolutely small holes compared to large holes, although the stress concentration factor K_t takes the same value, namely 3. With large holes, more material volume is highly stressed and thus physically-based the probability of failure due to more activated, material-inherent flaws is increased.

  Further known is in the case of discontinuities such as notch singularities with steep stress decays: only a *toughness + characteristic length*-*based energy balance condition* may form a sufficient set of two fracture conditions. Hence, a SFC is a necessary condition but might not be a sufficient condition for the prediction of ‘Onset-of-Cracking’.

  When applying SFCs usually ideal solids are viewed which are assumed to be free of essential micro-voids or microcrack-like flaws, whereas applying Fracture Mechanics tools the solid is considered to contain macro-voids or macro-cracks.

- <u>Transversely-isotropic material</u>

  It is also known for a long time from the so-called ‘Thin layer effect’ of UD-layer-composed laminate that the SFC-application is not sufficient to understand failure: *Due to being strain-controlled, the material flaws in a thin lamina cannot grow freely up to micro-crack size in the thickness direction, because the neighboring laminas act as* *micro-crack-stoppers*. In other words: Thin plies, embedded in a laminate, fail at a higher loading level than thick ones.

Employing here fracture mechanics, the strain energy release rate, responsible for the development of damage energy in the 90° plies - *from flaws into micro-cracks and larger cracks* -, increases with increasing ply thickness. Therefore, the actual absolute thickness of a lamina in a laminate is a driving parameter for initiation of cracks, i.e. [*Fla82*].

4.4 FFM modelling, isotropic material focused

The FFM concept is demonstrated here by the example "Uni-axially loaded symmetric open-hole plate strip". For this case, the coupled criterion can be simplified and can be analytically solved. Thereby no initial crack a_0 is to treat. Brittle fracture behavior is presumed.

The energy criterion postulates that the critical energy release rate $\mathcal{G}_{Ic} = K_{Ic}^{2} \cdot (1 - \nu^2) / E$, being proportional to the square of the fracture toughness, is met and that the stress criterion = SFC postulates that the concentrated stress within the net-section area, *averaged along the crack length Δa*, reaches a material strength value. This averaging is an assumption, which should to be checked.

Whereas the FM is more concerned about the full net section width, in the FFM the concern is basically just the net section length Δa, a portion of the width!

The coupled FFM criterion

Goal of the coupled FFM criterion is to derive two fracture conditions, a strength R-related one and a fracture mechanical one assuming a crack of the size Δa. Finally the two conditions are equated and deliver an equation for the unknown critical crack Δa_c being the crack level at which OoC would occur under a critical stress and fracture mechanical condition, simultaneously.

The establishment of the coupled model is to perform on basis of average properties in order to obtain the optimally achievable reliability of 50 %. This means model validation, whereas in the DV statistically based Design Allowables are to apply.

The two parts of the coupled criterion can be expressed by equalities from a Fracture Mechanics (FM) criterion and a Strength Failure Criterion (SFC):

$$\text{FM:} \quad \frac{1}{\Delta a} \cdot \int_r^a K_1^2(a) \cdot da = K_{1c}^{\,2} \quad \text{and} \quad \text{SFC:} \quad \frac{1}{\Delta a} \cdot \int_r^a \sigma(x, y=0) \cdot dx = R_{\mathrm{m}} .$$

For a simpler comparison, for the SFC the square usually is taken, whereby – advantageously - the remote stress σ cancels out in the coupled equation. Fracture failure occurs if both these criteria are simultaneously fulfilled. This leads to the required equation for the determination of the generated critical crack size Δa_{c} via

$$\frac{\frac{1}{\Delta a} \cdot \int_r^a K_I^2(x) \cdot dx}{\left(\frac{1}{\Delta a} \cdot \int_r^a \sigma_y(x) \cdot dx \right)^2} = \frac{K_{Ic}^{\,2}}{(R_m)^2} = c_{KR} .$$

Later, the author will use the upper single versions, because this better displays the parallel working of FM-condition together with the SF-condition.

As the two required resistance quantities are not fully clear and not given, it is sufficient for the following first numerical application of the FFM to apply the available values '*Plain Strain Fracture Toughness'* K_{Ic} (the inherent lowest material property, subject to certain minimum geometric test specimens requirements to achieve a plain strain condition), and tensile strength R_m. This will mean the application to a brittle metal. In general, the real critical fracture toughness should be termed '*Apparent Fracture Toughness'* K_{app} (to be understood as a component property, adapted to the current geometrical conditions). For K_{app} seldom a value is available. Hence, K_{Ic} will be used for the FFM here, despite of the necessity to consider small scale yielding at the crack tip when using structural metal materials, like shown in the chapter R-curve.

Validation of the FFM model is effort-fully to be performed by running isotropic test series for different w/d-ratios of panels.

.5 Design Verification of a 'Through center cracked Open hole Panel'

Presumptions and given data for geometry, loading from testing

Presumptions:

- Linear Structural Analysis permitted
- Not fully brittle materials which generate small scale yielding at the crack tip
- Worst case loading situation, no residual *stresses.*

Material resistance: Aluminum alloy 7475-T7351 in L(ength)-T(ransverse) direction, example from [3]

- R-curve: A = 55.7 mm, B = 0.75, $K_{as} = 246\,\text{MPa}\cdot\sqrt{\text{m}}$, K_b =29 $\text{MPa}\cdot\sqrt{\text{m}}$. R_m = 850 MPa
- Yield strength: R_{p02} = 425 MPa (B-value, for t = 6...38 mm), HSB 62232-03. Concluding the 445 MPa, as used in HSB 62232-01, can be seen an average value.
- $K_{Ic} = 48\ \text{MPa}\cdot\sqrt{\text{m}} = 1518\ \text{MPa}\cdot\sqrt{\text{mm}}$, $(K_{Ic} / R_m)^2 = 3.23\ \text{mm}^{-1}$.

Panel dimensions

- Width w =300 mm, thickness t = 8 mm, open hole radius d = 25 mm
- Initial crack size a_0 = 30 mm.

Loading Action with Design Factor of Safety (FoS)

- j =1, Design Limit Load representative
- Uni-axial stress state $\{\sigma\}_{\text{design}} = \{\sigma_L\}\cdot j$ with $\{\sigma\}_L = (\sigma_x, \sigma_y, \tau_{xy})^T \cdot j = (0,\ 250,\ 0)^T$ MPa.

5.1 *Application of FM, R-curve, concerning 'Open hole panel fracture', pre-crack a_0*

See *Fig.3* with the procedure attached. a_0 = 30 mm, d = 25 mm, w = 300 mm.

Design case: Remote loading stress $\sigma_{\text{design}} = 250\ \text{MPa} \equiv \sigma_I$.

5.1.1 Determination of the residual strength [HSB 62232-03] with the R-curve

The computation in Table 1 delivers the following values in the instability point (touch point)

FM-resistance: $K_{ec} = 180\ \text{MPa}\cdot\sqrt{\text{m}} = 180\cdot\sqrt{1000}\ \text{MPa}\cdot\sqrt{\text{mm}}$, proof in *Fig.3*

and further residual strength σ_c = 396 MPa and critical crack length a_{ec} = 55 mm.

Above remote failure stress = structural residual strength of the panel (plate strip) reads

$$\sigma_{\text{fail}} = R_{\text{res}} = R_{\text{struct}} = \sigma_c \,.$$

For comparison, the following analyses deliver the satisfactory information:

* Stress concentration: $\sigma_{\text{fail}} = R_{\text{m}} \,/\, K_{\text{t}}(d = \infty) = 850 / 3 = 283$ MPa $> \sigma_I = 250$ MPa.
* Fracture Mechanics: for a Quality Assurance-defined crack size such as $a_{\text{defined}} = 33$ mm,

$$\sigma_{\text{fail}}(33) > \sigma_{\text{fail}}(55)\,.$$

► <u>Computation of the *Reserve Factor* for Design Limit load level</u>, Design Load case $j = 1$

Linear analysis is sufficient (presumption of FFM model at hand): then $\sigma \sim$ load.

$$RF = \frac{\text{Structural strength Design Allowable } R_{\text{struct}}}{\text{Stress } \sigma \text{ at } j \cdot \text{Design Limit Loading}} = \frac{R_{\text{struct}}}{\sigma_{\text{design}}} = \frac{396}{250} = 1.58 \;>\; 1.$$

According to the regulations, R_{struct} has to be a Design Allowable too, which is assumed here due to R_{m} being a strength Design Allowable and K_{Ic} being statistically–based, too.

<u>Yielding Check</u> in the net-section: as a limit-of-usage check. One obtains:

$$\sigma_{\text{fail}} = \sigma_{\text{netyield}} = R_{\text{p02}} \cdot \left(1 - \frac{2 \cdot a_{\text{ec}}}{w}\right) = 425 \cdot \left(1 - \frac{2 \cdot 55}{300}\right) = 267 \text{ MPa} \;\rightarrow\; RF = \frac{267}{250} = 1.14 \;>\; 1.$$

<u>Result</u>: Due to the requirement $\sigma_{\text{netyield}} < \sigma_{\text{c}}$ net section yielding limits the loading here.

5.1.2 Determination of the critical crack length, touch point, considering 'no hole, 'with hole'

In the effective curve (*index e is written*) defined by $K_{ec} = 180 \text{ MPa} \cdot \sqrt{\text{m}}$ the plastic zone ω and the hole diameter are included.

The computation of the critical data set had to be still performed for the establishment of *Fig.3*.

► <u>Computation of the design stress-linked Touch Point + generated crack growth</u> $\Delta a_{\text{design loading}}$

Employing both the SIF functions from *§ 3.3.1*

$$K_{nh}(a) = \sigma \cdot \sqrt{\pi \cdot a} \cdot \sqrt{\sec \frac{\pi \cdot a}{w}} \;\;, \text{ and}$$

$$K_{e,wh}(a) = \sigma \cdot \sqrt{\pi \cdot a} \cdot \sqrt{1 - \frac{r}{a} \cdot (1 + 0.358 \cdot \frac{r}{a} + 1.425 \cdot \left(\frac{r}{a}\right)^2 - 1.578 \cdot \left(\frac{r}{a}\right)^3 + 2.156 \cdot \left(\frac{r}{a}\right)^4} \cdot \sqrt{\sec \frac{\pi \cdot r}{w} \cdot \sec \frac{\pi \cdot a}{w}}$$

the Mathcad computation in Table 2 was executed. (See [3]).

<u>Results:</u>

The crack grew under the design stress by $\Delta a_{\text{design loading}}$ *= 3 mm.*

→ *new* $a_0 = a_0 + \Delta a_{\text{design loading}}$ *= 30 +3 = 33 mm.*

Table 2 Derivation of a ductility-considering SIF K with improved associate crack a

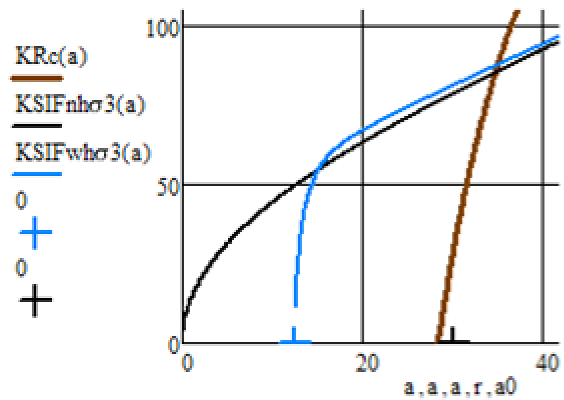
Kec = 180 Kas = 246 Kb = 29 A = 34.682 B = 0.75 a0 = 30 w = 300 Rp02 = 425

Vorgabe a := 33

Crack, no hole

$$sig3\cdot\sqrt{\pi\cdot a}\cdot\sqrt{Sek\left(\frac{\pi\cdot a}{w}\right)} = Kas-(Kas-Kb)\cdot\frac{1-B}{\exp\left(\frac{a-a0}{A}\right)-B}$$

Aa := Suchen(a) Aa = 32.734 a := Aa a = 32.7



Additional information: Determination of the (physical) K_p from the effective values a_e

There are two methods to determine data of a R-curve The Potential method is used to determine physical data and the Compliance-Method (*applied here*) effective data for the given initial crack length a_0 and the loading stress σ [13, 12, 11].

If necessary, physical data can be derived from effective data by inserting

$$a_p = a_e - 0.5\cdot\omega, \quad \omega = \frac{1}{\pi}\cdot\left(\frac{K_p}{R_{p0.2}}\right)^2 \quad \text{into} \quad K_p = \sigma\cdot\sqrt{\pi\cdot a_p}\cdot\sqrt{\sec\left(\pi\cdot a_p / w\right)}$$ solving the

generated implicit equation via

Vorgabe Kp := 180

$$Kp = \sigma\cdot\sqrt{\pi\cdot\left[ae-0.5\left[\frac{1}{\pi}\cdot\left(\frac{Kp}{Rp02}\right)^2\right]\right]}\cdot\sqrt{\frac{1}{\cos\left[\pi\cdot\frac{ae-0.5\left[\frac{1}{\pi}\cdot\left(\frac{Kp}{Rp02}\right)^2\right]}{w}\right]}}$$

D := Suchen(Kp)

.

Whether this might be important could be checked by inserting K_{pc} through K_{ec} calculating

$$\omega_{Kec} = \frac{1}{\pi}\cdot\left(\frac{K_{ec}}{R_{p0.2}}\right)^2 .$$

In order to present a good feeling for the difference between K_p and K_e the respective values shall be computed below for the critical case, indexed $_c$:

$\sigma fail := 12.5 \quad \sigma := \sigma fail \cdot \sqrt{1000} \quad \sigma = 395 \quad Rp02 := 425 \quad w := 300 \quad a0 := 30 \quad Kec := 180 \cdot (\sqrt{1000})$

$aec := 55.4 \quad \Delta aec := aec - a0 \quad \Delta aec = 25.4 \text{ mm} \quad Kec = 5692$

Vorgabe $\quad Kpc := 5555$

$$Kpc = \sigma \cdot \sqrt{\pi \cdot \left[aec - 0.5\left[\frac{1}{\pi}\cdot\left(\frac{Kpc}{Rp02}\right)^2\right]\right]} \cdot \sqrt{\frac{1}{\cos\left[\pi \cdot \frac{aec - 0.5\left[\frac{1}{\pi}\cdot\left(\frac{Kpc}{Rp02}\right)^2\right]}{w}\right]}}$$

$D := \text{Suchen}(Kpc) \quad D = 4479$

$\omega ec := 0.5\left[\frac{1}{\pi}\cdot\left(\frac{Kec}{Rp02}\right)^2\right] \quad \omega pc := 0.5\left[\frac{1}{\pi}\cdot\left(\frac{Kpc}{Rp02}\right)^2\right] \quad apc := aec - 0.5\cdot(\omega ec)$

$\omega ec = 28.5 \quad \omega pc = 27.2 \text{ mm} \quad apc = 41.1$

5.2 *Application of FFM*, *concerning 'Onset-of-Cracking' at a open hole edge, (no a_0)*

Determination of finite crack length Δa and failure stress of the panel: *Mathcad 15 application*

In this sub-chapter the 'classical' FFM-procedure with the square will be presented.

- The FM-linked failure portion: The equation reads:

$$\frac{1}{\Delta a}\cdot\int_r^a\left[K_I^2(a)\right]\cdot dx =$$

$$\frac{1}{\Delta a}\cdot\left[\int_r^a(\sigma\cdot\sqrt{\pi\cdot a}\cdot\sqrt{1-\frac{r}{a}}\cdot(1+0.358\cdot\frac{r}{a}+1.425\cdot\left(\frac{r}{a}\right)^2-1.578\cdot\left(\frac{r}{a}\right)^3+2.156\cdot\left(\frac{r}{a}\right)^4\cdot\sqrt{\sec\frac{\pi\cdot r}{w}\cdot\sec\frac{\pi\cdot a}{w}}\right]^2\cdot dx$$

- The SFC-linked failure portion: For details see *Annex1*

  For this portion a model for the stress distribution along the net section is to provide, namely,

$$\sigma_{\text{netsec}}(x) = \sigma\cdot c_{\text{wd}}\cdot[0.335+0.665\cdot(1+c_{11}\cdot\frac{x-r}{0.5\cdot w-r})^{c_{12}}+c_{13}\cdot(\frac{x-r}{0.5\cdot w-r})^4\ , \ [8]$$

with the abbreviation functions $c_{\text{wd}} = 3.215 - (\frac{w}{d})^{-0.5} + 4.294\cdot(\frac{w}{d})^{-1.5}$ and

$c_{11} = -3.765 + 2.148\cdot(\frac{w}{d})^{0.879}$, $c_{12} = -2.552 - 42.894\cdot(\frac{w}{d})^{-3.17}$, $c_{13} = -0.7497\cdot(\frac{w}{d})^{-1.858}$.

The equilibrium equation of the SFC-portion reads

$$\frac{1}{\Delta a}\cdot\int_r^{r+\Delta a}\sigma(x,y=0)\cdot dx = \frac{1}{\Delta a}\cdot\int_r^{r+\Delta a}\sigma_{\text{netsec}}(x)\cdot dx =$$
$$\frac{1}{\Delta a}\cdot\int_r^{r+\Delta a}\sigma\cdot c_{\text{wd}}\cdot[0.335+0.665\cdot(1+c_{11}\cdot\frac{x-r}{0.5\cdot w-r})^{c_{12}}+c_{13}\cdot(\frac{x-r}{0.5\cdot w-r})^4\cdot dx \ .$$

The implicit FFM-solution procedure of the Mathcad software in standard FFM-formulation is shown below:

Vorgabe

$$\Delta a := 1 \qquad \sigma := 11$$

$$\frac{\left[\frac{1}{\Delta a}\cdot\left[\int_{r}^{r+\Delta a}\left[\left[\sigma\cdot\sqrt{\pi\cdot x}\cdot\left[\sqrt{1-\frac{r}{x}}\cdot\left[1+0.358\cdot\frac{r}{x}+1.425\cdot\left(\frac{r}{x}\right)^{2}-1.578\cdot\left(\frac{r}{x}\right)^{3}+2.156\cdot\left(\frac{r}{x}\right)^{4}\right]\right]\right]\cdot\sqrt{\mathrm{Sek}\left(\frac{\pi\cdot r}{w}\right)\cdot\mathrm{Sek}\left(\frac{\pi\cdot x}{w}\right)}\right]^{2}dx\right]\right]}{\frac{1}{\Delta a}\cdot\left[\int_{r}^{r+\Delta a}\sigma\cdot crw\cdot\left[0.335+0.665\cdot\left(1+c11\cdot\frac{x-r}{0.5\cdot w-r}\right)^{c12}+c13\cdot\left(\frac{x-r}{0.5\cdot w-r}\right)^{4}\right]dx\right]} = \frac{KIc^{2}}{Rm}$$

$$\frac{1}{\Delta a}\cdot\left[\int_{r}^{r+\Delta a}\sigma\cdot crw\cdot\left[0.335+0.665\cdot\left(1+c11\cdot\frac{x-r}{0.5\cdot w-r}\right)^{c12}+c13\cdot\left(\frac{x-r}{0.5\cdot w-r}\right)^{4}\right]dx\right] = Rm$$

$$A := \mathrm{Suchen}(\Delta a,\sigma) \qquad A = \begin{pmatrix}1.77\\420.83\end{pmatrix} \qquad \Delta a := A_0 \qquad \Delta ac := \Delta a \qquad \sigma := A_1 \qquad \sigma fail := \sigma \qquad \Delta ac = 1.77 \qquad \sigma fail = 421$$

Results:

Within the FFM, two models from FM and from strength analysis are commonly employed to predict the failure event 'Onset-of-Cracking' at a non-cracked hole. In the case at hand, the instantaneously generated finite crack length reads $\Delta a_c = 1.77$ mm *and the associated remote average* structural *failure stress of the panel* σ_{struc} *reads* σ_{fail} *= 421 MPa.*

Fig.5 finally tries to illustrate the FFM hypothesis "*Both the conditions must be fulfilled*". It points out the failure-causing relationship and the dominated domains, where stress states may happen to be.

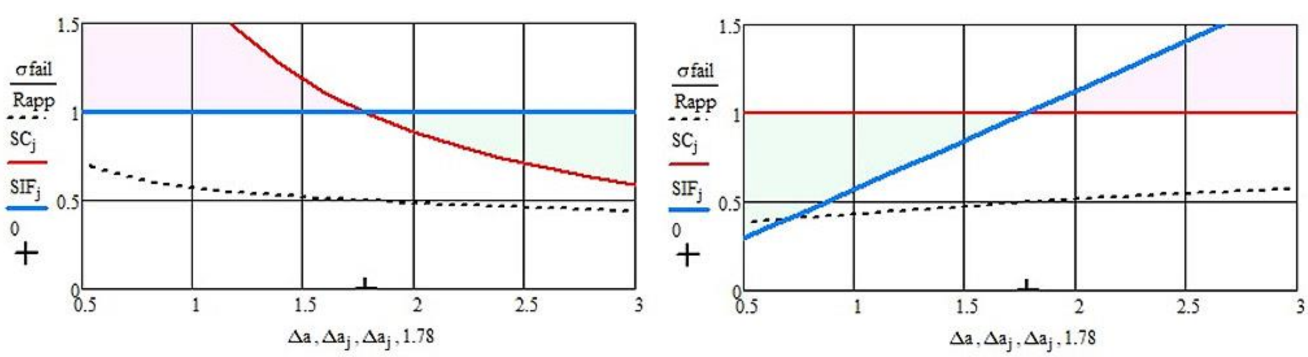


Fig.5: (left) 'SIF' assumed 100% with the question "When does the SC not show failure?
Vice versa: (right) SC assumed 100% with the question "When does the 'SIF' not show failure?

One basic interest is how a varying resistance ratio $c_{KR} = K_{Ic}^2/R_m^2$ affects critical **crack length** and **failure stress**. *Fig.6* shows the mapped numerical results for a number of ratios.

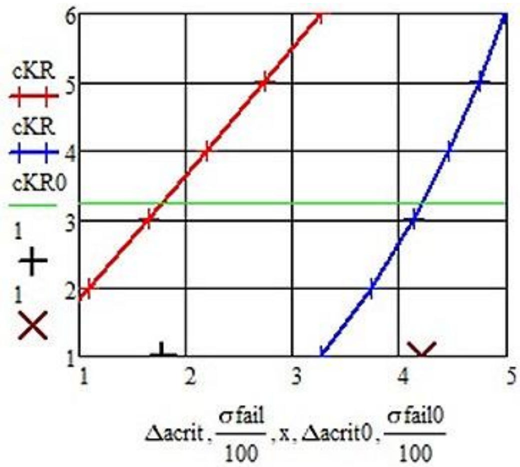


Fig.6, w=36mm , a_0 = 30 mm, d =6 mm: Effect of varying resistance ratio c_{KR} on Δa_c and σ_{fail}.

AA 7475-T7351: $c_{KR0} = (K_{Ic} / R_m)^2 = 3.23\ \text{mm}^{-1}$

Result: With increasing resistance ratio both critical crack size and failure stress naturally grow.

Of further interest might be how the FM-linked and the SC-linked portions change with the crack length. *Fig.7* depicts these courses after employing the two integrals, termed 'SIF' and SC, below.

$$\frac{\frac{1}{\Delta a}\cdot\int_r^a K_I^2(x)\cdot dx}{\left(\frac{1}{\Delta a}\cdot\int_r^a \sigma_y(x)\cdot dx\right)^2} = \frac{K_{Ic}^2}{(R_m)^2} = c_{KR} \quad \Rightarrow \quad \frac{\frac{1}{\Delta a}\cdot\int_r^a K_1^2(x)\cdot dx \,/\, K_{1c}^2}{\left(\frac{1}{\Delta a}\cdot\int_r^a \sigma_y(x)\cdot dx\right)^2 / R_m^2} = \frac{\text{'SIF'}}{\text{SC}}$$

with the components

$$\text{SIF}_i := \frac{\frac{1}{\Delta a_i}\cdot\int_r^{r+\Delta a_i}\left[\sigma\cdot\sqrt{\pi\cdot x}\cdot\left[\sqrt{1-\frac{r}{x}}\cdot\left[1+0.358\cdot\frac{r}{x}+1.425\cdot\left(\frac{r}{x}\right)^2-1.578\cdot\left(\frac{r}{x}\right)^3+2.156\cdot\left(\frac{r}{x}\right)^4\right]\right]\cdot\sqrt{\text{Sek}\left(\frac{\pi\cdot r}{w}\right)\cdot\text{Sek}\left(\frac{\pi\cdot x}{w}\right)}\right]^2 dx}{\text{KIc}^2}$$

$$\text{SC}_i := \frac{\frac{1}{(\Delta a_i)^2}\cdot\left[\int_r^{r+\Delta a_i}\sigma\cdot\text{crw}\cdot\left[0.335+0.665\cdot\left(1+\text{c11}\cdot\frac{x-r}{0.5\cdot w-r}\right)^{\text{c12}}+\text{c13}\cdot\left(\frac{x-r}{0.5\cdot w-r}\right)^4\right]dx\right]^2}{\text{Rm}^2}$$

Result: The critical point at a_c = 1.78 mm is clearly outlined at 'SIF' = SC.

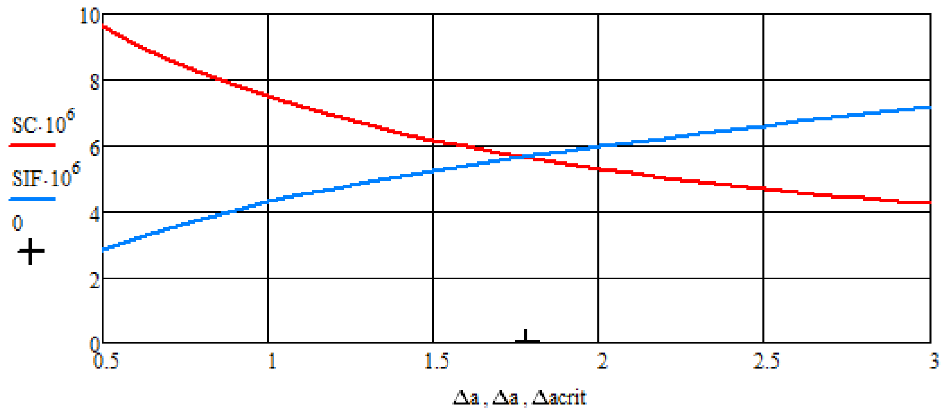


Fig.7, w =36mm, a_0 = 30 mm, d =6 mm: Representation of the course of the growing FM-portion (SIF) and the decaying Strength Mechanics portion (SC)

After having depicted the influence of the resistance ratio $c_{KR} = K_{Ic}^2/R_m^2$ in *Fig.6* the effect of a fixed ratio 'panel width/hole diameter' w/d shall be displayed for two widths in *Fig.8* presenting how the remote failure stress σ_{fail} of the panel changes with Δa.

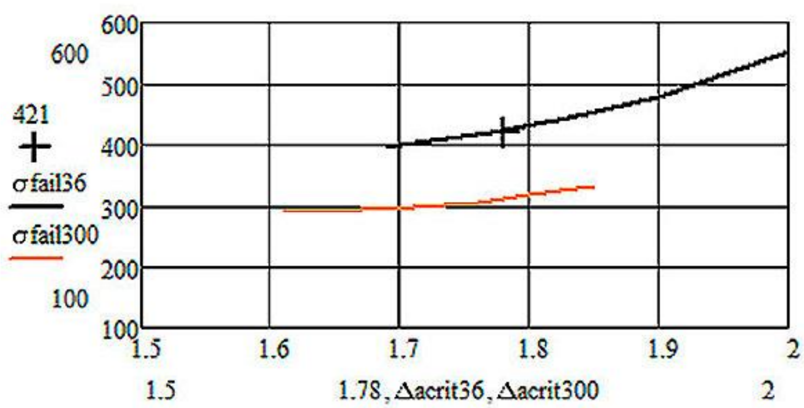


Fig.8, w=36mm, w = 300mm: Effect of different panel geometry, ratios w/d = 6.

Result:
For a given resistance ratio c_{KR}, for two panel widths, above stress failure curves are plotted as functions of the individually given critical crack size. The wider panel allows a lower stress only, because more volume is highly stressed.

► Computation of the *Reserve Factor* for Design Limit load level, *j* =1

Remote loading stress $\sigma = \sigma_I$ = 250 MPa, a_0 = 30 mm, d =25 mm.

Linear analysis is sufficient (presumption of FFM model): then $\sigma \sim$ load

Assumed σ_{fail}, to be a Design Allowable, the Reserve Factor against 'Onset-of-Cracking' at the hole edge is

$$RF = \frac{\sigma_{struct}}{\sigma_{design}} = \frac{421 \text{ MPa}}{250 \text{ MPa}} = 1.7 \; .$$

According to the regulations, a Design Allowable has to be applied, too, which is assumed here, because R_m is a Strength Design Allowable and K_{Ic} is assumed to be statistically based.

Yielding Check in the net-section:, as a limit-of-usage check. One obtains:

$$\sigma_{\text{fail}} = \sigma_{\text{netyield}} = R_{\text{p02}} \cdot \left(1 - \frac{2 \cdot a_{\text{ce}}}{w}\right) = 425 \cdot \left(1 - \frac{2 \cdot 55.4}{300}\right) = 268 \text{ MPa}.$$

$$RF = \frac{268}{250} = 1.14 \ > \ 1.$$

Result: Due to the requirement $\sigma_{\text{netyield}} < \sigma_c$ *net section yielding limits the loading here.*

7 Application of the FFM to an HSB-example

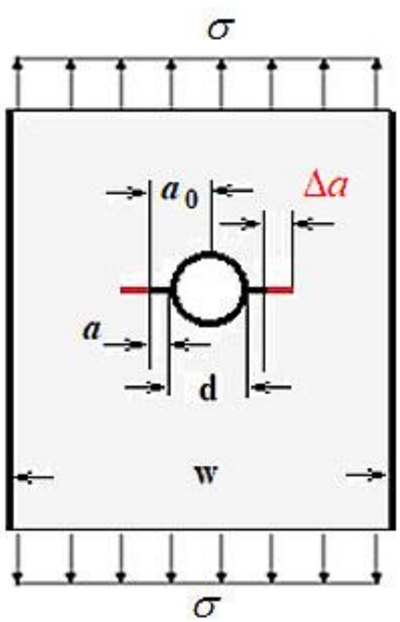
Task: Mapping of the critical stress 'σ_c-curve' as function of the running crack size a.

The course of just 3 test points of a fixed open hole panel (from HSB 62232-01 on '*Width dependency of the Feddersen-parameter'*, [10], is to map. These fracture values are given for the original $a_0 = a + \text{r}$, also depicted in the plot.

> *Note, please: The 3 test points with the different crack sizes are assumed average values. (1) In this context, in the HSB sheet the sample size number of tests belonging to one 'average' point was not given. (2) Further, an additional fitting process of the foreseen correction function was performed.*

Fig.9, left, displays the geometry and the loading of the envisaged HSB-panel. The coordinate x has its origin in the hole center.

Fig.9 right, presents the course of the SIF K and of the net section stress along x.



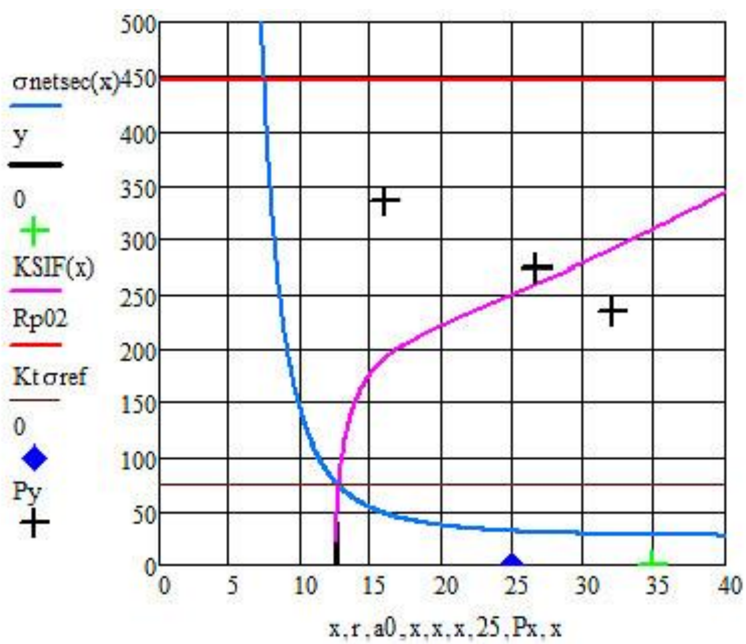


Fig.9: (left) Geometry of the fixed Open Hole Panel and its uniaxial loading.
(right) Test points with the courses of the 'SIF' and the net section stress in width x-direction
w =160 mm, t =2 mm, d =25 mm. *AA 7475-7761:* R_{p02}= 445 MPa, $K_{c,\infty}$ = 2500 MPa $\cdot \sqrt{\text{m}}$.
Abscissa points in mm: x = r = 12.5, x = 25, x= 35 $K_{t\sigma ref}$ = +

Result:
Shifting the FFM failure stress point by Δa gives a point a little far from the derived FM-curve, This crack size Δa defines the a_0 when analyzing future loading and crack growth.

In *Fig.10* for the given hole, d = 2·r, the computed FFM-linked failure stress point σ_{fail} (bold) is depicted with the generated crack size Δa. The Mathcad computations are presented in *Table 3*.

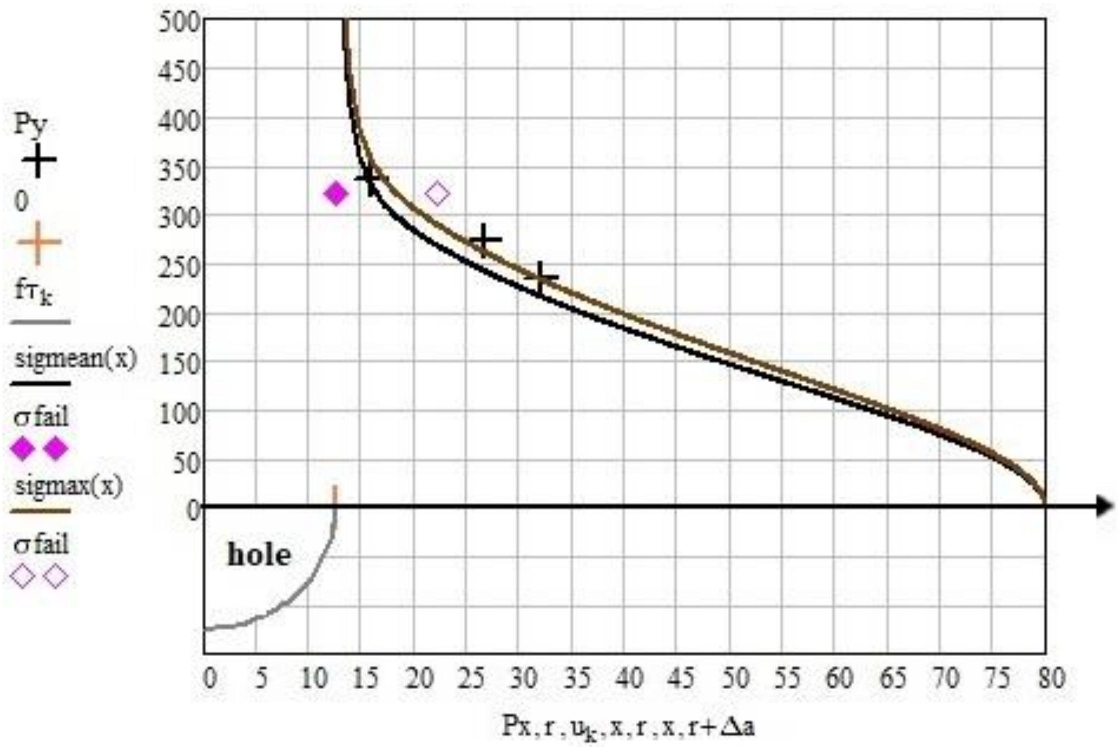


Fig.10: (a) Depiction of the FFM-based failure stress at 'Onset-of-Cracking' generating Δa.
(b) FM-based mapping of the course of the three test points with its initial different crack size a_0,
Application of two different K-values, r =12.5 mm

The Mathcad computation is presented in *Table 3*.
The upper part depicts the classical FFM procedure and the center the Cuntze procedure with directly using the single equations.

Result:
Both, the procedures end with the same numbers.

Also a FM-linked mapping of the three test point examples with its initial crack sizes a_0 was successfully performed, see the bottom of *Table 3*. Thereby the SIF *K* was varied, a mean and a maximum assumed value was applied.
This might be of interest for a rework of the 'Feddersen parameter sheet' HSB 63321-06.

Result:
Mapping was successful. The difference vanishes at both the ends.

Table 3, Mathcad computations:
*(up) Standard FFM procedure (*using the square*), however solved without necessary iterations, $a_0 = 0$*
(center) Cuntze's procedure: separate FFM- and SFC-equation, $a_0 = 0$
(down) FM-based mapping of the three test points with its individual initial cracks a_0

Vorgabe $\quad \Delta a := 11 \quad \boxed{\sigma := 111}$

$$\frac{\left[\frac{1}{\Delta a}\cdot\left[\int_r^{r+\Delta a}\left[\left[\sigma\cdot\sqrt{\pi\cdot x}\cdot\left[\sqrt{1-\frac{r}{x}}\cdot\left[1+0.358\cdot\frac{r}{x}+1.425\cdot\left(\frac{r}{x}\right)^2-1.578\cdot\left(\frac{r}{x}\right)^3+2.156\cdot\left(\frac{r}{x}\right)^4\right]\right]\cdot\sqrt{Sek\left(\frac{\pi\cdot r}{w}\right)\cdot Sek\left(\frac{\pi\cdot x}{w}\right)}\right]\right]^2 dx\right]\right]}{\left[\frac{1}{\Delta a}\cdot\int_r^{r+\Delta a}\sigma\cdot crw\cdot\left[0.335+0.665\cdot\left(1+c11\cdot\frac{x-r}{0.5\cdot w-r}\right)^{c12}+c13\cdot\left(\frac{x-r}{0.5\cdot w-r}\right)^4\right]dx\right]^2} = \frac{Kc^2}{Rp02^2}$$

$$\frac{1}{\Delta a}\cdot\left[\int_r^{r+\Delta a}\sigma\cdot crw\cdot\left[0.335+0.665\cdot\left(1+c11\cdot\frac{x-r}{0.5\cdot w-r}\right)^{c12}+c13\cdot\left(\frac{x-r}{0.5\cdot w-r}\right)^4\right]dx\right] = Rp02$$

$A := \text{Suchen}(\Delta a, \sigma) \quad A = \begin{pmatrix} 9.6 \\ 322.28 \end{pmatrix} \quad \Delta a := A_0 \quad \sigma := A_1 \quad \boxed{\Delta a = 9.6} \quad \boxed{\sigma = 322}$

Vorgabe $\quad \Delta a := 11 \quad \boxed{\sigma := 111}$

$$\frac{\left[\frac{1}{\Delta a}\cdot\left[\int_r^{r+\Delta a}\left[\left[\sigma\cdot\sqrt{\pi\cdot x}\cdot\left[\sqrt{1-\frac{r}{x}}\cdot\left[1+0.358\cdot\frac{r}{x}+1.425\cdot\left(\frac{r}{x}\right)^2-1.578\cdot\left(\frac{r}{x}\right)^3+2.156\cdot\left(\frac{r}{x}\right)^4\right]\right]\cdot\sqrt{Sek\left(\frac{\pi\cdot r}{w}\right)\cdot Sek\left(\frac{\pi\cdot x}{w}\right)}\right]\right]^2 dx\right]\right]}{Kc^2} = 1$$

$$\frac{\frac{1}{\Delta a}\cdot\int_r^{r+\Delta a}\sigma\cdot crw\cdot\left[0.335+0.665\cdot\left(1+c11\cdot\frac{x-r}{0.5\cdot w-r}\right)^{c12}+c13\cdot\left(\frac{x-r}{0.5\cdot w-r}\right)^4\right]dx}{Rp02} = 1$$

$A := \text{Suchen}(\Delta a, \sigma)$

$A = \begin{pmatrix} 9.6 \\ 322.3 \end{pmatrix} \quad \Delta a := A_0 \quad \sigma := A_1 \quad \boxed{\Delta a = 9.6} \quad \boxed{\sigma = 322}$

$$\text{sigmean}(x) := \frac{2500}{\left[\sqrt{\pi\cdot x}\cdot\left[\sqrt{1-\frac{r}{x}}\cdot\left[1+0.358\cdot\frac{r}{x}+1.425\cdot\left(\frac{r}{x}\right)^2-1.578\cdot\left(\frac{r}{x}\right)^3+2.156\cdot\left(\frac{r}{x}\right)^4\right]\right]\cdot\sqrt{Sek\left(\frac{\pi\cdot r}{w}\right)\cdot Sek\left(\frac{\pi\cdot x}{w}\right)}\right]}$$

Kc − Variants

$$\text{sigmax}(x) := \frac{2700}{\left[\sqrt{\pi\cdot x}\cdot\left[\sqrt{1-\frac{r}{x}}\cdot\left[1+0.358\cdot\frac{r}{x}+1.425\cdot\left(\frac{r}{x}\right)^2-1.578\cdot\left(\frac{r}{x}\right)^3+2.156\cdot\left(\frac{r}{x}\right)^4\right]\right]\cdot\sqrt{Sek\left(\frac{\pi\cdot r}{w}\right)\cdot Sek\left(\frac{\pi\cdot x}{w}\right)}\right]}$$

8 Conclusions, *concerning*

Strength criteria alone or energy-based fracture mechanical criteria alone cannot always lead to a reliable fracture failure prediction. Design Verification (DV) by using a coupled criterion will improve the situation and be an aid for understanding the stress state-depending Onset-of-Cracking. The so-called FFM concept should bring a solution to close the gap. It assumes the formation of cracks of finite size Δa at Onset-of-Cracking.

Fracture Mechanics

The crack-linked residual strength R_{res} is the gross-sectional tensile stress σ at failure of a structural component containing a crack. R of the last fatigue phase is to discriminate from R_{res} in the previous fatigue phase. Thereby, the crack length a_0 at the beginning of the static up-loading will increase to its critical value a_c in general.
A structural component will fail in the case of static loading if the SIF K of a brittle material reaches its critical value at $K = K_c$, termed fracture toughness, which depends on the material behavior. The determination of the K_c values requires in the so-called K-concept used above the fulfilment of a geometric bound in order to achieve a real minimum value by taking a minimum test specimen thickness of

$$t \;>\; 2.5 \cdot (K_{\mathrm{Ic}} \,/\, R_{0.2}^{t})^2 \;\rightarrow\; \sigma_c = K_{\mathrm{Ic}} \,/\, (\sqrt{\pi \cdot a_0} \cdot f\,(a_0)\,.$$

In the less brittle material case the limit reads Ɠ = Ɠc.
The influence of the geometry factor f decreases with the specimen thickness, resulting in fracture toughness independent of the specimen dimensions. For the same materials, the fracture toughness decreases with an increasing yield strength of 0.2 %.

Fig.11 shall illustrate how the failure stress is governed by the crack size. Plastic deformation plays a significant role.

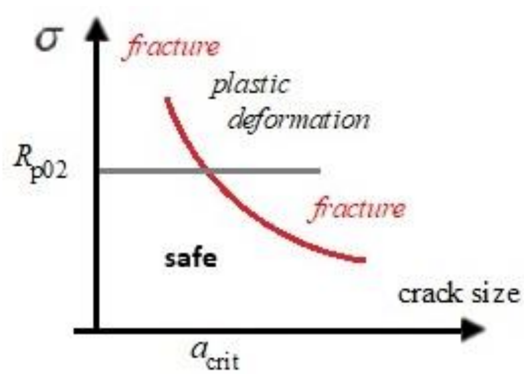


Fig.11: Illustration of the example with the concern plastic yielding

Strength

Dependent on the design requirements the average, the upper or a lower value of the property is used for the various physical properties.
In the case of the resistance property strength a statistically reduced value R is to apply and in order to achieve a reliable design a so-called Strength Design Allowable has to be applied. It is a value, beyond which at least 99% (“A”-value) or 90% (“B”-value) of the population of values is expected to fall, with a 95% confidence (*on test data achievement*) level, see MIL-HDBK 17.

In this context, note please: Measurement data sets are the result of a Test Agreement (norm or standard), that serve the desire to make a comparability of different test procedure results possible. The Test Agreement consists of test rig, test specification, test specimen, test procedure and the test data evaluation method. Therefore, one could only speak about '*exact test results and properties in the frame of the obtained test quality'*.

Test specimens shall be manufactured like the structure, 'as-built'.

Bearable load(ing)

The provision of bearable load(ing)s requires series tests of the distinctive structural component with statistical evaluation in order to determine a structural 'load-resistance design allowable'. This is valid for the FFM applications. See the 3 average open-hole dots in *Fig.10*.

Load-defined Reserve Factor *RF* and design Factor-of-Safety *FoS j*

* A *RF* is usually the result of worst case assumptions that does not take care of the joint actions of the stochastic design parameters and thereby cannot take care of their joint failure action and probability.

* The *RF* value does not outline a failure probability, and failure probability p_f does not dramatically increase if *RF* turns slightly below 1.

* A *FoS* is given and not to calculate such as a the Reserve Factor *RF* .

Application limits linked to FFM

In Design, as with each criterion, validity limits are faced, such as

- Application-extension of linear structural analysis and high brittleness
- Future task to capture small scale yielding at the crack tip which requires the provision of the associate statistically-based toughness K_c-values in order to master Design Verification
- The stress in the net-section of the panel should not exceed the tensile yield strength R_{p02}.
- 3D-application.

Many thanks to my friends, Prof. Dr.-Ing. habil. Wilfried Becker and Dr.-Ing. Jürgen Broede for the excellent exchange on this difficult novel topic FFM.

Annex

1. Course of net-section stress

In the context above and because it is necessary for understanding the FFM an illustration of the stress distribution along the net-section is to provide. In *Fig.12* the curves are depicted for the x- and an integration-simplifying normalized ξ-coordinate, proposed in HSB 34112-11. The relationship reads $x = d/2$:

$$\xi = \frac{2 \cdot x/d - 1}{w/d - 1} = \frac{x - r}{0.5 \cdot w - r} = 0 \quad \text{(hole edge)}$$

and $x = a = d/2 + \Delta a$:

$$\xi = \frac{2 \cdot x/d - 1}{w/d - 1} = \frac{2 \cdot \Delta a / d}{w/d - 1} = \Delta\alpha \text{ , abbreviated .}$$

In [10] was given

$$\boxed{\sigma_y = \sigma_{\text{netsec}}(\xi) = \sigma \cdot K_{\text{t},wd} \cdot [0.335 + 0.665 \cdot (1 + c_{11} \cdot \xi)^{c_{12}} + c_{13} \cdot \xi^4}$$

with the geometry-dependent stress concentration factor K_t (w, d)

$$K_t(\mathrm{w,d}) = 3.215 - (\frac{\mathrm{w}}{\mathrm{d}})^{-0.5} + 4.294 \cdot (\frac{\mathrm{w}}{\mathrm{d}})^{-1.5} \equiv c_{dw}$$

and the abbreviation functions

$$c_{11} = -3.765 + 2.148 \cdot (\frac{w}{d})^{0.879}, \quad c_{12} = -2.552 - 42.894 \cdot (\frac{w}{d})^{-3.17}, \quad c_{13} = -0.7497 \cdot (\frac{w}{d})^{-1.858}.$$

For the example w = 300 mm, d = 25 mm, c_{dw} = 3.03, c_{11} = 19.5, c_{12} = -2.56, c_{13} = -4.9· 10^{-3} follow after normalization by $K_{t,\infty}\ (w=\infty)$=3, and setting a reference stress σ = 100 MPa the following plots:

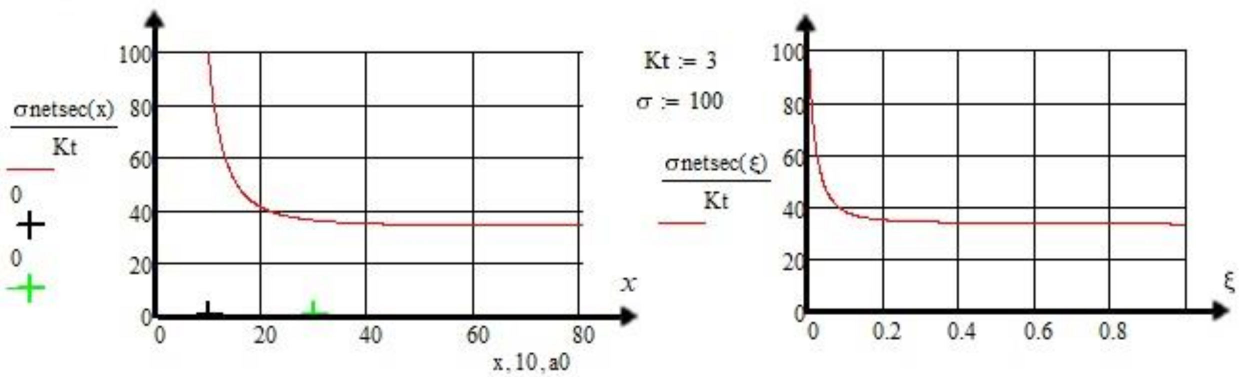


Fig.12: Contour of the stress along the net-section of the panel considering the coordinates x and ξ.

σ_{ref} = *100*MPa, *r =12.5* mm, *a0= 30*mm, $K_{t,\infty}$ =3, *x width coordinate (ligament),*

$$\xi = (x-r)/(0.5 \cdot w - r),$$

Results:

With increasing distance to the hole edge the stresses are monotonically descending whereas the incremental energy release rate $\mathcal{G}$ is monotonically ascending (see *Fig.12).*

2. Integration of net-section stress

HSB 34112-11 computation, retraced:

Applying the afore mentioned coordinate transformation x → ξ enables the following symbolic integration

$$\frac{1}{\Delta a} \cdot \int_r^{r+\Delta a} \sigma_y \cdot dx = \frac{\sigma \cdot c_{\mathrm{wd}}}{\Delta \alpha} \cdot \int_0^{\Delta \alpha} [0.335 + 0.665 \cdot (1 + c_{11} \cdot \xi)^{c_{12}} + c_{13} \cdot \xi^4] \cdot d\xi$$

$$= \frac{\sigma \cdot c_{\mathrm{wd}}}{\Delta \alpha} \cdot \left[0.335 + c_{14} \cdot \frac{(1 + c_{11} \cdot \Delta \alpha)^{c15} - 1}{\Delta \alpha} + c_{16} \cdot \Delta \alpha^4 \right]$$

with $$c_{14} = \frac{0.665}{c_{11} \cdot (c_{12} + 1)}, \quad c_{15} = c_{12} + 1, \quad c_{16} = \frac{c_{13}}{3}.$$

Variant Cuntze:

Despite of the more complicate integration limit $r + \Delta a$ instead of $\Delta \alpha$, the Mathcad solution process allows to stick to the x coordinate, avoiding a mixture of α with a within the solution process. Inserting into the equation above the relationship $\xi = (x-r)/(0.5 \cdot w - r)$ leads to

$$\frac{1}{\Delta a}\cdot\int_{r}^{r+\Delta a}\sigma_y\cdot dx=\frac{1}{\Delta a}\cdot\int_{r}^{r+\Delta a}\sigma\cdot c_{wd}\cdot[0.335+0.665\cdot(1+c_{11}\cdot\xi)^{c_{12}}+c_{13}\cdot\xi^4]\cdot dx$$

$$=\frac{1}{\Delta a}\cdot\int_{r}^{r+\Delta a}\sigma\cdot c_{wd}\cdot[0.335+0.665\cdot(1+c_{11}\cdot\frac{x-r}{0.5\cdot w-r})^{c_{12}}+c_{13}\cdot\left(\frac{x-r}{0.5\cdot w-r}\right)^4\cdot dx.$$

Result:

The solution of the coupled equation delivers the remote failure stress with its associated crack length size Δa , see Table 3, too.

Of interest could be the effect of a varying panel width geometry. *Finally Fig.13 plots the influence of the resistance ratio $c_{KR}= K_{app}^2/R_{app}^2$ on the critical crack size Δa_c . The c_{ik} are the variables:*

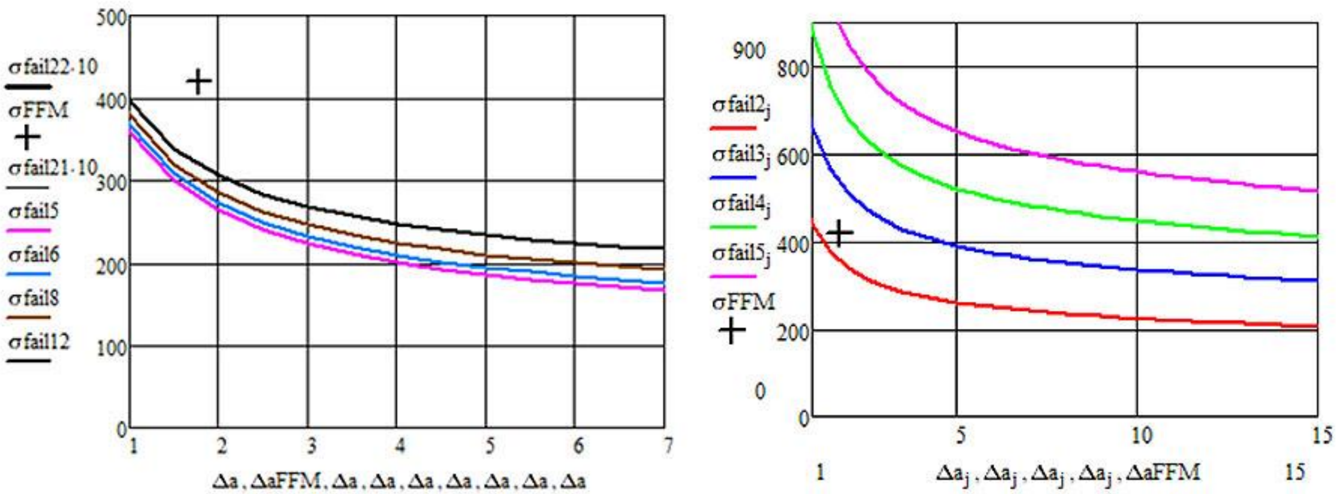


Fig.13 (l eft), general, w = 300mm: Effect of different panel geometry, ratios w/d=5, 6, 8, 12 as variables.
Fig.13 (right), general, w = 300mm: Effect of different resistance ratios KIc^2/Rm^2 = 2, 3, 4, 5

Lessons Learned on FFM and its two parts

FFM:

- *In the case of plain structural parts 'Onset-of-Cracking' in brittle and semi-brittle materials cannot be fully captured by the SFCs, because both a critical energy and a critical stress state must be fulfilled. Therefore, SFCs are 'just' necessary but not sufficient for the prediction of strength failure, onset of cracking. Basically, also due to internal flaws, an energy criterion is to apply*
- *The novel approach 'Finite Fracture Mechanics (FFM)' offers a 2D hybrid criterion to more realistically predict the stress-based 'Onset-of-Cracking' in brittle isotropic (the focus here) and UD materials.*
- *FFM enables to predict a hybrid (coupled) failure stress = a resistance quantity on basis of the resistances of the FFM-parts fracture mechanics (FM) and structural strength (SFC)*
- *FFM is advantageous for the analysis of notched structural parts and captures applications usually treated by the well-known Neuber theory. The coupled FFM-criterion 'SFC-FM' can be used with some confidence to predict onset of cracking (failure) in brittle materials in design situations as never could be done before.*

- *The FMC-application looks successful for the 'open hole panel' example, a realistic failure stress can be estimated.*
- *Unfortunately there is still a lack of test data sets for the validation of FFM*
- *Multi-axial stress states are captured by the principal stress σ_I*
- *Using a locally evenly distributed stress curve averaged over the finite length Δa is to check*

FM (R-curve):

- *It is to regard, when considering the formulations to be applied: Short Cracks behave differently to Large Cracks*
- *It is unbelievable (see the treated HSB example Feddersen concept) that no test results can be found in literature concerning panels with different ratios 'width/hole radius'. Such tests should have been performed when investigating the Neuber theory*
- *Notch surface quality and the metal homogeneity faced have its impacts on the results.*
- *The R-curve does not depend on a_0 and w.*
- *The fracture stress is to base on $a_e = a + \Delta a + \omega$.*
- *Principal stress-linked.*

SFCs Cuntze:

- *Full 3D- stress state-capable.*