EVALUATION OF MULTIAXIAL TEST DATA OF UD-LAMINAE BY SO-CALLED "FRACTURE-TYPE STRENGTH CRITERIA" AND BY SUPPORTING PROBABILISTIC MEANS

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SUMMARY

Overall objective of this elaboration is to calibrate fracture criteria resp. to define size and shape of the fracture body as cheaply as possible by executing the characteristic tests only.

Based on knowledge, achieved by investigating *v. Mises, Mohr - Coulomb* and the new physically based *Hashin - Puck* Strength Criteria for inter fibre fracture (IFF) of brittle unidirectional laminae, a new and general concept for the derivation of fracture criteria will be proposed and applied to fibre reinforced plastics (FRP).
The fracture body derived here is basically piecewise smooth, each piece representing a single *failure mode*. As interpolation functions the *invariants* associated to the material's symmetries are utilised. Physical basis is the reference to the 2 fracture-types in a material: normal fracture (NF) and shear fracture (SF). For the subsets of failure modes "fibre fracture (FF)" and "IFF" two conditions for FF and three for IFF are derived. These five conditions describe the five failure modes or mechanisms occurring, and five failure modes are the maximum number a transversally-isotropic material, modelled a crystal, can possess.

In the transition zone of two failure modes or domains of mixed fracture, respectively, a probabilistic modelling has to be applied. This finally leads again to a smooth surface of the complete fracture body.

As the most remarkable results of the elaboration have to be pointed out: Consider micromechanics resp. real material stresses in the constituents fibre and matrix (incl. interface) which only can fail, however, formulate and visualise in lamina stresses at composite resp. macromechanical level and think in Mohr's fracture stresses.

The application of the criteria to test results is very promising. Erroneous results, possible if applying the traditional global (stress) interaction criteria, should not be achieved.

Keywords: Fibre reinforced composites, strength criteria, test data evaluation

1 INTRODUCTION

"Hot spots" of load carrying isotropic structures have to be designed versus various (Fig. 1) strength failure modes including *onset of yielding* and *fracture* resp. *onset of cracking*.
In case of laminates built from brittle UD-laminae the stress man has to dimension each lamina (ply-by-ply) versus IFF and fibre-fracture (FF). Whereas IFF indicates the onset of *fibre-parallel cracking* in one lamina of the laminate FF will indicate the onset of *fibre cracking*. First FF of a lamina usually marks the final fracture of the laminate. A reliable prediction of IFF under 3D-states of stress is mandatory for the calculation of progressive failure.

As well in the stress analysis as in the failure criterion of the strength analysis there are scattering design parameters. The uncertainty of these parameters (loads, strengths, geometrical quantities, young's module, ...) is of *physical nature* or of *statistical nature* (shortage of information due to a too small sample size of a certain design parameter

measured). Besides this there is always some *uncertainty in the calculation model*.
This scatter is usually considered in the design by the use of fixed, deterministic *factors of safety*, better called *design factors of safety* (DFOS), which are based on experience with structural tests. In aerospace industry structures are dimensioned by using DFOS j which *increase* the so-called *design limit load* (DLL) to the design ultimate load (DUL) according to $j_{ult} \cdot$ DLL. This procedure includes the idea of *proportional loading* and that the reserve factor f_{Res} is related to the external load only. Above *"load factors"* are discriminating the *onset of yield* (e.g. $j_{po,2} = 1.1$) for isotropic material and *ultimate fracture* (e.g. $j_{ult} = 1.5$).
The design allowables to be used as strength values are statistics-based minimum values [Cun96/2]. This might be a so-called Mil-Hdbk 17 "B-value" with 90 % reliability and 95 % confidence probability, the latter number regards the confidence in the transfer of the finite number of sample test data to the parent distribution with its infinite number of data.

Proofs of Design or *design verifications* done by analysing and/or testing a structure have to be given for *failure modes* such as stability, deformation, vibration and strength. Target of the designer is to achieve a margin of safety MS ≥ 0 for each single failure mode and - in this paper - for fracture modes as a subset of them. In order to achieve a reliable margin of safety (MS = f_{Res} -1) in the Strength Proof of Design the engineer should accurately know the normally non-linear 3D-state of stress in the critical material element and its separation into load-induced and residual stresses!

Fracture criteria as a subset of *strength* criteria should enable the engineer to perform the Strength Proof of Design for any 3-dimensional state of stress resp. combination of stresses with a *minimum* amount of test data.
Verified fracture criteria as a subset of strength criteria (Fig. 1) enable to do this. They can be calibrated after execution of the *basic strength* tests, only.

The traditional criteria such as Tsai/Wu have some shortcomings which forces research to correct or replace them [VDI97]. However, in the last four years heavy progress has been made in Germany with the derivation of more physically-based fracture criteria for brittle unidirectional isolated laminae. On the basis of [Has80] *A. Puck* developed the "Hashin-Puck (Action-Plane) Strength Criteria" for IFF [Puc92] assuming 2 modes. Additionally, tests were performed under a German grant to verify the criteria [VDI97].
Parallel, the author tried to generate strength criteria, each of which should be related to one single failure mechanism resp. fracture *mode* (not *type*) and formulated by *invariants*, thus taking the material's symmetries into account. Basis are the *types of fracture in a material* which are normal stress induced fracture (NF) and shear stress induced fracture (SF). This

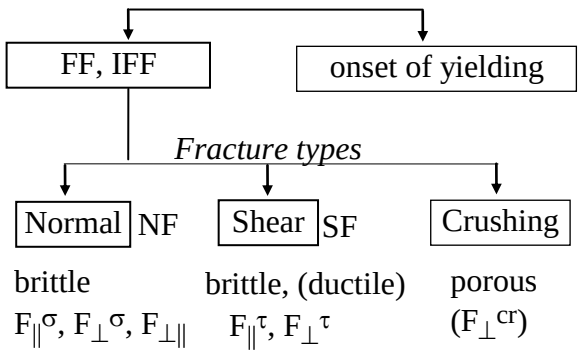


Fig. 1:
Strength Failure Modes (modi) of Composites.
(NF = normal fracture, SF = shear fracture.
$F_{\|}^{\sigma}$, ...,$F_{\perp}^{cr}$ are the UD-lamina's failure functions representing fracture modes)

fracture-type concept proposed may be applied to *any* brittle or ductile behaving material [Cun96/2].

In this paper brittle UD-laminae are described, only. The fracture criteria derived are *stress* criteria and have as foundations: The material is *homogeneous* (smeared or averaged material)

and can be modelled an *ideal crystal* with its symmetries and mutual independent basic strengths from a meso/macromechanical point of view. However, for the in reality heterogeneous composite (is a structure not a material) the micromechanical behaviour of the constituents (real materials) fibre, matrix and interphase will be considered too.

2 FRACTURE MECHANISMS IN UD-LAMINAE

Fracture is understood in this arcticle as a separation of material, which is free of damage such as cracks but not free of defects and flaws prior to loading. In this context *crushing* of a porous material has to be seen a separate *fracture mechanism* resp. *(fracture) failure mode* and *type*.

Idealising a material a crystal one can draw from Table 1 the information: The number of symmetries determines the number of strengths. The higher the *structural level* of the material the more basic strengths have to be measured.
Due to the fact 5 strengths being usually measured in case of a transversally-isotropic lamina the application of the crystal model is still general practice.

Table 1 : Number of Strengths and Fracture Types for Various Ideal Materials (t = tension, c = compression; ||, ⊥ = parallel, transversal to the fibre; W = warp, F = fill. Superscript c = critical)

material has	no structure	increasing structural level →	
	0	1	2
allocation to crystal type	isotropic	transversally isotropic	rombically anisotropic
symmetries	2	5	9
material example	resin matrix, mono. ceramics	UD-lamina, mat	fabrics
elasticity quantities	E, ν (2)	$E_{\parallel}$, $E_{\perp}$, $G_{\parallel\perp}$ $\nu_{\perp\parallel}$, $\nu_{\perp\perp}$ (5)	E_W, E_F, G_{WF}, ν_{FW}, E_3, ν_{3W}, ν_{3F}, G_{F3}, G_{W3}
strength quantities *(resistances)*	R^t, R^c or (R_σ , R_τ) (2)	$R_{\parallel}^t$, $R_{\parallel}^c$, $R_{\perp}^t$, $R_{\perp c}$, $R_{\perp\parallel}$ (5)	R_W^t, R_W^c, R_F^t, R_F^c, R_3^t, R_{3W}, R_{FW}, R_{3F}, R_3^c (9)
invariants	I_1, I_2, or J_2 ...	I_1, ..., I_5, ...	I_1, ..., I_7, ...
fracture toughnesses	K_{Ic}^t, K_{IIc}^c (2)	$K_{\parallel c}^t, K_{\parallel c}^c, K_{\perp c}^t$ $K_{\perp c}^c, K_{\perp\parallel c}$ (5)	

Fig. 2 shall give answer to the question: How many *types of fracture* are recognised in case of isotropic and transversally-isotropic *ideal materials*? These are just two: the shear stress induced fracture SF (Mohr's stress τ_n is acting at the "plane" of fracture) and normal stress σ_n induced fracture NF - as we know - for isotropic materials, but also two for the UD-material. However, the letter case comprises several SF and NF which belong to FF and IFF.

Also the physical fracture "planes" are pointed out in the figure.

A comparison of Fig. 2 with Table 1 leads to an essential conclusion*: One strength governs one distinct fracture type.*

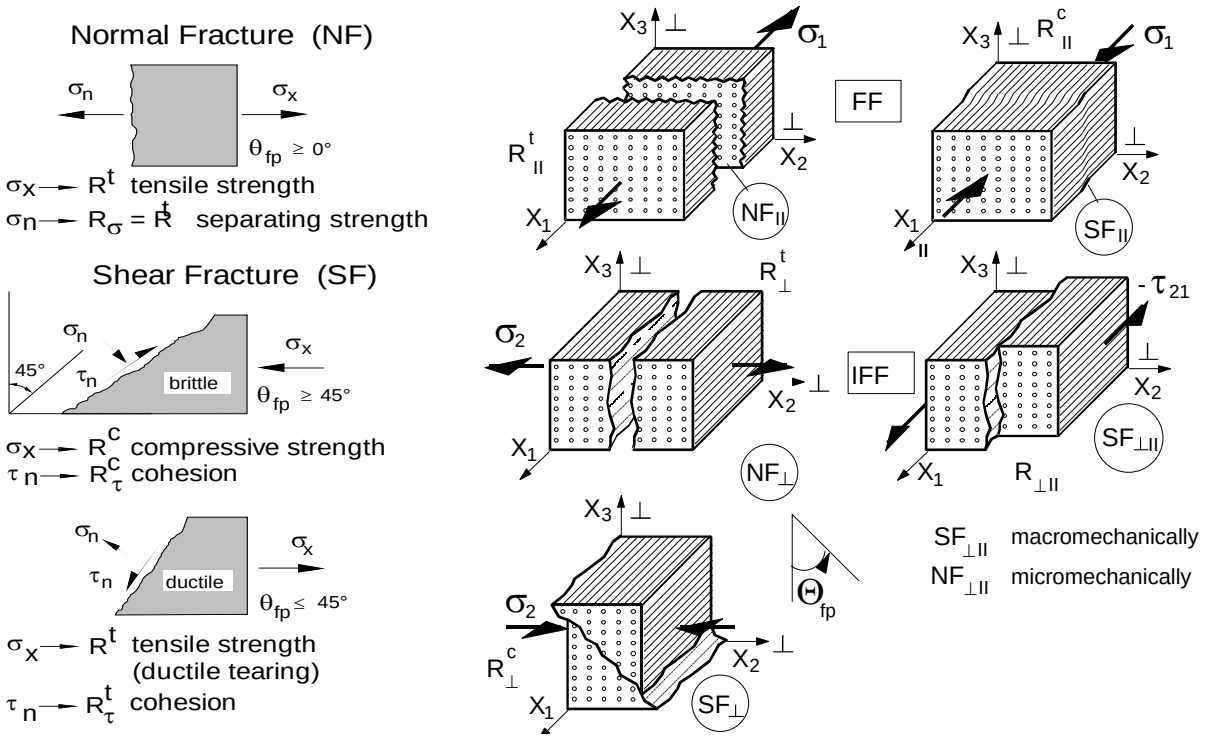


Fig. 2: Fracture Types (≡ modes) in case of Brittle and Ductile behaving Isotropic Material and Fracture Modes in case of Transversally-isotropic Material (σ_n^A ($\theta = \theta_{fp}$) = σ_n).

The characterisation of the strength of transversally-isotropic composites requires the measurement of five independent basic strengths: $R_{\parallel}^t$, $R_{\parallel}^c$ (fibre parallel tensile and compressive strength) as well $R_{\perp}^t$, $R_{\perp}^c$ (tensile, compressive strength perpendicular to the fibre direction) and $R_{\perp\parallel}$ (fibre parallel shear strength). $R_{\parallel}^t$ is determined by the strength of the constituent fibre, $R_{\parallel}^c$ is generally governed by *shear stability*. This includes different microfailure mechanisms: The matrix may shear under loading and does not stabilise the generally somewhat misaligned fibres embedded. Hence it comes to bending and "kinking" (structure behaviour). But, the load "grasping" fibre as the well-stabilised, stiffer constituent may shear (this is material behaviour) under $\sigma_{\parallel}^c$ and $\tau_{\perp\parallel}$, too. $R_{\perp}^t$ is determined as well by the relatively low strength properties of the matrix (cohesive failure) and the interphase material in the interface fibre-matrix (adhesive failure caused by a weak fibre-matrix bond), as by the fibres acting as embedded stress risers.

Well known from literature is the shift of the fracture limit of IFF strain resp. stress to higher values the absolutely thinner the lamina is and the stiffer in comparison the embedding laminae and the laminate are. Whereas the free isolated lamina (UD-specimen) if transversally tensile loaded is underlying a weakest-link failure (resulting in a Weibull distribution for $R_{\perp}^t$) the embedded lamina will be strain controlled by its neighbours. Inherent defects or microcracks as often reported can grow together to form mesocracks *only* in case they have a chance to open. However, these microcracks are arrested fracture-mechanically by the embedding laminae.

In order to consider the constraint effect on a "thin" embedded lamina following [Fla 82] and some own investigations the author recommends e.g. for CFRP ($t_{thr} \approx 0{,}35$ mm) as correction formula for the design allowable $R_{\perp}^t$ due to

$$R_{\perp}^t (t_{\perp} < t_{thr}) = R_{\perp}^t \sqrt{t_{thr} / t_{\perp}} \; . \qquad (2.1)$$

3 GENERAL FORMULATIONS OF STRENGTH CRITERIA AND CONDITIONS

Strength *criteria* $F \gtrless 1$ are generally formulated as (isotropic case shown for simplicity)

$$F(\{\sigma\}; \{R\}) = F(\sigma_x, \sigma_y, \sigma_z, \tau_{zy}, \tau_{zx}, \tau_{yx}; R^t, R^c) \gtrless 1 \quad \text{... criterion.} \qquad (3.1)$$

In the failure (here fracture) function F the vector $\{\sigma\}$ contains the six stresses in the co-ordinate system of the structure - as above - or the six lamina stresses. The vector $\{R\}$ contains all *basic strengths* to be applied such as R^t, R^c in isotropic case.

Failure surfaces are determined by the strength condition $F = 1$

$$F(f_{Res}\{\sigma\}; \{R\} = 1 \quad \text{.... condition} \qquad (3.2)$$

To achieve this, the stresses in equation 3.1 have to be multiplied by the reserve factor f_{Res}, which is usually dropped for reasons of a simpler writing.

Composites with a polymeric matrix exhibit brittle behaviour from a macroscopic point of view. Thus a formulation of strength criteria can be based on the fracture hypotheses of *Mohr* and *Coulomb*, adapted to UD-laminae. This was suggested by Hashin [Has80]. Hence, using Mohrs fracture stresses, which act together at the *same plane*, the isotropic equations read:

$$F_\sigma = F(\{\sigma^{Mohr}\}, R_\sigma) = 1 \quad \text{..... NF} \qquad (3.3a)$$
$$= \sigma_n/R_\sigma = 1 \quad (\textit{normal stress hypothesis})$$

$$F_\tau = F(\{\sigma^{Mohr}\}, R_\tau) = 1 \quad \text{... ..... SF} \qquad (3.3b)$$
$$= \tau_n/(R_\tau - \mu\sigma_n) = 1 \quad \text{e.g.}(\textit{shear stress hypothesis, Mohr-Coulomb})$$

with R_σ: = separating strength (resistance) R_τ: = cohesion (resist.), μ: = Coulomb friction.

This outlines: *one* condition is to be used for *each* fracture mechanism which is dominated by *one fracture resistance* R_σ or R_τ (s. also [Cun96/2] for isotropic materials. A resistance (of the fracture plane) corresponds with the basic strength associated if the action plane of a distinct stress comes to be the potential (load must be high enough, too) fracture plane: e.g. in brittle case $\max \sigma = R_\sigma = R^t$ (isotropic) or $R_\perp{}^t$ (transversally-isotropic). In case of brittle behaviour only R_σ can be measured and vice versa R_τ in case of ductile behaviour. Therefore, R_τ principally remains undeterminable until the fracture angle θ_{fp} is determined. This is also valid for the *fracture plane stresses* $\{\sigma^{Mohr}\}$.

The "curve fitting" of the course of test data can be performed much easier by not taking Mohr's stresses and the 2 resistances R_σ, R_τ but $\{\sigma\}$ and the 2 basic strengths. However, equation (3.1) will now be used in Mohrs sense meaning one basic strength governs one fracture mechanism NF or SF

$$F_\sigma = F(\{\sigma\}, R^{NF}) = F(\{\sigma\}, R^t) = 1 \quad \text{and} \quad F_\tau = F(\{\sigma\}, R^{SF}) = 1. \qquad (3.4)$$

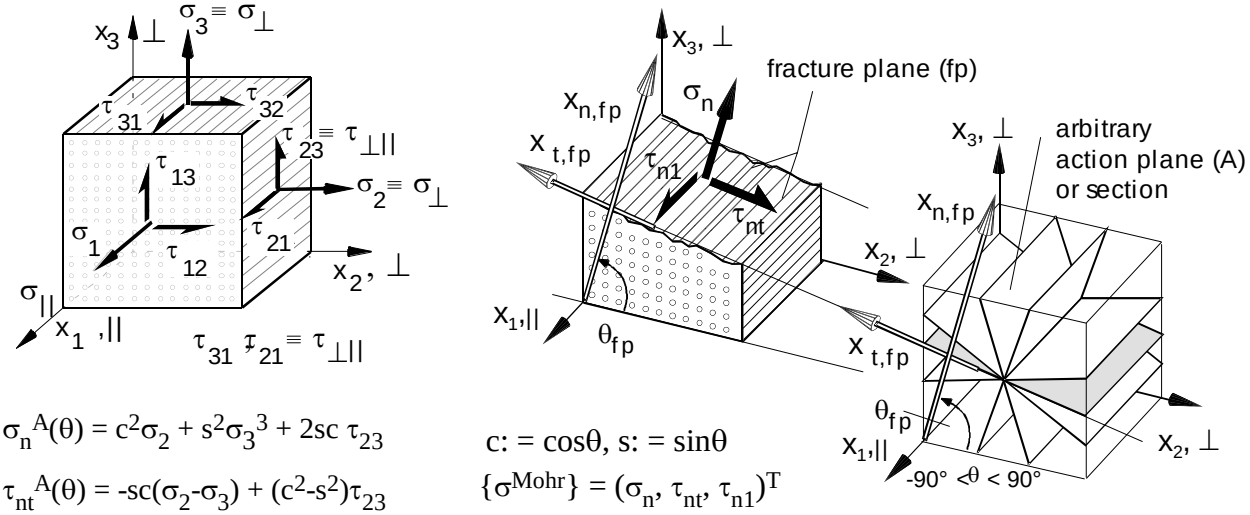
Both, the physical basis and the numeral handling are improved by using invariants as interpolation functions, a well known and classical approach.

4 DERIVATION OF "FRACTURE-TYPE STRENGTH CRITERIA"

4.1 Description of 3D-States of Stress

Three-dimensional states of stress in a lamina may be described by lamina stresses $\{\sigma\}$. A further possibility is given by using *Mohr's fracture stresses* mentioned above. They are acting in the potentially physical fracture "plane" (Fig. 3) and are decisive for fracture. In case of

normal (stress induced) fracture σ_n will be responsible for fracture and in case of shear (stress induced) fracture τ_n will be the fracture dominating one. Fig. 3 does also explain how



$\sigma_n^A(\theta) = c^2\sigma_2 + s^2\sigma_3^3 + 2sc\,\tau_{23}$

$\tau_{nt}^A(\theta) = -sc(\sigma_2-\sigma_3) + (c^2-s^2)\tau_{23}$

$\tau_{nl}^A(\theta) = c\tau_{21} + s\tau_{31}$

c: = cosθ, s: = sinθ

$\{\sigma^{Mohr}\} = (\sigma_n, \tau_{nt}, \tau_{n1})^T$

Fig. 3: 3D-states of Stress (lamina stresses and Mohr's stresses at the fracture plane)

Mohr's fracture stresses are derived by transformation of $\{\sigma\}$ into a fibre-parallel plane $\{\sigma(\theta)\} = [T(\theta)]\,\{\sigma\}$, ([Cun94]). Fracture plane will become that plane where the effort E under $\{\sigma(\theta)\}$ will be the maximum or f_{Res} will become a minimum, if $\sigma_n = \sigma_n^A(\theta_{fp})$ etc.

A comprehensive way to describe states of stress is given if using invariants formed by the lamina stresses (s. [Has80], [Boe85]). Table 2 depicts the invariants for a transversally-isotropic UD-lamina (I_3 = square of max $\tau_{\perp\|}$, $I_4 \approx$ square of max $\tau_{\perp\perp}$):

Table 2: Invariants of a UD-lamina
(v_f = fibre volume fraction; index f for fibre)

$I_{lf} = v_f \cdot \sigma_{lf}$

$I_2 = \sigma_2+\sigma_3 = \sigma_n + \sigma_t = \sigma_{II} + \sigma_{III}$

$I_3 = \tau_{31}^2+\tau_{21}^2 = \tau_{t1}^2 + \tau_{n1}^2$, $\quad I_5 = (\sigma_2 - \sigma_3)(\tau_{31}^2 - \tau_{21}^2) + 4\tau_{23}\tau_{31}\tau_{21}$.

$I_4 = (\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2 = (\sigma_n - \sigma_t)^2 + 4\tau_{nt}^2$ or $I_4 = \tau_{23}^2 - \sigma_2\sigma_3 = -\sigma_{II}\sigma_{III}$.

4.2 Fracture-type strength conditions

Table 3 summarises the author's proposal for fracture-type strength conditions being applied later to glass fibre reinforced plastics (GFRP).

Table 3: Fracture-type Strength Conditions for Transversally-isotropic Laminae.
(f_{Res} dropped in the equations; characteristic basic strength depicted; τ and σ indicate the responsible stress. The correction terms for IFF considering σ_1 are not depicted here, see [Cun96/1])

- Fibre Fracture (FF) -

$F_\|^\sigma$: $\quad a_\|^\sigma \cdot I_{1f} = R_\|^t$ and $F_\|^\tau$: $\quad a_\|^\tau \cdot I_{1f} + b_\|^\tau I_3 = R_\|^c$ .....$NF_\|$ and $SF_\|$

- Inter Fibre Fracture (IFF) - (matrix, interface) -

$F_\perp^\sigma$: $\quad a_\perp^\sigma I_2 + b_\perp^\sigma \sqrt{I_4} + c_\perp^\sigma I_2^2/R_\perp^t +$

$+[d_\perp^\sigma I_3 + (e_\perp^\sigma I_2 I_3 + f_\perp^\sigma I_5)R_\perp^t/R_{\perp\|}]/R_{\perp\|}^2 = R_\perp^t$ .....$NF_\perp$

$F_{\perp\|}$: $\quad a_{\perp\|} I_3 + (b_{\perp\|} I_2 I_3 + c_{\perp\|} I_5)/R_{\perp\|} = R_{\perp\|}^2$ .....$NF_{\perp\|}$

$F_\perp^\tau$: $\quad a_\perp^\tau I_2 + b_\perp^\tau I_4 / R_\perp^c + c_\perp^\tau I_3 R_\perp^c / R_{\perp\|}^2 = R_\perp^c$ .....$SF_\perp$

- Crushing - (porous matrix) -

$F_\perp{}^{cr}$: $a_\perp{}^{cr}I_2 + b_\perp{}^{cr}I_4 = R_\perp{}^{cr}$ .....$CrF_\perp$

The prediction of failure - made by whichever condition - is satisfied first. Terms in table 3 enveloped by dots are *local correction* terms to approximate (fit) the mixed fracture domains. They are not *global interaction* terms as e.g. in the global fracture criteria of Tsai-Wu.
The stresses τ_{21} and $\sigma_1{}^t$ do not have a common action plane, but marginally act micromechanically together. However, τ_{21} and $\sigma_1{}^c$ almost have the same action plane in case of brittle constituents fibre and matrix.
How the stress efforts are adding, linearly or quadratically or even cubically is determined by the course of test data.

The fracture type Crushing Fracture is added in Table 3 for reasons of completeness. It belongs to a material which due to its porosity will fail by internal *voluminous* degradation under compressive stresses and not in *"planes"* any more.

4.3 Determination of IFF Calibration Conditions besides the Basic Strengths

• Putting the compressive strength $R_\perp{}^c$ into $F_\perp{}^\tau$ delivers $a_\perp{}^\tau = b_\perp{}^\tau - 1.$ (4.1)

A second equation, if using the measurable fracture angle $\Theta_{fp}{}^c$ as the other obtainable information of the lateral compression test will be Mohr-Coulomb's condition ($\hat{c} := \cos\theta_{fp}{}^c$)

$$-\frac{\partial F_\perp^\tau}{\partial \sigma_n} / \frac{\partial F_\perp^\tau}{\partial \tau_n} = -\mu_{\perp\perp} = +\cot an\ 2\Theta_{fp}^c = 0{,}5\left(\frac{\hat{c}}{\hat{s}} - \frac{\hat{s}}{\hat{c}}\right) \quad \text{yielding} \quad b_\perp{}^\tau = 1/(4\,\hat{c}^{\,2}-1). \qquad (4.2)$$

• Trials to estimate the tensile fracture stress $\sigma_\perp{}^{tt}$ in an appropriate test had no success [VDI97]. However, a good estimation can be achieved by using the information $R_\perp{}^c$ being Weibull distributed. Following [Awa78] easily can be derived

$\sigma_\perp{}^{tt} = R_\perp{}^t / \sqrt[k]{2}$ with k = Weibull's module. (4.3)

• With respect to Fig. 6 a value for a point on the "bulge" $(\tau_{\perp\|}^{2D}, \sigma_\perp^{2D})$ can be estimated or directly a value for the linearized slope $m_{\perp\|} \approx (\tau_{\perp\|}^{2D} - R_{\perp\|})/\sigma_\perp^{2D}$ (4.4)

4.4 Analytical Derivation of Mohr's Envelope Curve

Resolving the fracture condition $F_\perp{}^\tau = 1$ for σ_2 delivers in case of $\tau_{23} = 0$

$$\sigma_2 \cdot 2b_\perp{}^\tau = -R_\perp^c\, a_\perp{}^\tau + 2\sigma_3 b_\perp{}^\tau + \sqrt{a_\perp^{\tau 2} R_\perp^{c2} - 8a_\perp^\tau R_\perp^c b_\perp^\tau \sigma_3 + 4b_\perp^\tau R_\perp^{c2}} = f(\sigma_3) \qquad (4.5)$$

with $\tilde{c} = \sqrt{(a_\perp^\tau R_\perp^c - 2b_\perp^\tau \sigma_3 + 2b_\perp^\tau \sigma_2)\,/\,(4b_\perp^c\,/\,(\sigma_2 - \sigma_3))},\ \sigma_3 = \text{variable}$ (4.6)

and $\tau_{nt} = |\,\tilde{s}\tilde{c}\ (-\sigma_2 + \sigma_3)\,|,\quad \sigma_n = \tilde{c}^{\,2}\sigma_2 + \tilde{s}^{\,2}\sigma_3,\quad \tilde{s} = \sqrt{1-\tilde{c}^{\,2}}\,.$ (4.7)

Then the cohesion can be calculated from $R_\perp{}^\tau = \tau_{nt}\ (\sigma_n = 0)$. This quantity corresponds to Puck's $R_{\perp\perp}^A$ [Puc96].

5 PROBABILISTIC MODELLING OF MULTIPLE FAILURE DOMAINS

The application of probabilistics will connect adjacent modes.
The influence of the FF modes on IFF can be taken into account very simply by practical correction terms (see [Cun96/1]) in the IFF conditions *or* by probabilistics as shown now.

To estimate the *joint probability of failure* of the different IFF modes as *logical model of the failure system* the series system or union U of $\ell = 3$ IFF failures will be taken. This delivers a failure probability p_f on the safe side (index S for sum, see [Cun96/2]), of

$$p_f \le \sum p_{f,\ell} = 1 - (1-f_{\perp}{}^{\sigma S})(1-f_{\perp\parallel}{}^{S})(1-f_{\perp}{}^{\tau S}) \qquad (5.1)$$

since small $p_{f,\ell}$ are expected. Correlation among the ℓ (component) failures leads to a smaller (system) failure probability p_f, above expression therefore is conservative. In case of a Weibull-distributed strengths $R_{\perp}{}^{t}, R_{\perp}{}^{c}, R_{\perp\parallel}$ as "sum" function

$$f^{S} = 1 - \exp[-(\frac{r}{w})^{k}] \qquad (5.2)$$

is valid (it represents the distribution of the extreme defects) with r = stress variable, k = Weibull's module (characterising the scatter of a strength) and w = shape parameter. A further parameter τ characterising the position of the distribution is put zero. This is in any case permitted for very brittle materials (e.g. monolithic ceramics, $k \ge 2$. An estimation of Weibull's parameter is possible [Pli95] on basis of the mean value m and the coefficient of variation of the parent distribution of each strength measured (bounds: cov < 25 %, k > 4,8) by

$$k \approx 1.2/\mathrm{cov}; \quad w = m/\Gamma(1+1/k); \quad m = \overline{R} \quad \text{with} \quad \Gamma = \text{Gamma function.} \qquad (5.3)$$

As estimations for m, v the values of a well sized sample are taken *without* considering the confidence interval.

Combining (5.1) and (5.2) yields the following equation for the IFF body

$$\left(\frac{r_{\perp}^{\sigma}}{w_{\perp}^{\sigma}}\right)^{k_{\perp}^{\sigma}} + \left(\frac{r_{\perp\parallel}}{w_{\perp\parallel}}\right)^{k_{\perp\parallel}} + \left(\frac{r_{\perp}^{\tau}}{w_{\perp}^{\tau}}\right)^{k_{\perp}^{\tau}} = -\ell_n\left(1-p_f\right). \qquad (5.4)$$

Instead of a uniaxial stress the *equivalent stress* as representation of the multiaxial state of stress will be put in (5.4): $r_{\perp}{}^{\sigma} = \sigma_{\perp}{}^{\sigma}$, $r_{\perp\parallel} = \sigma_{\perp\parallel}$, $r_{\perp}{}^{\tau} = \sigma_{\perp}{}^{\tau}$. It can be calculated in case of $F_{\perp}{}^{\tau}$ (shorter than for the IFF cases having cubic invariants)

$$F_{\perp}{}^{\tau} = f_{Res\perp}^{\tau}\,\sigma_{\perp}{}^{\tau}/R_{\perp}{}^{c} = 1 \quad \text{or} \quad \sigma_{\perp}{}^{\tau} = R_{\perp}{}^{c}/f_{Res\perp}^{\tau} \qquad (5.5)$$

$$f_{Res\perp}^{\tau} = [-a_{\perp}{}^{\tau} I_2 + \sqrt{a_{\perp}^{\tau 2} I_2{}^2 + 4b_{\perp}^{\tau} I_4 + 4c_{\perp}^{\tau} R_{\perp}^{c2}/R_{\perp\parallel}{}^2}\,/\,2[b_{\perp}^{\tau} I_4/R_{\perp}^{c} + c_{\perp}^{\tau} I_3 R_{\perp}^{c}/R_{\perp\parallel}^{2}].$$

For the IFF Proof of Design an engineering approach estimates f_{Res} in the mixed failure domain

$$(1/f_{Res})^{k} = (1/f_{Res\perp}^{\sigma})^{k} + (1/f_{Res\perp\parallel})^{k} + (1/f_{Res\perp}^{\tau})^{k} = f(f_{Res}{}^{(\mathrm{modes})}) \qquad (5.6)$$

(MFD) with $k \approx \min k$ (safe side) causing a maximum out-smoothing.

6 APPLICATION TO GLASS/EPOXIDE-FRP

6.1 Determination of Curve Parameters (applying MATHCAD)

For the application of the criteria achieved at first all informations on the basic strengths: The type of distribution, the mean value $\overline{R}$ and the coefficient of variation *cov* have to be provided. Table 4 represents an example set of data. The sources are various investigations [Kna72], [ZTL], [VDI97], due to the fact that a real complete test series of all interesting combinations of stresses including σ_1 for FRP is not existing.

Table 4: Strength Properties (v_f = 0.60) and data of Calibration Points (mean values)

$\overline{R}_{\parallel}^{t}$:	Weibull,	1500 MPa, cov =	6 %
$\overline{R}_{\parallel}^{c}$:	Weibull,	1200 MPa, cov =	7 %
$\overline{R}_{\perp}^{t}$:	Weibull,	40 MPa, cov =	12 %
$\overline{R}_{\perp}^{c}$:	Weibull,	144 MPa, cov =	7%
$\overline{R}_{\perp\parallel}$:	Weibull,	61 MPa, cov =	10 %

$\bar{\theta}_{fp}^{c} = 49°;\ \tau_{\perp}^{\tau} = 73\,\text{MPa};\ \ \sigma_{\perp}^{\tau} = -95\,\text{MPa}$

$\sigma_{\perp}^{tr} = -86\,\text{MPa},\ \tau_{\perp}^{\sigma} \approx 1.05\,R_{\perp\parallel}\ (\sigma_2 = 0)$

$(\sigma_2, \tau_{21}) \ = (40,0),(28,40),(0,58),(-90,72)$

6.2 Fracture Curves: Deterministic Approach incl. Effect of Correction Terms

Fig. 4 comprises four different cross-sections of the five-dimensional IFF-body (σ_2, σ_3, τ_{23}, τ_{31}, τ_{21}): The graph (τ_{21}, σ_2) represents stresses in the plane of the lamina which are of highest interest in the justification of the stress-output delivered from Classical Laminate Theory (CLT). Three curves $F_{\perp\parallel}^{\tau}$ (transversal SF), $F_{\perp\parallel}$ (transversal-parallel SF) and $F_{\perp\parallel}^{\sigma}$ (transversal-parallel NF) envelope the "50 % safe domain". The graph (τ_{31}, σ_2) outlines by "*Puck's corner"* that in contrary to (τ_{21}, σ_2^t) the shear stress τ_{31} has *not* the same action plane as σ_2^t. The remaining two graphs (σ_2, σ_3) and (τ_{23}, σ_2) are well-known from isotropic materials. In these isotropic cross-sections fracture can be described excellently by the homogenised lamina stresses (σ_2, σ_3, τ_{23}) or artificially smeared stresses, respectively. In the hydrostatic domain $\sigma_2^c = \sigma_3^c \approx -10\ R_{\perp}^c$ the IFF cureves will be closed by $F_{\parallel}^z$, due to Poisson's effect.

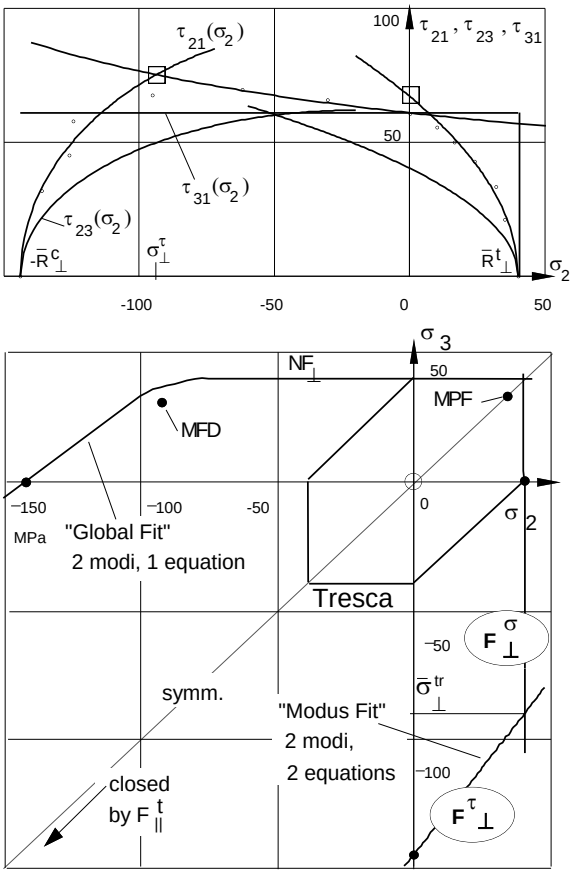


Fig. 4: IFF-curves (Glass FRP).(MFD = mixed failure domain = fracture due to 2 modes, MPF = multiplane fracture of the same mode NF⊥)

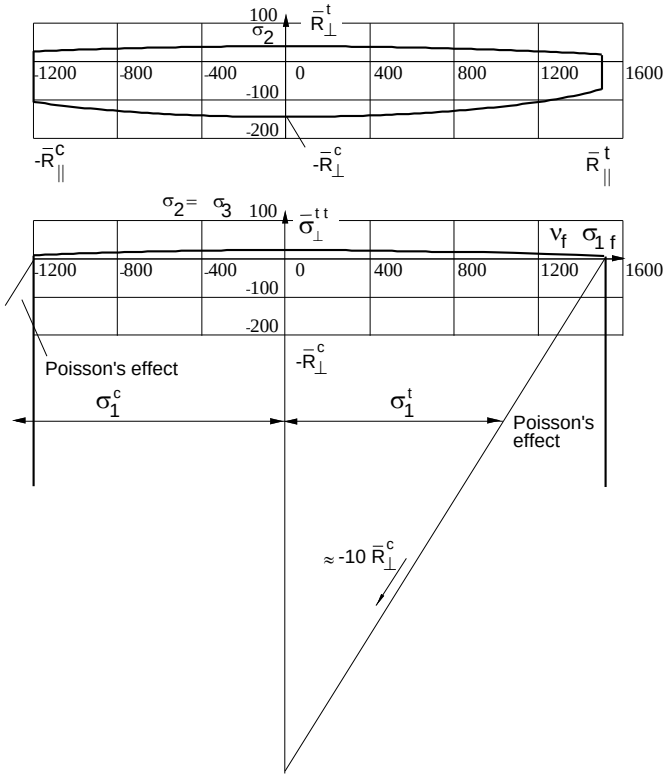


Fig. 5: FF-and IFF-curves (Glass FRP). (with correction terms, different scales)

Fig. 5 outlines the limited applicability of the homogenized lamina stresses. The lamina (composite) stress σ_1 is not the fracture active stress, however, the stress σ_{1f} of the constituent (material) fibre. In order to nevertheless remain in the graphical display on composite level the

fibre stress has to be multiplied by the fibre volume fraction ν_f. This should be an acceptable approach due to the extreme difference in the stiffness and thereby in the load carrying capacity of fibre and matrix. Tests performed, see [VDI97], show the effect of Poisson: Under biaxial compression the load can be only increased until tensile fracture stress or strain of the fibre will be met.

6.3 Simple 2D-Approximation of the (σ_2, τ_{21}) Fracture Curve

For an improved but nevertheless simple approximation of the (σ_2, τ_{21})-test results, which are mandatory for the evaluation of CLT stresses, the utilisation of the formulae

$$F_{\perp\parallel}^{\sigma}: \quad a_{\perp}^{\sigma}\sigma_2 / R_{\perp}^{t} + c_{\perp}^{\sigma}\sigma_2^2 / R_{\perp}^{t2} + d_{\perp}^{\sigma}\tau_{21}{}^2 / R_{\perp\parallel}{}^2 + e_{\perp}^{\sigma}\sigma_2\tau_{21}{}^2 / R_{\perp\parallel}^{3} = 1 \qquad (6.1a)$$

$$F_{\perp}^{\tau}: \quad a_{\perp}^{\tau}\sigma_2 / R_{\perp}{}^{c} + b_{\perp}^{\tau}\sigma^2 / R_{\perp}^{c2} + c_{\perp}^{\tau}\tau_{21}{}^2 / R_{\perp\parallel}{}^2 = 1 \qquad (6.1b)$$

is recommended with $a_{\perp}^{\tau}, b_{\perp}^{\tau}$ from the equations (4.1, 4.2) and the calibration points indicated in Fig. 6 by a square. Due to micromechanical interaction caused by the matrix the author *micro*mechanically combines $NF_{\perp\parallel}$ and $NF_{\perp}$ in his approach (Puck: $SF_{\perp}$ and $SF_{\perp\parallel}$ *macro*mechanically).

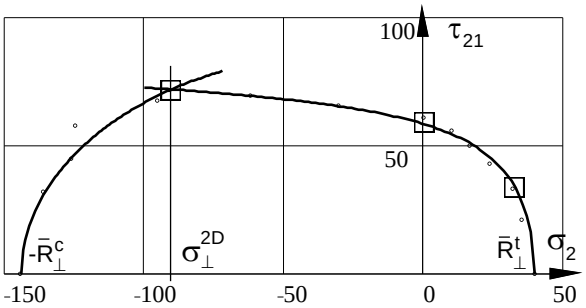


Fig. 6: 2D-Approximation (50%, Mean Curve. ○ Test Data from [Kna72]. □ calibration points

6.4 Fracture Curves: Probabilistic Approach

The probabilistic out-smoothing in the MFD or transition zone, resp., of the piecewise smooth fracture-type related domains of the IFF-body shall be visualised by Fig. 7.

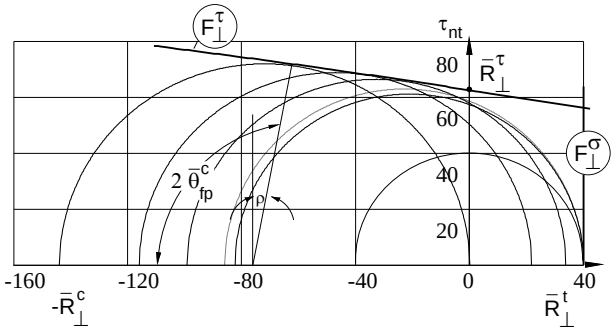


Fig. 8: An analytically derived Mohr-Coulomb "Envelope" Curve (bold) with MFD-circles and Transition Circle (dashed).

($R_{\perp}^{\tau}$ = friction cohesion, $\rho = \tan \mu_{\perp\perp}$,

$\mu_{\perp\perp}^{c} = -\cot an\, 2\theta_{fp}^{c}$*)*

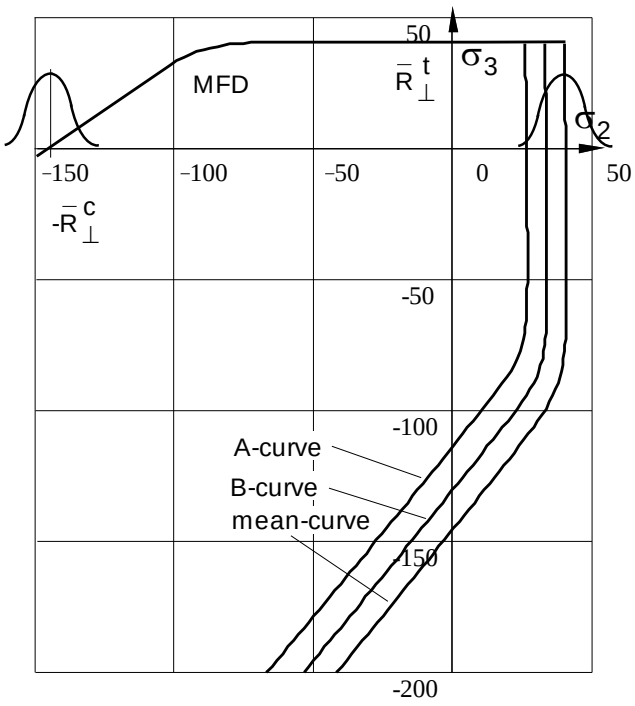


Fig. 7: "Global Fit" versus "Modus Fit". Design curves showing Probabilistic Out-smoothing in the Mixed Failure Domain (MFD)

Further the traditional *"global fit"* by polynomial interaction failure models (applied by most of the traditional strength criteria) is compared with the *"modus fit"* proposed in this article. The application of the probabilistic out-smoothing leads again back to one equation for the description of the global course of test data. Based on the numerical investigations for Fig. 7 one further conclusion can be drawn that

multiple fracture in a mode, e.g. $NF_{\perp}$ caused by biaxial tensile stresses, will be not covered by the simple modus fit ($c_{\perp}^{\sigma} = 0$). This correction term is necessary to approximate the test data in the MPF regime. Beside this, the term also has an effect in the MFD regime ($SF_{\perp}$, $NF_{\perp}$), which is not adverse because such brittle materials with a high contents of defects show both sorts of out-smoothing.
Essential for the dimensioning of FRP-structures is the "B-value Design Curve", most often.

Fig. 8 eventually shows (probably for the first time) an analytically derived Envelope Curve. To remind us of the former enveloping procedure the compression circle and the transition (the fracture mechanism jumps from $NF_{\perp}$ to $SF_{\perp}$) circle are included. Also the relationship of fracture angle, friction angle ρ and the local Coulomb friction coefficient $\mu_{\perp\perp}^{c}$ derived from a $R_{\perp}^{c}$-loading are given.
In the MFD no realistic fracture angle will be determined. Consequently the τ_{nt} (σ_n) curve again has to be an evelope of the circles (σ_2^d, σ_3) or (σ_2, σ_3^d) the values of which are the curve points of the probabilistic function (5.4) describing the transition from $SF_{\perp}$ to $NF_{\perp}$.
And, looking at the FEM codes the fracture conditions offered for a so-called "linear or parabolic Mohr-Coulomb Material" is nothing but a continuous mathematical description of an envelop of Mohr's circles in the MFD. It just has relation to Mohr's fracture angle outside the MFD where a "pure" $SF_{\perp}$ or $NF_{\perp}$ mode is dominating.

CONCLUSIONS, OUTLOOK AND REMARKS

Test Data Fit and Failure Modes:

- The author has the same point of view as L.J. Hart-Smith: It is scientifically incorrect to employ polynomial interaction failure models whenever the (micromechanical) failure mechanism of the critical constituent of the composite changes with the state of stress. And - valid for any material - the main shortcoming of global criteria is: A change in one strength (which belongs to a distinct fracture mode) has an effect on the *whole failure surface* (*surface of fracture body*) which represents all modes ([Har93, Cun94]).
Interactions between *different* failure modes are incorrect. Interactions between stresses affecting the *same* failure mode in the *same* constituent of the composite are permitted.

- Separate characterisations are needed for *each* failure mechanism in *each* constituent of a composite of materials. The fit has to be performed for each mode separately.

- It does not matter whether a state of stress comes from a one-, a two- or a three-dimensional stressing as long it belongs to one fracture mechanism or mode. Consequently all these states of stress can be described by the *same* fracture condition.

- In the isotropic planes test data excellently can be described by the homogenized stresses σ_2, σ_3, τ_{23} (composite = smeared or averaged material). However, if the UD-lamina is e.g. subjected to (σ_1, σ^2, σ^3) the constituent (real isotropic or anisotropic material) stress σ_{1f} will be responsible for fracture and not the composite stress (σ_1).

Strength Criteria:

- There are 3 categories of failure criteria: stress-based, strain-based and energy-based. The criteria can be grouped into independent, partly (stress) interactive and globally interactive ones. Their development may be based on micromechanical (constituent level) and/or (meso/) macromechanical investigations (lamina resp. composite level).

• The author prefers stress criteria in the proof of design because: stresses fracture the material, residual stresses are taken into account and strain history is considered in the strength values.

• The concept of strength criteria presented exhibit a physical basis, a good fit of test data course, invariance with respect to coordinate transformations, a relatively easy application involving numerical stability and applicability to ductile behaving composites as well.

• From the several applications a preliminary assessment of the concept is possible:
- the fracture conditions F = 1 determine the shape of the failure surface with its five piece-wise smooth areas, representing the five modes
- a separation of mechanical and probabilistic modelling would be advantageous for an accurate interpretation of test results but is not completely possible. The probabilistic tool has to be taken to smooth-out in the mixed failure domain (MFD) resp. transition domain of two or more modes the adjacent mode-related pieces of the "failure surface"
- the out-smoothing approximately can be done by applying correction terms instead of performing real probabilistic calculations

• The application of the concept to various isotropic test results was very successful and is very promising in case of transversally-isotropic UD-laminae, too. A big challenge will be an application to the various woven fabric laminae.

• Because "high tech" composite parts normally have notches and often have to be designed to damage tolerance they were qualified more or less by tests. So, the physical drawbacks of the traditional strength criteria applied to unnotched (and notched) structures have not been revealed.

Design (dimensioning) and Proof of Design:

• There are 5 basic strengths (or 5 resistances, if Mohr would be strictly applied) and as much modes or fracture mechanisms, respectively.

• To establish 3D-design curves the concept just needs for the "fitting" of the 5 pure modes the 5 basic strength allowables, the fracture angle $\theta_{fp}{}^{c}$ (pertaining Coulomb's friction) and a calibration point at the (σ_2, τ_{21})-bulge. If micromechanical interaction will be taken into account 3 more calibration points have to be provided or can be assumed from experience with a FRP family in order to determine the remaining curve parameters belonging to the correction terms. The curve parameters of each failure function in this paper are computed- beside the basic strength points - from further distinct calibration points, only. Generally of course, a root mean square-fit is foreseen to achieve their values.

• For the dimensioning the designer needs the coloured contour plots or profiles of the stresses as output of the FEM stress analysis and for the Proof of Design he should get offered by the FEM codes coloured profiles of $f_{Res}{}^{mode}$ too. This is indirectly still practiced when plotting the equivalent stress of v. Mises ($3\,J_2/\sigma_{yield}{}^2 = 1 \quad \rightarrow f_{Res}{}^{mode} = (\sigma_{yield}{}^2/\sigma_{equiv}{}^2)^{0,5} = \sigma_{yield}{}^2/3J_2)^{0,5}$) plots for the mode "onset of yielding" in case of isotropic material or for NF with the principal tensile stress ($\sigma_I/R^t = 1 \quad \rightarrow f_{Res}{}^{mode} = R^t/\sigma_I$). The lowest $f_{Res}{}^{mode}$ computed for IFF and FF drives the design.

• In the mixed failure domain equ. (5.6) delivers an estimation for $f_{Res} = f(f_{Res}(modes)$, therewith bypassing together with the correction terms a real probabilistic out-smoothing.

• The determination of f_{Res} in case of cubic invariants is a little more laborious. If residual stresses have to be taken into account, due of $f_{Res} \cdot \{\sigma\}^{(load)} + \{\sigma\}^{(residual)}$, see [VDI97].

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EVALUATION OF MULTIAXIAL TEST DATA OF UD-LAMINAE BY SO-CALLED "FRACTURE-TYPE STRENGTH CRITERIA" AND BY SUPPORTING PROBABILISTIC MEANS

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Keywords: Fibre reinforced composites, strength criteria, test data evaluation

OVERVIEW

Overall objective of this elaboration is to calibrate fracture criteria resp. to define size and shape of the fracture body as cheaply as possible by executing the characteristic tests only.

Based on knowledge, achieved by investigating *v. Mises, Mohr - Coulomb* and the new physically based *Hashin - Puck* Strength Criteria for inter fibre fracture (IFF) of brittle unidirectional laminae, a new and general concept for the derivation of fracture criteria will be proposed and applied to fibre reinforced plastics (FRP).

The fracture body derived here is basically piecewise smooth, each piece representing a single *failure mode*. As interpolation functions the *invariants* associated to the material's symmetries are utilised. Physical basis is the reference to the 2 fracture-types (Fig. 1) in a material: normal fracture (NF) and shear fracture (SF). For the subsets of failure modes "fibre fracture (FF)" and "IFF" two conditions for FF and three for IFF are derived. These five conditions describe the five failure mechanisms occurring, and five failure modes are the maximum number a transversally-isotropic material, modelled a crystal, can possess. There are also 5 strengths, only, and each strength governs one mode.

In the transition zone of two failure modes or domains of mixed fracture, respectively, a probabilistic modelling has to be applied. This finally will lead again to a smooth surface of the complete fracture body and smooth fracture curves (Fig. 2).

RESULTS AND PRINCIPAL CONCLUSIONS

As one remarkable result of the elaboration has to be pointed out: Consider micromechanics resp. real material stresses in the constituents fibre and matrix (incl. interface) which only can fail, however, formulate and visualise in lamina stresses at composite resp. macromechanical level, and think in Mohr's fracture stresses.

The application of the criteria to test results is very promising (Fig. 3). Erroneous reserve factors, possible if applying the traditional global (stress) interaction criteria, should not be achieved.

To establish 3D-design curves the concept just needs for the "fitting" of the 5 pure modes the 5 basic strength allowables, the fracture angle $\theta_{fp}{}^{c}$ under $\sigma_2{}^{c}$ and a calibration point at the (σ_2, τ_{21})-bulge. If micromechanical interaction will be taken into account 3 more calibration points have to be provided or can be assumed from experience with a FRP family in order to determine the remaining curve parameters.

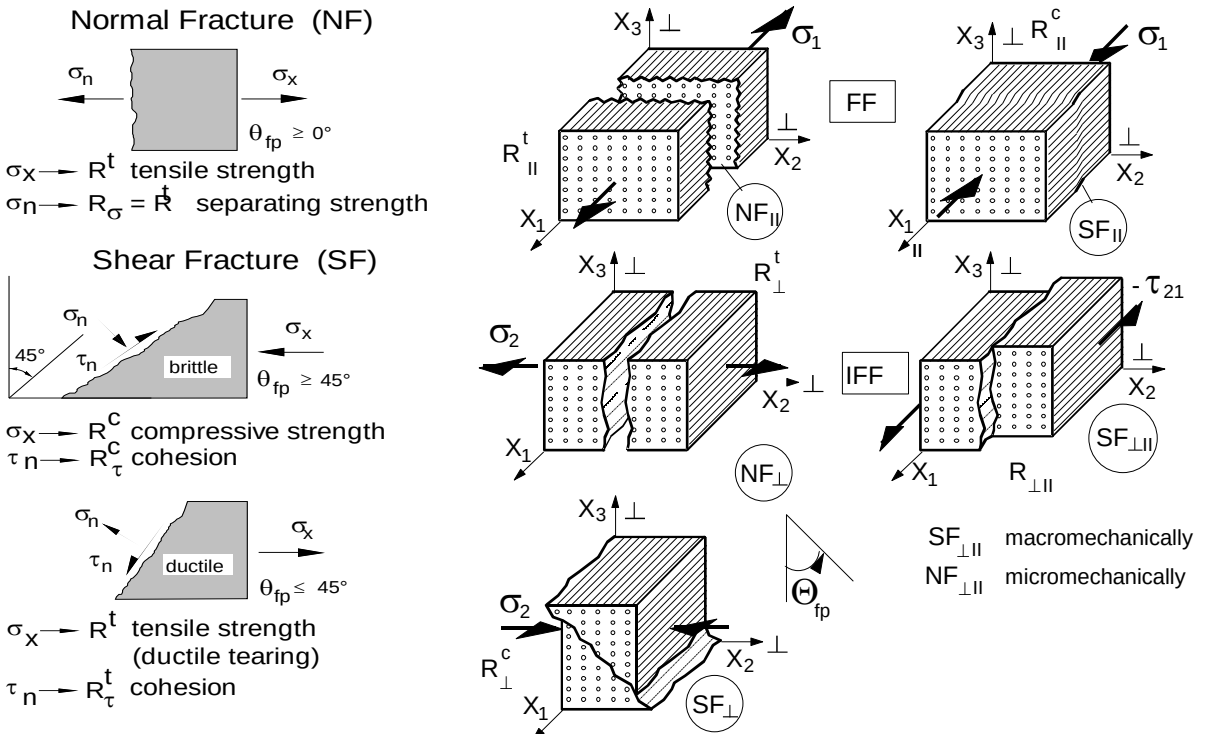

Fig. 1: Fracture Types (≡modes) in case of Brittle and Ductile behaving Isotropic Material and Fracture Modes in case of Transversally-isotropic Material ($\sigma_n^A(\theta = \theta_{fp}) = \sigma_n$). (c: = compression, t: = tension, $\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$

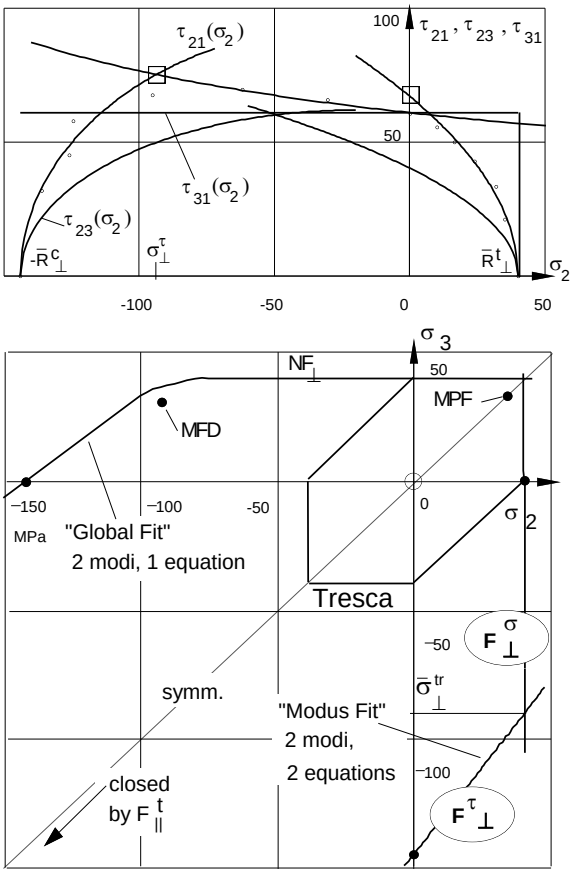

Fig. 3: IFF-curves (Glass FRP).(MFD = mixed failure domain = fracture due to 2 modes, MPF = multiplane fracture of the same mode $NF_\perp$

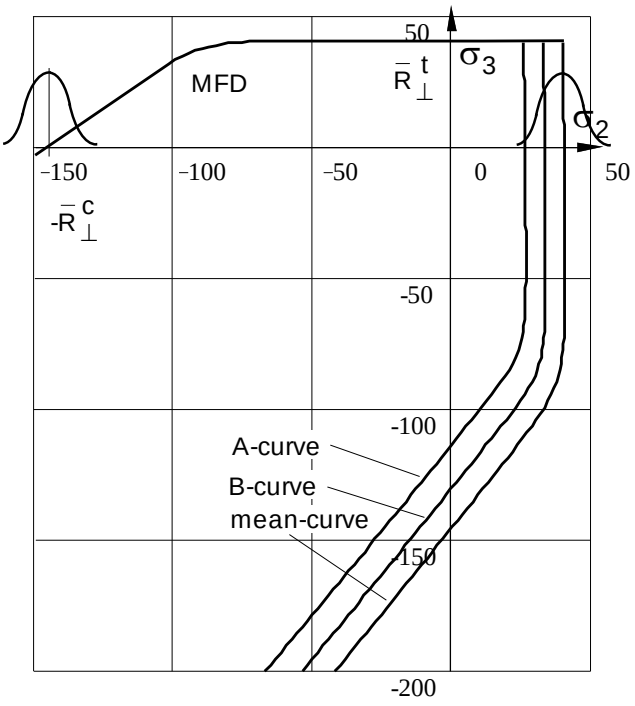

Fig. 2: "Global Fit" versus "Modus Fit". Design curves showing Probabilistic Out-smoothing in the Mixed Failure Domain (MFD) of the (σ_2 , σ_3) cross section of the fracture body