8. CFK-Valley Stade Convention, Juni 2014

Es zeigte Dirk Roosen, Head of Rohacell Aerospace bei Evonik, einen neuen PMI-Strukturschaum mit deutlich verbesserter Reißdehnung, der speziell für die Luftfahrt entwickelt wurde.

Er heißt Rohacell Hero = "Held der Lüfte" .

Für strukturelle Flugzeugbauteile mit hoher Steifigkeit werden derzeit Honigwabenkerne verwendet, deren Herstellung sehr teuer ist. Mit Rohacell Hero haben wir jetzt einen schadenstoleranten Schaum, der die teuren Wabenkerne ersetzen kann."

Schaumkern anstatt Wabenkern!

CARBON COMPOSITES

The Fracture Failure Surface of Foams

derived on basis of the author's Failure-Mode-Concept

1 Introduction to Strength Failure Conditions (SFCs)
2 Fundamentals when generating SFCs (criteria)
3 Attempt for a Systematization of Material Behaviour
4 Short Derivation of Cuntze's Failure-Mode-Concept (FMC)
5 Visualizations of some FMC-based *Failure Conditions*
6 Application to an Isotropic Foam (Rohacell 71 G)
Conclusions

Results of a time-consuming never funded „hobby"

Prof. Dr.-Ing. habil. Ralf Georg Cuntze VDI

Formulations of 3D Strength Failure Conditions

- Isn’t it basically just *Beltrami* and *Mohr-Coulomb* ? -

Hencky-
Mises-
Huber

Richard von Mises	Eugenio Beltrami	Otto Mohr	Charles de Coulomb
1883-1953	**1835-1900**	**1835-1918**	**1736-1806**
Mathematician	***Mathematician***	***Civil Engineer***	***Physician***

‘Onset of Yielding‘ **‘Onset of Cracking‘**

Motivation for my Investigations

Existing Links in the Mechanical Behaviour show up: ***Different structural materials***

- *can possess similar material behaviour* ***or***
- *can belong to the same class of material symmetry*

> **similarity aspect**

Welcomed Consequence:

- **The same strength failure function F can be used for different materials**
- **More information is available for pre-dimensioning + modelling**

in the case of a newly applied material

from experimental results of a similarly behaving material.

DRIVER:

Author‘s experience with structural material applications, range 4 K - 2000 K .

MESSAGE: **Let's use these benefits!**

What is a …???

Material: homogenized (macro-)model of the envisaged solid

Failure: structural part does not fulfil its functional requirements such as onset of yielding, brittle fracture, FF, IFF, leakage, deformation limit, delamination size limit, frequency bound

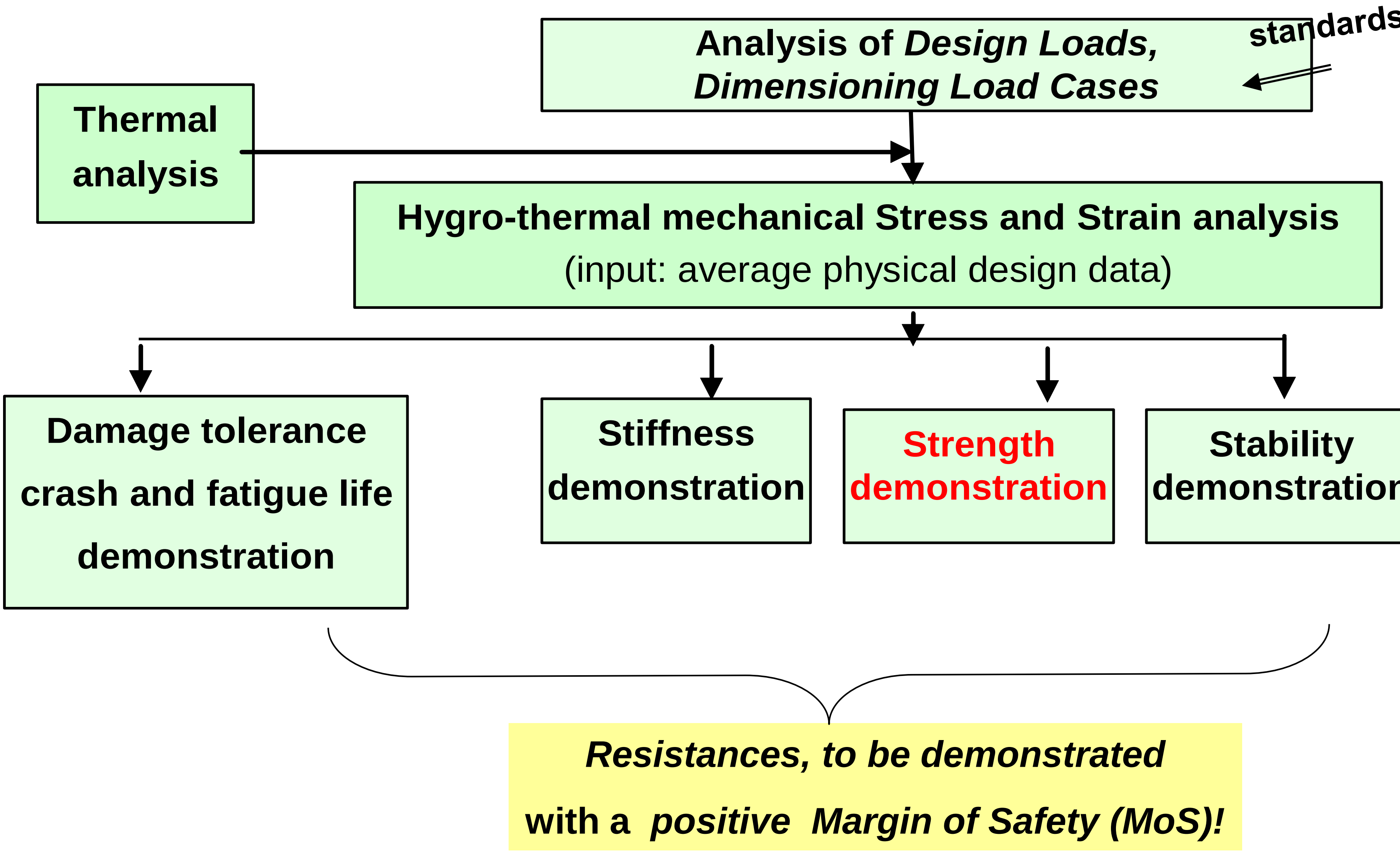
= project-fixed Limit State

Failure Theory: tool to predictive failure of a structural part

Strength Failure Condition: subset of a failure theory (in WWFE at least) to assess

a ‘multi-axial failure stress state ‘ in a critical location of the material

1. Introduction
Analyses in Structural Design and Design Verification



Worüber reden wir?

Ausschluß von Kerben und Delaminationen

- **Spannung (**lokaler Werkstoff ‘punkt‘): **Verifizierung mit Festigkeit**
- **Spannungskonzentration (**Neuber**) : Verifizierung mit Kerbfestigkeit**
- **Spannungsintensität (**Delamination, Riss**): Verifizierung mit Bruchzähigkeit**

… weitere zu liefernde Nachweise

Design Objective: Achievement of a Reserve against a Limit State

Reserve Factor is load-defined : ***RF = Failure Load / applied Design Load***

Material Stressing Effort : ***Eff = 100% if RF = 1*** **(Anstrengung)**

Material Reserve Factor : f_{Res} = ***Strength / Applied Stress***

If linear situation: $f_{Res} = RF = 1 / Eff$

Testdatenabbildung und Festigkeitsnachweis

- **Validierung der Festigkeitsbedingungen**

durch sog. Abbilden des Verlaufs der Bruch-Testdaten,

d.h. durch eine mittlere Bruch-Kurve

topic here

mit der späteren Abgabe eines zuverlässigen

- **Festigkeitsnachweises**

durch Berechnung einer Sicherheitsmarge (Last-Reservefaktors)

$$MoS > 0 \quad \text{oder} \quad RF = MoS + 1 > 1$$

auf Basis einer statistisch abgeminderten Bruch-Kurve.

Strength Failure Conditions are for

Prediction of ***Onset of Yielding*** + ***Onset of Fracture*** for non-cracked materials

Assessment of multi-axial stress states in a critical material location,

by **utilizing the uniaxial strength values R** and an
equivalent stress σ_{eq}, representing a distinct actual multi-axial stress state.

for *** dense & porous,**
*** ductile & brittle behaving materials,**

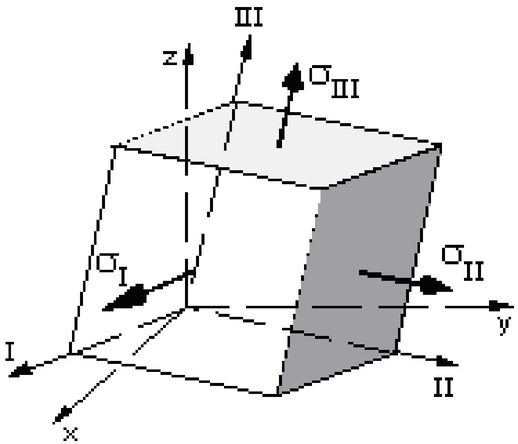
ductile : $R_{p0.2} \cong R_{c0.2}$ brittle, dense : $R_m^{\ c} \geq 3R_m^{\ t}$

for *** isotropic material**
*** transversally-isotropic material (UD := uni-directional material)**
*** rhombically-anisotropic material (fabrics) + 'higher' textiles etc.**

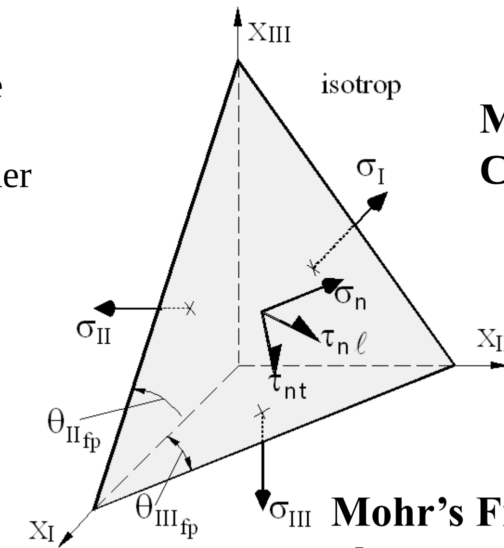
Shall allow for inserting stresses from the utilized various coordinate systems into stress-formulated failure conditions, -and if possible- invariant-based.

2 Fundamentals when generating Strength Failure Conditions

Isotropic Material (3D stress state), Stresses & Invariants



The stress states in the various COS can be transferred into each other



Mohr's COS

Principal Stresses

$$\{\sigma\}_{principal} = (\sigma_I, \ \sigma_{II}, \ \sigma_{III})^T$$

$$I_1 = (\sigma_I + \sigma_{II} + \sigma_{III})^T = 3\sigma_{oct} \equiv f(\sigma),$$

$$6J_2 = (\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2$$

$$= 4(\tau_{III}{}^2 + \tau_{II}{}^2 + \tau_I{}^2) = 9\tau_{oct}{}^2 \equiv f(\tau)$$

Structural Component Stresses

$$\{\sigma\}_{comp} = (\sigma_x, \ \sigma_y, \ \sigma_z, \ \tau_{yz}, \ \tau_{xz}, \ \tau_{xy})^T$$

'isotropic' invariants !

$$I_1 = (\sigma_x + \sigma_x + \sigma_z)^T$$

$$6J_2 = (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + (\sigma_x - \sigma_y)^2$$

$$+6(\tau_{yz}{}^2 + \tau_{zx}{}^2 + \tau_{xy}{}^2) \ \ (Mises, HMH)$$

Mohr's Fracture plane Stresses

$$\{\sigma\}_{Mohr} = (\sigma_\ell, \ \sigma_n, \ \sigma_t, \ \tau_{nt}, \ \tau_{t\ell}, \ \tau_{\ell n})^T$$

$$I_1 = (\sigma_\ell + \sigma_n + \sigma_t)^T$$

$$6J_2 = (\sigma_n - \sigma_t)^2 + (\sigma_t - \sigma_\ell)^2 + (\sigma_\ell - \sigma_n)^2$$

$$+6(\tau_{nt}{}^2 + \tau_{t\ell}{}^2 + \tau_{n\ell}{}^2)$$

$$27J_3 = (2\sigma_I - \sigma_{II} - \sigma_{III})(2\sigma_{II} - \sigma_I - \sigma_{III})(2\sigma_{III} - \sigma_I - \sigma_{II}), \quad I_\sigma = 4J_2 - I_1{}^2/3, \quad \sigma_{mean} = I_1/3$$

Invariant := Combination of stresses –powered or not powered- the value of which does not change when altering the coordinate system.

Types of Strength Failure Conditions – Global and Modal

1 Global ***strength*** **failure condition** : **F ({σ}, {R}) = 1 (usual formulation)**

Set of Modal **strength failure conditions: F ({σ}, R^{mode}) = 1 (addressed in FMC)**

Example: UD

vector of 6 stresses (general)

$$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$$

vector of 5 strengths

$$\{R\} = (R_{\parallel}^t, R_{\parallel}^c, R_{\perp}^t, R_{\perp}^c, R_{\perp\parallel})^T$$

needs an Interaction of Failure Modes: performed by a

probabilistic-based 'rounding-off' approach (series failure system model)

directly delivering the (material) reserve factor in linear analysis

Experience with Failure Prediction:

A Strength Failure Condition (SFC) is a necessary but not a sufficient condition to predict Strength Failure (i.e. thin-layer problem).

Kritik an den sog. ‘Globalen‘ Festigkeitsbedingungen

***Globale Festigkeitsbedingungen zwangsverbinden**, wie* **z. B. bei Drucker-Prager (isotrop), Tsai-Wu (transversal-isotrop, UD)**

die einzelnen Modi in einer Formel,

was generell nachteilig ist und sogar zu Ergebnissen auf der unsicheren Festigkeits-Seite führen kann,

weil eine Änderung in einem Modusbereich (z. B. Zugbruch), der durch die Formel insgesamt (global) beschriebenen Bruchversagensoberfläche, zwangsläufig Änderungen in unabhängigem anderen Modusbereich nach sich zieht.

Dies ist physikalisch nicht korrekt!

***Ein modales Konzept - wie bei z.B. Cuntze (generell) und Puck (UD) - baut die Bruchversagensoberfläche hingegen modus-bereichsweise auf*.**

Material Symmetry Requirements *(helpful, when generating SFCs)*

1 If a material element can be homogenized to an ideal (= frictionless) crystal,

then, material symmetry demands for the transversely-isotropic UD-material

- 5 *elastic 'constants' , 5 strengths, 5 fracture toughnesses* and

- *2 physical parameters (such as CTE, CME, material friction, etc.)*

(for isotropic materials the respective numbers are 2 and 1)

2 Mohr-Coulomb requires for the real crystal another inherent parameter,

- the *physical parameter 'material friction' : UD* $\mu_{\perp\parallel}$, ;$\mu_{\perp\perp}$ **, *Isotropic*** μ

3 Fracture morphology witnesses:

- Each strength corresponds to a distinct *failure mode*

and to a *fracture type* as Normal Fracture (NF) or Shear Fracture (SF).

Above Facts and Knowledge gave reason

why the FMC strictly employs single *independent* failure modes

by its failure mode–wise concept.

if brittle: failure = fracture failure

Which failure types (brittle or ductile) are observed ?

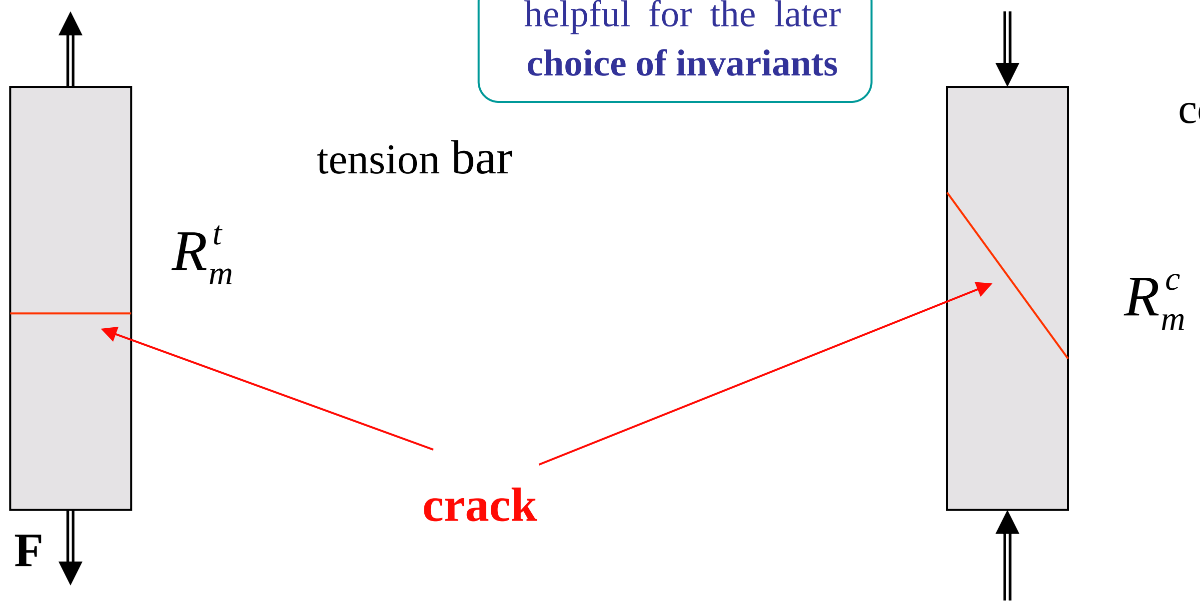
Cleavage fracture (NF) (Spaltbruch, Trennbruch) :

- poor deformation before fracture
- ‘smooth’ fracture surface

Shear fracture (SF) :

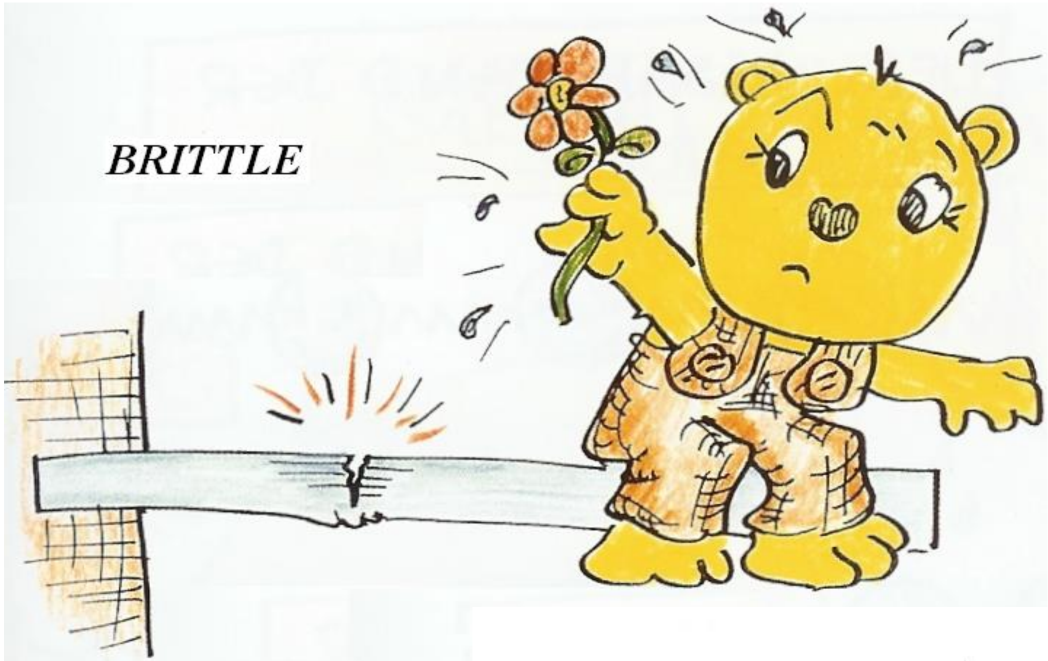
- shear deformation before fracture

knowledge is helpful for the later **choice of invariants**



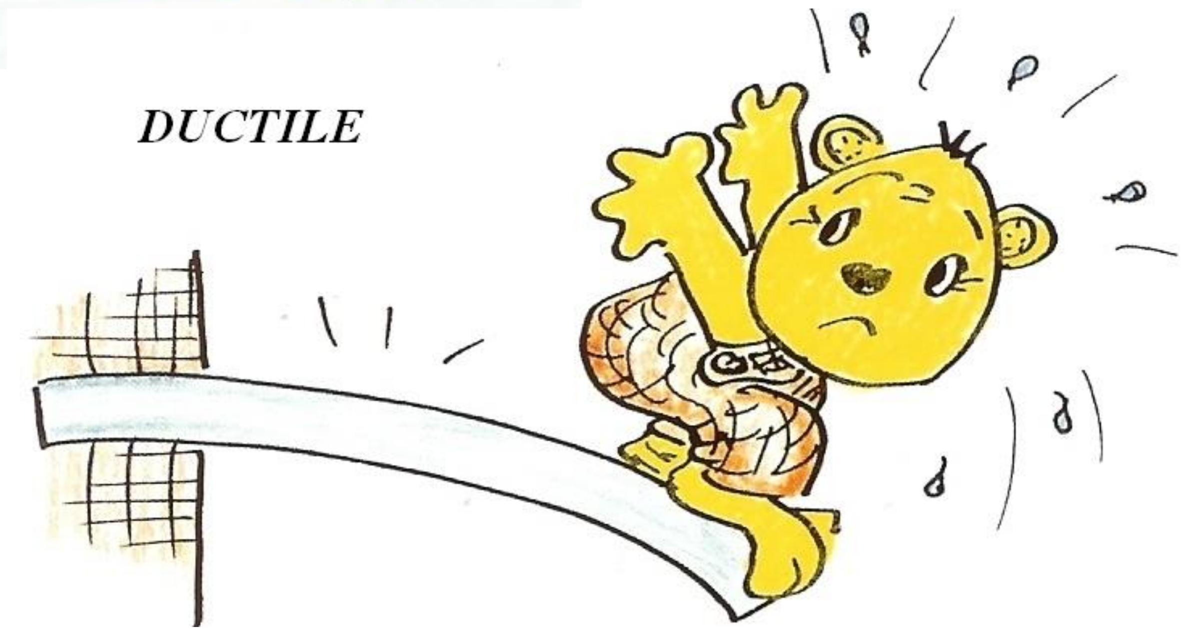
conclusion: ▶ **2 strengths** to be measured

How may one principally discriminate *material behaviour* ?



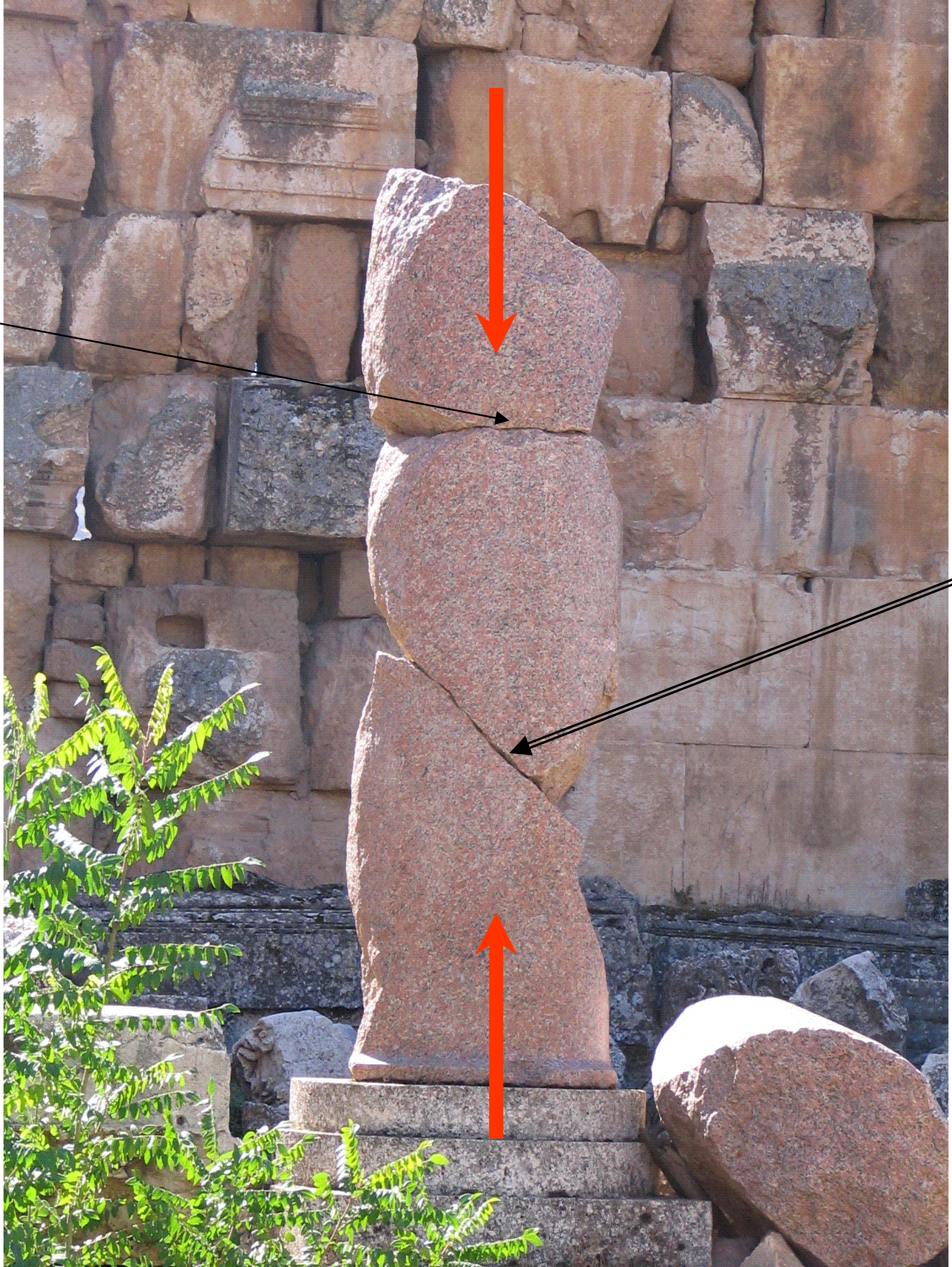
One feels good until sudden fracture occurs

Ductile Fracture = *type of a failure mode in a material or structure generally preceded by a large amount of plastic deformation*



Courtesy: Prof. C. Mattheck

just a joint



Example SF : R_m^c

Shear Fracture plane under compression

(**Mohr-Coulomb,** acting ***at a rock material*** *column,*

at Baalbek, Libanon)

Isotropic Material *brittle*, *porous*

if brittle: failure = fracture failure

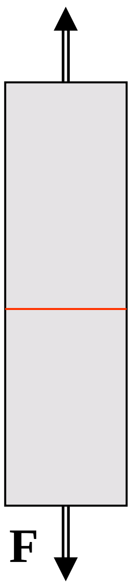
Normal Fracture (NF) (Spaltbruch, Trennbruch) :

- poor deformation before fracture
- rough fracture surface

Crushing Fracture (CrF): ⇐ **SF**

- volumetric deformation before fracture

helpful for the l
choice of invariants



Tension

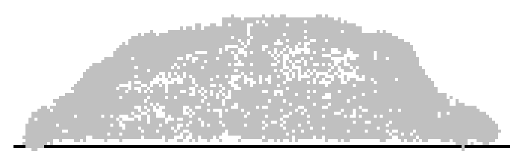
R_m^t

F

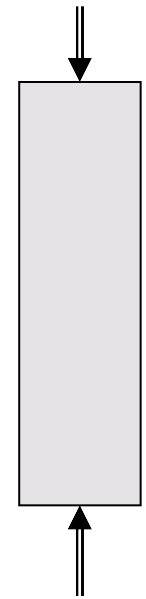
Compression

result of the compression test

= *hill of fragments* (*crumbs*)



= decomposition of texture

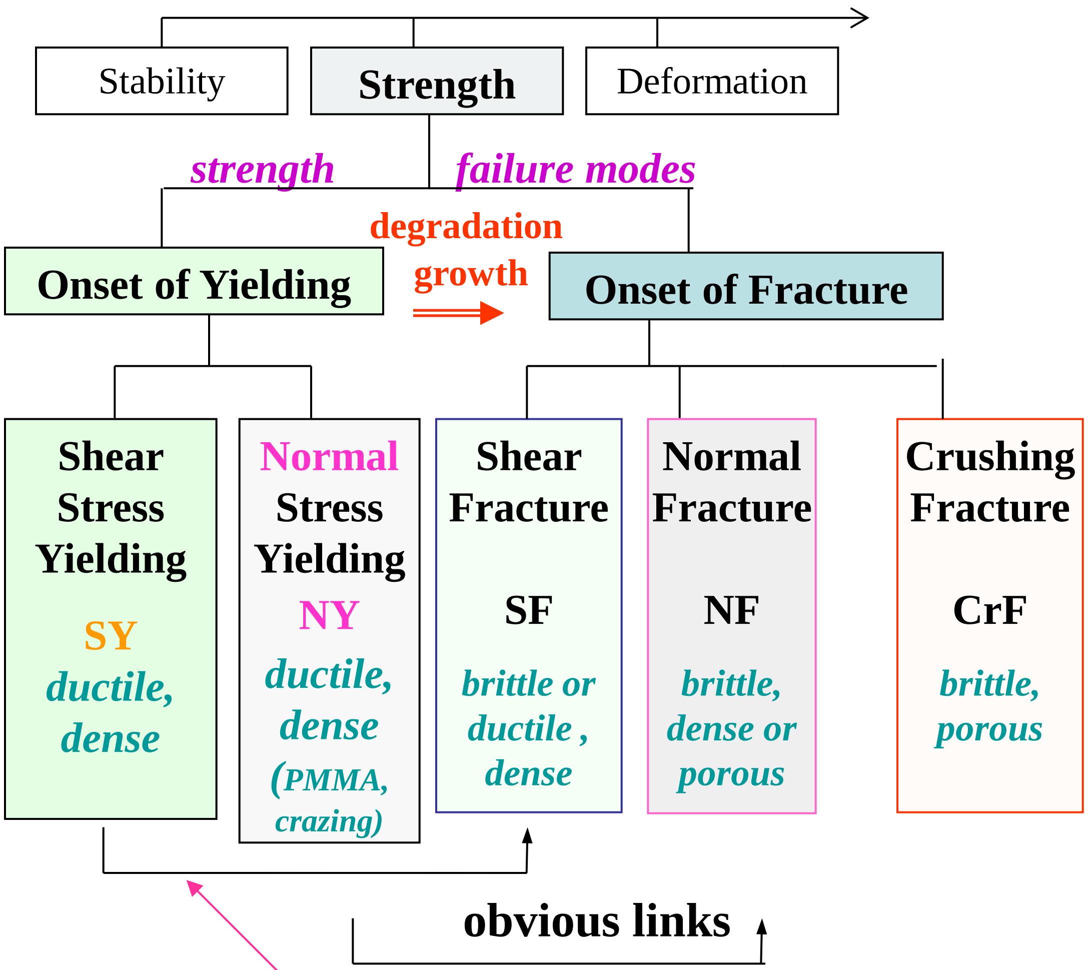


R_m^c

▶ **2 strengths** to be measured

3 Attempt for a Systematization of Material Behaviour

Scheme of Strength Failures for *isotropic materials*



The growing yield body (SY or NY)

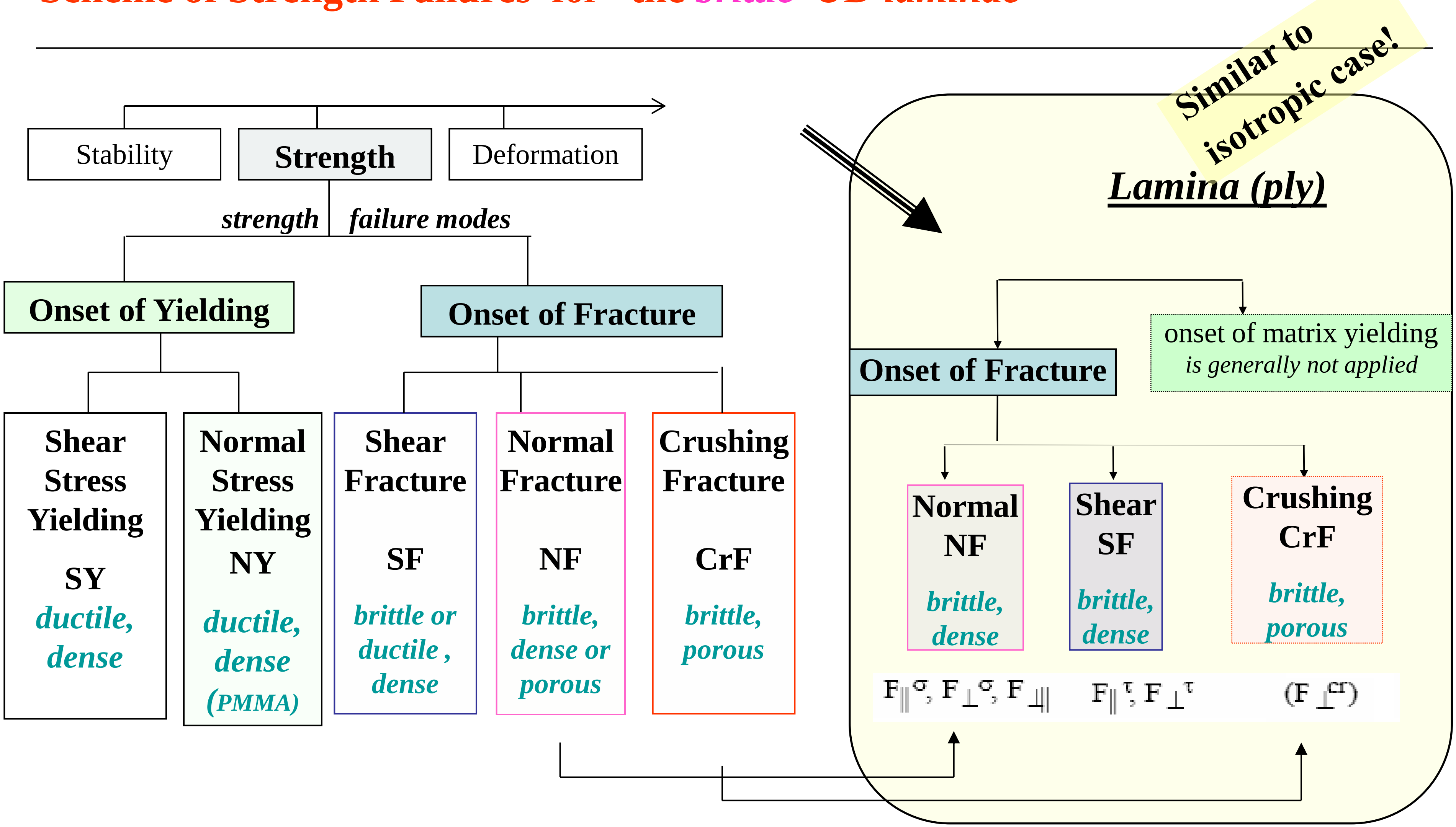
is confined by the fracture surface (SF or NF)**!**

◄ = ***kinds of fracture***

Lesson learned from Mapping Test Data:

The same mathematical form of a failure condition holds - from 'onset of yielding' to 'onset of fracture' - if the physical mechanism remains !

Scheme of Strength Failures for the *brittle* *UD laminae*

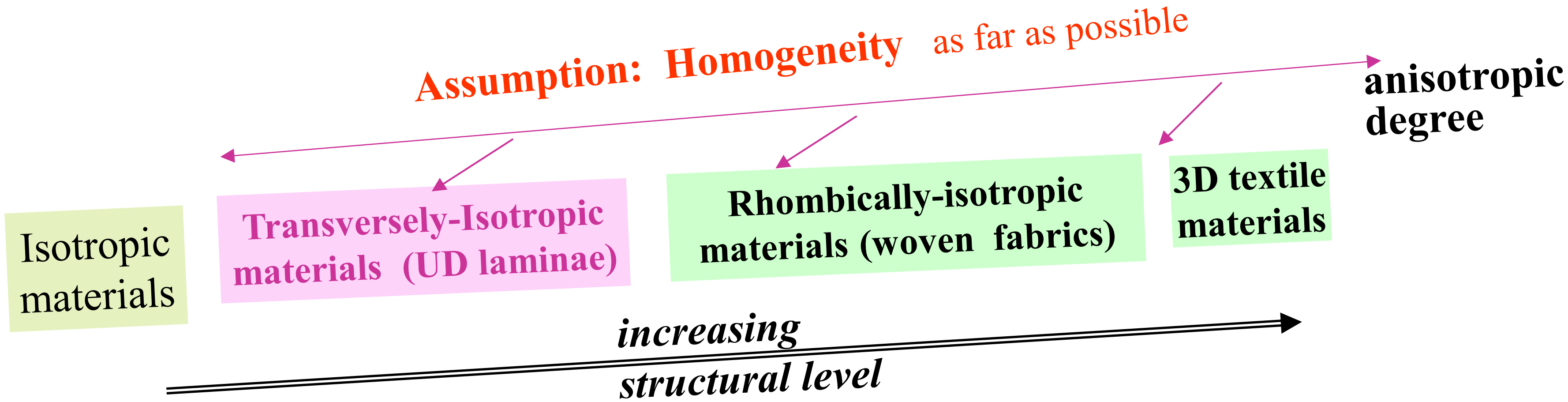


+ **delamination failure of *laminate***

Lessons learned:

* *There are coincidences between brittle UD laminae and brittle isotropic materials*
* *Increased degradation occurs in the* laminate *beyond onset of the first IFF*

Material Homogenizing (smearing) + Modelling



Material symmetry shows:

Number of strengths ≡ number of elasticity properties !

Application of material symmetry knowledge:

- ***Requires that homogeneity is a valid assessment for the <u>task-determined</u> model ,***

but, if applicable

- **A *minimum number of properties has to be measured, only* (cost + time benefits) !**

Proposed Classification of Homogenized (assumption) Materials

A Classification helps to structure the Modelling Procedure:

Failure Type / *Consistency*	**brittle, semi-brittle** Design Ultimate Load	**(quasi-) ductile** Design Yield Load
dense	fibre re-inforced plastics, mat, woven fabrics, grey cast iron, matrix material, amorphous glass C90-1,.	Glare, ARALL, metal alloys braided textiles
porous	foam, fibre re-inforced ceramics	sponge

design Driving Load (→ Design Yield Load)

failure: fracture (brittle) — functional or usability limit (ductile)

e.g. limiting strain

Lesson Learnt:

Modelling, Structural Analysis + Design Verification strongly depend on material behaviour + consistency

Self-explaining Notations for Strength Properties (homogenised material)

		Fracture Strength Properties									required by material symmetry
	loading	**tension**			**compression**			**shear**			
	direction or **plane**	1	2	3	1	2	3	12	23	13	
9	**general orthotropic**	R_1^t	R_2^t	R_3^t	R_1^c	R_2^c	R_3^c	R_{12}	R_{23}	R_{13}	**comments**
5	**UD, ≅ non-crimp fabrics**	$R_{\parallel}^t$ NF	$R_{\perp}^t$ NF	$R_{\perp}^t$ NF	$R_{\parallel}^c$ SF	$R_{\perp}^c$ SF	$R_{\perp}^c$ SF	$R_{\parallel\perp}$ SF	$R_{\perp\perp}$ NF	$R_{\parallel\perp}$ SF	$R_{\perp\perp} = R_{\perp}^t / \sqrt{2}$ (compare Puck's modelling)
6	**fabrics**	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	*Warp = Fill*
9	**fabrics general**	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	*Warp ≠ Fill*
5	**mat**	R_{1M}^t	R_{1M}^t	R_{3M}^t	R_M^c	R_{1M}^c	R_{3M}^c	R_M^τ	R_M^τ	R_M^τ	$R_M^\tau (R_M^t)$
2	**isotropic**	R_m SF	R_m SF	R_m SF	*deformation-limited*			R_M^τ	R_M^τ	R_M^τ	*ductile, dense* $R_M^\tau = R_m / \sqrt{2}$
		R_m NF	R_m NF	R_m NF	R_m^c SF	R_m^c SF	R_m^c SF	R_m^σ NF	R_m^σ NF	R_m^σ NF	*brittle, dense* $R_M^\sigma = R_m^t / \sqrt{2}$

NOTE: *As a consequence to isotropic materials (European standardisation) the letter R has to be used for strength. US notations for UD material with letters X (direction 1) and Y (direction 2) confuse with the structure axes' descriptions X and Y . *Effect of curing-based residual stresses and environment dependent on hygro-thermal stresses. *Effect of the difference of stress-strain curves of e.g. the usually isolated UD test specimen and the embedded (redundancy) UD laminae. R_m := 'resistance maximale' (French) = tensile fracture strength (superscript t here usually skipped), R:= basic strength. Composites are most often brittle and dense, not porous! SF = shear fracture

4 Short Derivation of the Failure Mode Concept (FMC)

Failure Theory and Failure Conditions

A **3D Failure Theory** has to include:

1. Failure Conditions to ***assess multi-axial states of stress***
2. Non-linear Stress-strain Curves of a material as input
3. Non-linear Coding for structural analysis

A Failure Condition is the mathematical formulation of the failure surface !

Pre-requisites for the establishment of failure conditions are:

- simply formulated, numerically robust,

- physically-based, and therefore, need only few information for pre-dimensioning

- shall allow for a simple determination of the design driving reserve factor.

Driving idea behind the FMC

A possibility exists to *more generally* formulate failure conditions

- **failure mode-wise** *(shear yielding etc.)*
- **stress invariant-based** *(J_2 etc.)*

Mises, Hashin, Puck etc.

Mises, Tsai, Hashin, Christensen, etc.

Basic Features of the FMC

- **Each failure mode represents 1 independent failure mechanism and 1 piece of the complete *failure surface***

- **Each failure mechanism is governed by 1 basic strength**

- **Each failure *mechanism* is represented by 1 failure *condition*** (interaction of acting stresses).

- **Interaction of Failure Modes:**

  Probabilistic-based 'rounding-off' approach (series model)

  directly delivering the reserve factor in linear analysis.

Fundamentals of the FMC (*example*: UD material)

Remember:

- **Each of the observed fracture failure modes was linked to one strength**
- **Symmetry of a material showed :** ***Number of strengths =*** $R_{||}^{t}, R_{||}^{c}, R_{\perp||}, R_{\perp}^{t}, R_{\perp}^{c}$

number of elasticity properties ! $E_{\parallel}, E_{\perp}, G_{\parallel\perp}, \nu_{\perp\parallel}, \nu_{\perp\perp}$

Due to the facts above the

FMC postulates in its '*Phenomenological Engineering Approach*' :

► Number of failure modes = number of strengths, too !

e.g.: isotropic = 2 or above transversely-isotropic (UD) = 5

t:= tensile, c: = compression, || : = parallel to fibre, := transversal to fibre

Physical-based Choice of Invariants when generating Failure Conditions

* Beltrami : "At 'Onset of Yielding' the material possesses a distinct ***strain energy*** composed of *dilatational energy* (I_1^2) and *distortional energy* ($J_2 \equiv Mises$) ".

* So, from Beltrami, Mises (HMH), and Mohr / Coulomb (friction) can be concluded:

Each invariant term in the *failure function* ***F*** may be dedicated to one physical mechanism in the solid = cubic material element:

- volume change : I_1^2 … ***(dilatational energy)***
- shape change : J_2 (**Mises**) … ***(distortional energy)***

and - friction : I_1 … ***(friction energy)***

Mohr-Coulomb

Interaction of Single Strength Failure Modes in the FMC

Interaction of adjacent Failure Modes by a *series failure system* model

= 'Accumulation' of interacting *failure danger portions* Eff^{mode}

$$Eff = \sqrt[m]{(Eff^{\text{mode 1}})^m + (Eff^{\text{mode 2}})^m + \ldots} = 1 = 100\%, \text{ if failure}$$

with mode-interaction exponent *m from mapping experience*

and

$$Eff^{\text{mode}} = \sigma_{eq}^{\text{mode}} / \overline{R}^{\text{mode}}$$

***modal* material stressing effort**

(Werkstoffanstrengung)

equivalent mode stress

mode associated average strength

Basis des Versagensmoduskonzepts (Failure-Mode-Concept FMC)

- **Invariantenformuliert (analog Fließhypothese von Hencky-Huber-Mises (HMH))**
- **Benutzung der Hypothesen von Beltrami (Zuordnung von Invarianten, ob sich ein Werkstoff-Element verzerrt (HMH) oder das Volumen ändert) und Mohr-Coulomb (innere Reibung eines sich spröd verhaltenden Werkstoffs) zur Wahl der richtigen Invariante**
- **Verwendung der Forderungen der Werkstoffsymmetrie an einen Werkstoff.**

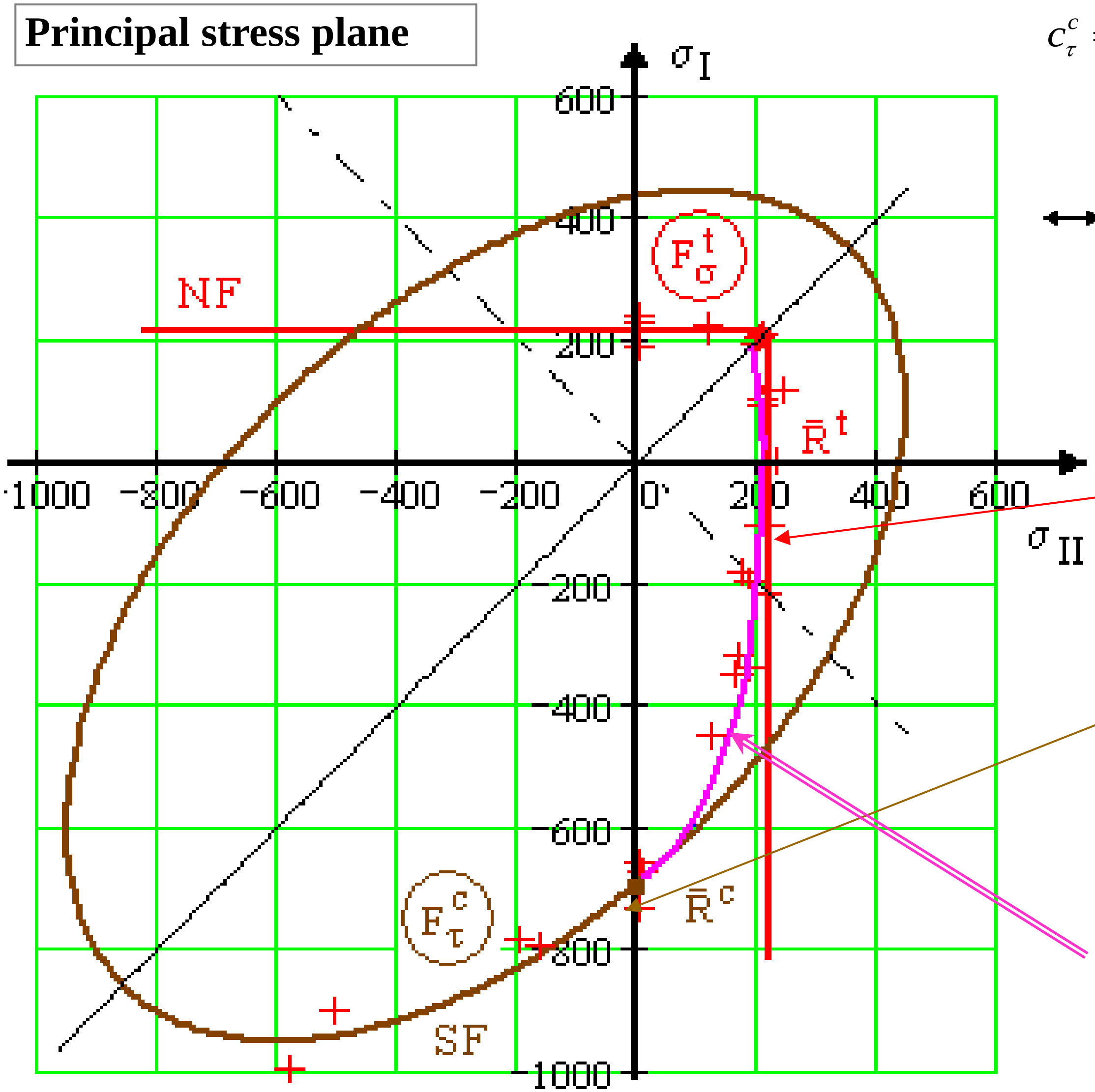
  Es sind anzuwenden : isotrop

  2 Festigkeitsversagensmodi, 2Basis-Festigkeiten und 2 Basis-Invarianten.

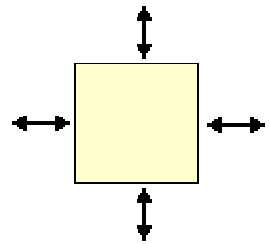
  Die Kennzahl für den transversal-isotropen UD-Werkstoff ist 5 !
- **Anwendung von Vergleichsspannung σ_{eq} und von Werkstoffanstrengung *Eff***

5 Visualisation of some Derived Failure Conditions

Grey Cast Iron (brittle, dense, microflaw-rich), *Principal stress plane*



$c_\tau^c = a_\tau^c - 1, \quad a_\tau^c = 1.58 \qquad m = 3.1 \qquad \bar{R}^t = 215 MPa; \quad \bar{R}^c = 690 MPa$



$\{\sigma\}_{principal} = (\sigma_I, \sigma_{II}, 0)^T$

$$F_\sigma^t = \frac{\sqrt{I_\sigma} + I_1}{2 \cdot \bar{R}^t} = 1$$

NF deformation poor

$$F_\tau^c = a_\tau^c \frac{3J_2}{\bar{R}^{c2}} + c_\tau^c \frac{I_1}{3\bar{R}^c} = 1$$

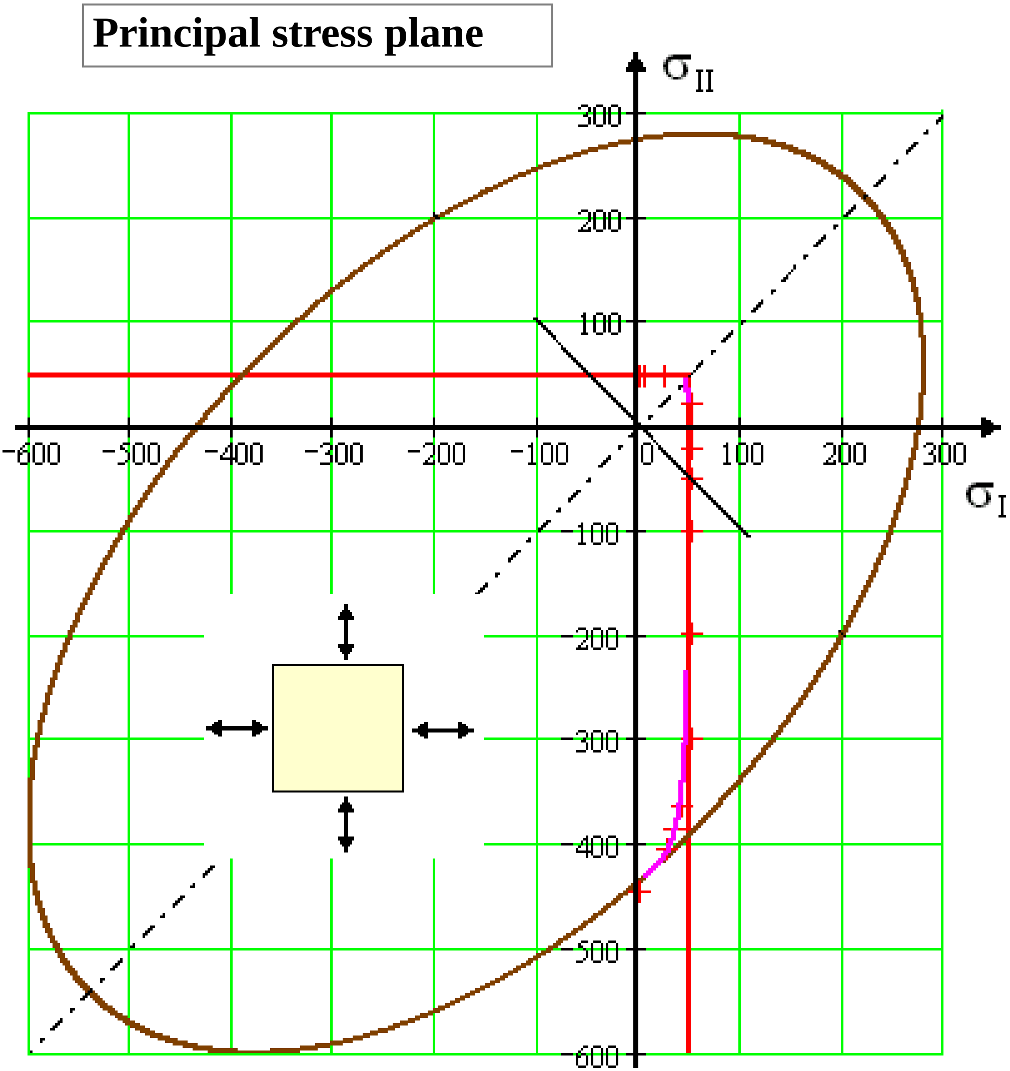
shear change — friction — **SF**

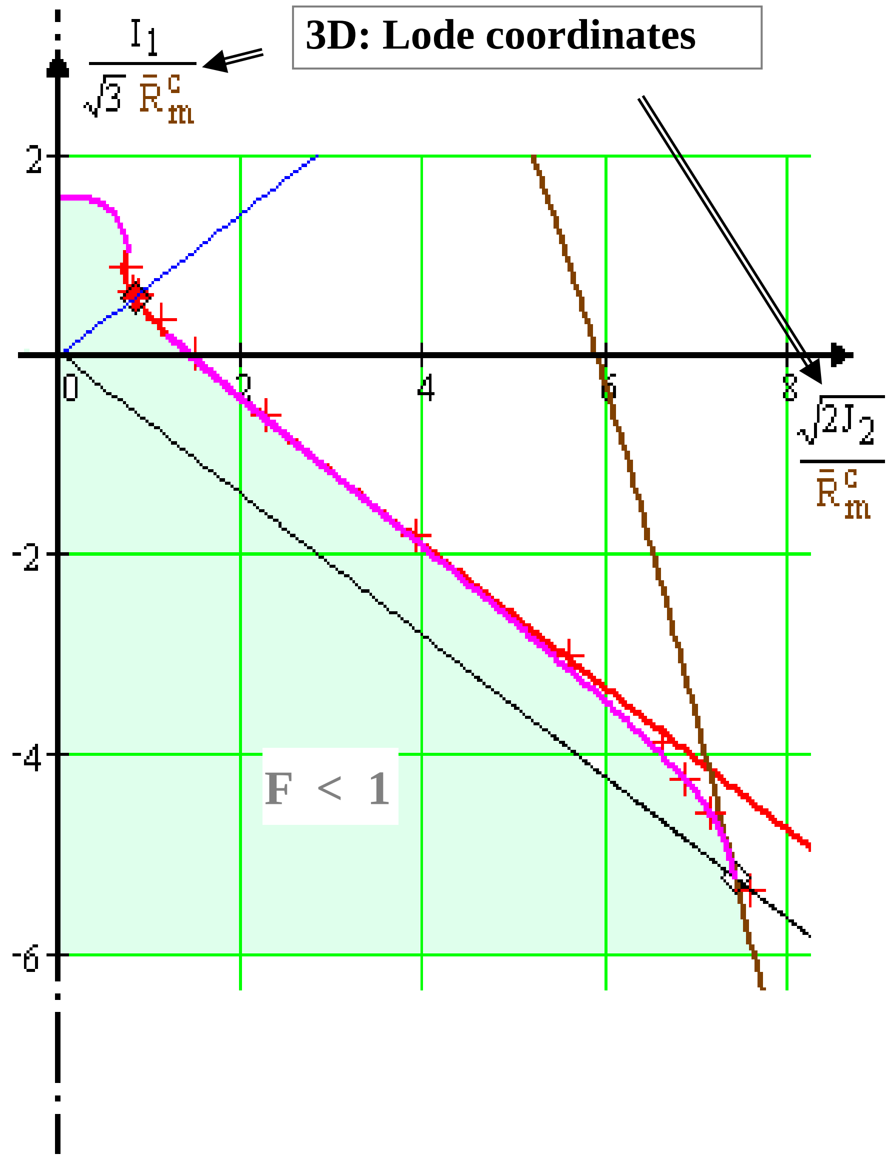
= 2 Mode Failure Conditions

Interaction zone

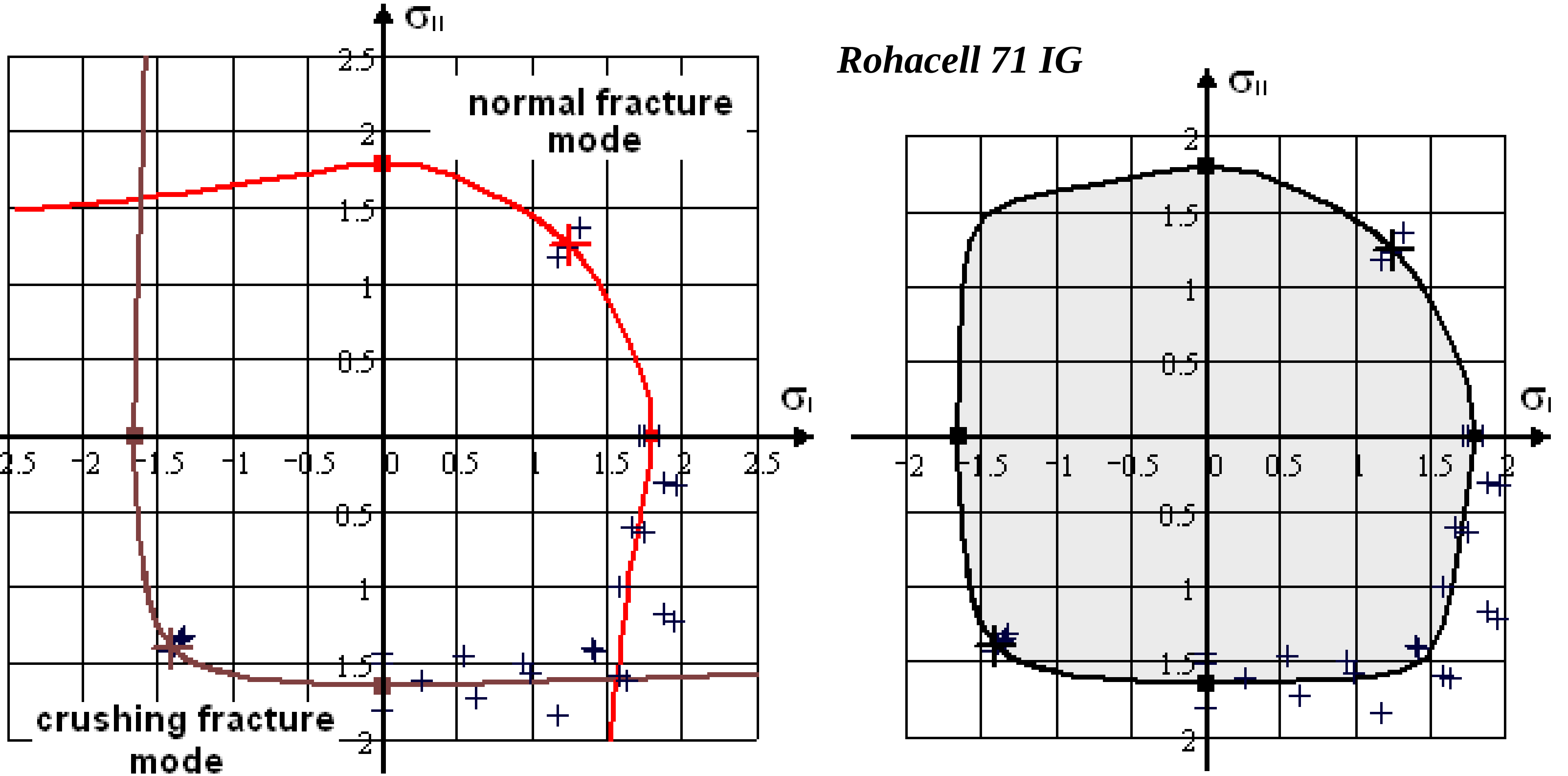
Lessons learned: Basically, *Dense concrete and Glass C 90 will have same failure condition*

Glass C 90 (brittle, dense isotropic material) *ISS window pane*





2D Foam Test Data and its Mapping in the Principal Stress Plane (brittle, porous)

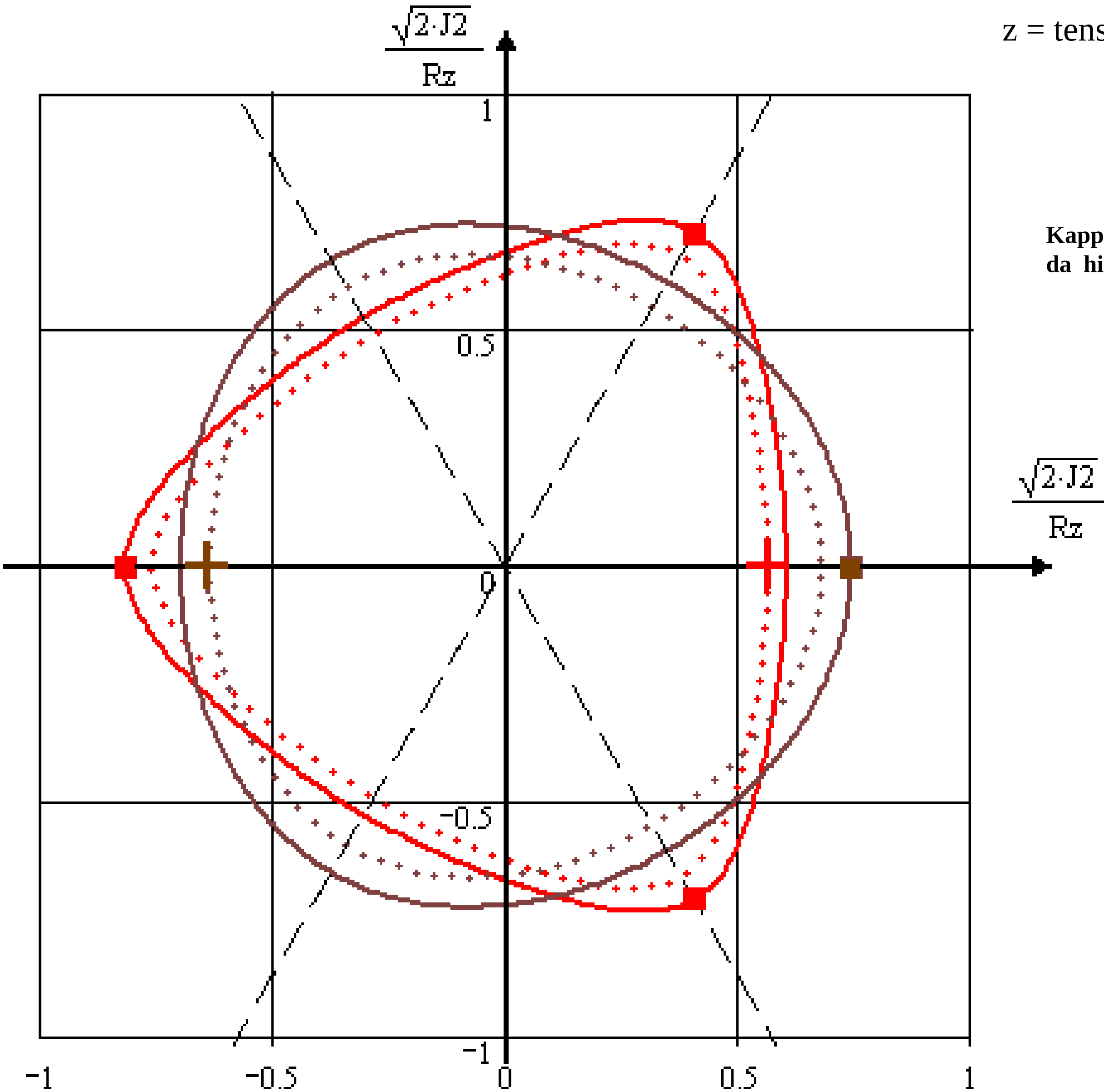


- **Mapping must be optimal in the 2D-plane because fracture data are given there**
- **The 2D-mapping uses the 2D-subsolution of the 3D-strength failure conditions**
- **The 3D-fracture failure surface (body) is based on the 2D-derived model parameters.**

Courtesy: LBF-Darmstadt, Dr. Kolupaev

2D Test Data and Mapping in the Orthogonal Stress Plane (brittle, porous)

z = tensile, d = compressive ***Rohacell 71 IG***



I1 = 0, interaction domain: Is about a circle.

wwith characteristic uni-axial and bi-axial strength points

Kappen: Kegel gewählt, da hier keine Testdaten

$$\frac{I1}{\sqrt{3} \cdot Rz} = scap \cdot \left(\frac{\sqrt{2J2 \cdot \Theta\sigma zz}}{Rz} \right) + \frac{maxI}{\sqrt{3} \cdot Rz}$$

$$F\sigma = c1\Theta\sigma \cdot \frac{\sqrt{4 \cdot J2 \cdot \Theta\sigma - \frac{1}{3} \cdot I1^2} + I1}{2Rz} = 1$$

crushing

$$Fcr = c1\Theta cr \cdot \frac{\sqrt{4 \cdot J2 \cdot \Theta cr - \frac{1}{3} \cdot I1^2} - I1}{2Rd} = 1$$

$$\frac{I1}{\sqrt{3} \cdot Rz} = sbot \cdot \left(\frac{\sqrt{2J2 \cdot \Theta crdd}}{Rz} \right) + \frac{minI1}{\sqrt{3} \cdot Rz}$$

interaction of modes

$$Eff = \sqrt[m]{Eff\Theta\sigma^m + Eff\Theta cr^m}$$

$$\Theta\sigma = \sqrt[3]{1 + D\sigma \cdot \sin(3\theta)} = \sqrt[3]{1 + D\sigma \cdot 1.5 \cdot 3^{0.5} \cdot J3 \cdot J2^{-1.5}}$$

$$\Theta cr = \sqrt[3]{1 + Dcr \cdot \sin(3\theta)} = \sqrt[3]{1 + Dcr \cdot 1.5 \cdot 3^{0.5} \cdot J3 \cdot J2^{-1.5}}$$

+ + consideration of a ‘twofold mode‘ by above well-known approach

Lode-angle, here set as <u>sin</u> (3 *θ*) :

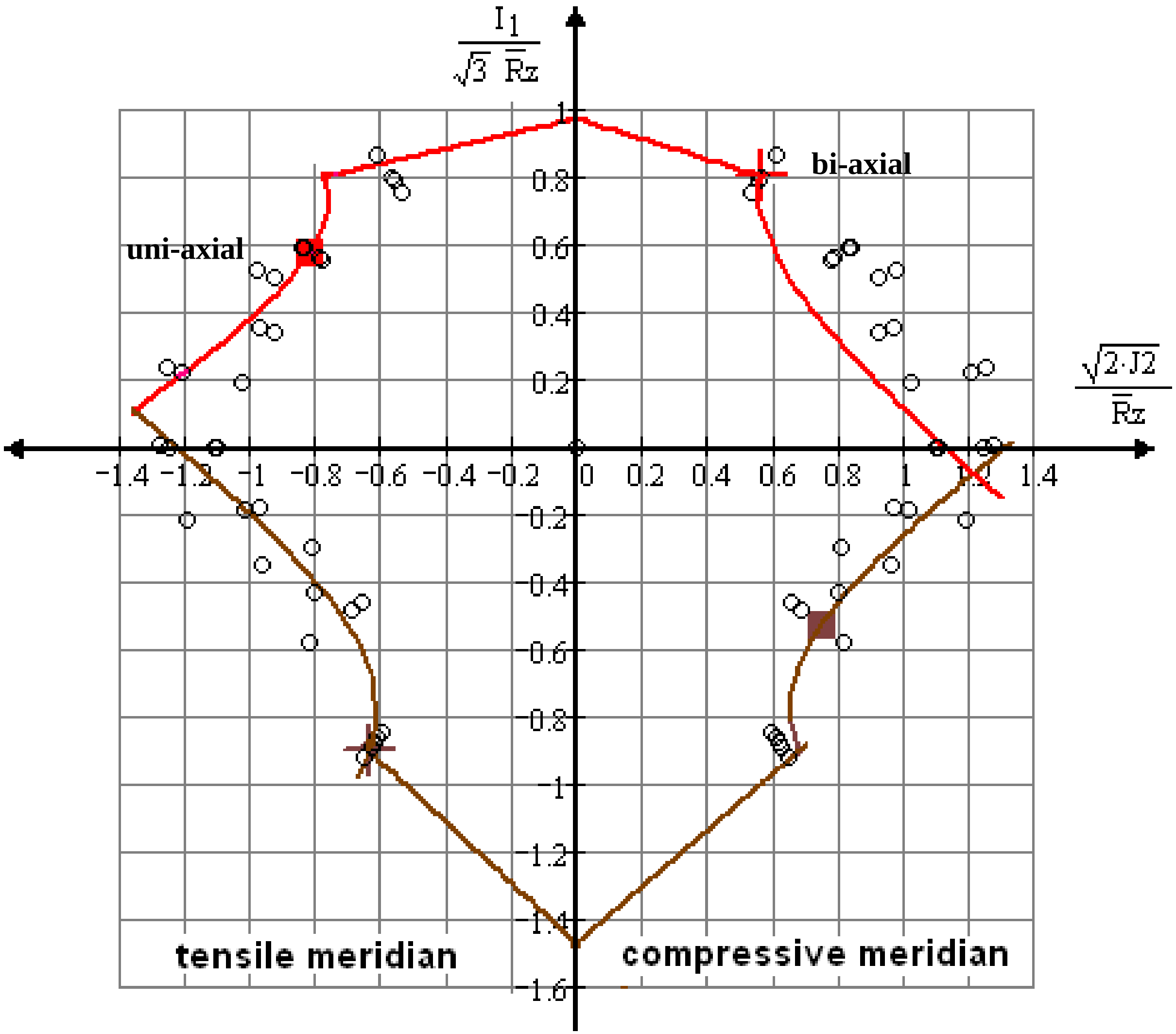
shear meridian angle = 0°

tensile meridian +30°

compressive meridian -30°

Tensile and Compression Meridian of the Fracture Failure Surface

Rohacell 71 IG



$$Eff = \sqrt[m]{Eff\circledast\sigma^{m} + Eff\circledast cr^{m}}$$

$Rz = 1.8$ $Rzz = 1.25$ $Rzzz = 1.01$

$Rd = 1.65$ $Rdd = 1.4$ $Rddd = 1.53$

$maxI1 = 3.03$ $minI1 = -4.58$

$D\sigma = -0.71$ $Dcr = 0.21$ $c1\circledast cr = 1.03$

$scap = -0.27$ $sbot = 0.87$

$\theta\sigma zz = -0.52$ $\theta crdd = 0.52$

$\circledast\sigma zz = 1.2$ $\circledast crdd = 1.07$

in **Lode-Westergaard coordinates**

The fracture test data are located at a distinct Lode angle of its associated ring o, 120°-symmetry of the isotropic failure surface (body) .

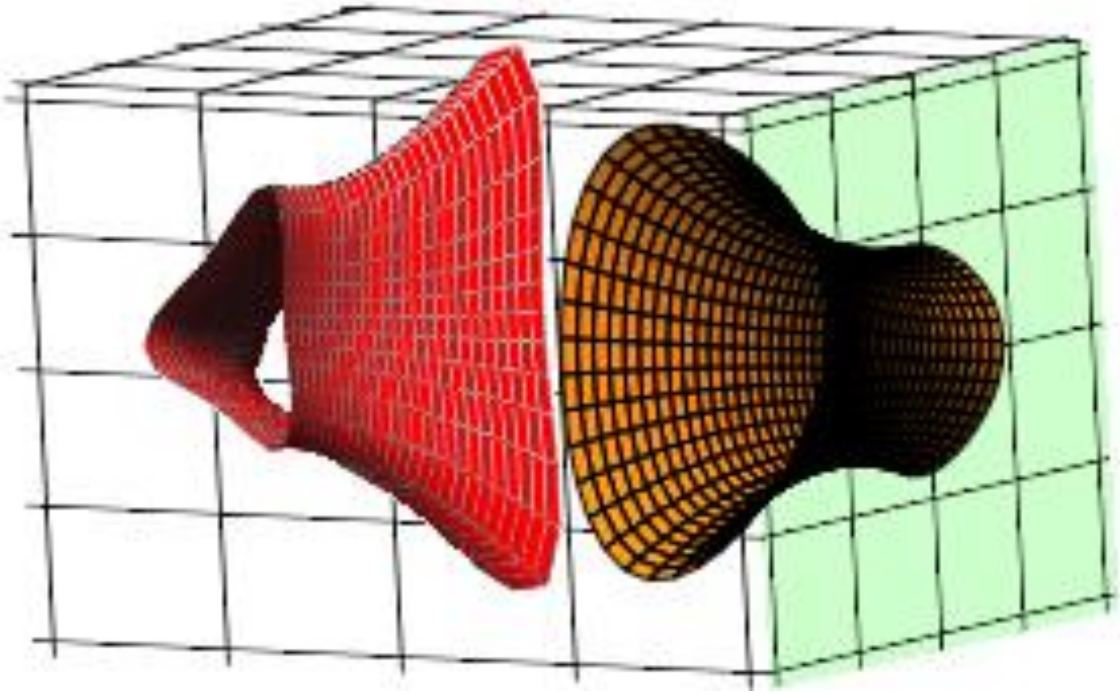
Cap and bottom are closed by a cone-ansatz, a geometry being on the safe side.

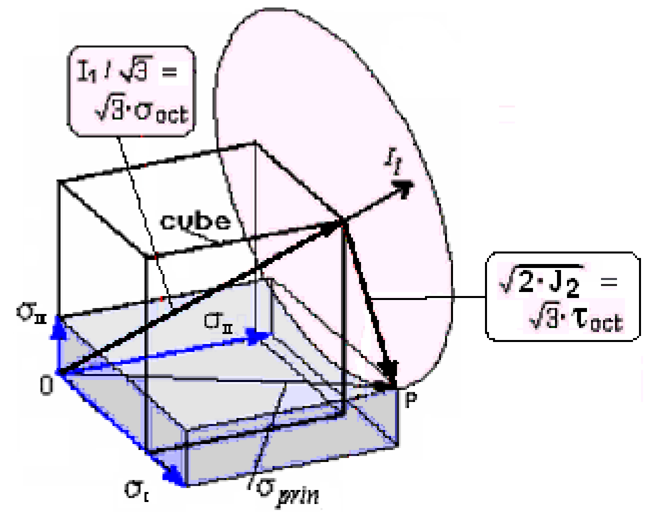
The 3D-strength failure condition enables to predict the 120°-symmetric failure body and to judge a 3D- stress state

Fracture Failure Surface of *Rohacell 71 IG*

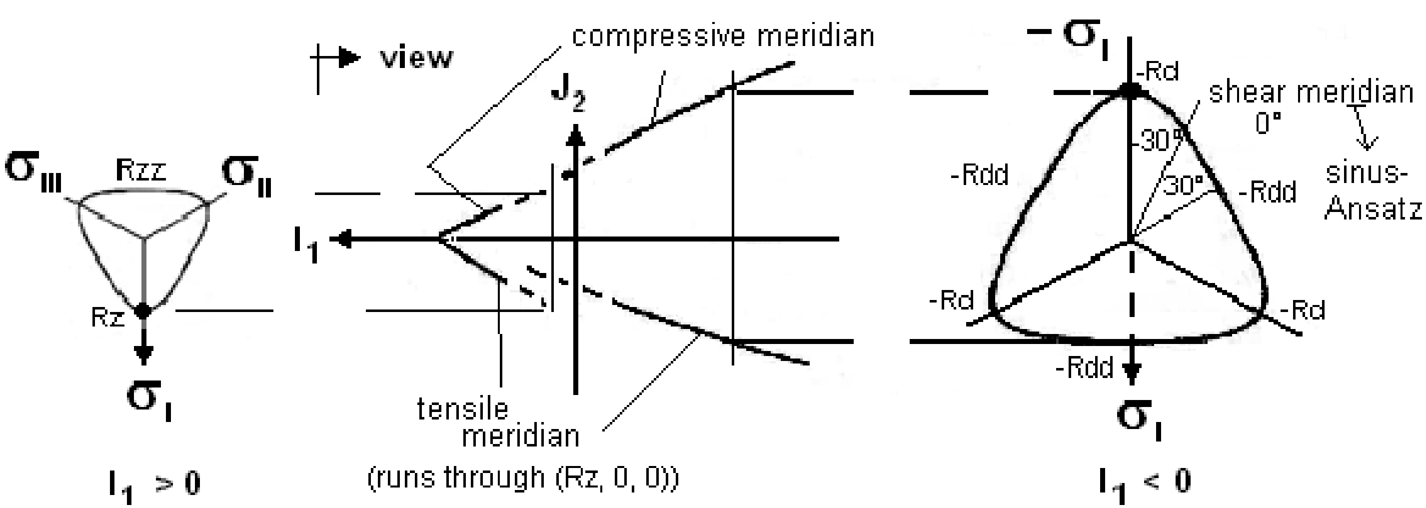
(cap, interaction, bottom missing*)*

The dent turns !





visualization of the Lode-Westergaard coordinates



Determination of the Load-defined Reserve Factor RF

Linear elastic problem for this brittle behaving material

Residual stresses = 0

RF $= f_{Res}$ (material reserve factor) $= Eff^{-1}$

Stress state:

$\sigma I := 0.9 \qquad \sigma II := -0.4 \qquad \sigma III := 0.5$

Statistically reduced Strengths:

$Rz := 0.9 \cdot \bar{Rz} \qquad Rd := 0.85 \cdot \bar{Rd}$

Shape parameters: $D\sigma = -0.71 \qquad D\tau = 0.21 \qquad c1\Theta\sigma = 1.15 \qquad c1\Theta\tau = 1.03$

$I1 := \sigma I + \sigma II + \sigma III$

$I1 = 1$

$$J2 := \frac{\left[(\sigma I - \sigma II)^2 + (\sigma II - \sigma III)^2 + (\sigma III - \sigma I)^2\right]}{6}$$

$J2 = 0.44$

$$J3 := \frac{\left[(2 \cdot \sigma I - \sigma II - \sigma III) \cdot (2 \cdot \sigma II - \sigma I - \sigma III) \cdot (2 \cdot \sigma III - \sigma II - \sigma I)\right]}{27}$$

$J3 = -0.07$

$$Eff\Theta\sigma := c1\Theta\sigma \cdot \frac{\sqrt{4 \cdot J2 \cdot \sqrt[3]{1 + D\sigma \cdot 1.5 \cdot 3^{0.5} \cdot J3 \cdot J2^{-1.5}} - \frac{1}{3} \cdot I1^2} + I1}{2Rz}$$

$$Eff\Theta\tau := c1\Theta\tau \cdot \frac{\sqrt{4 \cdot J2 \cdot \sqrt[3]{1 + D\tau \cdot \left(1.5 \cdot 3^{0.5} \cdot J3 \cdot J2^{-1.5}\right)} - \frac{1}{3} \cdot I1^2} - I1}{2 \cdot Rd}$$

$$Eff := \sqrt[9]{Eff\Theta\sigma^{m_{int}} + Eff\Theta\tau^{m_{int}}} \qquad Eff = 0.802 \qquad RF := \frac{1}{Eff} \qquad RF = 1.25$$

The loading may be monotonically increased by the factor RF !

Conclusions

- **The FMC is an efficient concept,**

  that improves prediction + simplifies design verification

  is applicable to brittle and ductile, dense and porous,
  isotropic, transversely-isotropic and orthotropic materials

  if clear failure modes can be identified and the material element homogenized.

  Formulation basis is whether the material element experiences
  a *volume* change, a *shape change* and *friction* .

  Builds not on the ***material*** but on ***material behaviour !***

- **Delivers a <u>combined formulation</u> of *independent modal failure modes*,**

  without the well-known drawbacks of <u>global</u> SFC formulations

  (which *mathematically combine in-dependent failure modes*) .

- **The FMC-based Failure Conditions are simple but**
  describe physics of each single failure mechanism pretty well.

- **Mapping of the brittle behaving porous foam was successful and with new findings !**