

 21. Münchner Leichtbauseminar, 2024, Keynote Lecture, 50 min + 10 min

Comparison of four UD Strength Criteria – including a **'Numerical Review'**

Prof. Dr.-Ing. habil. Ralf Cuntze VDI, formerly MAN-Technologie, linked to Composites United

- **1 Concerns when Generating Strength Failure Criteria (***SFC***)**
- **2 Terminology, Laminate Description, Material Stressing Effort** *Eff*
- **3 'Global' SFCs versus 'Modal' SFCs**
- **4 Background of Cuntze's Failure-Mode-Concept (***FMC***)**
- **5 3D /2D SFCs** (*harmonized due to VDI 2014*)
	- **5.1 SFC Cuntze** with **3 Examples from the UD World-Wide-Failure-Exercise**
	- **5.2 SFC Hashin**
	- **5.3 SFC Puck**
	- **5.4 SFC Tsai-Wu**
- **6 Comparison of the different SFC Failure Envelopes** $\tau_{21}(\sigma_2)$, $\sigma_2(\sigma_1)$, $\tau_{21}(\sigma_1)$
- **7 Computation of a SFC-linked Reserve Factor**

Streamlining the presentation:

Which structural component surfaces are faced by the Designing Engineer?

Depending on stress state and environment a brittle material may behave brittle or ductile .

A UD-material might be defined brittle for a strength ratio

$$
R_{\perp}^{c} / R_{\perp}^{t} \geq 2.5
$$

Streamlining the presentation: *Which is the Material Behaviour to be Discriminated?*

Same Terminology against misinterpretations

"A general system of signs and symbols is of high importance for a logically consistent universal language for scientific use !"

 Gottfried Wilhelm Leibniz (about 1800)

Validated Theoretical Model

"Theory is the Quintessence of all Practical Experience"

 A. Föppl

Daher benötigt man gute Werkstoffmodelle, wozu die Festigkeitsbedingungen gehören.

ähnlich der versuchsbegeisterte Kreiseltheoretiker Karl Magnus
ahnlich der versuchsbegeisterte Kreiseltheoretikerungs-Zent ähnlich der versuchsbegeisterte Kreiseltheoretiker Karl Magnus
bei einer Projektbesprechung 1972 zur Urananreicherungs-Zentrifuge. bei einer Projektbesprechung 1972
Für die Version mit CFK-Rohr noch einen Geschwindigkeits-Weltrekord gefahren.
Für die Version mit CFK-Rohr noch einen Geschwindigkeits-Weltrekord gefahren. Für die Version mit CFK-Rohr noch einen Geschwinger-
Allerdings dann das Aussteigen der MAN aus diesem Geschäft.

Objective = Product Certification by achieving Design Verification (here strength)

Failure function *F* : mathematical formulation of the failure event by $F = 1$, characterizing the Limit State

Tool Bricks:

Model with Modelling: Structure and Material

Model: Theoretical conception of a real process

(Strength) Failure Criterion SFC (mathematically accurate: *condition*): Condition on which a failure becomes effective, meaning $F = 1$ for one limit state.

Analysis: Computation that uses fixed model parameters (*e.g. Design Verification of the final design*)

Simulation: Process, that consists of several analysis loops and lasts until the system is imitated in the Design Dimensioning process. Model parameters are adjusted hereby to the 'real world' parameter set.

'Generic' number: Witnessed material symmetry knowledge seems to tell: There might exist a 'generic' (*term was chosen by the author*) material inherent number for the UD material family,, namely 5 for this transversely-isotropic material, where the plane 2-3 is quasi-isotropic and due to that UD is termed transversely-isotropic

Validation of a model: 'qualification' of a created model by well mapping physical test results with the derived model (here the material failure model SFC)

Design Verification: fulfillment of a design requirement data set (for a deformation, a frequency, design load, etc) in the final design

Material Stressing Effort *Eff* (corresponds to Puck's stress exposure): see definition later.

Terminology Basis: VDI 2014 guideline (*Author was editor 2006 and co-author*)

<u>Properties</u>: symbolic indices are dedicated to measurable properties in order to bypass misinterpretation, R_\perp^t . Theoretical or model parameters, running variables: numbers are dedicated according to mechanics Model parameters (basically the focus here) are average values and marked by a bar over.

**Such a 'global'' formulation is mathematically elegant*

- **Prediction* of a non-feasible domain in quadrant III of $\sigma_1(\sigma_2)$, whereas the 'modal' SFCs of *Puck and the FMC-based one of Cuntze map the test data*
- **Treatment of σ³ like σ² , which is not accurate but model-inevitably*
- **Cannot map for instance the humb in , because the material inherent internal friction cannot be directly considered in the global SFC. Hence, the computed Reserve Factor RF may not be on the safe side in this domain* **erminology of the Tsai-Wu:**
atically elegant
quadrant III of $\sigma_1(\sigma_2)$, whereas the
ntze map the test data
ccurate but model-inevitably
 $\sigma_2(\sigma_2^c)$, because the material inher
e global SFC. Hence, the computed Form **Terminology of the Tsai-Wu 3D-UD**
 catically elegant
 cation quadrant III of $\sigma_1(\sigma_2)$, whereas the 'mode
 cation map the test data
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 $\sigma_2(\sigma_2^c)$, because the material inherent inter **Example 15 and Allen Scheme Scheme Scheme Allen SFCs**
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 Example 16 Blue Section
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 Example 16 Blue Section
 Example 16 Blue Scheme Sche
- **Difficult determination of the model parameters in the 3D-formulation.The stress interaction* term $F_{12} = F_{13}$ (if UD) needs additional bi-axial (σ_1 , σ_2)-tests. The bi-axial material parameter *F*₁₂ *is 'principally' obtained by bi-axial compression tests. Usually it is applied* $F_{12} = -0.5$ *.*
- **For application just strength values are necessary, but this is not sufficient!*

**No information on the prevailing failure mode FF or IFF is received*

**Tsai's Strength Ratio R corresponds to 1 / Eff = fre (Altair also uses Strength Ratio, marked by the letters SR , corresponding to fRF)*

(However: Since many decades, in mechanics the letter R is dedicated in Standards to Strength (Resistance) *R.* Further it reads valid the Strength Ratio R = compressive strength /tensile strength (better term is SR). *In fatigue is forever practice (straight letter)* R *= minσ / maxσ, termed Stress Ratio.*

- ** On top, unfortunately in manuals, using Tsai-Wu, his computed 1/R value is called Safety Factor but a safety factor j is given and a reserve factor RF is to compute!*
- Question: The differently termed out-of-plane shear strength $S_{\infty}, \tau_{\infty}^*$, $R_{\infty} = R_{\infty}$ is how to *measure??*

Dear Ralf, July 3,

Thank you for your very important point. Too many undefined, or duplicate terms. Will have to clean them up. *Thanks. Steve*

Hamonizing Composite Terminology

Since the author is looking at all 3 material families at the same time, (*Which author has done this before***?) he used a self-explanatory, symbolic indexing,**

as he sensibly defined it as Editor and Co-author

of the VDI 2014, Sheet 3 'Analysis' 2006,

on the basis of already well-known applied designations in mechanics

together with his working group colleagues, such as A. Puck.

This only will make an understanding over the material & discipline fences possible and was the

"Conditio sine qua non" for the elaboration of this comparison!

Example: Strength terms

2 Terminology, Laminate Description, Material Stressing Effort

On basis of investigations for the VDI-2014 and the formerly planned novel ESA Materials Handbook, Cuntze proposed internationally not confusing designations for the strength properties (+physical properties). $t =$ tension, $c =$ compression

Notes on some designations:

* As a consequence to isotropic materials (*European standardization*) the letter *R* has to be used for strength !! US notations for UD material with letters X (*direction 1*,) and Y (*direction 2*,) confuse with the structural axes' descriptions X and Y. * R_m := 'resistance maximale' (French) = tensile fracture strength (superscript ^t is usually skipped because design runs in tensile domain), *R* is basic strength. Composites are most often brittle and only slightly porous! SF is shear fracture, NF Normal Fracture.

number of independent properties due to material symmetry of isotropic materials

Clear Laminate Descriptions for the Designer

Modeling laminates is a challenge. In this context, essential for the interpretation of the failures faced after testing, is the knowledge about the lay-up (stack) of the envisaged laminate, because crimped fabrics and Non-Crimped Fabric (NCF) -materials behave differently. It is further extremely necessary to provide the material-modeling design engineer and his colleague in production (for the Ply Book) with a clear, distinguishing description of UD-lay-ups being Non Crimp Fabrics NCFs (stitched multi-UD-layer) or Fabric layers (crimped). **example 16**
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F) -materials behave

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 Clear Laminate Descriptions for the Designer
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One could distinguish the various types by a clear optical designation, a square bracket [..] and a wavy bracket $\{\ldots\}$, in order to enable a realistic material modelling in the case of ply-by-ply analyses, that optically helps to distinguish NCF **{**stitched UD-stack**}** from woven fabrics, where one practically cannot mechanically separate the single woven layers within one fabric layer as in the case of *plain weave* binding, which therefore is 'globally' symmetric in itself. Applied this means:

deposited stack $\left[0/90\right]_{\text{S}} = \left[0/90/90/0\right]$ -lay-up, prepregs. *sited* stack $[0/90]_S = [0/90/90/0]$ -lay-up, prepregs.
stitched NCF: {0/90} + {90/0} symmetrically stacked, dry;

deliverable 'building blocks' are $\{0/45/-45/90\}$,

and the novel 'doble-double' C-plies $\{\varphi/\neg \psi/\neg \varphi/\psi\},\$

as DD building block and sub-laminate i.e.
$$
\{75 / -75 / -15 / 15\}_r
$$
 with r = repetitions.
\n* Semi-finished product, *woven fabric* $\begin{bmatrix} 0 \\ 90 \end{bmatrix}$, symmetric in itself.

Due to unclear descriptions unfortunately one can often not use the seldom available valuable test results of fiber-reinforced materials.

Cuntze's System (1990 proposed**) of Macro-scopic Fracture Failure Modes NF , SF**

Pre-requisites, when Generating FMC-based Strength Failure Criteria (SFC)

Pre-requisites for the establishment of the *F*ailure function *F* are:

- **simply formulated, numerically robust**,
- **physically-based,** and therefore, need only few information for pre-dimensioning
- shall allow for a simple determination of the design driving mode
- all **model parameters shall be measurable (**is not standard in SFC theories**)**
- a SFC must become zero if its driving stress becomes zero.

A SFC,
$$
F = 1
$$
, is the mathematical description of the failure surface !

Pre-requisites, especially required for UD Material Modelling and **Model Validation**

- **The UD-lamina is homogenized to a macroscopically homogeneous solid** or the lamina can be seen as a 'smeared' material
- The UD-lamina is transversely-isotropic:

On planes transverse to the fiber direction it behaves quasi-isotropically

• **For validation** of the strength material model **a uniform stress state** about the critical stress 'point' (location**) in the test specimen is mandatory**.

The presentation shall give an overarching comparative understanding
^{On the linear analysis level} ^{on the linear analysis level.}
resent of of the 4 present Strength Failure Criteria from
Tsai-Wu, Hashin, Puck and a Tsai-Wu, Hashin, Puck and Cuntze.
(LaRC from NASA LaRC from NASA was not included)

Steve Tsai:

- * Zusammen mit Steve war ich 1986 Chairman einer ICCM-session. "*Mach Du den chair, ich mache das Licht an und aus, was damals sehr wichtig war.*"
- * Seit Jahren bin ich mit Steve im Rahmen seiner neuen Ideen verbunden. Meine Übertragung in übliche Bezeichnungen sind in einem Beitrag enthalten, den ich mit Erik Kappel im Frühjahr 2024 veröffentlicht habe *" Benefits, applying Tsai's Ideas 'Trace', 'Double-Double' and 'Omni Failure Envelope' to UD-plies composed Laminates?*" Für letztere praktische Vor-Auslegungsidee habe ich eine formelmäßige Lösung anstatt der bisherigen numerischen Lösung gefunden, die ich Steve letztes Jahr als Weihnachtsgeschenk überreichen konnte.

Zvi Hashin:

Hashin saß vor mir in einer traditionsreichen Konferenz in Brüssel und sagte in etwa:

* *Wir werden kaum in der Lage sein, mit Bruchkriterien jemals Nachweise für UD-Bauteile führen zu können***"**

 Protest: Als Industriemann darf man diese Aussage nicht tolerieren, weil wir Nachweise führen müssen, um Strukturintegrität für unser Produkt belegen zu können, um es verkaufen zu können.

(Sir) Alfred Puck:

* Zusammen mit Puck suchten Michael Gädke, DLR und Cuntze MAN in vielen Besprechungen seit etwa 1985 kontinuierlich nach einer Verbesserung der UD-SFCs, was dann ja auch gelungen ist.

Mohr's Statement for isotropic materials:

" The strengths of a material are determined by the stresses σ_n , τ_{nt} on the fracture plane" (the fracture plane is usually inclined with respect to the action of the external stresses) Paul's modification of the Mohr-Coulomb Hypothesis:

 " Brittle (behaving) material will fracture in either that plane where the shear stress *τnt* reaches a critical value which is given by the shear resistance of a fiber-parallel plane increased by a certain amount of friction caused by the simultaneously acting compressive stress σ_n on that plane. Or, it will fracture in that plane, where the maximum principal (tensile) stress reaches the transverse tensile strength $R_{\perp}^{ \text{t}}$ (in the quasi-isotropic plane)".

* Hashin (1980):

 Proposed a modified Mohr-Coulomb IFF approach but did not pursue this idea due to numerical difficulties (*Puck succeeded on this way*). Also into this paper he included an invariant-based global quadratic approach (*Cuntze's invariant way*)

* Puck's Action Plane IFF Conditions (1990):

 Based his IFF conditions on Mohr-Coulomb and Hashin, Puck interacts the 3 Mohr stresses *σn , τnt , τn1* on the IFF fracture plane. He uses simple polynomials (*parabolic or elliptic*) to formulate a so-called master fracture body in the $(\sigma_n$, $\tau_{nt}, \, \tau_{n1})$ space. A compressive $\sigma_{\!}$ cannot cause fracture on its action plane

* Cuntze Failure-Mode-Concept – based IFF conditions (1993):

 Used 3 different invariant IFF conditions, based on his idea that each fracture condition is governed by 1 strength.

History of Cuntze's Failure Mode Concept (FMC), Collaboration with Puck

- * Since 1985 common search *with Prof. A. Puck (Uni Gh Kassel) and Dr. M. Gädke (DLR Braunschweig)* of improved fracture criteria to more reliably dimension UD lamina-composed laminates.
	- Results of this personal collaboration:
		- * Puck originally presented his stress-angle relationships by an excellent wording. For a discussion with the DLR in 1991 Cuntze recommended to use the present matrix formulation.
		- * Puck delivered in the author's WWFE-I PartA thankfully a *Comparison of Puck's and Cuntze's failure theories, considering the first and more complicate version of Cuntze's FMC-based SFC. His there used 'addition theorems' to combine invariant scalar formulation with Puck's – Mohr's vector formulation later enabled Cuntze to replace fictitious friction parameters of his scalar SFC model by 2 directly measurable friction values, good to estimate input values.*
- * From 1992-1997, investigation of the '*Hashin-Puck Action Plane Strength Criterion*' (*see VDI Fortschrittbericht 1997, project leader R. Cuntze, MAN Technologie*).
- * Since 1993, in parallel the elaboration of the FMC began.

 This is based on von Mises invariant idea, who describes by his criterion (just) 1 failure mode, namely yielding. As describing function an isotropic invariant J_2 he used. It should be possible to transfer this idea from the yield mode of ductile isotropic materials to fracture modes of brittle materials. Of course, the invariants (which reflect material symmetry) to be applied for the transversely–isotropic UD materials are different.

** *Note on the VDI-Guideline (1980-2006): Puck's Hashin-based SFC-model was at the finalization-time of the VDI 2014,sheet 3, the best validated SFC and therefore Puck was kindly invited to include his SFC into the Guideline!*

 Primary Objective in *Structural Design Verification*of the Structural Part is a Reserve Factor *RF* > 1 against a Limit State in order to achieve Certification for the Production

> **For each designed structural part it is to compute** *for each distinct 'Load Case' with its various Failure Modes*

*R***eserve** *F***actor** (load-defined) **:** *RF = Failure Load / applied Design Load*

Material Reserve factor : fRF = Strength / Applied Stress

if linear analysis: $f_{RF} = RF = 1/Eff$

*Material Stressing Effort *:* $Eff = \sigma / R = 100\%$ if $RF = 1$

** in German: Werkstoff-Anstrengung, a very expressive term.*

***** *equivalent artificial English term , being created in 2003 together with QinetiQ as organizer of the World-Wide-Failure-Exercises on UD-SFCs.*

stress exposure: körperliche Stressbelastung und Beanspruchungslimit (Belastbarkeit).
Werkstoffanstrengung, Beanspruchungsniveer. Korperliche Stressbelastung und Beanspruchung
material stressing effort: Werkstoffanstrengung, Beanspruchungsniveau
Anstrengung Anstrengung.

Advantageous Use of the Equivalent Stress-linked Material Stressing Effort

Brittle materials possess a set of fracture failure modes

"*Which SFC-Types are used?"* **So-called 'Modal' and 'Global'** (pauschal) **SFCs**

Cuntze's 'Play on Words' All modes are married in the Global formulation. Any change hits all mode domains NF and SF of the fracture body surface

Drucker-Prager, Ottosen, Willam-Warnke, Tsai-Wu, Altenbach/Bolchun/Kulupaev, Yu, etc.

Mises.Puck.Cuntze

All modes are separately formulated.

Any change hits only the relevant domain of the fracture body surface

$$
F\left(\{\sigma\}, \left\{R^{\text{mode}}; \mu^{\text{mode}}\right\}\right) = 1 \quad \text{more precise formulation} \quad \text{Now,}
$$

by direct introduction of the friction value considering Mohr-Coulomb for brittle materials under compression

$$
UD: \quad \left\{\sigma\right\} = \left(\sigma_{1}, \sigma_{2}, \sigma_{3}, \tau_{23}, \tau_{31}, \tau_{21}\right)^{T}, \quad \left\{\overline{R}\right\} = \left(\overline{R}_{\parallel}^{t}, \overline{R}_{\parallel}^{c}, \overline{R}_{\perp}^{t}, \overline{R}_{\perp}^{c}, \overline{R}_{\perp\parallel}^{t}; \mu_{\perp\parallel}, \mu_{\perp\perp}\right)^{T}
$$

Isotrop: $\{\sigma\} = (\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{zx}, \tau_{xy})^T = (\sigma_y, \sigma_y, \sigma_{yy})^T$, $\{\overline{R}\} = (\overline{R}', \overline{R}^c; \mu)^T$

3 'Global' SFCs versus 'Modal' SFCs

Consequences of mathematically forcibly-married 'Global' SFCs

'Modal':

The figure visualizes for a distinct global SFC, termed ZTL (Zukunft Technik Luftfahrt), still used in the Airbus-linked structural HSB Handbook, how dramatically a change of the tensile strength affects the failure curve in the compression domain, although no physical impact can be! In the figure the word initially refers to the originally ZTL-mapped curve and finally to the ZTL-mapped curve considering the reduced tensile strength.

 A change in IFF1 affects just IFF1 and not the two other modes as a global SFC (red dotted) !!

4 Basics of the FMC

performed by the author analogously to :

 failure mode-wise *(shear yielding failure, etc.)*

 \triangleright **stress invariant-based** (*J₂ etc.*) using *physical content of the distinct Invariant*

use of material symmetry demands

application of equivalent stresses

Mises, Hashin, Puck etc.

Mises, Tsai, Hashin (also), **Christensen, etc.**

Christensen

Mises for shear yielding, Rankine for fracture

How can the Driving Ideas be realized?
Details of the first 3 points above Details of the first 3 points above \rightarrow

Failure mode-wise based Features of the FMC (1995)

It could be found:

- **Each failure mode represents 1 independent failure mechanism** and thereby 1 piece of the complete *failure surface*
- **Each failure mechanism is governed by 1 basic strength**
- **Each failure** *mode* **can be represented by 1 strength failure** *criterion (SFC). Therefore, equivalent stresses can be computed for each mode !!*

Observation \rightarrow

Physical Observation: Which UD Strength Fracture Failure Modes are given ?

[7]

Stress Invariants-based

* Invariants remain unchanged under coordinate transformations

* Invariants (see Mises) can be dedicated to a physical mechanism of the deforming solid ! $\frac{1}{2}$

4 Basics of the FMC

There seems to exist (after intensive investigations of the author) a 'generic' (term was chosen by the author) material inherent number for the envisaged 3 Material Families:

Isotropic Material: 2

Transversely-Isotropic UD Material: 5

- 5 e*lastic 'constants' E,ν; 5 strengths R ; 5 strength failure modes* (NFs with

SFs); *5 fracture mechanics modes K*

Orthotropic Material: 9

The existence of a 'generic' number * will significantly simplify the Structural Mechanics Building
* determines the necessary Test Amount I * determines the necessary Test Amount !

→ This involves 2 aspects for the author:

(1) σeq **captures the common action** *Eff* **(Werkstoffanstrengung) of a multi-axial stress state, active in a distinct failure mode** *is equal to an action: a multi-axial stress state as in * Mises σeq**: ductile, Mode 'Shear stress Yielding',*

 ** Maximum σeq : brittle, Mode 'Normal Fracture' etc.*

(2) The value of *σ*^{*eq*} is

comparable to a resistance: a strength value R

belonging to the activated failure mode.

Visualization for an isotropic material experiencing structural stresses:

Choice of Modal Concept → **requires** an **interaction formula for the Modal SFC set**

Multi-axial stress states usually activate more than one failure mode → interaction is to apply. This **Interaction in the 'mode transition zones'** of

 adjacent Failure Modes *is captured by a series failure system* **model**

 = 'Accumulation' of interacting *failure danger portions* mode *Eff*

$$
Eff = \sqrt[m]{(Eff^{mode 1})^m + (Eff^{mode 2})^m + \dots} = 1 = 100\% , if failure
$$

with a mode-interaction exponent *2.5 < m < 2.9 , from a long mapping experience*

It is assumed engineering-like: *m* takes the same value for all mode transition zones !

 $Note:$ Modal SFCs need an interaction of Failure Modes.
This is performed by a probabilistic This is performed by a probabilistic approach (was a so-called series failure system)
in the transition zone between neighboring modes NF and SF in the transition zone between neighboring modes NF and SF.

Applying an interaction equation to consider all micro-damage causing portions of all activated modes makes to move from the absolute value of the Failure Function |*F|* **to** *Eff !*

* For a mathematically homogeneous Failure Function F using
 $Eff = \sigma / R$ it reads

homogeneous Failure Function F using
 $\frac{df}{dt} = \frac{\sigma}{R}$ it reads
 $\frac{1}{2}$ / $R = \sqrt{3 \cdot 2\sigma^2 / 6}$ / $R = \frac{\sigma}{R}$ = Eff = 1 \Rightarrow F = Eff σ \sim σ $=\sigma/R$ it reads
 $R = \sqrt{3 \cdot 2\sigma^2 / 6} / R = \frac{\sigma}{\epsilon} = Eff = 1 \implies F = Eff$

* For a mathematically non-homogeneous *F* such as
\n
$$
F = c_1 \cdot \frac{\sigma^2}{R^2} + c_2 \cdot \frac{\sigma}{R}
$$
\nor $F = c_1 \cdot Eff^2 + c_2 \cdot Eff$ \Rightarrow $F \neq Eff$.

- **What von Mises reached by the invariant description of the single failure mode yielding shall be performed for the set of 5 fracture failure modes faced with UD materials**
- **Applying Poisson ratio** *ν* **directly in the SFC formula**
- **Enabling the necessary interaction of all activated single FF and IFF modes, when** *'Modal'*
- **A UD Strength Failure Criterion should directly capture the fracture of the fiber, the matrix, fiber-matrix interface and of the delamination of a layer as a subpart of the laminate**
- **WWFE-II requirements:**

 - To consider that Poisson's ratio *ν* **may cause micro-mechanically axial tensile failure of the constituent filament under bi- axially compressive stressing without any external tension loading** σ_1 **(considered in** Cuntze's FMC)

based Mathcad-program, however then was not interrogated by the organizer, who was familiar with this topic !) **- To capture weakening of the matrix under pressures > 200 bar.** (effortfully considered in Cuntze's FMC- **Invariant Formulations of Cuntze's SFC set**

invariant Formulations of Cuntze's SFC set
\n
$$
FF1: F_{\parallel}^{\sigma} = \frac{I_{1}}{R_{\parallel}^{\epsilon}}, \qquad FF2: F_{\parallel}^{\tau} = \frac{-I_{1}}{R_{\parallel}^{\epsilon}},
$$
\n
$$
IF1: F_{\perp}^{\sigma} = \frac{I_{2} + \sqrt{I_{4}}}{2\overline{R}_{\perp}^{\epsilon}}, \quad IFF2: F_{\perp}^{\tau} = a_{\perp\perp} \frac{I_{2}}{\overline{R}_{\perp}^{\epsilon}} + \frac{b_{\perp1}\sqrt{I_{4}}}{\overline{R}_{\perp}^{\epsilon}}, \quad IFF3: F_{\perp\parallel} = \frac{I_{3}^{3/2}}{\overline{R}_{\perp\parallel}^{\epsilon}} + b_{\perp\parallel} \frac{I_{2} \cdot I_{3} - I_{5}}{\overline{R}_{\perp\parallel}^{\epsilon}},
$$
\nwith $a_{\perp\perp}(\mu_{\perp\perp}), b_{\perp\perp}(\mu_{\perp\perp}), b_{\perp\parallel}(\mu_{\perp\parallel})$ and
\n
$$
I_{1} = \sigma_{1}, \quad I_{2} = \sigma_{2} + \sigma_{3}, \quad I_{3} = \tau_{31}^{2} + \tau_{21}^{2}, \quad I_{4} = (\sigma_{2} - \sigma_{3})^{2} + 4 \cdot \tau_{23}^{2},
$$
\n
$$
I_{5} = (\sigma_{2} - \sigma_{3}) \cdot (\tau_{31}^{2} - \tau_{21}^{2}) - 4\tau_{23}\tau_{31}\tau_{21}, \quad \text{(countesy: obtained from Boehler, 1995)}
$$
\n
$$
I_{2} \cdot I_{3} - I_{5} \rightarrow I_{23-5} = 2\sigma_{2} \cdot \tau_{21}^{2} + 2\sigma_{3} \cdot \tau_{31}^{2} + 4\tau_{23}\tau_{31}\tau_{21}.
$$
\n
$$
I_{2} = \frac{7\overline{\sigma}_{1} \cdot \frac{1}{\sigma_{1} \cdot \sigma_{2}}}{\sigma_{1} \sigma_{1} \cdot \frac{1}{\sigma_{2} \cdot \sigma_{3}}} = \frac{7\overline{\sigma}_{2} \cdot \frac{1}{\sigma_{2} \cdot \sigma_{3}}}{\sigma_{2} \
$$

$$
I_{1} = \sigma_{1}, \quad I_{2} = \sigma_{2} + \sigma_{3}, \quad I_{3} = \tau_{31}^{2} + \tau_{21}^{2}, \quad I_{4} = (\sigma_{2} - \sigma_{3})^{2} + 4 \cdot \tau_{23}^{2},
$$
\n
$$
I_{5} = (\sigma_{2} - \sigma_{3}) \cdot (\tau_{31}^{2} - \tau_{21}^{2}) - 4 \tau_{23} \tau_{31} \tau_{21},
$$
\n(Contresy: obtaine
\n
$$
I_{2} \cdot I_{3} - I_{5} \rightarrow I_{23-5} = 2 \sigma_{2} \cdot \tau_{21}^{2} + 2 \sigma_{3} \cdot \tau_{31}^{2} + 4 \tau_{23} \tau_{31} \tau_{21}.
$$
\n(10.10)

Invariants can be formulated in structural stresses and in Mohr stresses as the author had to execute.

 $||$ **tze's 3D SFC: 5 Mode F**
 $\begin{aligned}\n\mathbb{I}^{\sigma} &= \sigma_1 / \bar{R}_{\parallel}^t = \sigma_{eq}^{\parallel \sigma} / \bar{R}_{\parallel}^t \\
\mathbb{I}^{\tau} &= -\sigma_1 / \bar{R}_{\parallel}^c = +\sigma_{eq}^{\parallel \tau} / \bar{R}_{\perp}^t\n\end{aligned}$ $\begin{aligned}\n\mathbb{I}^{\tau} &= -\sigma_1 / \bar{R}_{\parallel}^c = +\sigma_{eq}^{\parallel \tau} / \bar{R}_{\perp}^t \\
\mathbb{I}^{\tau$ tze's 3D SFC: 5 Mode Fo
 $\overline{P} = \sigma_1 / \overline{R}_1^t = \sigma_{eq}^{\parallel \sigma} / \overline{R}_1^t$
 $\overline{P} = -\sigma_1 / \overline{R}_1^c = +\sigma_{eq}^{\parallel \sigma} / \overline{R}_1^t$
 $\overline{P} = -\sigma_1 / \overline{R}_1^c = +\sigma_{eq}^{\parallel \sigma} / \overline{R}_1^t$
 $\overline{P} = 0.5 \cdot [(\sigma_2 + \sigma_3) + \sqrt{\sigma_2^t}]$
 $\overline{P} = [\sigma_1 \$ $\overline{\text{FF1}}\colon\thinspace\text{Eff}^{\text{d}\sigma}=\thinspace\sigma_{\scriptscriptstyle 1}^{\scriptscriptstyle -\prime}\,\bar{R}_{\scriptscriptstyle \parallel}^{\scriptscriptstyle t} \qquad \,=\hskip.2cm \sigma_{\scriptscriptstyle eq}^{\scriptscriptstyle \parallel\sigma} \,\,/\,\bar{R}_{\scriptscriptstyle \parallel}^{\scriptscriptstyle t}$ **FF2:** $Eff^{||\tau} = -\sigma_1 / \bar{R}_{||}^c = +\sigma_{eq}^{||\tau} / \bar{R}_{||}^c$ FC: 5 Mode Formulations for FF1, FF2 and IFF
 $= \sigma_{eq}^{|\sigma|}/\overline{R}'_1$ Invariant SFC,
 $\sigma_{eq}^{|\sigma|}/\overline{R}'_1$ Invariant SFC,
 $\sigma_2 + \sigma_3$) + $\sqrt{\sigma_2^2 - 2\sigma_2 \cdot \sigma_3 + \sigma_3^2 + 4\tau_{23}^2}$] / $\overline{R}'_1 = \sigma$
 $(\sigma_2 + \sigma_3) + 4_{1\perp} + 1$ 2: 5 Mode Formulations for FF1, FF2 and IFF
 $= \sigma_{eq}^{||\sigma} / \overline{R}_{||}$ Invariant SFC
 $= +\sigma_{eq}^{||\sigma} / \overline{R}_{||}$ Invariant SFC
 $= +\sigma_{eq}^{||\sigma} / \overline{R}_{||}$ Invariant SFC
 $\sigma_2 + \sigma_3$) + $\sqrt{\sigma_2^2 - 2\sigma_2 \cdot \sigma_3 + \sigma_3^2 + 4\tau_{22}^2}$] **FF1, FF2 and IFF**
 Invariant SFC
 Invariant SFC
 $\frac{1}{2}$
 $\$ **2 3 Mode Formulations for FF1, FF2 and IF**
 $= \sigma_{eq}^{||\sigma} / \overline{R}_{||}$ Invariant SP
 $= + \sigma_{eq}^{||\sigma} / \overline{R}_{||}$ Invariant SP
 $+ \sigma_3) + \sqrt{\sigma_2^2 - 2\sigma_2 \cdot \sigma_3 + \sigma_3^2 + 4\tau_{23}^2}$] / $\overline{R}_{||}^t$ =
 $+ \sigma_3) + a_{\perp \perp} + 1 \cdot \sqrt{(\sigma_2 - \$ **Cuntze's 3D SFC: 5 Mode Formula (EXECT)**

FF1: $Eff^{||\sigma} = \sigma_1 / \overline{R}_\parallel^t = \sigma_{eq}^{||\sigma} / \overline{R}_\parallel^t$

FF2: $Eff^{||\tau} = -\sigma_1 / \overline{R}_\parallel^c = +\sigma_{eq}^{||\tau} / \overline{R}_\parallel^t$

IFF1: $Eff^{\perp \sigma} = 0.5 \cdot [(\sigma_2 + \sigma_3) + \sqrt{R}_\parallel^t]$

IFF2: $Eff^{\perp \tau} = [a_{$ **Cuntze's 3D SFC:** 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3
 $E(f^{x|x} = \sigma_1 / \overline{R}_1^x = \sigma_{eq}^{||\sigma} / \overline{R}_1^x$ for the three sets of the three states of the three subsets functions
 $E(f^{x|x} = -\sigma_1 / \overline{R}_1^x = +\sigma_{eq}^{$ **Cuntze's 3D SFC: 5 Mode Fc**
 $Eff^{\parallel \sigma} = \sigma_1 / \overline{R}_{\parallel}^t = \sigma_{eq}^{\parallel \sigma} / \overline{R}_{\parallel}^t$
 $Eff^{\parallel \tau} = -\sigma_1 / \overline{R}_{\parallel}^c = +\sigma_{eq}^{\parallel \tau} / \overline{R}_{\parallel}^t$
 $Eff^{\perp \sigma} = 0.5 \cdot [(\sigma_2 + \sigma_3) + \sqrt{6}$
 $Eff^{\perp \tau} = [a_{\perp \perp} \cdot (\sigma_2 + \sigma_3) + a_{\perp \perp}$
 Cuntze's 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1,

F1: $E[f]^{1/\pi} = \sigma_1 / \overline{R}_1^{\ell} = \sigma_{\overline{N}}^{1/\pi} / \overline{R}_1^{\ell}$
 $\frac{I_{\text{H}^* \text{trivial}}}{I_{\text{H}^* \text{tr}} \text{trivial}} \frac{E_{\text{C}} f_{\text{H}^*}}{S_{\text{H}^*}} = -\sigma_1 / \overline{R}_1^{\ell} = +\sigma_{\overline{$ IFF1: $Eff^{\perp \sigma} = 0.5 \cdot [(\sigma_2 + \sigma_1)]$ **Cuntze's 3D SFC: 5**

FF1: $Eff^{\parallel \sigma} = \sigma_1 / \overline{R}_{\parallel}^t =$

FF2: $Eff^{\parallel \tau} = -\sigma_1 / \overline{R}_{\parallel}^c =$

IFF1: $Eff^{\perp \sigma} = 0.5 \cdot [(\sigma_2 +$

IFF2: $Eff^{\perp \tau} = [a_{\perp \perp} \cdot (\sigma_2 +$

IFF3: $Eff^{\perp \parallel} = \{0.5 \cdot [b_{\perp \parallel} \cdot I_2\}$

Insertion: Compr **FF1**: $Eff^{\perp \sigma} = 0.5 \cdot [(\sigma_2 + \sigma_3) + \sqrt{\sigma_2^2 - 2\sigma_2 \cdot \sigma_3 + {\sigma_3}^2 + 4\tau_{23}}]/R_{\perp}^t = {\sigma_{eq}^{\perp \sigma}}/R_{\perp}^t$
 FF2: $Eff^{\perp \tau} = [a_{\perp \perp} \cdot (\sigma_2 + \sigma_3) + a_{\perp \perp} + 1 \cdot \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}}]/\overline{R}_{\perp}^c = +{\sigma_{eq}^{\perp \tau}}/\overline{R}_{\per$ *eq* $\begin{array}{lll} \sigma^c_1 &=& +\sigma^{\parallel\tau}_{eq}\ /\ \ \bar{R}^c_\parallel \ \sigma_2 + \sigma_3) & +& \sqrt{\sigma_2^{\;\;2} - 2\sigma_2\cdot\sigma_3 + {\sigma_3}^2 + 4{\tau_{23}}^2}\]/\ \bar{R}^t_\perp &=& \sigma^{\perp\sigma}_{eq}\ /\ \bar{R}^t_\perp \end{array}$ *t c* **Example 25 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3**
 Eff ${}^{\circ} = \sigma_1 / \overline{R}_1^{\circ}$ = $\sigma_{nq}^{\circ} / \overline{R}_1^{\circ}$ from the properties π_1 for π_2 *Eff* ${}^{\circ} = -\sigma_1 / \overline{R}_1^{\circ}$ = $+ \sigma_{nq}^{\circ} / \over$ **Eff**<sup> $||\sigma = \sigma_1 / \overline{R}_\parallel^t = \sigma_{eq}^{||\sigma} / \overline{R}_\parallel^t$
 Eff<sup> $||\sigma = \sigma_1 / \overline{R}_\parallel^t = \sigma_{eq}^{||\sigma} / \overline{R}_\parallel^t$
 Eff<sup> $||\sigma = -\sigma_1 / \overline{R}_\parallel^c = +\sigma_{eq}^{||\sigma} / \overline{R}_\parallel^c$
 Eff<sup> $||\sigma = 0.5 \cdot [(\sigma_2 + \sigma_3) + \sqrt{\sigma_2^2 - 2\sigma_2 \cdot c}$
 Eff<sup> $||\sigma = 1.5$ **Luntze's 3D SFC:** 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3
 Eff^{1/p} = $\sigma_1/\overline{R_1}$ = $\sigma_m^{107}/\overline{R_1}$ *Eff R*²
 Eff^{1/p} = $-\sigma_1/\overline{R_1}$ = $+\sigma_m^{107}/\overline{R_1}$ *E*
 *Eff*¹¹ = $-\sigma_1/\overline{R_1}$ = **Last 2 3D SFC: 5 Mode Form**
 Eff^{$||\sigma = \sigma_1 / \overline{R}_\parallel^t = \sigma_{eq}^{||\sigma} / \overline{R}_\parallel^t$
 Eff^{$||\sigma = -\sigma_1 / \overline{R}_\parallel^c = +\sigma_{eq}^{||\sigma} / \overline{R}_\parallel^t$
 Eff^{$||\sigma = 0.5 \cdot [(\sigma_2 + \sigma_3) + \sqrt{\sigma_2^2}]$
 Eff^{$||\sigma = [a_{\perp\perp} \cdot (\sigma_2 + \sigma_3) + a_{\perp\perp} +$
}}}} R_{\shortparallel}^{t} R_{\shortparallel}^c **ze's 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3
** $\sigma = \sigma_1 / \overline{R}_1^t = \sigma_{eq}^{lp} / \overline{R}_1^t$ **Invariant SEC-formulas
** $\tau = -\sigma_1 / \overline{R}_1^c = +\sigma_{eq}^{lp} / \overline{R}_1^c$ **Invariant SEC-formulas
 \tau = -\sigma_1 / \overline{R}_1^c = +\sigma_{ ze's 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3
** $\sigma = \sigma_1/\overline{R}_1^i = \sigma_{eq}^{||\sigma}/\overline{R}_1^i$ **from that SEC-formulas
** $r = -\sigma_1/\overline{R}_1^c = +\sigma_{eq}^{||\sigma}/\overline{R}_1^c$ **from regulated by their stress formulations.
 \sigma e's 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3

=** $\sigma_1/\overline{R_1}$ **=** $\sigma_{qq}^{1g}/\overline{R_1}'$ **from the** *morreplaced by maristality sFC: formulations***.

=** $-\sigma_1/\overline{R_1}$ **=** $+\sigma_{eq}^{1g}/\overline{R_1}'$ **from** $\overline{R_1}$ **f BD SFC:** 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3
 $\overline{I} \cdot \overline{R}^i_{\parallel} = \sigma_{eq}^{|\pi} \cdot \overline{R}^i_{\parallel}$ *mentions sec-jomulations*
 $\sigma_1 / \overline{R}^c = +\sigma_{eq}^{|\pi} \cdot \overline{R}^c_{\parallel}$ *mentions secon formulations*
 $0.5 \cdot [(\sigma$ **C:** 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3

= $\sigma_{eq}^{[pr}/\overline{R}_{||}^{L}$ *Invariant SEC-formaties*
 $\sigma_{eq}^{[pr}/\overline{R}_{||}^{L}$ *Invariant SEC-formaties*
 $\sigma_{2} + \sigma_{3}$) + $\sqrt{\sigma_{2}^{2} - 2\sigma_{2} \cdot \sigma_{3} + \sigma_{3}^{2} + 4\tau_{2$ **C:** 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3

= $\sigma_{eq}^{[r]} / \overline{R}_{\parallel}^{i}$ *Invariant SFC-formidas*

= $+\sigma_{eq}^{[r]} / \overline{R}_{\parallel}^{i}$ *Invariant SFC-formidas*
 $\sigma_{2} + \sigma_{3}$) + $\sqrt{\sigma_{2}^{2} - 2\sigma_{2} \cdot \sigma_{3} + \sigma_{3}^{2} + 4$ **Lze's 3D SFC:** 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3
 ${}^{\text{tr}}\mathbf{z} = \sigma_1 / \overline{R}_1^r = \sigma_{eq}^{\parallel \text{tr}} / \overline{R}_1^r$
 ${}^{\text{invariant SFC: Symulus}}$
 ${}^{\text{tr}} = -\sigma_1 / \overline{R}_1^r = +\sigma_{eq}^{\parallel \text{tr}} / \overline{R}_1^r$
 ${}^{\text{invariant SFC: Symulus}}$
 ${}^{\text{tr}}$ \perp and the set of \perp $f^{\perp \tau} = [a_{11} \cdot (\sigma_2 + \sigma_3) + a_{11} + 1 \cdot \sqrt{(\sigma_2 - \sigma_3)^2 + 4 \tau_{23}^2}]/\overline{R}_1^c = + \sigma_{ea}^{\perp \tau}/\overline{R}_1^c$ **SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3**
 $\overline{R}^{\ell}_{\parallel} = \sigma^{\parallel \sigma}_{\infty} / \overline{R}^{\ell}_{\parallel}$ for $\overline{R}^{\ell}_{\parallel}$ for $\overline{R}^{\ell}_{\parallel}$ for $\overline{R}^{\ell}_{\parallel}$ for $\overline{R}^{\ell}_{\parallel}$ for $\overline{R}^{\ell}_{\parallel}$ for $\overline{R}^{\ell}_{\$ **B'S 3D SFC:** 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3
 $= \sigma_1 / \overline{R}_\parallel^i$ $= \sigma_{\alpha i}^{1\sigma}/\overline{R}_\parallel^i$ from the *now replaced by their wees formulations*
 $= -\sigma_1 / \overline{R}_\parallel^i$ $= +\sigma_{\alpha i}^{1\sigma}/\overline{R}_\parallel^i$ from \over **e's 3D SFC:** 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3

= $\sigma_i / \overline{R}_i^i$ = $\sigma_{eq}^{bf} / \overline{R}_i^i$ from the properties $\sigma_i / \overline{R}_i^i$ from the properties $\sigma_i / \overline{R}_i^i$ from σ_i functions $-\sigma_i / \overline{R}_i^i$ = 's 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3
 $\therefore \sigma_1 / \overline{R}_1^i = \sigma_{\alpha_1}^{i\sigma} / \overline{R}_1^i$
 $\frac{Imurian SFC-formulas}{invvreplacot Ay under stress formulations}$
 $\therefore -\sigma_1 / \overline{R}_1^c = +\sigma_{\alpha_1}^{i\sigma} / \overline{R}_1^c$
 $= 0.5 \cdot [(\sigma_2 + \sigma_3) + \sqrt{\sigma_2^2 - 2\sigma_2$ **S 3D SFC:** 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3
 $\sigma_1 / \overline{R}'_1 = \sigma_{eq}^{||\sigma|} / \overline{R}'_1$ Invariant SEC-formulas
 $-\sigma_1 / \overline{R}_1'' = +\sigma_{eq}^{||\sigma|} / \overline{R}_1''$ Invariant SEC-formulas
 $-\sigma_1 / \overline{R}_1'' = +\sigma_{eq}^{||\sigma|} / \overline$ **e's 3D SFC:** 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF;
 $= \sigma_1/\overline{R}_1^t = \sigma_{eq}^{1/2}/\overline{R}_1^t$
 $= -\sigma_1/\overline{R}_1^c = +\sigma_{eq}^{r}/\overline{R}_1^c$
 $= 0.5 \cdot [(\sigma_2 + \sigma_3) + \sqrt{\sigma_2^2 - 2\sigma_2 \cdot \sigma_3 + \sigma_3^2 + 4\tau_{23}^{-2}}] / \overline{R}_2^t = \sigma_{eq$ **Cuntze's 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3

FF1:** Eff^{or} = $\sigma_1/\overline{R}_1^c = \sigma_{eq}^{10}/\overline{R}_1^c$

FF2: Eff^{or} = $-\sigma_1/\overline{R}_1^c = +\sigma_{eq}^{10}/\overline{R}^c$

IFF1: Eff^{or} = $-\sigma_1/\overline{R}_1^c = +\sigma_{eq}^{10}/\overline{$ **Cuntze's 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3
** $Eff^{*/-} = \sigma_1 / \overline{R}_1^{\prime} = \sigma_{eq}^{||\sigma} / \overline{R}_1^{\prime}$ **from thus** $\mu_0 = \sigma_0$ **
** $Eff^{*+} = -\sigma_1 / \overline{R}_1^{\prime} = +\sigma_{eq}^{||\sigma} / \overline{R}_1^{\prime}$ **from replaced by their we** Formulations for FF1
 $\overline{R}_{\parallel}^{c}$
 $\sqrt{\sigma_2^2 - 2\sigma_2 \cdot \sigma_3 + \sigma_3^2 + 4}$
 $a_{\perp \perp} + 1 \cdot \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau}$
 $b_{\perp \parallel}^2 \cdot I_{23-5}^2 + 4 \cdot \overline{R}_{\perp \parallel}^2 \cdot (\tau_{31}^2)$

(0, $-\overline{R}_{\perp}^{c}$) with the effort
 $\mu_{\perp \perp}$), IFF3: $Eff^{\perp \parallel} = \{0.5 \cdot [b_{\perp \parallel} \cdot I_{23-5} + (\sqrt{b_{\perp \parallel}^2} \cdot I_{23-5}^2 + 4 \cdot R_{\perp \parallel}^2 \cdot (\tau_{31}^2 + \tau_{21}^2)^2] / R_{\perp \parallel}^3\}$
Insertion: Compressive strength point $(0, -\overline{R}_\perp^c)$ with the effortfully derived fri **utions for FF1, FF2 and**
 Invariant
 now repla
 $\frac{m_{\text{variant}}}{m_{\text{varient}}}$
 $\frac{1}{\sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2}} \frac{1}{\sqrt{R_1^t}} =$
 $\frac{1}{2}$
 $\frac{1}{$ $\left(\begin{array}{c} c \\ l \end{array}\right)$ with the effortfully *eq* $\sqrt{K_{\perp}}$ *eq* \mathbf{R}_{\perp} . \overline{R}^c_+ **Example 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3**
 Eff $|F^2 = \sigma_1 / \overline{R}_1^k = \sigma_{\nu_1}^{1/2} / \overline{R}_1^k$ *Eff I R Eff Eff I R R R R Eff E Eff Eff Eff Eff Eff Eff Eff* $\perp \tau$ / $\overline{D}C$ $(\sigma_2 + \sigma_3)$ + $a_{\perp\perp}$ + $1 \cdot \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}}$ | / $R_{\perp} = +\sigma_{eq}^2$ / R_{\perp}
 $I_{\perp\parallel} \cdot I_{23-5} + (\sqrt{b_{\perp\parallel}^2 \cdot l_{23-5}^2 + 4 \cdot \overline{R}_{\perp\parallel}^2 \cdot (\tau_{31}^2 + \tau_{21}^2)^2}) / \overline{R}_{\perp\parallel}^3$ }^{0.5} = $\sigma_{eq}^{\perp\parallel}$ / $\$ **EP1, FF2 and IFF1, IFF2, IFF3**
 Invariant SFC-formulas
 now replaced by their stress formulations.
 $+4\tau_{23}^2$] / $\overline{R}_\perp^t = \sigma_{eq}^{\perp \sigma} / \overline{R}_\perp^t$
 $-4\tau_{23}^2$] / $\overline{R}_\perp^c = +\sigma_{eq}^{\perp \tau} / \overline{R}_\perp^c$
 ze's 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3
 $\vec{r} = \sigma_i / \vec{R}_i' = \sigma_{eq}^{1/\sigma} / \vec{R}_i'$
 $\vec{r} = \sigma_i / \vec{R}_i' = +\sigma_{eq}^{1/\sigma} / \vec{R}_i'$
 $\vec{r} = -\sigma_i / \vec{R}_i'' = +\sigma_{eq}^{1/\sigma} / \vec{R}_i''$
 $\vec{r} = 0.5 \cdot [(\sigma_2 + \sigma_3) + \sqrt{\$ Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3
 $\sigma_{eq}^{\text{IF}} / \overline{R}_{\parallel}^{\text{f}}$ $\frac{Invariant~STC-formula}{\text{non-replaced by their stress formulation}}$
 $+\sigma_{eq}^{\text{IF}} / \overline{R}_{\parallel}^{\text{f}}$
 σ_{3}) + $\sqrt{\sigma_{2}^{2} - 2\sigma_{2} \cdot \sigma_{3} + \sigma_{3}^{2} + 4\tau_{23}^{2}}$] / $\overline{R}_{\perp}^{\text{f}}$ \int_{\perp}^{c}) with the effortfully derived friction μ relation SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3
 $\overline{R}_1^{\nu} = \sigma_{eq}^{\pi}/\overline{R}_1^{\epsilon}$
 $\overline{R}_1^{\nu} = +\sigma_{eq}^{\mu\nu}/\overline{R}_1^{\epsilon}$
 $\overline{R}_1^{\epsilon} = +\sigma_{eq}^{\mu\nu}/\overline{R}_1^{\epsilon}$
 $\overline{R}_1^{\epsilon} = +\sigma_{eq}^{\mu\nu}/\overline{R}_1^{\epsilon}$
 \overline{R}_1 **'s 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3
** $\vec{C} = \sigma_{\vec{q}}^{\alpha}/\vec{R}_{\parallel}^{\prime}$ **=** $\sigma_{\vec{q}}^{\alpha}/\vec{R}_{\parallel}^{\prime}$ *Invertent SFC-formulas***
** $-\sigma_{\vec{l}}/\vec{R}_{\parallel}^{\prime} = +\sigma_{\vec{q}}^{\alpha}/\vec{R}_{\parallel}^{\prime}$ *Invertent by thet* From mapping experience obtained typical FRP-ranges: $0 < \mu_{\perp} < 0.3$, $0 < \mu_{\perp} < 0.2$. **ode Formulations for FF1, FF2 and IF**
 $\int_{\pi}^{\pi}/\overline{R}_{\parallel}^{t}$ $\int_{\pi}^{\pi}/\overline{R}_{\parallel}^{t}$ $\int_{\pi}^{\pi}/\overline{R}_{\parallel}^{t}$ $\int_{\pi}^{\pi}/\overline{R}_{\parallel}^{c}$
 $+ \sqrt{\sigma_{2}^{2}-2\sigma_{2}\cdot\sigma_{3}+\sigma_{3}^{2}+4\tau_{23}^{2}}$] / \overline{R}_{\perp}^{t} = $+$
 $+ (\sqrt{b_{\$ $\mu_{\parallel} \cong Z \cdot \mu_{\perp \parallel}.$ **Cuntze's 3D SFC: 5 Mode Formulations for FF**

FF1: $Eff^{|\sigma|} = \sigma_1/\overline{R}_1^{\prime} = \sigma_{\alpha q}^{|\sigma}/\overline{R}_1^{\prime}$

FF2: $Eff^{|\sigma|} = -\sigma_1/\overline{R}_1^{\sigma} = +\sigma_{\alpha q}^{|\sigma}/\overline{R}_1^{\sigma}$

IFF1: $Eff^{\perp\sigma} = 0.5 \cdot [(\sigma_2 + \sigma_3) + \sqrt{\sigma_2^2 - 2\sigma_2 \cdot \sigma_3 + \sigma_3^$ Two-fold failure danger in the σ_2 - σ_3 -domain (quadrant I) is mod **Cuntze's 3D SFC:** 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3

FF1: $E_i B^{\text{eff}} = \sigma_i / R_i^t = \sigma_{eq}^{10} / R_i^t$
 σ_{re}^{10}

FF2: $E_i B^{\text{eff}} = -\sigma_i / \overline{R}_i^c = +\sigma_{eq}^{10} / \overline{R}_i^c$
 σ_{re}^{10}

IFF1: $E_i B^{\text{eff}} = -\sigma_i / \overline$ **ze's 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3**
 $\sigma = \sigma_1 / \overline{R}_1^i$ = $\sigma_{eq}^{1/\overline{G}} / \overline{R}_1^i$ from the state is the proposed by their state formulations
 $r = -\sigma_1 / \overline{R}^c$ = $+\sigma_{eq}^{1/\overline{G}} / \overline{R}_1$ Formulations for FF1, FF2 and IFF1, IFF2, IFF3
 \overline{R}_1^i
 $\sqrt{\sigma_2^2 - 2\sigma_2 \cdot \sigma_3 + \sigma_3^2 + 4\tau_{23}^2}$ $]/ \overline{R}_\perp^i$ = $\sigma_{eq}^{1\sigma} / \overline{R}_\perp^i$
 $\sqrt{\sigma_2^2 - 2\sigma_2 \cdot \sigma_3 + \sigma_3^2 + 4\tau_{23}^2}$ $]/ \overline{R}_\perp^i$ = $\sigma_{eq}^{1\sigma} / \over$ **e's 3D SFC:** 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3

= $\sigma_i / \overline{R}_i^i$ = $\sigma_{eq}^{1g} / \overline{R}_i^i$ from the previous for σ_i from the properties $\sigma_i / \overline{R}_i^i$ from the properties of productions.

= $-\sigma_i / \over$ **3D SFC:** 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3
 $\sigma_1 / \overline{R}_\parallel^t = \sigma_{\text{eq}}^{\text{IV}} / \overline{R}_\parallel^t$ for $\sigma_1 / \overline{R}_\parallel^t$ for $\sigma_2 / \overline{R}_\parallel^t$ for $\sigma_1 / \overline{R}_\parallel^c = + \sigma_{\text{eq}}^{\text{IV}} / \overline{R}_\parallel^c$ for $\sigma_2 / \overline{R}_\parallel^$ $\overline{R}^{\text{MfFd}}_{\perp} = (\sigma_2^t + \sigma_3^t) / 2 \overline{R}^{\text{tt}}_{\perp}$, and $\overline{R}^{\text{tt}}_{\perp} \approx \overline{R}^{\text{t}}_{\perp} / \sqrt[m]{2}$ after [*Awa* 78] considering **D SFC: 5 Mode F**
 $\sqrt{\overline{R}}_||^t = \sigma_{eq}^{||\sigma|}/\overline{R}_{||}^t$
 $\frac{1}{2}$ $\sqrt{\overline{R}}_||^c = +\sigma_{eq}^{||\sigma|}/\overline{R}_{||}^t$
 $\frac{3}{2}$ $\frac{1}{2}$ $\frac{1$ **Cuntze's 3D SFC: 5 Mode Formulation**
 $Eff^{\text{tr}} = \sigma_1 / \overline{R}_1^t = \sigma_{eq}^{\text{tr}} / \overline{R}_1^t$
 $Eff^{\text{tr}} = -\sigma_1 / \overline{R}_1^c = +\sigma_{eq}^{\text{tr}} / \overline{R}_1^c$
 $Eff^{\text{tr}} = 0.5 \cdot [(\sigma_2 + \sigma_3) + \sqrt{\sigma_2^2 - 2\sigma_2 \cdot \frac{(\sigma_2 + \sigma_3)}{2}} + \sqrt{\sigma_2^2 - 2\sigma_2 \cdot \frac{$ Two-fold failure danger in the σ_2 - σ_3 -domain (quadrant I) is modelled by (bi-axial fracture stress R_{\perp}^u)
 $Eff_{\perp}^{\text{MfFd}} = (\sigma_2^t + \sigma_3^t)/2\overline{R}_{\perp}^u$, and $\overline{R}_{\perp}^u \approx \overline{R}_{\perp}^t/\sqrt[m]{2}$ after [Awa78] co is Formulations for FF1, FF2 and IFF1, IFF2, IFF3

invariant SEC-formulas
 $\sqrt{R_1'}$
 $\sqrt{R_2^c}$
 $+\sqrt{\sigma_2^2 - 2\sigma_2 \cdot \sigma_3 + \sigma_3^2 + 4\tau_{23}^2}$] / $\overline{R}_1' = \sigma_{eq}^{+\sigma}/\overline{R}_1'$
 $+ a_{\perp \perp} + 1 \cdot \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2}$] $\overline{)}$ **Cuntze's 3D SFC: 5 Mode Formulations for FIFI:** Eff^{tle} = $\sigma_1 / \overline{R}_1^{\mu}$ = $\sigma_{eq}^{\mu} / \overline{R}_1^{\mu}$

FF2: Eff^{tle} = $-\sigma_1 / \overline{R}_1^{\mu}$ = $+\sigma_{eq}^{\mu} / \overline{R}_1^{\mu}$

IFF2: Eff^{tle} = $-\sigma_1 / \overline{R}_1^{\mu}$ = $+\sigma_{eq}^{\mu} / \overline{$ **b SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3
** $\sqrt{R_i'} = \sigma_{eq}^{\text{df}} / \overline{R_i'}$ *the promulations***
** $\sqrt{R_i'} = +\sigma_{eq}^{\text{df}} / \overline{R_i}$ **
** $\int \sqrt{R_i'} = +\sigma_{eq}^{\text{df}} / \overline{R_i}$ **
** $\int \sqrt{R_i'} = \sigma_{eq}^{\text{df}} / \overline{R_i}$ **
 \int \sqrt{R_i'} = \sigma_{ Cuntze's 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3**
 $E(f^{(0)} = \sigma_1 / \overline{R_1} = \sigma_{eq}^{(0)} / \overline{R_1}$
 $F(f^{(0)} = -\sigma_1 / \overline{R_1} = +\sigma_{eq}^{(0)} / \overline{R_1}$
 $F(f^{(0)} = -\sigma_1 / \overline{R_1} = +\sigma_{eq}^{(0)} / \overline{R_1}$
 $F(f^{(1)} = -\sigma_1 /$ **Cuntze's 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2
** *Eff* ${}^{\sigma} = \sigma_1 / \overline{R}_1^{\sigma}$ **=** $\sigma_{eq}^{\sigma} / \overline{R}_1^{\epsilon}$ *Invariant SPC-formulations***
** *Eff* ${}^{\tau} = -\sigma_1 / \overline{R}_1^{\epsilon}$ **=** $+\sigma_{eq}^{\tau} / \overline{R}_1^{\epsilon}$ *Reformul* **Eig^{rig}** = $\sigma_1/\overline{R}_1^c$ = $\sigma_m^{\parallel r}/\overline{R}_1^c$ = $\sigma_m^{\parallel r}/\overline{R$ **Cuntze's 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3

:** $Ef^{[n]'} = \sigma_1/\bar{R}_1^i = \sigma_{eq}^{\text{lift}}/\bar{R}_1^i$
 $\frac{I^{(n)min_{\text{diff}}} \mathcal{SC}|f^{(n)min_{\text{diff}}} \mathcal{SC}|f^{(n)min_{\text{diff}}} \mathcal{SC}|f^{(n)min_{\text{diff}}} \mathcal{SC}|f^{(n)min_{\text{diff}}} \mathcal{SC}|f^{(n)min_{$ **Iode Formulations for FF1, FF2 and IFF1, IFF2, IFF3**
 $\frac{1}{\sqrt{a_{eq}}} \times \overline{R_1^k}$ *Invariant SFC-formulas*
 $\int_{\alpha q}^{\alpha q} / \overline{R_1^k}$ *Invariant SFC-formulas*
 $\int_{\alpha q}^{\alpha q} / \overline{R_1^k}$ $\int_{\alpha q}^{\alpha q} / \overline{R_1^k}$ $\int_{\alpha q}^$ **SFC:** 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3
 $\vec{R}_{\parallel}^{\epsilon} = \sigma_{eq}^{\parallel \sigma} / \vec{R}_{\parallel}^{\epsilon}$ $\vec{R}_{\parallel}^{\epsilon} = +\sigma_{eq}^{\parallel \epsilon} / \vec{R}_{\parallel}^{\epsilon}$ $\vec{R}_{\parallel}^{\epsilon} = +\sigma_{eq}^{\parallel \epsilon} / \vec{R}_{\parallel}^{\epsilon}$ $\vec{R}_{\parallel}^{\epsilon} = +\sigma_{eq}^{\parallel \epsilon} / \vec{R}_{\parallel}^{\$ **B'S 3D SFC:** 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3

= $\sigma_1 / \overline{R}_1^i$ = $\sigma_{eq}^{1\sigma} / \overline{R}_1^i$ from the *normalisa SEC*, formulations

= $-\sigma_1 / \overline{R}^c$ = $+\sigma_{eq}^{1\sigma} / \overline{R}_1^i$

= $0.5 \cdot 1(\sigma_2 + \sigma_3) + \sqrt{\$ **Lattice's 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3**
 $\mathcal{L}(\vec{F})^{\text{tr}} = \sigma_1 / \overline{R}_{\parallel}^* = \sigma_{\epsilon q}^{\text{tr}} / \overline{R}_{\parallel}^*$
 $\mathcal{L}(\vec{F})^{\text{tr}} = -\sigma_1 / \overline{R}_{\parallel}^* = +\sigma_{\epsilon q}^{\text{tr}} / \overline{R}_{\parallel}^*$
 $\mathcal{L}(\vec{F})^{\text{$ From mapping experience obtained typical range of interaction exponent $2.5 < m < 2.9$. $)$ **IDENTIFY AND IFF1, IFF2, IFF3**

Invariant SFC-formulas

now replaced by their stress formulations.
 $\frac{1}{2} \frac{\sigma_3 + \sigma_3^2 + 4\tau_{23}^2}{\sigma_3^2 + 4\tau_{23}^2}$ | / $\overline{R}_\perp^t = \sigma_{eq}^{\perp \sigma}$ / \overline{R}_\perp^t
 $\frac{1}{4} \cdot \overline{R}_{\perp$ **FF2 and IFF1, IFF2, IFF3**
 Invariant SFC-formulas
 now replaced by their stress formulation
 $\frac{1}{2}$
 $\$ **and IFF1, IFF2, IFF3**
 cow replaced by their stress formulations.
 $\overline{R}_{\perp}^{t} = \sigma_{eq}^{\perp \sigma} / \overline{R}_{\perp}^{t}$
 $\overline{R}_{\perp}^{c} = + \sigma_{eq}^{\perp \tau} / \overline{R}_{\perp}^{c}$
 $\frac{1}{2}$ $\overline{R}_{\perp}^{c} = + \sigma_{eq}^{\perp \tau} / \overline{R}_{\perp}^{c}$
 $\overline{R}_{$ **and IFF1, IFF2, IFF3**
 Pow replaced by their stress formulations.
 $\overline{R}_{\perp}^{t} = \sigma_{eq}^{\perp\sigma}/\overline{R}_{\perp}^{t}$
 $\overline{R}_{\perp}^{c} = +\sigma_{eq}^{\perp\tau}/\overline{R}_{\perp}^{c}$
 $\overline{R}_{\perp}^{c} = +\sigma_{eq}^{\perp\tau}/\overline{R}_{\perp}^{c}$
 $\overline{R}_{\perp}^{c} = \sigma_{eq}$ Formulations for FF1, FF.
 $\overline{R}_{\parallel}^{t}$
 $\sqrt{\sigma_2^2 - 2\sigma_2 \cdot \sigma_3 + \sigma_3^2 + 4\tau_{23}^2}$
 $-a_{\perp\perp} + 1 \cdot \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2}$
 $\sqrt{b_{\perp\parallel}^2 \cdot I_{23-2}^2 + 4 \cdot \overline{R}_{\perp\parallel}^2 \cdot (\tau_{31}^2 + \tau_{21}^2)}$

(t) (0, $-\overline{R}_{\perp}^{c}$ ntze's 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3
 $e^{r/a} = \sigma_1 / \overline{R}_1^i = \sigma_{eq}^{||\sigma} / \overline{R}_1^i$ *Invertiant SFC-formulas*
 $e^{rz} = -\sigma_1 / \overline{R}_1^c = +\sigma_{eq}^{||\sigma} / \overline{R}_1^c$ *Invertiant SFC-formulas*
 $r^{+x} =$ ie's 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3

= $\sigma_1/\overline{R_1}$ = $\sigma_{nn}^{15}/\overline{R_1}$ from the morphoed by the section of $\overline{R_1}$ from $\sigma_1/\overline{R_1}$ from σ_2 from σ_2 from $\overline{R_1}$ from $\$ **Cuntze's 3D S**

FF1: $Eff^{\parallel \sigma} = \sigma_1 / \overline{R}_{\parallel}^t$

FF2: $Eff^{\parallel \tau} = -\sigma_1 / \overline{R}_{\parallel}^t$ **cuntze's 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3

FF1:** $Eff^{\parallel \sigma} = \sigma_1 / \overline{R}_{\parallel}^t = \sigma_{eq}^{\parallel \sigma} / \overline{R}_{\parallel}^t$

FF2: $Eff^{\parallel \tau} = -\sigma_1 / \overline{R}_{\parallel}^c = +\sigma_{eq}^{\parallel \tau} / \overline{R}_{\parallel}^c$

FF2: $Eff^{\parallel \tau} = -\sigma_1 / \overline{R}_{\parallel$ **Cuntze's 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3**

FF1: $Eff^{\parallel \sigma} = \sigma_1 / \overline{R}_{\parallel}^t = \sigma_{eq}^{\parallel \sigma} / \overline{R}_{\parallel}^t$
 Invariant SFC-formulas
 now replaced by their stress formulations 2, IFF3

ss formulations.

formulations.

delation $\begin{array}{lll} \textbf{F} & \textbf{2, IFF3} & \ \textbf{f} & \textbf{f} & \ \textbf{f} & \textbf{f}$ $F3$
 mulations.
 \perp \parallel \cdot
 \therefore \parallel \cdot 2 2 **Solution 1988**
 Example 1988
 Example 1989
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 Example 2
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 Example 2
 Example 2
 Example 1
 Ex ulations for FF1, FF2 and IFF1, IFF2, IFF3

Invariant SFC-formulas

now replaced by their stress formulations.
 $-2\sigma_2 \cdot \sigma_3 + \sigma_3^2 + 4\tau_{23}^2$ $1/\overline{R}_\perp^t = \sigma_{eq}^{\perp \sigma}/\overline{R}_\perp^t$
 $\cdot \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2}$ $1/\$ Formulations for FF1, FF2 and IFF1, IFF2, IFF3
 R^{*c*}
 R^{*c*}
 c E^{*c*}
 c n^{*c*} *P*^{*c*}
 c n^{*c*} *R*^{*c*}
 *c*_{*c*}² - 2*c*₂ · *c*₃ + *c*₃² + 4*c*₂₃²] / \overline{R} ^{*f*}₁ = $\sigma_{eq}^{1-\sigma}$ tions for FF1, FF2 and IFF1, IFF2, IFF3

Invariant SEC-formulas

now replaced by their stress formulations.
 $\sigma_2 \cdot \sigma_3 + \sigma_3^2 + 4\tau_{23}^2 / R_1^t = \sigma_{eq}^{\perp \sigma} / \overline{R}_1^t$
 $(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2 / \overline{R}_1^e = + \sigma_{eq}^{\perp \tau} / \overline$ **Dynamize 10.1 FF1, FF2 and IFF1, IFF2, IFF3**
 Invariant SFC-formulas
 now replaced by their stress formulations
 $\overline{C_1^c}$
 $\overline{C_2^c}$
 $\overline{C_3^c}$
 $\overline{C_2^c}$
 $\overline{C_3^c}$
 $\overline{C_1^c}$
 $\overline{C_1^c}$
 $\overline{$ **nulations for FF1, FF2 and IFF1, IFF2, IFF3**
 Invariant SFC-formulas
 Invariant SFC-formulas
 $\frac{Imvariance}{B}$ $\int \vec{R}_{\perp}^{\prime} = \sigma_{eq}^{1-\sigma}/\vec{R}_{\perp}^{\prime}$
 $\frac{1}{2} - 2\sigma_{2} \cdot \sigma_{3} + \sigma_{3}^{2} + 4\tau_{23}^{2}$] $\sqrt{R}_{\perp}^{\prime} = \sigma_{eq$

What is really required for the Pre-design using Cuntze's 3D UD SFCs ?

Eff = 1 : 2D and 3D Fracture Body (after Replacement of σ , τ by $\sigma_{eq}^{\text{mode}}$) $\sigma_{eq}^{\rm mod}$ σ , τ by $\sigma_{eq}^{\text{mode}}$)

Interaction of UD Failure Modes = Outsmoothing in the Mode Transition Zones

*in-plane Mohr-simplification (2004***)**

World-Wide-Failure-Exercises-I and –II on UD laminas (1991-2013) **'***Mapping Round Robin Test***': Comparing theoretical UD-SFC-predictions with test results**

Organizer : *QinetiQ , UK (Hinton, Kaddour, Soden, Smith, Shuguang Li)*

Aim: "*Testing Predictive UD Failure Theories*

 = SFC of the UD lamina material including programming, however

 + non-linearity treatment + programming of the structure **laminate**

of Fiber–Reinforced Polymer Composites to the full !"

Procedure of the WWFE-I (2D test data**) and WWFE-II** (**3D test data):**

Part A : *Blind Predictions* **with average strength data , only.** (a necessary friction value information *µ* was not provided !)

Part B : *Comparison Theory-Test* **with Test data sets,** which were

not applicable or even involved false failure points. More

than 50% could not be initially used without specific care!

Cuntze's invariant-based strength criteria mapped the provided **accurate** test Cuntze's invariant-based strength criteria mapped the provised and current current data sets best. One third of the provided test data sets were finally not usable ! ata sets best. One time or SFC Model Validation.

SThe UD-examples serve for SFC Model Validation. \rightarrow The UD-examples serve for SFC <u>Model Validation.</u>
 \rightarrow The WWFE-II laminate examples serve as <u>Design Verification of</u> the Laminate.

A The WWFE-II latitude shallers and
In WWFE-I the author was officially the winner and In WWFE-II he ranked at the top!

The author tried to ask the Russian originator whether the two different data sets might belong to different tests. Unfortunately he had passed way.

Part A: Data of 4 strength points were provided, only Part B: Test data in quadrant IV show discrepancy, testing? No data for quadrants II, III was provided !

Furtrier Test Cases are assessed in [CUN 22 Life compilation]
5.1 SFC Cuntze with essential WWFE information

Mapping in the 'Tsai-Wu non-feasible domain', quadrant III $\sigma_2(\sigma_1)$

Part A, prediction: Data of elasticity properties, UD strengths \bar{R} provided, only. No friction value μ for IFF3 slope provided! Lowest value for μ in Part A assumed (safe side)

Part B:* the 3 strength points were altered!* Two doubtful (?) failure stress points were provided. Nevertheless, QinetiQ asked to map the course of PartB-test data (magenta curve) despite of the fact that a Poissons ratio μ_{011} had to be taken which is much too large, compared to values of own and other test campaigns !

* *The author knew one originator (R. Aoki) of the experiments at the DLR Stuttgart and asked him whether the question-mark*indicated, unusually high 2 test points might stem from a false evaluation. Unfortunately, a check of the data base was not *possible anymore. With the provided PartA inforation no reasonable Part A-mapping was possible!*

IFF Cross–section $\tau_{21}(\sigma_2)$ of the Fracture Failure Body

"Test results can be far away from the reality like an inaccurate theoretical model, like the 'global' SFC, partly.

Theory creates a model of the reality, one experiment shows one realization of the reality".

Mind: Being an *embedded* layer reduces the scatter in comparison to the measured technical strength results from the standard *isolated* specimens.

3rd Example <mark>WWFE-I,</mark> Test Case 2: CFRP, T300/BSL914C Ep $\tau^{}_{21}(\sigma^{}_{\!1})$

Provision of a mixture of 90°-tube test data and not accurately evaluated 0°-tube data. **The author corrected the 0°-tube data such that these could be also advantageuosly used together with the 0°-ones !**

Part A, prediction: Strength data only provided, no friction value (slope) μ .

- Part B, comparison: Strength data sets were provided, partly from 0°-test specimens (axial fiber direction) and partly from the traditional 90°- tube test specimen! After transformation, the two chosen ■◆, by executing a non-linearly CLT-computed shear strain analysis, these two 0°-points exemplarily could be shifted onto the magenta envelope. The shear strength point (blue) had to be adjusted according to new B information.
- Data from 0°- tube test specimens \Diamond cannot be used like hoop-wound 90°-tube test specimens, physically non-sense.
- The coordinate system of the 0°-test specimen lamina twists under torsion by y. Hence, 0°-test data must be transferred into the twisted material coordinate system.
- ► TC2 test data set could serve for material model validation after correction by the author and choosing just 90°- wound tubes.

6 Comparison of failure envelopes

defined by

Accurate 3D-analyses require *to Unlock* **the '***Mystery'* **behind** R^A , (better R^A_{∞} due to not being a measurable property UD-plane -isotropic
|ane
| i-isotropic
›lane Quasi-isotropic
 *ry*UD-plane Quasi-isotropic

In future we will have to analyze 3D stress states !

The strength quantity R_{23} is 'formally' linked to the associated ply stress $τ_{23}$

$$
max \tau_{23} = max \sigma_{\perp}^{t} = R_{23} = R_{\perp}^{t}
$$

alyses <u>require</u> to *Unlock* the '*Myste*
 $_{23} = R_{23} = \tau_{23}^*$; $R_{\perp\perp}^A$ (better R_{23}^A due to not being a
 1 future we will have to analyze 3D stress stat

tity R_{23} is 'formally' linked to the associate **Accurate 3D-analyses <u>require</u>** to *Unlock* the 'Mystery' behind

Tsai, Hashin/Puck : $S_{23} = R_{23} = r_{23}^2$; $R_{\perp\perp}^A$ (better R_{23}^A due to not being a measurable prop

In future we will have to analyze 3D stress **Accurate 3D-analyses <u>require</u> to Unlock the 'Myst**

Tsai, Hashin/Puck : $S_{23} = R_{23} = r_{23}^*$; $R_{\perp\perp}^A$ (better R_{23}^A due to not being
 In future we will have to analyze 3D stress state
 A The strength quan **analyses <u>require</u>** to Unlock the 'Myste
 $\frac{1}{2}$: $S_{23} = R_{23} = r_{23}^*$; $R_{\perp\perp}^A$ (better R_{23}^A due to not being a

In future we will have to analyze 3D stress state

uantity R_{23} is 'formally' linked to t alyses <u>require</u> to
 $R_{23} = R_{23} = \tau_{23}^*$; $R_{\perp\perp}^A$
 c if tuture we will have

ity R_{23} is 'formally

for relationship connect

or stresses $\binom{6}{p}$ = failure
 $\binom{2}{p}$ = failure
 $\binom{2}{p}$ = failure
 \bin the associated Mohr stresses ϵ_{fp} = failure plane) **EXECUTE: EXECUTE: COUPLOCK the 'Myst**

In the sum of $\frac{1}{2}$ once to $\frac{1}{2}$ once to **Example 3D-analyses <u>require</u>** to **Unlock the 'Myst**

Hashin/Puck : $S_{23} = R_{23} = r_{23}^*$; R_{\perp}^A (better R_{23}^A due to not being
 In future we will have to analyze 3D stress st

strength quantity R_{23} is 'fo **9 3D-analyses <u>require</u> to Unlock the**

VPuck : $S_{23} = R_{23} = \tau_{23}^*$; $R_{\perp1}^A$ (better R_{23}^A due to

In future we will have to analyze 3D s

gth quantity R_{23} is "formally" linked to the a

max $\tau_{23} = m$

ow **require to Unlock the 'Mys:**
 τ_{23}^* ; $R_{\perp\perp}^A$ (better R_{23}^A due to not being
 re will have to analyze 3D stress st
 s 'formally' linked to the associat
 $max \tau_{23} = max \sigma_{\perp}^t$
 $\begin{pmatrix} \sigma_2 \\ \rho_1 \\ \rho_2 \end{pmatrix}$ f_p \prime $\begin{array}{ccc} & C & S \end{array}$ $\begin{bmatrix} \n\frac{\partial}{\partial p} & \frac{\partial}{\partial p} \\ \n\frac{\partial}{\partial p} & \frac{\partial}{\partial p} \end{bmatrix} = \begin{bmatrix} \nc & s \\ -sc & sc \\ 0 & 0 \end{bmatrix}$ **rally ses <u>require</u>** to Unlock the $S_{23} = R_{23} = \tau_{23}^*$; $R_{\perp\perp}^A$ (better R_{23}^A due in future we will have to analyze 3D

n future we will have to analyze 3D

stitly R_{23} is 'formally' linked to the $max \tau_{23$ **alyses <u>require</u> to Unlock**
 $23 = R_{23} = \tau_{23}^*$; $R_{\perp L}^A$ (better R_{23}^A du
 1 future we will have to analyze

tity R_{23} is 'formally' linked to t
 $max \tau_{23}$

sfer relationship connects between the sum stress $\{\theta_{c}\}\right\} = |-sc \quad sc \quad c^{2} - s^{2} \quad 0$ 2 | \blacksquare $3 \mid$ $\sigma_{\rm c}$ and $\sigma_{\rm c}$ are the set of $\sigma_{\rm c}$ and $\sigma_{\rm c}$ are the set of $\sigma_{\rm c}$ and $\sigma_{\rm c}$ are the set of $\sigma_{\rm c}$

23

belongs to a point in the transition zone between IFF2 and IFF1.

 \div The strength quantity Cohesive shear Strength

 f_p) $\begin{pmatrix} - \end{pmatrix}$ sc sc

 $n(\mathcal{O}_{fp})$ \vert - \vert -sc sc

 $n\ell$ (ν_{fp}) | \lfloor \circ \cdot \cdot

 $\tau_{\rm nt}^{\rm fail}(\sigma_{\rm n}=0) = R_{23}^{\;\;\tau}$

Accurate 3D-analyses require to Unlock the 'Mystery' behind Quasi-isotropic
\nTsai, Hashin/Puck :
$$
S_{23} = R_{23} = r_{23}^*
$$
; $R_{\perp\perp}^A$ (better R_{23}^A due to not being a measurable property)UD-plane
\nIn future we will have to analyze 3D stress states !
\n
\n• The strength quantity R_{23} is 'formally' linked to the associated ply stress r_{23}
\n $max r_{23} = max \sigma_{\perp}^t = R_{23} = R_{\perp}^t$
\nThe following transfer relationship connects between the structural ply stresses and
\nthe associated Mohr stresses $(p_0 = \text{failure plane})$
\n
$$
\begin{bmatrix} \sigma_n(\theta_p) \\ \tau_n(\theta_p) \end{bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc & 0 & 0 \\ -sc & sc & c^2 - s^2 & 0 & 0 \\ 0 & 0 & 0 & s & c \end{bmatrix} \begin{bmatrix} \sigma_2 \\ \tau_3 \\ \tau_4 \\ \tau_5 \end{bmatrix}, c = cos \theta_p \text{ and } s = sin \theta_p
$$
\n
\n• The strength quantity Cohesive shear Strength
\ndefined by $\tau_{\text{int}}^{\text{fail}}(\sigma_n = 0) = R_{23}^{\text{T}}$
\nbelongs to a point in the transition zone between IFF2 and IFF1.

$$
\sum_{\sigma_{\perp}^{t}}\sum_{\nu_{\perp}}\sum_{\nu_{\perp}}^{T_{23}}
$$

 $\sigma_{\rm n} = 0$

Quasi-isotropic plane: Mohr Stresses

Display of two Cohesive Strength Determinations with fracture angle growth [Cun23a]

Extrapolation using just the SF (IFF2). Accurate derivation using SF and NF (IFF!)

Usual determination of a cohesive strength value (in the *transition zone between IFF2 and IFF1*) means an extrapolation from the compression strength point in the mode domain IFF2. Puck's '*Action plane resistance'* corresponds to a quantity termed '*Cohesive Shear Strength*' Such a quantity is linked to the chosen linear or non-linear 'Mohr model' ! Mohr Half Circles included for a better visualization

 $c =$ songt – 0.21 Ω f_p – const – 0. 21, \cup f_p $c \circ$ - 51° σ - 52 MD₀ *R*²³

 $(C+1)+\sigma_3\cdot (1-C)$, $\tau_{nt} = -0.5\cdot (1-C)^2\cdot (\sigma_2-\sigma_3)$.

Hashin's four invariant-based 3D-SFCs Set (4 modes, shear mode IFF3 is missing) r mode IFF3 is missing)
————————————————————

 $\overline{R} = \{\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21}\}^T$, $\overline{R} = (\overline{R}_{ij}^t, \overline{R}_{ij}^c, \overline{R}_{\perp}^t, \overline{R}_{\perp}^c, \overline{R}_{\perp}^t, \overline{R}_{\perp}^c)$, \overline{R}_{\perp} is trengths principally employed $(\sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$, $\{\overline{R}\} = (\overline{R}_{//}^t, \overline{R}_{//}^c, \overline{R}_{\perp}^t, \overline{R}_{\perp}^c, \overline{R}_{\perp}^t, \overline{R}_{23})^T$; 6 strengths principally employed
 $F\left(\{\sigma^A\}, \{\overline{R}^A\}, \theta_{fp}\right) = 1$, Puck's way \iff Hypothesis 2: $F\left$ Hashin does not formulate the (shear) IFF3,

Hyp. $1:$ " In the event that a failure plane under a distinct fracture angle can be identified, the failure is produced by the normal and shear stresses on that plane".

Hyp.2: "For UD – material the SFCs should be invariant under any rotation around the fiber direction."

If **invariant-based 3D-SFCs Set** (4 modes, shear mode
 \overline{R}_3 , \overline{r}_{21} , \overline{r}_1 , $\overline{R}_1^2 = (\overline{R}_N^t, \overline{R}_2^c, \overline{R}_1^t, \overline{R}_2^c, \overline{R}_{1,ji}; \overline{R}_{2,3})^T$; 6 strengths principal
 \overline{R}^A , $\overline{\theta}_P$) = 1, Puck ashin's four invariant-based
 $\frac{1}{1!}\sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21}$, \overline{I} , \overline{R} = $(\overline{R}_{ij}^t, \overline{R}_{ij}^t)$
 \therefore $F(\{\sigma^A\}, {\overline{R}^A\}, \theta_{fp}) = 1$, Puck's w

esses on the Action plane θ , failure if $\theta =$

the event des, shear mode IFF3 is missing)

strengths principally employed

2: $F\left(\{\sigma\}, {\overline{R}\}\right) = 1$, Cuntze's way

variants

oot formulate the (shear) IFF3,

nsiders τ in the FF1-model !

can be identified,

plane".

ot pursue **Hashin's four invariant-based 3D-SFCs Set** (4 modes, shear mode IFF3 is missing)
 $\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{31})^T$, $\{\overline{R}\} = (\overline{R_1}, \overline{R_2}, \overline{R_3}, \overline{R_3}, \overline{R_3}, \overline{R_3})^T$; 6 strengths principally employ **Hashin's four invariant-based 3D-SFCs Set** (4 modes, shear m
 $(\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$, $[\overline{R}] = (\overline{R}'_3, \overline{R}'_3, \overline{R}'_1, \overline{R}_{1, 3}, \overline{R}_{2, 3})^T$; 6 strengths prin

s 1: $F([\sigma^A], {\overline{R}^A}, \theta_{ip}) = 1$, Puck's **ed 3D-SFCs Set** (4 mc
 t_i, \overline{R}_{ij}^c , \overline{R}_{1}^t , \overline{R}_{1}^c , \overline{R}_{2}^t , \overline{R}_{2}^t , \overline{R}_{2}^t , \overline{R}_{2}^t , \overline{R}_{2}^t , \overline{R}_{23}^t)^T; (

's way ⇔ Hypothesis

'θ = θ_{*fp*} ⇔ use of in

Hashin **Four invariant-based 3D-SFCs Set** (4 modes, shear mode IFF3 is m
 $\frac{1}{\tau_{23}, \tau_{31}, \tau_{21}, \tau_{12}, \tau_{23}, \tau_{13}}$, $\frac{1}{\sqrt{R}}$ = ($\overline{R}_{\text{eff}}^{\text{V}}$, $\overline{R}_{\text{eff}}^{\text{V}}$, $\overline{R}_{\text{eff}}^{\text{V}}$, $\overline{R}_{\text{eff}}^{\text{V}}$, \overline{R}_{\text **nin's four invariant-based 3D-SFCs Set** (4 modes, shear mode IFF3 is missing)
 $\frac{1}{2}, \sigma_3, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_1, \mathbf{r}_2, \mathbf{r}^3$, $\sqrt{R_2}, \overline{R_2}, \overline{R_2}, \overline{R_2}, \overline{R_2}, \overline{R_2}, \overline{R_2}, \overline{R_2}, \overline{R_2}, \overline{R_2}, \overline{R$ **Shin's four invariant-based 3D-SFCs Set** (4 modes, shear mode IFF3 is missing)
 $\overline{P_2, P_3, T_3, T_4, T_2, Y}$. $\overline{R_1} = (\overline{R_2^2}, \overline{R_3^2}, \overline{R_1^2}, \overline{R_1^2}, \overline{R_2^2}, Y_3)$; 6 strengths principally employed
 $F\left[\left(\sigma$ **Hashin's four invariant-based 3D-SFCs Set** (4 modes, shear mode IFF3 is missing)
 σ] = ($\sigma_1, \sigma_2, \sigma_3, \tau_3, \tau_3, \tau_3, \tau_1^T$)⁷, $\{R\}$ = ($R_2^T, R_3^T, R_1^T, R_{21}^T, R_{32}^T$)⁷; 6 strengths principally employed
 ant-based 3D-SFCs Set (4 modes, shear mode IFF3 is missing)
 $\{\overline{R}\} = (\overline{R}_0^i, \overline{R}_0^i, \overline{R}_1^i, \overline{R}_{\perp 0}^i, \overline{R}_{\perp 0}$ **Hashin's four invariant-based 3D-SFCs Set** (4 modes, shear mode IFF3 is missing)
 $-(\sigma_1, \sigma_2, \sigma_1, \tau_2, \tau_3, \tau_1)^T$, $\{\overline{R}\}^2 - (\overline{R}_2^*, \overline{R}_2^*, \overline{R}_1^*, \overline{R}_1^*, \overline{R}_1^*, \overline{R}_2^*, \overline{Y}_3^*)$; 6 steengths principally **Hashin's four invariant-based 3D-SFCs Set** (4 modes, shear mode IFF3 is missing
 $\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{32})'$, $\{\overline{R}\} = (\overline{R_0}, \overline{R_2}, \overline{R_3}, \overline{R_4}, \overline{R_3}, \overline{R_4})$. It strengths principally employed
 Hashin's four invariant-based 3D-SFCs Set (4 modes, shear mode IFF3 is missing)
 $\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{32})'$, $\{\overline{R}\} = (\overline{R_0}, \overline{R_2}, \overline{R_3}, \overline{R_3}, \overline{R_3}, \overline{R_3})'$; 6 strengths principally employed
 Hashin's four inv.

 Hypothesis 1: $F(\lbrace \sigma^A \rbrace, \lbrace \overline{R}^A \rbrace, \theta$

in Mohr stresses on the Action pla

 Hyp. 1: " In the event that a fail

the failure is produced

Hashin herewith proposed a mande to numerical dif **Hashin's four invariant-based 3D-SFCs Set (4 modes, shear mode IFF3 is missing)**
 $\boxed{(\sigma)=(\sigma_1,\sigma_2,\sigma_3,r_{23},r_{31},r_{31})^T$, $\boxed{k}=(\overline{R_2^k},\overline{R_2^k},\overline{R_3^k},\overline{R_4^k},\overline{R_2^k},\overline{R_3^k},\overline{R_4^k},\overline{R_4^k})$; 6 strength **Hashin's four invariant-based 3D-SFCs Set** (4 modes,
 $\{\overline{\sigma}\} = (\overline{\sigma_1}, \sigma_2, \sigma_3, \overline{r_{23}}, \overline{r_{31}}, \overline{r_{41}})^T$, $\{\overline{R}\} = (\overline{R'_n}, \overline{R'_n}, \overline{R'_n}, \overline{R'_n}, \overline{R'_n}, \overline{R'_n}, \overline{R'_n}, \overline{R'_n}, \overline{R'_n}\}$. (Fypothesis 1: $F([\sigma^$ nin's four invariant-based 3D-SFCs Set (4 modes, shear mode 1
 $\frac{1}{2}, \sigma_3, \tau_{33}, \tau_{31}, \tau_{21}$)^T, $\{R\} = (\overline{R}_3^T, \overline{R}_3^T, \overline{R}_1^T, \overline{R}_{1, N_2}^T, \overline{R}_{2, N_2}^T)$; 6 strengths principally
 $F(\{\sigma^A\}, {\{\overline{R}^A\}}, \theta_{10$ **Hashin's four invariant-based 3D-SFCs Set** (4 modes, shear mode IFF3 is missing)
 $\frac{1}{2} = (\sigma_1, \sigma_2, \sigma_3, r_{23}, r_{31}, r_{21})^T$, $\frac{1}{\{R\}} = (\overline{R_3'}, \overline{R_2'}, \overline{R_3'}, \overline{R_4'}, \overline{R_4'}, \overline{R_5'}, \overline{R_5'}, \overline{R_5'}, \overline{R_5'}, \overline{R_5'}, \over$ $I_5^{Hashin} = 4\tau_{23}\tau_{31}\tau_{21} - \sigma_2 \cdot {\tau_{31}}^2 - \sigma_3 \cdot {\tau_{21}}^2$. Invariants now replaced in the generated SFCs by their stress relationships Hashin used the 5 stress invariants $I_1 = \sigma_1$, $I_2 = \sigma_2 + \sigma_3$, $I_3 = \tau_{31}^2 + \tau_{21}^2$, $I_4^{Has} = \tau_{23}^2 - \sigma_2 \cdot \sigma_3 \neq I_4^{Boeller}$, **Hashin's four invariant-based 3D-SFCs Set** (4 modes, shear mode IFF3 is missing)
 $\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{32})'$, $\{\overline{R}\} = (\overline{R_0}, \overline{R_2}, \overline{R_3}, \overline{R_4}, \overline{R_3}, \overline{R_4})$. It strengths principally employed *,* **If thin's four invariant-based 3D-SFCs Set** (4 modes, shear mode IFF3 is missing)
 $\frac{1}{T_2(T_2, T_2, T_3, T_1, T_1)}$, $\frac{1}{R_1} = (\overline{R_2}, \overline{R_2^2}, \overline{R_2}, \overline{R_3}, \overline{R_2}, \overline{R_3}, \overline{R_2}, \overline{R_3}, \overline{P_3}, \overline{P_4}, \overline{P_5}, \overline{$ ashin's four invariant-based 3D-SFCs Set (4 modes, shear mode IFF3 is missing)
 $\overline{r_1, \sigma_2, \sigma_3, r_{23}, r_{31}, r_{21}\}$, $\overline{R_1^1} = \overline{(R_2^1, \overline{R_2^2}, \overline{R_3^2}, \overline{R_4^2}, \overline{R_4^2}, \overline{R_5^2}, \overline{R_4^2}, \overline{R_5^2}, \overline{R_4^2}, \$ **Notariant-based 3D**

21)^T, $\{\overline{R}\} = (\overline{R}_{ij}^t, \overline{R}_{ij}^c, \overline{R}_{ij}^c)$
 $,\theta_{fp} = 1$, Puck's way

plane θ , failure if $\theta = \theta_{fp}$

ailure plane under a diverse the burn of the normal and s

modified Mohr-Coulomb

iffi in's four invariant-based 31
 $\overline{\mathbf{R}}$, $\overline{\sigma}_3$, $\overline{\mathbf{r}}_{23}$, $\overline{\mathbf{r}}_{31}$, $\overline{\mathbf{r}}_{21}$)^T, $\overline{\{\mathbf{R}\}}$ = $(\overline{\mathbf{R}}_n^U, \overline{\mathbf{R}}_n^U, \overline{\mathbf{R}}_n^U)$,
 $\overline{\mathbf{R}}$ = (σ^A) , $\{\overline{\mathbf{R}}^A\}$, $\theta_{\text{fp$ t-based 3D-SF
 \overline{R} = $(\overline{R}^t_{ij}, \overline{R}^c_{ij}, \overline{R}^t_{ij}, \overline{R}^t_{ij})$
 \overline{R} , Puck's way
 \overline{R} ailure if $\theta = \theta_{fp}$ \Leftarrow
 \overline{R} and \overline{R} and \overline{R} and \overline{R} and \overline{R} and \overline{R} areally intera $31¹$ $21²$ $4¹$ $2¹$ $2¹$ $2¹$ $2¹$ $2¹$ $2¹$ 2 $\frac{1}{2}$ 3D-SFCs: FF1, $\sigma_1 > 0$: $\left| \frac{\sigma_1}{\sigma_1} \right| + \frac{\tau_{31} + \tau_{21}}{\sigma_2}$ directly interacted !; FF2, $\sigma_1 < 0$: $\left| \frac{\sigma_1}{\sigma_2} \right| = 1$, 1 **Cour invariant-based 3D-SFCs Set** (4 modes, shear modes, $\overline{(x_2, x_3, x_3, x_2)}$, $\overline{(x_3, x_3, x_3)}$, $\overline{(x_3, x_3)}$, $\overline{($ whin's four invariant-based 3D-SFCs Set (4 modes, s
 $\overline{\sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21}}$)^T, \overline{R} } = $(\overline{R}_n^i, \overline{R}_n^c, \overline{R}_n^i, \overline{R}_n^c, \overline{R}_{1i}^i, \overline{R}_{23})^T$; 6 streng
 $F(\{\sigma^A\}, {\overline{R}^A\}, \partial_{\overline{r}})$ = 1, Pu Let invariant-based 3D-SFCs Set (4 modes, shear mode IFF;
 $[\overline{R}_3]$, F_{21})^T, $\{\overline{R}\} = (\overline{R}_0^i, \overline{R}_3^c, \overline{R}_1^c, \overline{R}_2^c, \overline{R}_3^c, \overline{R}_4^c, \overline{R}_2^c, \overline{R}_3^c, \overline{R}_4^c, \overline{R}_2^c, \overline{R}_3^c, \overline{R}_2^c, \overline{R}_3$ In's four invariant-based 3D-SFCs Set (4 modes, shear mode IFF3 is missing)
 $\overline{f(x_1, x_{31}, x_{31}, x_{31})}$, $\overline{f(x)} = (\overline{R'_x}, \overline{R'_y}, \overline{R'_1}, \overline{R'_3}, \over$ σ , $\tau_{\alpha} + \tau_{\alpha}$ σ > 0 : \pm + \pm \pm direction $rac{\tau_{31}^2 + \tau_{21}^2}{\overline{R}_{\perp/\!/}}$ directly interacted !; Fig. **I** invariant-based 3D-SFCs Set (4 modes, shear mode IFF3 is missing)
 $\overline{x_1x_1}^T$, $\overline{[R]} = (\overline{R'_0}, \overline{R'_2}, \overline{R'_2}, \overline{R'_2}, \overline{R'_3}, \overline{R$ **four invariant-based 3D-SFCs Set** (4 modes, shear mode IFF3 is missing)
 $\frac{1}{2z_1r_3_1r_{21}}\int^T$, $\frac{1}{R}$, $\frac{1}{R}$, $\frac{1}{R_0^2}$, $\frac{1}{R_0^2}$, $\frac{1}{R_1^2}$, $\frac{1}{R_0^2}$, $\frac{1}{R_0^2}$, $\frac{1}{R_0^2}$, $\frac{1}{R_$ **four invariant-based 3D-SFCs Set** (4 modes, shear mode IFF3 is missing)
 $\frac{r_{23}, r_{31}, r_{21}\overline{r_3}, \overline{r_4}\overline{r_5}, \overline{R'_2}, \overline{R'_3}, \overline{R'_1}, \overline{R'_2}, \overline{R'_3}, \overline{R'_1}, \overline{R'_2}, \overline{R'_3}, \overline{R'_3}, \overline{R'_3}, \overline{R'_3}, \overline{R'_3}, \overline{R'_3}, \over$ \overline{R}_{23} $\overline{R}_{\perp\parallel}$ $\overline{R}_{\perp\parallel}$ $\overline{R}_{\perp\parallel}$ $\overline{R}_{\perp\parallel}$ \overline{R}_{23} $\overline{R}_{\perp\parallel}$ sometimes)
 $(\sigma_2 + \sigma_3)$ $(\sigma_2 + \sigma_3)^2$ $(\tau_{23}^2 - \sigma_2 \cdot \sigma_3)$ $(\tau_{31}^2 + \tau_{21}^2)$ \overline{R}_{33} \overline{R}_{23}/R_{23} \overline{R}_{23} $1 \quad -1 \quad \cdot$ **Notariant-based 3D-SFCs Set** (4

²¹₂₁)^T, { \overline{R} } = (\overline{R} _{*if*}, \overline{R} _{*ij*}, \overline{R} _{*ij*}, \overline{R} _{*ij*} **DUI invariant-based 3D-SFCs Set** (4)
 $\overline{R}_{11}, \overline{R}_{31}, \overline{R}_{21}$)^T, $\{\overline{R}\} = (\overline{R}_{jj}^H, \overline{R}_{jj}^E, \overline{R}_{11}^E, \overline{R}_{12}^E, \overline{R}_{23})^T$;
 $\{\overline{R}^A\}$, $\theta_{\overline{1p}} = 1$, Puck's way \Leftrightarrow Hypothes

Action plane θ **Hashin's four invariant-based 3D-SFCs Set** ($\overline{(a_1, a_2, a_3, \tau_{23}, \tau_{31}, \tau_{21})^T}$, $\overline{\{R\}} = (\overline{R'_y}, \overline{R''_y}, \overline{R'_1}, \overline{R'_1}, \overline{R'_1}, \overline{R'_1}, \overline{R_{23}})^T$

is 1: $F(\overline{a^A}, \overline{R'_1}, \overline{a^A}, \overline{b_1}) = 1$, Puck's way \Leftrightarrow Let invariant-based 3D-SFCs Set (4 modes, shear mode IFF
 $\overline{r}_{31}, \overline{r}_{21}$)^T, $\{\overline{R}\} = (\overline{R}_{ij}^x, \overline{R}_{ij}^x, \overline{R}_{\perp}^x, \overline{R}_{\perp}^x, \overline{R}_{\perp}^x)$; 6 strengths principally en
 $[\overline{R}^A], \theta_{ip}] = 1$, Puck's way \Rightarrow **ant-based 3D-SFCs Set** (4 modes, shear mode IFF;
 $\{\overline{R}\} = (\overline{R}_{ij}^L, \overline{R}_{ij}^C, \overline{R}_{1}^C, \overline{R}_{1, ij}^C, \overline{R}_{2, j})^T$; 6 strengths principally em
 $= 1$, Puck's way \Leftrightarrow Hypothesis 2: $F(\{\sigma\}, {\{\overline{R}\}}) = 1$, C

9, fail **Hashin's four invariant-based 3D-SFCs Set** (4 modes, shear mode IFI
 $=(\sigma_1, \sigma_2, \sigma_3, \tau_{33}, \tau_{31}, \tau_{31})^T$, $\overline{[R]}_2 = (\overline{R_0}, \overline{R_0'}, \overline{R_1'}, \overline{R_1'}, \overline{R_1}, \overline{R_3}, \overline{Y}^T$; 6 strengths principally

sis 1: $F\left(\{\sigma^A$ 23 \mathbf{R} $\frac{1}{R_{23}} + \frac{(2S - 2S)^2}{R_{33}} + \frac{(3S - 2S)^2}{R_{11}} = 1,$ $\left(R_{23} = R_{11}$ sometimes 23 \mathbf{R}_{23} \mathbf{R}_{\perp} 2 1 **Hashin's four invariant-based 3D-SFCs**
 $\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$, $\{R\} = (\overline{R_n^b}, \overline{R_n^c}, \overline{R_1^c}, \overline{R_1^c}, \overline{R_1^c}, \overline{R_{1ij}})$

Hypothesis 1: $F([{\sigma}^A], {\overline{R}^A}, {\partial_{1p}}] = 1$, Puck's way \Leftrightarrow H

i **Hashin's four invariant-based 3D-SFCs Set** (4 modes, shear mode IFF3 is missing)
 $\sigma = (\sigma_1, \sigma_2, \sigma_3, r_{2x}, r_{31}, r_{21})^T$, $\{\overrightarrow{R}\} = (\overrightarrow{R_x^0}, \overrightarrow{R_x}, \overrightarrow{R_x}, \overrightarrow{R_y}, \overrightarrow{R_z}, \overrightarrow{R_z})^T$; σ strengths principally employed
 Hy four invariant-based 3D-SFCs Set (4 mo
 $\tau_{23}, \tau_{31}, \tau_{21}$)^T, $\{\overline{R}\} = (\overline{R}'_j, \overline{R}''_j, \overline{R}'_1, \overline{R}''_1, \overline{R}''_2, \overline{R}'_{23}, \overline{R}_{33}$)^T; ϵ
 $A'_1, \{\overline{R}^A\}, \theta_{IP}$ = 1, Puck's way \Leftrightarrow Hypothesis :

he *|| ||* $\left(\frac{-\sigma_1}{\overline{R}_{\parallel}^c} \right) = 1,$ c_2 \qquad \qquad \qquad \qquad c ^T \overline{D} ² *s* **four invariant-based 3D-SFCs Set (4 modes, shea**
 i, $\overline{R_3}$, $\overline{r_{33}}$, $\overline{r_{33}}$, $\overline{r_{31}}$, $\overline{r_{41}}$, $\overline{R_1}$ = $(\overline{R_1^d}, \overline{R_2^e}, \overline{R_3^e}, \overline{R_1^e}, \overline{R_2^e}, \overline{R_3^e}, \overline{R_3^e}, \overline{R_1^e}, \overline$ $R_1^{c_2}$ $(\sigma_2 + \sigma_3)$ $(\sigma_2 + \sigma_3)$ **n's four invariant-based 3D-SFCs Set** (4 modes, shear mode IFF3 is $\overline{(\sigma_3, \tau_{33}, \tau_{31}, \tau_{21})^T}$, $\overline{R}_1^3 = (\overline{R}_3^i, \overline{R}_1^i, \overline{R}_1^i, \overline{R}_{12}^i, \overline{R}_{23}^i)^T$; 6 strengths principally employ
 $(\{\sigma^A\}, {\{\overline{R}}^A\$ $\frac{1}{R_{23}}$ $\frac{1}{R_{33}}$ $\frac{1}{R_{41}}$ $\frac{1}{R_{11}}$ $\frac{1}{R_{12}}$ $\frac{1}{R_{13}}$ $\frac{1}{R_{14}}$ $\frac{1}{R_{14}}$ **Hashin's four invariant-based 3D-SFCs Set** (4 modes, shear mode IFF3 is missing)
 $-(\sigma_1, \sigma_2, \sigma_3, \tau_3, \tau_3, \tau_1)^\top$, $\{\overline{R}\} = (\overline{R_1}, \overline{R_2}, \overline{R_1}, \overline{R_1}, \overline{R_1}, \overline{R_1}, \overline{R_2}, \overline{R_3})^\top$; 6 steengths principall **Hashin's four invariant-based 3D-SFCs Set** (4 modes, shear mode IFF3 is missing)
 $\frac{1}{2} = (\sigma_1, \sigma_2, \sigma_3, r_{23}, r_{31}, r_{21})^T$, $\frac{1}{\{R\}} = (\overline{R_0^0}, \overline{R_1^0}, \overline{R_2^0}, \overline{R_2}, \overline{R_2}, \overline{R_3})^T$; 6 strengths principally \perp // $\frac{R_{\perp}}{R_{\perp}}$ -1). $\frac{(\sigma_2 + \sigma_3)}{\overline{R}_{\perp}^c}$ + $\frac{(\sigma_2 + \sigma_3)^2}{4 \cdot \overline{R}_{23}^2}$ + $\frac{(\tau_{23}^2 - \sigma_2 \cdot \sigma_3)}{\overline{R}_{23}^2}$ + $\frac{(\tau_{31}^2 + \tau_{21}^2)}{\overline{R}_{23}^2}$ = 1, $\frac{(\tau_{31}^2 + \tau_{21}^2)}{\overline{R}_{\perp}}$ as $\tau_{23/2}$
 \perp 1 $(\frac{0}{2} + \frac{0}{3})$ $(\frac{0}{2} + \frac{0}{3})$ \perp $4 \cdot \Lambda_{23}$ $= 1,$ ur invariant-based 3D-SFCs Set (4 modes, shear mode IFF3 is missing)
 $\overline{r_{31}, r_{21}}$, $\overline{R_1^3} = (\overline{R_2^3}, \overline{R_3^3}, \overline{R_1^3}, \overline{R_{22}^3}, \overline{R_3}, \overline{R_3^3})^2$; 6 strengths principally employed
 $[R^A], \theta_{\overline{p_2}}$ = 1, Hashin's four invariant-based 3D-SFCs Set (4 modes, shear mode IFF3 is missing)
 $\overline{\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21} \}^T$, $\overline{R}_1^3 = (R_0', R_0'', R_1', R_2', R_3', R_2, \overline{Y})^T$. 6 strengths principally employed

1: $F\left(\left{\sigma^A\right$ nt-based 3D-SFCs Set (4 modes, shear mode IFF3 is missing)
 \overline{R} } = ($\overline{R}^{i}_{ij}, \overline{R}^{i}_{ij}, \overline{$ **Hashin's four invariant-based 3D-SFCs Set** (4 modes, shear mode IFF3 is missing)
 $(\sigma_1, \sigma_2, \sigma_3, \tau_3, \tau_1, \tau_2)^T$, $\{\overline{R}\} = (\overline{R'_2}, \overline{R'_3}, \overline{R'_1}, \overline{R'_1}, \overline{R'_2}, \overline{R'_3}, \overline{R'_1}, \overline{R'_2}, \overline{R'_3}, \overline{R'_1}, \overline{R'_2}, \overline{R$ **four invariant-based 3D-SFCs Set** (4 modes, shear mode IFF3 is missing)
 $\frac{1}{125}, F_{21}, F_{21}$)^T, $\{R\} = (R_5^T, R_5^T, R_1^T, R_{23}, R_{23})^T$; 6 surengths principally employed
 $\{\{R^A\}, \theta_{\text{fp}}\} = 1$, Puck's way
 \Leftrightarrow near mode IFF3 is missing)

In principally employed
 σ , $\{\overline{R}\}$ = 1, Cuntze's way

and σ (sear) IFF3,

and τ in the FF1-model!

identified,

in the FF1-model!

identified,

cuntainty and t es, shear mode IFF3 is missing)

trengths principally employed
 $F(\{\sigma\}, {\overline{R}}) = 1$, Cuntze's way

ariants

at formulate the (shear) IFF3,

siders *x* in the FF1-model !

an be identified,

plane".

the pursue this idea

y **S** four invariant-based 3D-SFCs Set (4 modes, shear mode IFF3 is missing)
 $\frac{1}{2}, \frac{1}{2}, \frac{1$ **SS Set (4 modes, shear mode IFF3 i**
 $\overline{R}_{\perp/f}$; \overline{R}_{23})^T; 6 strengths principally empl

Hypothesis 2: $F\left(\lbrace \sigma \rbrace, \lbrace \overline{R} \rbrace \rbrace\right) = 1$, Cun

use of invariants

Hashin does not formulate the (shear) IF

however c FCS Set (4 modes, shear mode IFF
 $\frac{1}{k_1R_{\perp j}R_{\perp j}}$, $\overline{R_{2j}}$ = 1, d 3D-SFCs Set (4 modes, shear mode IFF3
 \overline{R}_{ij}^c , \overline{R}_{1}^i , \overline{R}_{1}^c , \overline{R}_{1}^c , \overline{R}_{23} , \overline{Y} ; 6 strengths principally emp

way \Leftrightarrow Hypothesis 2: $F(\{\sigma\}, {\overline{R}\}) = 1$, Ct
 $\theta = \theta_{jp} \Leftrightarrow$ use of **Hashin's four invariant-based 3D-SFCs Set** (4 modes, shear mode IFF3 is missing)
 $\[\sigma\] = (\sigma_1, \sigma_2, \sigma_3, \tau_4, \tau_5, \tau_7)$ ⁷. $[\overline{R}] = (\overline{R}_3, \overline{R}_2^*, \overline{R}_2^*, \overline{R}_3^*, \overline{R}_2^*, \overline{R}_3^*, \overline{R}_3^*, \overline{R}_3^*, \overline{R}_3^*, \overline{R}_3$ **D-SFCs Set (4** modes, shear mode IFF3 is missing)
 \overline{R}_1^r , \overline{R}_2^r , \overline{R}_{2i} , \overline{R}_{3i} , \overline{R}_{33} , \overline{R}_{1i} + 6 strengths principally employed
 \Leftrightarrow Hypothesis 2: $F(\{\sigma\}, {\overline{R}}_i) = 1$, Cuntze's way **SFCs Set** (4 modes, shear mode IFF3 is missing)
 $\overline{R_1^c, \overline{R}_{2i}, \overline{R}_{23}}$, \overline{r} , 6 strengths principally employed
 \Leftrightarrow Hypothesis 2: $F(\{\sigma\}, {\{\overline{R}\}}) = 1$, Cuntze's way
 \Leftrightarrow use of invariants

Hashin does ed 3D-SFCs Set (4 modes, shear mode IFF3 is missing)
 $\frac{1}{k_0 R_0 R_1 R_2 R_1 R_2 R_3 R_3}$, \overline{R}_{20} is of strengths principally employed
 $\overline{R}_0 R_1 R_2 R_3 R_4 R_5$, \overline{R}_{20} is expected in the set of the set of the se \mathcal{L} \mathbf{r}_1 \mathbf{r}_2 \mathbf{r}_3 \mathbf{r}_4 \mathbf{r}_2 \mathbf{r}_3 \mathbf{r}_2 \mathbf{r}_3 \mathbf{r}_4 \mathbf{r}_2 \mathbf{r}_3 \mathbf{r}_4 \mathbf{r}_5 \mathbf{r}_5 \mathbf{r}_6 \mathbf{r}_7 \mathbf{r}_8 \mathbf{r}_7 \mathbf{r}_8 \mathbf{r}_9 \mathbf{r}_9 $\$ **3D-SFCs Set** (4 modes, shear mode IFF3 is missing)
 $\frac{\overline{R}_{ij}^C \cdot \overline{R}_{1,i}^T \cdot \overline{R}_{1,ij}; \overline{R}_{2,ij}; \overline{R}_{2,ij}; \overline{R}_{3,ij}; \overline{R}_{3,ij};$ SFCs Set (4 modes, shear mode IFF3 is missing)
 \overline{R}_{\perp}^n , $\overline{R}_{\perp ij}$, $\overline{R}_{\perp 3}$, \overline{r} ; 6 strengths principally employed
 \Leftrightarrow Hypothesis 2: $F(\{\sigma\}, {\overline{R}}) = 1$, Cuntze's way
 \Leftrightarrow use of invariants

H

5.2 SFC Hashin

(1) The strength $\overline{R}_{\perp 1}$ is seen to equal the failure shear stress of τ_{23} . No further detail. **2D** – **SFC**: Equations without the suffix ₃ remain. \Rightarrow 5 strengths + R_{23} , not real definition found **SFCs Set** (sheal
remain. \Rightarrow 5 strengths
equal the failure shear straction of the 4 modes
 \therefore , $\sigma_1 < 0$: $\left(\frac{-\sigma_1}{\overline{R}c}\right)^2 = 1$, **Hashin's four 2D-SFCs Set** (shear mode IFF3 is missing)
 D-SFC: Equations without the suffix ₃ remain. \Rightarrow 5 strengths + R_{23} , not real definition for *Questions*: (1) The strength \overline{R}_{\perp} is seen to equal t *|| || c* **r 2D-SFCs Set** (shear mode IFF3 is missing)

uffix ₃ remain. \Rightarrow 5 strengths + R_{23} , not real definition found

seen to equal the failure shear stress of τ_{23} . No further detail.

12D-SFC: Equations without the suffix ₃ remain.
$$
\Rightarrow
$$
 5 strengths + R_{23} , not real definition found
\n*Questions* : (1) The strength \overline{R}_{11} is seen to equal the failure shear stress of τ_{23} , No further detail.
\n(2) How does the mandatory interaction of the 4 modes to determine FPF read ??
\nFF1, $\sigma_1 > 0$: $\left(\frac{\sigma_1}{\overline{R'_1}}\right)^2 + \left(\frac{\tau_{21}}{\overline{R}_{14}}\right)^2$; FF2, $\sigma_1 < 0$: $\left(\frac{-\sigma_1}{\overline{R''_1}}\right)^2 = 1$,
\nIFF1, $\sigma_2 > 0$: $\frac{(\sigma_2)^2}{\overline{R'_1}^2} + \frac{(\tau_{21}^2)}{\overline{R}_{14/2}^2} = 1$, IFF2, $\sigma_2 < 0$: $\left(\frac{-\sigma_1}{\overline{R''_1}}\right)^2 = 1$,
\nwith a recommendation to use for \overline{R}_{23} the value from Christensen's formula below
\nStrength Ratio $SR = R'_1 / R'_1 \rightarrow R_{23} = R'_1$: $\sqrt{\frac{1+1/SR}{3+5/SR} \cdot \frac{1}{SR}}$.
\nOrder to remain compatible in the SFC-comparison the interaction of the two IFF- with the two FF-modes,
\nwill be performed like with Cuntze, by applying 'proportional stressing'.
\n
\n**Eff**^m = (*Eff*^{n/r})^m + (*Eff*^{n/r})^m

the value from Christensen's formula below

Strength Ratio
$$
SR = R_{\perp}^c / R_{\perp}^i \rightarrow R_{23} = R_{\perp}^c \cdot \sqrt{\frac{1 + 1 / SR}{3 + 5 / SR}} \cdot \frac{1}{SR}
$$
.

In order to remain compatible in the SFC-comparison the interaction of the two IFF- with the two FF-modes, will be performed like with Cuntze, by applying '*proportional stressing*'.. *Eff (Eff) (Eff) (Eff) (Eff) () Eff Eff R Eff , Eff R R*

2 2 2 2 2 2 23 23 FF1, 0 FF2, 0 1 IFF1, 0 1 IFF2, 0 1 1 4 4 with a recommendat *|| || t c : ; : , R R R : , : , R R R R R R* ion to use for 1 1 1 Strength Ratio = . 3 5 *c t c R / SR SR R / R R R / SR SR* 1 The strength s seen to equal the fai i .No further lure she en + Equations without the suffix remain. , *Questions : R* 2 H ow does the mandatory interaction of the 4 modes to determine FPF read ?? 1 21 1 2 2 2 2 2 2 2 21 2 2 m m m m m m with with , = () + () + () + () + () = 1 0 *|| || t || c t || c || || R R R Eff Eff R Eff , Eff F R R Eff Eff Eff Eff Eff* 2 2 2 2 21 2 2 2 23 23 1 4 4 *^c R R R R ||* 5 strengths + (represents due to the Moh R . r modelling the 'Cohesive strength' *A*

(Hashin)-**Puck's Mohr Stresses/Coulomb Friction-based 3 IFF-SFCs**

 $\sigma_{\rm n}$ -domain the IFF formulations re

(Hashin)-Puck's Mohr Stresses/Coule
\nFF1 and FF2 : Taken are the usual *Maximum Stress Circle*
\nIFF: Due to the Mohr IFF approach, two different equations are p
\nusing Mohr's 'Action plane' stresses and 'Action plane' resistance
\nIn the tensile σ_n-domain the IFF formulations read: [*Puc*96, *p*.14:
\nIFF: σ_n > 0:
\n
$$
\left(\frac{\tau_{nt}}{\overline{R}_{\perp\perp}}\right)^2 + \left(\frac{\tau_{n1}}{\overline{R}_{\perp\parallel}}\right)^2 + \left(\frac{\sigma_n}{\overline{R}_{\perp}^A}\right) = 1,
$$
\nIn the compressive σ_n-domain friction
\nis to consider
\nIFF: σ_n < 0:
\n
$$
\left(\frac{\tau_{nt}}{\overline{R}_{\perp\perp}^A - p_{\perp\perp}^C \cdot \sigma_n}\right)^2 + \left(\frac{\tau_{n1}}{\overline{R}_{\perp\parallel}^A - p_{\perp\parallel}^C \cdot \sigma_n}\right)^2 + \left(\frac{\sigma_n}{\infty}\right) = 1,
$$
\nThe Mohr approach-dependent action plane resistance $\overline{R}_{\perp\perp}^A$ is model-
\n⇒ Practically 5 technical strengths are addressed only, which support
\nSFC Puck

In the compressive σ_n -domain friction $-R_{\perp}$ is to consider

(Hashin)-**Puck's Mohr Stresses**

and FF2 : Taken are the usual *Maximum Str*

Due to the Mohr IFF approach, two different eqs

Mohr's 'Action plane' stresses and 'Action plane'

tensile σ_n -domain the IFF formulations r 2 $($ τ_{nt} τ_{n1} σ_{n} τ_{n2} and action p in)-**Puck's Mohr Stresses/C**

Taken are the usual *Maximum Stress*

Mohr IFF approach, two different eqatio

tion plane' stresses and 'Action plane' res

-domain the IFF formulations read: [*Puc9*

1:
 $\frac{\tau_{n1}}{\overline{R}_{\perp\$ (Hashin)-Puck's Mohr Stresses/Courry

FF1 and FF2 : Taken are the usual *Maximum Stress Cr*

IFF: Due to the Mohr IFF approach, two different eqations a

using Mohr's 'Action plane' stresses and 'Action plane' resistant

Master fracture body with Puck's IFF modes and action plane stresses $(\sigma_n, \tau_{nt}, \tau_{n1}).$ *[courtesy H. Schürmann]*

The Mohr approach-dependent action plane resistance $\overline{R}_{\perp\perp}^A$ is model-fixed.
 \rightarrow Practically 5 technical strengths are addressed only, which supports Cuntze's 'generic' nur \rightarrow Practically 5 technical strengths are addressed only, which supports Cuntze's 'generic' number 5 for UD materials.

5.3 SFC Puck

3D-Search of the Fracture Plane Angle θ_{fn} by variation of θ in a program with determining the minimum failure danger.

the 'Cohesive strength' R_{23}^{\prime} , a result of two acting modes, the 'Cohesive strength' R_{23}^r , a result of two acting modes,
namely IFF2 with IFF1 causing a range $R_{\perp}^c > R_{\tau} > R_{\perp}^t$. would represent due to the Mohrstresses-based Approach Puck can derive the *general* formulation :

3D-'Global' UD-SFC of Tsai-Wu (there is also an orthotropic version)

$$
\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T, \ \{\overline{R}\} = (\overline{R}_{\parallel}^t, \overline{R}_{\parallel}^c, \overline{R}_{\perp}^t, \overline{R}_{\perp}^c, \overline{R}_{\perp\parallel}^t; \ \overline{R}_{\perp\perp})^T = (X, X', Y, Y', S; S_{23})^T
$$

$$
F(\{\sigma\}, \{\overline{R}\}) = 1, \ \text{6 strengths principally}
$$

A general anisotropic tensor polynomial expression of Zakharov and Goldenblat-Kopnov with the parameters F_i , F_{ij} as strength model parameters was the basis of the Tsai-Wu SFC, see [14],

3D-'Global' UD-SFC of Tsai-Wu (there is also an orthotropic version)
\n
$$
\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T, \{\overline{R}\} = (\overline{R}_n^r, \overline{R}_n^r, \overline{R}_1^r, \overline{R}_{1ij}; \overline{R}_{1ij})^T = (X, X', Y, Y', S; S_{23})^T
$$
\n
$$
F(\{\sigma\}, \{\overline{R}\}) = 1, \text{ 6 strengths principally}
$$
\n
$$
\text{and anisotropic tensor polynomial expression of Zakharov and Goldenblat-Kopnov with the parameters } F_i, F_{ij} \text{ as } H_i \text{ is the basis of the Tsai-Wu SFC, see [14],}
$$
\n
$$
\sum_{i=1}^6 (F_i \cdot \sigma_i) + \sum_{j=1}^6 \sum_{i=1}^6 (F_{ij} \cdot \sigma_i \cdot \sigma_j) = 1 \quad F_i \cdot \sigma_i + F_{ij} \cdot \sigma_i \cdot \sigma_j = 1 \quad \text{with} \quad (i,j = 1,2,...,6)
$$
\n
$$
F_i \cdot \sigma_i + F_{ij} \cdot \sigma_i \cdot \sigma_j = 1 \quad \text{with} \quad (i,j = 1,2.6) \quad \text{or executed} \quad \sigma_3 \text{ treated like } \sigma_2
$$
\n
$$
F_{i1} \cdot \sigma_1^2 + F_{i1} \cdot \sigma_1 + 2F_{i2} \cdot \sigma_1 \cdot \sigma_2 + 2F_{i3} \cdot \sigma_1 \cdot \sigma_3 + F_{i2} \cdot \sigma_2^2 + F_{i2} \cdot \sigma_2 + 2F_{i3} \cdot \sigma_3 + F_{i3} \cdot \sigma_3^2 + F_{i3} \cdot \sigma_3 + F_{i4} \cdot \tau_{23}^2 + F_{i5} \cdot \tau_{13}^2 + F_{i6} \cdot \tau_{12}^2 = 1
$$
\n
$$
\text{with the strength model parameters = coefficients of the criterion}
$$
\n
$$
F_i = 1/\overline{R}_i^r - 1/\overline{R}_i^r, F_{i1} = 1/(\overline{R}_i^r \cdot \overline{R}_i^r), F_2 = 1/\overline{R}_i^r - 1/\overline{R}_i^r, F_{i2}
$$

and - in order to avoid an open failure surface - the so-called interaction term

$$
F_{12} = \overline{F}_{12} \cdot \sqrt{F_{11} \cdot F_{22}}
$$
 with $-1 \le \overline{F}_{12} \le 1$; usually it is applied $F_{12} = -0.5$.

- The bi-axial material parameter F_{12} is 'principally' to obtain from aequi-biaxial compression tests to close the fracture body. This would mean, that *F*12 cannot consider the stress signs (question: effect in the domain σ_1^t , σ_2 ? surface - the so-called :
 $\frac{1}{2} \leq 1$; usually it is appli

F₁₂ is 'principally' to obta

This would mean, that F₁
 $\frac{1}{1}, \sigma_2$?

[*Li17*].
- For details considering *F*12 , see [*Li17*] .

Transformation of the 2D-SFCs into an Eff-formulated Interaction-capable shape

$$
F\left(\{\sigma\},\{\overline{R}\},\mu\right) = 1: \Rightarrow \mathbf{Eff}^{\mathrm{m}} = [(\mathbf{Eff}^{1/\sigma})^{\mathrm{m}} + (\mathbf{Eff}^{1/\tau})^{\mathrm{m}} + (\mathbf{Eff}^{\perp \sigma})^{\mathrm{m}} + (\mathbf{Eff}^{\perp \prime})^{\mathrm{m}} + (\mathbf{Eff}^{\perp \tau})^{\mathrm{m}}] = 1 = 100\%
$$

Template

transformation of the 2D-SFCs into an Eff-formulated Interaction-capable shape

\n
$$
F(\{\sigma\}, \{\overline{R}\}, \mu) = 1: \Rightarrow E f f^m = [(E f f^{s/\sigma})^m + (E f f^{s/\sigma})^m + (E f f^{-1/\sigma})^m + (E f f^{-1/\sigma})^m + (E f f^{-1/\sigma})^m] = 1 = 100\%
$$
\nmplate

\ndirectly including the friction value μ, with mode portions, formulated to avoid physically senses, negative *E f f* modes

\n
$$
E f f^{\text{obs}} = \frac{(\sigma_1 + |\sigma_1|)}{2 \cdot \overline{R}_s}, \quad E f f^{\text{obs}} = \frac{(-\sigma_1 + |\sigma_1|)}{2 \cdot \overline{R}_s}, \quad E f f^{\text{obs}} = \frac{\sigma_2 - |\sigma_3|}{2 \cdot \overline{R}_s}, \quad E f f^{\text{obs}} = \frac{-\sigma_2 + |\sigma_3|}{2 \cdot \overline{R}_s}, \quad E f f^{\text{obs}} = \frac{|\tau_3|}{\overline{R}_{1/2} - \mu_{1/2} \cdot (\sigma_2^c)}.
$$
\nuntze: 2 FF + 3 IFF

\nis, 2 FF with 3 Mohr-coupled IFFs

\nai-Wu: Modes still globally interacted.

\nComputation code: Mathematical 15

\nComputation code: Mathematical 15

Cuntze: 2 FF + 3 IFF

Hashin: 2 FF + 2 IFF

Puck: 2 FF with 3 Mohr-coupled IFFs

Tsai-Wu: Modes still globally interacted.

 $F((\sigma), {\overline{R}}, \mu) = 1$ \Rightarrow $Eff^{m} = [(Eff^{1/\sigma})^{m} + (Eff^{1/\tau})^{m} + (Eff^{\perp \sigma})^{m} + (Eff^{\perp \prime})^{m} + (Eff^{\perp \tau})^{m}] = 1$ = 100% **IDENTIFY AT EXACT CONSTRANT ANTIFY (FIGURE 100)**

Interaction with $\{\overline{R}\} = (\overline{R}_1^*, \overline{R}_2^*, \overline{R}_1^*, \overline{R}_1^*, \overline{R}_2^*, \overline{R}_1^*, \overline{R}_1^$ directly including the friction value μ , with mode portions, formulated to avoid physically senseless negative *Eff* modes
 Eff^{|| σ} = $\frac{(\sigma_1 + |\sigma_1|)}{2 \cdot \overline{R}_{\mu}^t}$, *Eff*^{|| τ} = $\frac{(-\sigma_1 + |\sigma_1|)}{2 \cdot \overline{R}_{\mu$ **2D (in-plane)- Formulations of the four envisaged SFCs** (mo
 $\frac{1}{2} = (\sigma_1, \sigma_2, \sigma_3, r_{23}, r_{31}, r_{12})^T \rightarrow \{\sigma\} = (\sigma_1, \sigma_2, r_{21})^T$ requiring all 5 technical strengths $\{\overline{R}\}$ = nte 's **FMC-based Set of Modal SFCs: FF1**, **ations of the four envisaged SFC**
 $(\sigma_1, \sigma_2, \tau_{21})^T$ requiring all 5 technical strengths
 25: FF1, FF2, IFF1, IFF2, IFF3 $\{\overline{R}\} = (\overline{R}_N^t, \overline{R}_N^c)$
 $\forall m + (Eff^{l/r})^m + (Eff^{l-r})^m + (Eff^{l-r})^m + (Eff^{l-r})^m + (Eff^{l-r})^m + (Eff^{l-r})^m$
 2D (in-plane)- Formulations of the four envisaged SFCs (model mapping, bar)
 $F(G_1, G_2, G_3, G_3, G_3, G_1, G_1)$ ⁷ \rightarrow $\{\sigma\} = (G_1, G_2, G_3)$ ⁷ requiring all 5 technical steregits $\{\overline{R}\} = (\overline{R}_2^*, \overline{R}_2^*, \overline{R}_3^*, \overline{$ **2D (in-plane)- Formulations of the four envisaged SFCs** (model mapping, bar)
 $z_1, \sigma_2, \sigma_3, \tau_{31}, \tau_{31}, \tau_{32} \}$ \Rightarrow $\{\sigma\} = (\sigma_1, \sigma_2, \tau_{21})$ ^T requiring all 5 technical strengths $\{R\} = (R_0^T, R_0^T, R_1^T, R_1^T, R_1^$ **2 (in-plane)- Formulations of the four envisaged SFCs (model mapping, ba
** σ_3 **,** τ_{33} **,** τ_{31} **,** τ_{12} **)²** $\rightarrow \{\sigma_1^k = (\sigma_1, \sigma_2, \tau_{21})^T$ **requiring all 5 (cchnical strengths \{\overline{R}\} = (\overline{R}_a^i, \overline{R}_a^i, \overline{R}_a^ 2D (in-plane)- Formulations of the** 1
 $\sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{12}$)^T $\rightarrow \{\sigma\} = (\sigma_1, \sigma_2, \tau_{21})^T$ req

MC-based Set of Modal SFCs: FF1, FF2, IFF
 $\{\overline{R}\}, \mu\} = 1: \Rightarrow Eff^{\text{m}} = [(Ef^{t/\sigma})^{\text{m}} + (Ef^{t/\tau})^{\text{m}} + (i\text{ including the friction value }\mu, \$ **2D (in-plane)- Formulations of the four envisaged SFCs** (model mapping, bar)
 $r_2^1 = (G_1, G_2, G_3, T_{23}, r_{31}, r_{31})^T \rightarrow \{\sigma\}^1 = (G_1, G_2, r_{21})^T$ requiring all 5 rechained strengths $\{R_1^1 \in \{R_1^2, R_1^2, R_1^2, R_1^2, R_1^2$ **in-plane)- Formulations of the four envisaged SFCs** (model
 r_{23}, r_{31}, r_{12})^T $\rightarrow \{\sigma\} = (\sigma_1, \sigma_2, r_{21})^T$ requiring all 5 technical strengths $\{R\} = (R_0^2, \overline{R}_1^*, \overline{R}_1^*, \overline{R}_2^*, \overline{R}_2^*, \overline{R}_2^*, \overline{R}_2^*, \overline{R}_$ \int_{0}^{∞} $(0.1 + |0.1|)$ \int_{0}^{∞} \int_{0}^{∞} $(0.1 + |0.1|)$ \int_{0}^{∞} \int_{0}^{∞} **S** (model mapp
 $\{\overline{R}\} = (\overline{R}_{\parallel}^t, \overline{R}_{\parallel}^c, \overline{R}_{\parallel}^c, \overline{R}_{\parallel}^c, \overline{R}_{\perp}^c, \overline{R}_{\perp}^c, \overline{R}_{\perp}^c, \overline{R}_{\perp\parallel}^c)^T, \dots$
 m] = 1 = 100%

eless negative *Ef*
 $|\tau_{21}|$
 ${}_{\parallel} - \mu_{\perp\parallel} \cdot (\sigma_2^c)$
 $(\overline{X$ *|* (in-plane)- Formulations of the four envisaged SFCs
 $\sum_{i} r_{z,i}, r_{z,i}, r_{z,i} \in \mathbb{F} \rightarrow \{\sigma\} = (\sigma_1, \sigma_2, r_{z,i})^T$ requiring all 5 technical strengths $\{\overline{R}\}$

passed Set of Modal SFCs: FF1, FF2, IFF1, IFF2, IFF3 $\{\overline{R}\} = (\$ **he four envisaged SFCs** (model mapping, bar)

^F requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}_n^T, \overline{R}_n^T, \overline{R}_n^T, \overline{R}_n^T, \overline{R}_n^T, \overline{R}_{\perp n}^T)^T$
 FF1, IFF2, IFF3 $\{\overline{R}\} = (\overline{R}_n^T, \overline{R}_n^T, \overline{R}_$ **Eff** $E = \frac{E}{(R_1, R_2, R_3, R_4, R_5)}$ **Eff Eff E** $\lfloor \sigma \rfloor$ $\lfloor 2 \rfloor$ $\lfloor \sigma \rfloor$ $\lfloor \sigma \rfloor$ **10 four envisaged SFCs** (model mapping, bar)

requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}'_s, \overline{R}''_s, \overline{R}'_s, \$ (in-plane)- Formulations of the four envisaged SFCs (model mapping, bar)
 $\tau_3, \tau_3, \tau_{31}, \tau_{12} \rightarrow \{\sigma\} = (\sigma_1, \sigma_2, \tau_{21})^T$ requiring all 5 technical strengths $\{R\} = (R_0^T, R_0^T, R_1^T, R_1^T, R_1^T, R_2^T, \mathbf{F}^T, \mathbf{F}^$ **2D (in-plane)- Formulations of the four envisaged SFCs (model mapping, bar)**
 $\overline{G_2, G_3, \overline{L}_{31}, \overline{L}_{13}, \overline{L}_{13}}$ \Rightarrow $\{\sigma\} = (\sigma_1, \sigma_2, \tau_3, \tau_1^T)$ requiring all 5 iechnical stengths $\{R\} = (R_0^C, R_0^C, R_1^C, R_1$ (in-plane)- Formulations of the four envisaged SFCs (model mapping, bar)
 $\tau_3, \tau_3, \tau_{31}, \tau_{12} \rightarrow \{\sigma\} = (\sigma_1, \sigma_2, \tau_{21})^T$ requiring all 5 technical strengths $\{R\} = (R_0^T, R_0^T, R_1^T, R_1^T, R_1^T, R_2^T, \mathbf{F}^T, \mathbf{F}^$ $\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{12})^T \rightarrow \{\sigma\} = (\sigma_1, \sigma_2, \tau_{21})^T$ requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}_{\parallel}^t, \overline{R}_{\parallel}^c, \overline{R}_{\perp}^t, \overline{R}_{\perp}^c, \overline{R}_{\perp\parallel}^t)^T$
Cuntze's FMC-based Set of Modal SFCs Formulations of the four envisaged SFCs (model mapping, bar)
 $T \rightarrow \{\sigma\} = (\sigma_1, \sigma_2, \tau_{21})^T$ requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}_g^t, \overline{R}_g^c, \overline{R}_g^c, \overline{R}_g^c, \overline{R}_g^c, \overline{R}_g^c, \overline{R}_g^c, \overline{R}_g^c, \$ **2D (in-plane)- Formulati**
 $\frac{1}{1!}\sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{12} \rightarrow \{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{12} \rightarrow \{\sigma\} = (\sigma_2, \sigma_3, \mu) = 1: \Rightarrow \text{Eff}^m = [(Eff^{1/\sigma} \text{Hly including the friction value } \mu, \text{ with } \sigma_1 = \frac{(\sigma_1 + |\sigma_1|)}{2 \cdot \overline{R}_{\beta}^{\prime}}, \text{ Eff}^{\prime\prime\prime} = \frac{(-\sigma$ **2D (in-plane)- Formulations of the four envisaged SFCs** (model mapping, bar)
 $(\sigma_1, \sigma_2, \sigma_3, \tau_2, \tau_3, \tau_4, \tau_5)^T \rightarrow [\sigma] = (\sigma_1, \sigma_2, \tau_2)^T$ requiring all 5 ecclinical strengths $\{\overline{R}\} = (\overline{R}_a^T, \overline{R}_a^T, \overline{R}_a^T, \overline{$ **2D (in-plane)- Formulations of the four envisaged SFCs** (model mapping, bar)
 $\sigma_{2}^{1} - (\sigma_{1}, \sigma_{2}, \sigma_{3}, \tau_{2}, \tau_{3}, \tau_{12})^{T} \rightarrow |\sigma| = (\sigma_{1}, \sigma_{2}, \tau_{21})^{T}$ requiring all 5 rechnical strengths $\{R\} - (R_{2}^{2}, R_{2}^{2}, R_{2}^{2}, R_{2}^{2$ $_{\perp}, \mathbf{R}_{\perp}, \mathbf{R}_{\perp \parallel}/\mathbf{R}$ **Tsai-Wu, global SFC** (interaction inherent) $\{R\}=(R^t_\parallel,R^c_\parallel,R^c_\perp,R^c_\perp,R^c_\perp,R^c_\perp)^\mathrm{T}=(X,~X'~,Y~,Y'~,S_{12}~;S_{23})^\mathrm{T}$ **2D (in-plane)- Formulations of the four envisaged SFCs (model mapping, bar) F1, IFF2, IFF3** $\left\{\overline{R}\right\}=(\overline{R}_{\shortparallel}^{t},\overline{R}_{\shortparallel}^{c},\overline{R}_{\shortparallel}^{t},\overline{R}_{\shortparallel}^{c},\overline{R}_{\shortparallel}^{c})^{\text{T}},\,\,\mu_{\perp\parallel},\,m\cong2.6$ **2D (in-plane)- F(**
 $(\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{12})^T$ –
 2's FMC-based Set of M(
 $\{ \{\sigma\}, {\{\overline{R}\}}, \mu \} = 1 : \Rightarrow E f f^T$

irectly including the friction v
 $\widetilde{f}^{||\sigma} = \frac{(\sigma_1 + |\sigma_1|)}{2 \cdot \overline{R}_j}, \quad E f f^{\parallel \tau} = \frac{(-1)^{\sigma_1}}{$ **2D (in-plane)- Formulations of the four envisaged SFCs** (model mapping, bar)
 $=(\sigma_1, \sigma_2, \sigma_3, \tau_3, \tau_4, \tau_6)^T \rightarrow \{\sigma\} = (\sigma_1, \sigma_2, \tau_2, \tau)^T$ requiring all S technical steengths $\{\overline{R}\} = (\overline{R}_2^*, \overline{R}_2^*, \overline{R}_1^*, \overline{R}_1$ **2D (in-plane)- Formulations of**
 $\sigma_2, \sigma_3, r_{23}, r_{31}, r_{12})^T \rightarrow \{\sigma\} = (\sigma_1, \sigma_2, r_{21})^T$

WC-based Set of Modal SFCs: FF1, FF
 $\{\overline{R}\}, \mu\} = 1: \Rightarrow Eff^m = [(Eff^{t/\sigma})^m + (Eff^{t/\sigma})^m + (Eff^{t/\sigma})^m]$

including the friction value μ , **2D (in-plane)- Formulat**
 $=(\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{12})^T \rightarrow \{\sigma\} = (\sigma_1 + \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{12})^T \rightarrow \{\sigma\} = (\sigma_2 + \sigma_3),$
 $F\left(\{\sigma\}, \{\overline{R}\}, \mu\right) = 1: \Rightarrow Eff^m = [(Ef^{\#}]^k$

directly including the friction value μ , with
 Hashin, modal SFCs, FF1, FF2, IFF1, IFF2, no IFF3 $\{ \overline{R} \} = (\overline{R}_u^t, \overline{R}_v^c, \overline{R}_v^t, \overline{R}_v^c, \overline{R}_v, \overline{R}_z^s)^T$, determination of \overline{R} **2D (in-plane)- Formulations of the four envisaged SFCs (model mapp
 (\sigma_1, \sigma_2, \sigma_3, r_{21}, r_{12})^T \rightarrow (\sigma_2^2 = (\sigma_1, \sigma_2, r_{21})^T \text{ requiring all 5 technical strengths } {\{\bar{R}\} = (\bar{R}_a^2, \bar{R}_a^2, \bar{R}_a^2, \bar{R}_a^2, \bar{R}_a^2, \bar{R}_a^2, \bar{R}_a^2, \bar{R}_a^2, \bar{R}_a^2 of the four envisaged SFCs** (model map)
 r_{21})^T requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R_g}, \overline{R_g})$
 FF2, IFF1, IFF2, IFF3 $\{\overline{R}\} = (\overline{R_g}, \overline{R_g}, \overline{R_i}, \overline{R_g}, \overline{R_g})$
 $\overline{R}f^{(j/r)}$ ^m + $(Eff^{-1,r})$ ^m **1 of the four envisaged SF**
 $\vec{F}_{1}, \vec{F}_{21}$ \vec{F} requiring all 5 technical strengt
 $\vec{F}_{1}, \vec{F}_{21}$ \vec{F} requiring all 5 technical strengt
 $\vec{F}_{2}, \vec{F}_{1}, \vec{F}_{1}, \vec{F}_{2}, \vec{F}_{1}, \vec{F}_{2}, \vec{F}_{2}, \vec{F}_{2}, \vec{F}_{2}, \vec{F}_{$ **2D (in-plane)- Formulations of the four envisaged SFCs** (model mapping, bar)
 $\sigma_2, \sigma_3, \tau_2, \tau_3, \tau_4, \gamma^Y \rightarrow \{\sigma\} = (\sigma_1, \sigma_2, \tau_2, \gamma^Y)$ requiring all 5 technical sterengths $\{R\} = (R_2', R_3', R_4', R_4', R_5', R_6', \gamma^Y)$
 $\mathbb{$ **10 four envisaged SFCs** (model mapping, bar)

requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}_{ij}^t, \overline{R}_{ij}^c, \overline{R}_{i}^t, \overline{R}_{i}^c, \overline{R}_{iij}^t)^T$
 IFF1, IFF2, IFF3 $\{\overline{R}\} = (\overline{R}_{ij}^t, \overline{R}_{ij}^c, \overline{R}_{i}^t, \overline{R}_{$ T $T = (\overline{V} \quad \overline{V} \quad \overline{V}) \quad \overline{V} \quad \overline{V}$ **E four envisaged SFCs** (model mapping, bar)

requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}_x^i, \overline{R}_y^c, \overline{R}_z^c, \overline{R}_z^c$ $\mathcal{L}_1, \mathbf{\Lambda}_{\perp}, \mathbf{\Lambda}_{\perp\parallel}, \mathbf{\Lambda}_{\perp\perp}) = (\mathbf{\Lambda}, \mathbf{\Lambda}, \mathbf{\Lambda}, \mathbf{\Lambda})$ **he four envisaged SFCs** (model mapping, bar)

^T requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}_g^T, \overline{R}_g^T, \overline{$ **10 four envisaged SFCs** (model mapping, bar)

requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}'_n, \overline{R}''_n, \overline{R}''_n, \overline{R}''_1, \overline{R}''_1, \overline{R}''_1, \overline{R}''_1, \overline{R}''_1, \overline{R}''_1, \overline{R}''_1, \overline{R}''_1, \overline{R}''_1, \overline{R}''_1$ **2D (in-plane)- Formulations of the four envisaged SFCs (model mapping, bar)**
 $=(\sigma_1, \sigma_2, \sigma_3, \tau_2, \tau_3, \tau_4, \tau^7)$ ⁷ $\rightarrow |\sigma_1^2 = (\sigma_1, \sigma_2, \tau_2, \tau^7)$ ⁷ requiring all 5 ischinical surengths $\{\overline{R}\} = (\overline{R}_1^2, \overline{R}_2^$ $12 \begin{array}{ccc} 12 & 0 & 2 \end{array}$ 0 **Ilations of the four envisaged SFCs** (model
 $=(\sigma_1, \sigma_2, \tau_{21})^T$ requiring all 5 uchnical strengths $\{\overline{R}\} = (\overline{R}'_3, \overline{R}'_2) = (\overline{R}''_3, \overline{R}''_3) = (\overline{R}''_3, \overline{R}''_4, \overline{R}''_5, \overline{R}'_4, \overline{R}''_5, \overline{R}'_4, \overline{R}''_5)$ $1 \overline{b}t = \overline{b}c \overline{c}t + \overline{c}$ $1 \cdot 0^2$ 0^2 2 -1 2 $\overline{F}_{12} \cdot \overline{\sigma}_1 \cdot \overline{\sigma}_2$ σ_2^2 σ_3 $\left(1 \quad 1\right)$ $\overline{\tau}_{12}^2$ $\overline{\tau}_{13}^2$ (in-plane)- Formula
 Γ_3 , τ_{23} , τ_{31} , τ_{12})^T $\rightarrow \{\sigma\}$ =

based Set of Modal SFC
 μ) = 1 : \Rightarrow Eff^m = [(Eff'

uding the friction value μ , wi
 $+\vert \sigma_1 \vert$), Eff'^{|f'} = $\frac{(-\sigma_1 + |\sigma_1|)}{2 \cdot \overline{R}_{\parallel$ **ne)- Formulations of the four envisaged SFCs** (model mapping, bar)
 $\frac{1}{1+i_1} \sum_{i=1}^{T} \frac{1}{\sqrt{t_i}} = (G_1, G_2, T_{2,i})^T$ requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}'_2, \overline{R}'_2, \overline{R}'_1, \overline{R}'_2, \overline{R}'_1, \overline{R}'_2, \$ **tr envisaged SFCs**

ag all 5 technical strengths {
 \overline{F} **7.** \overline{F} **7.** \overline{R} \overline{R} = $(\overline{R}^t_{ij}, \overline{R}^c_{ij}, \overline{R}^c_{ij})$
 \overline{R} = $(\overline{R}^t_{ji}, \overline{R}^c_{ij})^m$
 \overline{R} = $\frac{\sigma_2 + |\sigma_2|}{2 \cdot \overline{R}^c_{j}}$, $Eff^$ **IFF2, IFF3**
 $\frac{1}{\sigma}$ $\frac{1}{m}$ + $(Eff^{\perp}$
 I and $\frac{1}{m}$ + $(Eff^{\perp}$
 I and Eff^{\perp}
 I and Eff^{\perp}
 I and $\frac{1}{R_{\parallel}^{t}}$, $\overline{R}_{\parallel}^{c}$, $\overline{R}_{\parallel}^{c}$, $\overline{R}_{\parallel}^{c}$, $\overline{R}_{\parallel}^{c}$, $\overline{R}_{\parallel}^{c}$, **IULATIONS Of the four envisaged SFCs** (model mapping, bar)
 $\vec{r}_1 = (\sigma_1, \sigma_2, \vec{r}_{21})^T$ requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R'_2}, \overline{R'_2}, \overline{R'_1}, \overline{R'_1}, \overline{R'_2}, \overline{R'_1}, \overline{R'_2}, \overline{R'_1}, \overline{R'_2}, \overline{R'_2}, \overline{$ **RUILATIONS of the four envisaged SFCs** (model mapping, bar)
 $\sigma_j^1 = (\sigma_1, \sigma_2, \tau_{2,1})^T$ requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}_s^T, \overline{R}_s^T, \overline{R}_s^T, \overline{R}_s^T, \overline{R}_s^T, \overline{R}_s^T, \overline{R}_s^T, \overline{R}_s^T, \overline{R$ $Eff = \frac{61}{5t} + \sigma_1$. R_{\perp}^c $R_{\perp}^c \cdot R_{\perp}^c$ $\qquad R_{\perp}^c$ R_{\perp}^c R_{\perp}^c $R_{\perp\parallel}^c$ **2D (in-plane)- Formulations of the four**
 $\frac{1}{2} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{12})^T \rightarrow \{\sigma\} = (\sigma_1, \sigma_2, \tau_{21})^T$ requiring
 fitze's FMC-based Set of Modal SFCs: FF1, FF2, IFF1, IFF
 $F[(\sigma_1, {\overline{R}}, \mu) = 1 : \Rightarrow E f f^m = [(E f f^{i$ **ILLERED ASSOCIATE:** $\overline{F}_1 = (\overline{G}_1, \overline{G}_2, \overline{F}_1, \overline{F}_1)$ **From Example 18 FCs** (model mapping, bar)

F($\overline{G}_1 = (\overline{G}_1, \overline{G}_2, \overline{F}_1)$ **FF2**, **IFF2**, **IFF2 [F3** $[\overline{R}] = (\overline{R}_2^0, \overline{R}_2^0, \overline{R}_2^0, \overline{R}_2^0,$ **ONS Of the four envisaged SFCs** (model mapping, bar)
 $\overline{r_1, \sigma_2, \overline{r_2}}$, $\overline{T_1}$ requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R'_n}, \overline{R'_n}, \overline{R'_1}, \overline{R'_1}, \overline{R'_1}, \overline{R'_2}, \overline{R'_2}, \overline{R'_1}, \overline{R'_1}, \overline{R'_2}, \overline{R$ **EXECTS (model mapping, bar)**

II 5 technical strengths $\{\overline{R}\} = (\overline{R}'_n, \overline{R}^c_n, \overline{R}'_n, \overline{R}'_1, \overline{R}^c_1, \overline{R}^c_n, \overline{R}'_{1n})^T$
 IFF3 $\{\overline{R}\} = (\overline{R}'_n, \overline{R}^c_n, \overline{R}'_1, \overline{R}^c_1, \overline{R}^c_n, \overline{R}'_1, \overline{R}^$ σ , $\{\bar{R}\}\cdot\bar{S}_{22}, F_{12}$ = 1 : $F = Eff = \frac{\sigma_1^2}{\sigma_1^2} + \sigma_1 \cdot \left(\frac{1}{\sigma_1} - \frac{1}{\sigma_2}\right) + 2 \frac{\bar{F}_{12} \cdot \sigma_1 \cdot \sigma_2}{\sigma_1^2 \cdot \sigma_2} + \frac{\sigma_1^2 \cdot \sigma_2 \cdot \sigma_1 \cdot \sigma_2}{\sigma_2^2 \cdot \sigma_2^2 \cdot \sigma_1^2 \cdot \sigma_2^2}$ **risaged SFCs** (model mapping, bar)

technical strengths $\{\overline{R}\} = (\overline{R}_{y}^{r}, \overline{R}_{y}^{c}, \overline{R}_{1}^{r}, \overline{R}_{1}^{c}, \overline{R}_{1}^{c}, \overline{R}_{1,y}^{c})^{\mathrm{T}}$

F3 $\{\overline{R}\} = (\overline{R}_{y}^{r}, \overline{R}_{y}^{c}, \overline{R}_{1}^{r}, \overline{R}_{1,y}^{c}, \overline{R}_{1,y}^{r}, \overline{R}_{1,y}$ **IS Of the four envisaged SFCs** (model mapping, bar)
 $\sigma_{2}, \tau_{2,1}$)^T requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}'_g, \overline{R}'_g, \overline{R}'_g, \overline{R}'_g, \overline{R}'_g, \overline{R}'_g, \overline{R}'_g)$ ^T
 $=$ **7, FF2, IFF1, IFF2, IFF3** $\{\$ **Fruitainns of the four envisaged SFCs** (model mapping, bar)
 $\{\sigma\} = (\sigma_1, \sigma_2, \tau_2)^T$ requiring all 5 rechnical strengths $\{R\} = (R'_2, R'_3, R'_1, R_1', R_1', R'_2, R'_3, R'_4, R'_5, R_{1,2})^T$
 Hal SFCs: FF1, FF2, IFF1, IFF2, IFF3 $\{\$ **Liations of the four envisaged SFCs** (model mapping, bar)
 $\frac{1}{2} = (\sigma_1, \sigma_2, \tau_{21})^T$ requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}_g^1, \overline{R}_g^2, \overline{R}_h^1, \overline{R}_h^2, \overline{R}_h^1, \overline{R}_h^1, \overline{R}_h^1, \overline{R}_h^1)$

FFC **6** (model mapping, bar)
 \overline{R} } = $(\overline{R}_{ij}^t, \overline{R}_{ij}^c, \overline{R}_{i}^t, \overline{R}_{i}^c, \overline{R}_{j}^t)^T$
 $\overline{R}_{i}^t, \overline{R}_{i}^c, \overline{R}_{jij}^t)^T$, $\mu_{\perp ||}, m \cong 2.6$

n₁ = 1 = 100%

eless negative *Eff* modes
 $|\tau_{21}|$
 $|\tau_{\mu_{j|i$ **IS Of the four envisaged SFCs** (model mapping, bar)
 $\sigma_{2}, \tau_{2,1}$)^T requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}'_g, \overline{R}'_g, \overline{R}'_g, \overline{R}'_g, \overline{R}'_g, \overline{R}'_g, \overline{R}'_g)$ ^T
 $=$ **7, FF2, IFF1, IFF2, IFF3** $\{\$ $=1: F = Eff = \frac{\sigma_1^2}{\overline{R}_1^t \cdot \overline{R}_2^c} + \sigma_1 \cdot \left(\frac{1}{\overline{R}_1^t} - \frac{1}{\overline{R}_2^c} \right) + 2 \cdot \frac{\overline{F}_{12} \cdot \sigma_1 \cdot \sigma_2}{\sqrt{\overline{R}_1^t \cdot \overline{R}_2^c \cdot \overline{R}_1^t \cdot \overline{R}_2^c}} + \frac{\sigma_2^2}{\overline{R}_1^t \cdot \overline{R}_2^c}$ **IS Of the four envisaged SFCs** (model mapping, bar)
 $\sigma_{2}, \tau_{2,1}$)^T requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}'_g, \overline{R}'_g, \overline{R}'_g, \overline{R}'_g, \overline{R}'_g, \overline{R}'_g, \overline{R}'_g)$ ^T
 $=$ **7, FF2, IFF1, IFF2, IFF3** $\{\$ **four envisaged SFCs (model mapping, bar)**

uiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}_{ij}^t, \overline{R}_{ij}^c, \overline{R}_{ij}^t, \overline{R}_{i,j}^c, \overline{R}_{i,j}^t)^T$
 1, IFF2, IFF3 $\{\overline{R}\} = (\overline{R}_{ij}^t, \overline{R}_{ij}^c, \overline{R}_{1,j}^c, \overline{R}_{1,j}^c, \$ **fthe four envisaged SFCs** (model mapping, bar)

^T requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}_{ij}^L, \overline{R}_{ij}^c, \$ **2D (in-plane)- Formulations of the four envisaged SFCs (n

{** $\sigma_1^1 = (\sigma_1, \sigma_2, \sigma_3, r_{33}, r_{31}, r_{13})^T \rightarrow {\sigma_1}^1 = (\sigma_1, \sigma_2, r_{24})^T$ **requiring all 5 technical strengths** $\{R\}$ **:

Cuntze's FMC-based Set of Modal SFCs: FF1, FF 2D (in-plane)- Formulations of the four envisaged SFCs** (model mapping, bar)
 $\{\sigma\} = (\sigma_1, \sigma_2, \sigma_1, \tau_3, \tau_3, \tau_4, \tau_5)^T \rightarrow \{\sigma\} = (\sigma_1, \sigma_2, \tau_3, \tau)^T$ requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}_2^*, \overline{R}_3^*, \overline{R}_$ **Fractain Sof the four envisaged SFCs**

{ σ } = ($\sigma_1, \sigma_2, \tau_{21}$)^T requiring all 5 technical strengths { \overline{R}
 tal SFCs: FF1, FF2, IFF1, IFF2, IFF3 { \overline{R} } = (\overline{R} *i*, \overline{R} } = [\overline{R} *ii*, $\overline{R$ **2D (in-plane)- Formulations of the four envisaged SFCs** (model mapple)
 $(\sigma_1, \sigma_2, \sigma_3, \tau_3, \tau_3, \tau_{13})^T \rightarrow \{\sigma\} = (\sigma_1, \sigma_2, \tau_2)^T$ requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}_i^i, \overline{R}_i^i, \overline{R}_i^i, \overline{R}_i^i, \over$ **nulations of the four envisaged SFCs** (model mapping, bar)
 $\sigma_1^1 = (\sigma_1, \sigma_2, \tau_{21})^T$ requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}_s^T, \overline{R}_s^T, \overline{R}_s^T, \overline{R}_s^T, \overline{R}_s^T, \overline{R}_s^T, \overline{R}_s^T, \overline{R}_s^T, \overline{R}_$ *F , R , R* **2D (in-plane)- Formulations of the four envisaged SFCs (model mapping, bar)**
 $\frac{1}{2} = (G_1, \sigma_2, \sigma_1, r_{a_1}, r_{a_2}, r_{a_3})^T \rightarrow [\sigma_1^1 = (G_1, \sigma_2, r_{a_3})^T$ requiring all 5 rechnical strengths $\{R\} = (\mathcal{R}_a^1, \mathcal{R}_a^2, \mathcal{R}_$ 1 1 1 1 2 2 **2D (in-plane)- Formulations of the four envisaged SFCs (model mapping,** $\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{33}, \tau_{33})^T \rightarrow \{\sigma\} = (\sigma_1, \sigma_2, \tau_{23})^T$ **. requiring all 5 rechuiced strengths \{R\} = (R_c^T, R_c^T, R_c^T, R_c^T, R_c^T, R_c^T, R_c^T, R_c^T, 2D (in-plane)- Formulations of the four envisaged SFC**
 $\sigma_1, \sigma_2, \sigma_3, \tau_{33}, \tau_{31}, \tau_{12}$)^T $\rightarrow \{\sigma\} = (\sigma_1, \sigma_2, \tau_{21})^T$ requiring all 5 technical strengths
 S FMC-based Set of Modal SFCs: FF1, FF2, IFF1, IFF2, IFF3 2D (in-plane)- Formulations of the four envisaged SFCs (model mapping, bar $\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_3, \tau_4, \tau_5, \tau_7) \rightarrow \{\sigma\} = (\sigma_1, \sigma_2, \tau_2, \tau_3)^\top$ requiring all 5 sechnical stangules $\{\overline{R}\} = (\overline{R}_a^T, \overline{R}_a^T, \overline{$ **2D (in-plane)- Formulations of the four envisaged SFCs (model mapping, bar)**
 $| = (\sigma_1, \sigma_2, \sigma_3, \tau_{31}, \tau_{32}, \tau_{33})^T \rightarrow [\sigma_1^1 = (\sigma_1, \sigma_2, \tau_{31})^T$ requiring all 5 rechained strengths $\{R\} = (\mathcal{R}_3^1, \mathcal{R}_3^2, \mathcal{R}_1^2, \math$ **ID (in-plane)- Formulations of the four en**
 $\frac{1}{2} \sigma_3, \tau_{23}, \tau_{31}, \tau_{12}$)^T $\rightarrow \{\sigma\} = (\sigma_1, \sigma_2, \tau_{21})^T$ requiring all 5
 IC-based Set of Modal SFCs: FF1, FF2, IFF1, IFF2, IF
 \overline{R} , μ) =1 : \Rightarrow *Eff* $^m =$ **plane)- Formulations of the four envisaged SFCs (model m**
 $r_{x_1}, r_{x_2}^T$)^{τ} $\rightarrow \{\sigma\} = (\sigma_1, \sigma_2, r_{z_1})^T$ requiring all 5 technical strengths $\{\overline{R}\} = (\overline{R}_x^{\prime}, \overline{R}_y^{\prime}, \overline{R}_z^{\prime}, \overline{R}_z^{\prime}, \overline{R}_z^{\prime}, \overline{R}_z$ **2D (in-plane)- Formulations of the four envisaged SFCs** (model mapping, bar)
 $\sigma_1^1 = (\sigma_1, \sigma_2, \sigma_3, \tau_{2k}, \tau_{1k})^T \rightarrow \{\sigma_1^1 = (\sigma_1, \sigma_2, \tau_{3k})^T\}$ requiring all 5 technical strengths $\{R\} = (R_2^1, R_2^2, R_1^2, R_1^2, R_2^2$ **2D (in-plane)- Formulations of the four envisaged SFCs** (model mapping, bar)
 $E(f_1, G_2, G_3, G_3, G_3, G_3, G_3, G_3, G_3, G_3)$
 $E(f_2, G_3, G_3, G_3, G_3, G_3, G_3)$
 $E(g_2^T, G_3^T, G_3^T, G_3^T, G_3^T, G_3^T, G_3^T, G_3^T, G_3^T, G_3^T, G_$ **2D (in-plane)- Formulations of the four envisaged SFCs (model mapping, bar)**
 $(\sigma_1, \sigma_2, \sigma_3, r_4, r_4, y^T \rightarrow \{\sigma\} = (\sigma_1, \sigma_2, r_5)^\text{T}$ requiring all 5 sechmical strengths $\{\overline{R}\} = (\overline{R}_0^2, \overline{R}_2^2, \overline{R}_4^2, \overline{R}_4^2, \$ **2D (in-plane)- Formulations of the four envisaged SFCs (model mapping, bar)**
 $\sigma_2, \sigma_3, \tau_3, \tau_3, \tau_4, \tau_5$ ^T $\rightarrow \{\sigma_1^1 = (\sigma_1, \sigma_2, \tau_3)^T$ requiring all 5 ischinical stengths $\{R\} = (R_2, R_3', R_4', R_4', R_5')$
 $MC-based Set of Moded SFCs:$ **S)- Formulations of the four envisaged SFCs** (model mapping, bar)
 $\sum y^T \rightarrow \{\sigma\} = (\sigma_1, \sigma_2, \tau_{21})^T$ requiring all 5 sechnical strengths $\{\overline{R}\} = (\overline{R}_g^T, \overline{R}_g^T, \overline{R}_g^T, \overline{R}_g^T, \overline{R}_g^T, \overline{R}_g^T, \overline{R}_g^T, \overline$

Hyp.2: $F\left(\{\sigma\}, {\{\overline{R}\}, \overline{S}_{23}}\right) = 1$: invariant way (like Cuntze), mo
 $\overline{R}_{23} = \overline{S}_{23}$ Interaction will be performed as wi S_{23} Interaction will be performed as with Cuntze's *Effs*

Puck's Action Plane IFF SFCs, (Mohr-based globally combining the *3 IFF*-domains) with *the 2 modal FF1, FF2* $\left(\left\{\sigma^A\right\},\left\{\bar{R}^A\right\},\theta_{\text{fp}}\right)=1$ with $\left\{\bar{R}\right\}=(\bar{R}_{\parallel}',\bar{R}_{\parallel}',\bar{R}_{\perp}^c=\bar{R}_{\perp}',\bar{R}_{\perp\perp}^A,\bar{R}_{\perp\parallel}^A=\bar{R}_{\perp\parallel}\right)^{\text{T}}$ including $\bar{R}_{\perp\perp}^A\neq\bar{R}_{\perp\perp}^A$

IFF-globally(\overline{R}_{\perp}^c , μ)) given \Rightarrow FF-modes are to interact with the given IFF-mode formulations, domain-wise

$$
Eff^{\parallel \sigma} = \frac{(\sigma_1 + |\sigma_1|)}{2 \cdot R_{\parallel}^t}, \quad \underline{Eff}^{\parallel \tau} = \frac{(-\sigma_1 + |\sigma_1|)}{2 \cdot R_{\parallel}^c}, \quad \text{IFFl: } F^{\perp \sigma} = \underline{Eff}^{\perp \sigma} = F = \frac{1}{\overline{R}_{\perp \parallel}} \cdot \left(\sqrt{\left(\frac{\overline{R}_{\perp \parallel}}{\overline{R}_{\perp}^t} - p_{\perp \parallel}^t \right)^2} \cdot \sigma_2^2 + \tau_{21}^2 + p_{\perp \parallel}^t \cdot \sigma_2 \right), \quad \text{IFFl: } \underline{Eff}^{\perp \tau}, \quad \text{IFFl: } \underline{Eff}^{\perp \parallel \tau}.
$$

6 Comparison of failure envelopes

Figure, after taking away numerically produced other curve branches of the envelope

Notes:

- The differences of the four models are obvious and the reader is asked for an assessment
- CFRP test results (MAN Technologie research project with A. Puck, IKV Aachen et al.)
[VDI 97] $\{\bar{R}\} = (1280, 800, 51, 230, 97)^TMPa, \mu_{\perp\parallel} = 0.3$ [VDI 97]

The reader is asked to assess the resulting failure envelopes

6 Comparison of failure envelopes

Comparison of the Failure Envelopes

- * The differences of the four models are clearly visualized and the reader asked for assessment.
- * In the WWFE the 4 strength values were provided together with the not R_1^c -matching test data in the fourth quadrant However, the tendency of the two different (*to assume*) test sets can be used for validation.

6 Comparison of failure envelopes *

Comparison of the Failure Envelopes $\tau_{21}(\sigma_1)$

Notes on this theoretical example:

- The differences of the four models are obvious.
- See Example 3, please. Another data set.

Task of the designer: To prove that a reserve factor *RF* > 1 could be obtained for the structural component.

Linear analysis is sufficient (presumption):
$$
\sigma \sim \text{load} \implies RF = f_{RF} = 1 / Eff
$$

Material Reserve Factor $f_{RF, \text{ult}} = \frac{\text{Strength Design} \text{Allowable } R}{\text{Stress at } j_{\text{ult}} \cdot \text{Design Limit} \text{ Load}} > 1,$

on-linear analysis required:
$$
\sigma
$$
 not proportional to load
Reserve Factor (load-defined) $RF_{ult} = \frac{\text{Predicted Failure Load at } Eff = 100\%}{j_{ult} \cdot \text{Design Limit Load}} > 1.$

A very simple example for a Design Verification of an applied stress state in a critical UD lamina location of a distinct laminate wall design shall depict the *RF*-calculation as most essential task in design which streamlines every procedure when generating a design tool in the following chapters:

terial Reserve Factor f_{RF}

reserve factor $RF > 1$ could be obtain

presumption): $\sigma \sim \text{load} \Rightarrow RF = \hat{f}R$
 $f_{RF,ult} = \frac{\text{Strength Design}.\text{Allowable}}{\text{Stress at } j_{ult}} \cdot \text{Design Limit} \cup \sigma$ not proportional to load
 $RF_{ult} = \frac{\text{Predicted Failure Load at } Eff}{j_{ult}} \cdot \text{Design Limit Load}$

ca **Computation of a Material Reserve Factor**

the designer: To prove that a reserve factor $RF > 1$ could

Linear analysis is sufficient (presumption): $\sigma \sim \text{load} \Rightarrow$

Material Reserve Factor $f_{\text{RF}, alt} = \frac{\text{Strength Design A}}{\text{Stress at } j_{alt$ **Computation of a Material Reserve Factor**

the designer: To prove that a reserve factor $RF > 1$ could b

Linear analysis is sufficient (presumption): $\sigma \sim \text{load} \Rightarrow R$

Material Reserve Factor $f_{RF, \text{ult}} = \frac{\text{Strength Design Allo}}{\text{Stress at }$ *n* **of a Material Reserve Factor fax (linear analysis permitted)**
 force that a reserve factor $RF > 1$ *could be obtained for the structural component.*
 sufficient (presumption): $\sigma - \text{load} \implies RF = \text{fixF} - 1/kf$
 Factor f **mputation of a Material Rese**
designer: To prove that a reserve factor
ar analysis is sufficient (presumption):
laterial Reserve Factor $f_{RF,ult} = \frac{Stres}{Stress}$
-linear analysis required: σ not proporti
eserve Factor (load**omputation of a Material Reserve Factor** f_{RF} (linear analyse designer: To prove that a reserve factor $RF > 1$ could be obtained for the stear analysis is sufficient (presumption): $\sigma \sim \text{load} \Rightarrow RF = f_{RF} = 1 / E_{ff}$
Material R *ult ult Eff RF .* **II Reserve Factor** far (linear analysis permitted)

we factor RF > 1 could be obtained for the structural component.

mption): $\sigma \sim$ load \Rightarrow RF = f k F = 1 / Eff

Stress at j_{adv} . Design Limit Load > 1,

proporti A very simple example shall depict the *RF*-calculation in design which streamlines the procedure when generating a design tool which is further to be applied such as in the following chapters. \rightarrow A stress state assessment is usually based on an agreement to apply the so-called 'Proportional Loading (stressing) Concept'. If linear, all stresses alter proportionally. Margin-of-Safety *MoS* = *RF* -1.

Design Load cases: $design Limit Load = dLL$, Design Ultimate Load = DUL

.

Numerical example UD Design Verification by a material *fRF* **> 1** *2D-Design Verification of a critical UD lamina in a distinct laminate wall design*

* Design loading (action): $\{\sigma\}_{\text{design}} = \{\sigma\} \cdot j_{\text{ult}}$, design FoS $j_{\text{ult}} = 1.25$ **erical example UD Design Verificism**

ign Verification of a critical UD lamina i.

ition Thear analysis permitted' = good eno

(action): $\{\sigma\}_{\text{design}} = \{\sigma\} \cdot j_{\text{ult}}$, design FoS j
 $\{\sigma\}_{\text{L}} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \$ **Numerical example UD Design Verification by a material far-**

2D-Design Verification of a critical UD lamina in a distinct laminate wall desi

PUT: Assumption Timear analysis permitted'= good enough

Design boding (actio **fication by a material fit**
 j_{in} in a distinct laminate wall

nough
 $j_{\text{ult}} = 1.25$

0, -76, 0, 0, 0, 52)^TMPa

cracking)

a average from mesurement,

725, 32, 112, 79)^TMPa.

ion exponent: $m = 2.7$

88, 0, 0, 0 INPUT: Asssumption 'Linear analysis permitted' = good enough **Numerical example UD Design**

2D-Design Verification of a critical UL

INPUT: Asssumption Linear analysis permitted

* Design loading (action): $\{\sigma\}_{\text{design}}^1 = \{\sigma\} \cdot j_{\text{ult}}$, de

* 2D-stress state: $\{\sigma\}_{\text{L}} = (\sigma_1, \sigma_2,$ **Numerical example UD Design Verification by a materix**

2D-Design Verification of a critical UD lamina in a distinct laminate v

NPUT: Assumption Linear analysis permitted "= good enough

* Design loading (action): $\{\sigma\$ **Numerical example UD Design Verification by a material for > 1**
 NPUT: Assumption Then analysis permitted UD lamina in a distinct laminate wall design

NPUT: Assumption Then analysis permitted = good enough

^{*} Design **rical example UD Design Verification by a material firs > 1**
 in Verification of a critical UD lamina in a distinct laminate wall design

ion Tinear malysis permitted = good cnough
 σ_1^1 , σ_2^1 , σ_3 , σ_4 , **I example UD Design Verification by a material for > 1**

enffication of a critical UD lamina in a distinct laminate wall design

Linear analysis permitted = good enough

1: { σ }, σ _{*s*}, σ ₃, τ ₃, τ ₃, (effect vanishes with increasing micro – cracking) $\{\overline{R}\}$ = (1378, 950, 40, 125, 97)^TMPa average statistically reduced $\{R\} = (R_{ij}^t, R_{ij}^c, R_{\perp}^t, R_{\perp}^c, R_{\perp ij}^t)^T = (1050, 750, 750)$ **n** by a material $f_{RF} > 1$

stinct laminate wall design

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112, 79)^{T} \text{ MPa}\n\end{bmatrix}$
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* Residual stresses: 0 *(effect vanishes with increasing micro – cracking)* **Numerical example UD Design Verifition**

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INPUT: Assumption Tinear analysis permitted' = good enc

* Design loading (action): $\{\sigma\}_{\text{design}} = \{\sigma\} \cdot J_{\text{uft}}$, design FoS j

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= { σ } $\cdot j_{ult}$, design FoS $j_{ult} = 1.25$
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** $2D-Design \; Verification$ **of a critical UD lamina in a distinct laminate wall design

Asssumption Tincar analysis permitted "good cnough

moduling (action): \{\sigma\}_{\text{long}} = rification by a material f_{RF} > 1**

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2D-Design Verification of a critical UD lamina in a distinct

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* Design loading (action): $\{\sigma\}_{\text{design}} = \{\sigma\} \cdot f_{\text{ul$ **Numerical example UD Design Verification by a materia

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^{*} D **cal example UD Design Verification by a material** $f_{RF} > 1$
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ion): $\{\sigma\}_{\text{design}} = \{\sigma\} \cdot f_{\text{win}}$, des **Ie UD Design Verification by a material far > 1**

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lysis permitted = good enough
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 $\frac{1}{3}$, r_{21} , r_{31} , r **Imerical example UD Design Verification by a material** $\hbar \kappa > 1$ **

Design Verification of a critical UD lamina in a distinct laminate wall design

ling (action):** $\{\sigma\}_{\text{asym}} = \{\sigma\} \cdot j_{\text{at}}$ **, design Fr6** $j_{\text{at}} = 1.25$ **
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2D-Design Verification of a critical UD

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loading (action): $\{\sigma\}_{\text{design}} = \{\sigma\} \cdot j_{\text{ult}}$, de

ss state: $\{\sigma\}_{L} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T \cdot j$ $Eff^{\perp} = \frac{|\tau_{21}|}{\sqrt{2\pi}} = 0.55$ $(f^{\perp \sigma})^m + (Eff^{\perp \tau})^m + (Eff^{\perp \parallel})^m \]^{1/m}.$ **Numerical example UD Design Verification by a n**

2D-Design Verification of a critical UD lamina in a distinct lamin

INPUT: Asssumption Timear analysis permitted – good enough

* Design loading (action): $\{\sigma^2\}_{\text{design}} = \$ **Numerical example UD Design Verificatio**

2D-Design Verification of a critical UD lamina in a distribution

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* Design Dading (action): $\{\sigma\}_{\text{design}} = \{\sigma\} \cdot j_{\text{ult}}$ **Numerical example UD Design Verification by a match-**
 LD-Design Verification of a critical UD lamina in a distinct laminat

Asssumption Tinear analysis permitted' = good enough

oading (action): $\{\sigma\}_{\text{design}} = \{\sigma\} \cdot j_{\$ $Eff^{\perp \sigma} = \frac{z_2 - |z_2|}{2 \overline{R}^t} = 0$, $Eff^{\perp \tau} = \frac{z_2 - |z_2|}{2 \overline{R}^c} = 0.60$, *T* **Numerical example UD Design Verification by a material far**
 ZD-Design Verification of a critical UD lamina in a distinct laminate wall de
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ssumption Tinear analysis permitted' = good e

dding (action): $\{\sigma\}_{\text{design}} = \{\sigma\} \cdot j_{\text{ult}}$, design Fot

state: $\{\sigma\}_{\text{L}} = (\sigma_1, \sigma_2, \sigma_3, \tau_2, \tau_3, \tau_1,$ **Numerical example UD Design Verification b.**

2D-Design Verification of a critical UD lamina in a distince

PUT: Asssumption Linear analysis permitted = good enough

Design loading (action): $\{\sigma\}_{\text{design}} = \{\sigma\} \cdot j_{\text{ult}}$, R_+^c and \Box *Eff* = $[(Ef^{i/\sigma})^m + (Ef^{i/\tau})^m + (Ef^{i-\sigma})^m + (Eff^{i-\sigma})^m + (Eff^{i-\sigma})^m]$

The results deliver the following failure danger portio

* *Eff* $\frac{d\sigma}{dt} = \frac{\sigma_2 + |\sigma_2|}{\sigma_2} = 0$, *Eff* $\frac{d\sigma}{dt} = \frac{-\sigma_2 + |\sigma_2|}{\sigma_2} = 0.60$, *.* **Numerical example UD Design Verification by a material fee > 1**

2D-Design Verification of a critical UD lamina in a distinct laminate wall design

Assumption Linear analysis permited $\frac{1}{2}$ to design For $\frac{1}{2}$ to \perp and the set of th \perp and the set of th **Numerical example UD Design Verification by a material far > 1**
 **2D-Design Verification of a critical UD lamina in a distinct laminate wall design

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ftion of a critical UD lamina in a distinct laminate wall design

ranalysis permitted = good enough
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 $\sigma_2, \sigma_3, \tau_$ **Numerical example UD Design Verification by a material far > 1**

2D-Design Verification of a critical UD lamina in a distinct iaminate wall design

ign bushing (action): [σ_1^1 , $=$ (σ_1 , σ_2 , σ_3 , τ_{21} , **Numerical example UD Design Verification by a material far > 1**

2D-Design Verification of a critical UD larmina in a distinct laminate wall design

Assumption Theorem unalysis permitted 'good enough

m loading (action): $\frac{1^{2}211}{\overline{R}_{\perp\!/\!}/-\mu_{\perp\!/\!/\!}/\sigma_{2}} = 0.55$ and the material reserve factor $f_{RF} = 1/Eff$ $\left| \int ds \, ds \right|$ $\Rightarrow f_{\text{RF}} = 1 / \text{Eff} = 1.25$ $f_{\text{RF}} \rightarrow \text{MoS} = RF - 1 = 0.25 > 0$! **11 example UD Design Verifica**
 Cerification of a critical UD lamina in

Thear analysis permitted' = good enough): $\{\sigma\}_{\text{design}}^1 = \{\sigma\} \cdot j_{\text{ult}}$, design FoS j_{ult}
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design FoS $j_{ult} = 1.25$
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* Design loading (action): { $\sigma^1_{\text{design}} = {\sigma^1_j \cdot j_{\text{ult}}}$, design

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* Design loading (action): $\{\sigma\}_{\text{design}} = \{\sigma\} \cdot j_{\text{ult}}$, design FoS

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Design loading (action): $\{\sigma\}_{\text{stsign}} = \{\sigma\} \cdot J_{\text{ult}}$, design FoS $J_{\text{ult}} = 2D$ -stress state: $\{\sigma\}_{\text{L}} = (\sigma_1, \sigma_2,$ **ble UD Design Verifi**

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2D-Design Verification of a critical UD lamina in a distinct iaminate wall design

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sign booking (acti 7 Computation Reserve Factor

Conclusions regarding the Comparison of the 4 SFCs

- The 4 SFCs differently well map the sparsely available **reliable** test data
- \triangleright 'Modal' SFCs are less dangerous in application than the forcibly modesmarrying 'global' ones
- \triangleright The full capacity of the fracture conditions cannot be fully verified. Derivation of more representative experimental test data is necessary to really make a better judgement even of the 2D UD failure conditions possible
- \triangleright Learning from WWFEs: Careful test data evaluation is of highest priority
- \triangleright Cuntze and Puck seem to map the course of the reliable test points best. This is essential, when computing for Design Verification the Reserve Factor of the Final Design
- Each SFC can only describe a one-fold occurring failure mode. Multi-fold failure, such if $\sigma^2 = \sigma^2$, is additionally to consider
- \triangleright For multi-directionally reinforced laminates, well-designed by netting theory, linear analysis is a good approximation on the safe side.
	- ► For 2D-analyses, after carefully looking at the comparison results, the designer is now enabled:
	- " *To not take the worst result to achieve a relatively conservative design" !*
- * The full capacity of the fracture conditions could not be fully verified. Derivation of more representative experimental test data is necessary to really make a judgement of the 2D UD failure conditions possible.
- UD-SFCs are presently not 3D-failure stress states-validated? We use *the SFCs* without any questioning ! **Therefore, the future challenge will be 3D-analyses of joints etc.**

Conclusions, Findings regarding Cuntze's SFC set

- \triangleright In the frame of his material symmetry-driven thoughts the author could derived an 'Engineering-practical strength criteria set' SFC on basis of measurable parameters, only.
- First direct use of the measurable friction value *µ* in a SFC formulation (*after effortful investigations possible*)
- \triangleright Clear notations to identify material properties and the observed laminate stack
- 'Generic' numbers found, simplify theoretical and test tasks: Isotropic (2), transversely-isotropic UD (5), Orthotr. (9)
- \triangleright Mapping (fitting) of the courses of provided and of own test data is very good.
- \triangleright Simple determination of the lowest mode reserve factor = design driving mode. Reliable computation of the material Reserve Factor f_{RF} . This requires *Eff* in the case of a non-homogeneous Failure function F or SFC
- At 1D, or 2D and 3D failure states *Eff* (*Werkstoff-Anstrengung)* can maximally achieve 100% ! This explains, that an axial failure stress under 3D-compression is higher than the Standard-fixed technical strength value (axial) *R*
- \triangleright For laminates, well-designed by netting theory, linear analysis is a good approximation on the safe side
- \triangleright Cuntze does not require the non-measurable cohesive shear strength design value as a sixth strength quantity
- The physical different action of laminate-embedded (more benihn) and isolated layers in test specimens is to consider.

Unfortunately, structural engineers believe in what is presented in the FE-code Manuals without questioning the quality of the given SFC or of any provided test results !?

Learning from my Pre-digital FE-Age (1960-1968**), some personal Reminders**

Lessons to Learn from Bronstein-Semendjajev today again: he wrote in 1960

A short arithmetic stick (slide rule, Rechenschieber) and a longer one for instance for static dimensioning in constructions and a more accurate output. A stick made parametric dimensioning possible! The book Bronstein-Semendjajew retained his excellent value over all the decades.

**About the purpose of the arithmetic stick*

In constructions and a more accounate output. A stock mase parameter currents of the and the purpose of the arithmetic state. The simplest calculations, in which multiplications, division, extraction of square and cube roo "*The simplest calculations, in which multiplications, division, extraction of square and cube roots, exponentiation and operations with trigonometric functions occur, can be carried out approximately with* the help of the arithmetic stick. The accuracy of the calculations varies from case to case, but it can be said, that on average the arithmetic stick of 25 cm length gives results on three decimals with an error of *less than 1%".*

**About the calculation accuracy*:

"When calculating, it is always important to consider the accuracy that you either have to achieve or can *achieve.*

- It is quite inadmissible to calculate with great accuracy if the nature of the task either does not allow *it or does not require it*
- *If an approximate value contains superfluous decimal numbers, it must be rounded. In some cases, more realistic but more complicated models can be replaced by simpler ones that give a result with an acceptable error*."

FEA-Tools for practicing engineering at 1960-1968

- * I remember that I could not finish my dissertation in 1967 because o and 0 where not clearly marked in the coding which cost two months filled with despair.
- * Later in the sixties Finite Element Analysis came up, however, there was no commercial tool available. The author had to beg for the building blocks of his intended program for a radial impeller at the DLR at his old university in Hannover. To punch cards was a must.

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Ich freue mich, dass Prof. D. Sabine Pfeiffer, Lehrstuhl für Soziologie der Uni Erlangen-Nürnberg, - als gelernte Werkzeugmacherin - Sprecherin des DFG-Schwerpunktprogrammns 'Digitalisierung der Arbeitswelten' bekundet:

"*KI kann nicht alles leisten, wir müssen uns stärker auf bewährtes Ingenieurwissen besinnen und mehr vom Ziel her denken.*

Dies bedeutet zum Beispiel auch:

Welche Tools sind wirklich vom 'Niveau her' notwendig zum Erreichen des Ziels und vor allem robust?"

Der Gebrauch des Rechenschiebers lehrt es uns.

Besten Dank für's Zuhören und Zusehen.

Ihr Ralf Cuntze

CUNTZE, Ralf

- **1964, Dipl.-Ing. Civil Engineering (**structural eng., TU Hannover**)**
- 1968, Dr.-Ing. Structural Dynamics (TU Hannover)
- 1968 1970, DLR FEA-programming
- **1970 2004, MAN-Technologie: Head 'Structural and Thermal Analysis' ARIANE 1-5, GROWIAN, Uranium Enrichment centrifuges, Solar Plants, Pressure Vessels, etc.**
- 1978, Dr.-Ing. habil. Mechanics of Lightweight Structures (TU Munich)
- 1980 2002 **Lecturer UniBw on Fracture Mechanics (construction), Lightweight**
- 1987, Full Professorship, *not started in favor of interesting industry tasks*
- 1998, Honorary Professorship at Universität der Bundeswehr München UniBw
- **1972 – 2018 contributor to the German** Aerospace **Hdbk HSB**
- **2006, VDI Guideline 2014 "Development of FRP-Components" (***editor sheet 3***)**
- 2019, GLOSSAR "Technical terms for composite parts". Springer
- 1972 2004 working on multiple ESA/ESTEC Standards and 2004 - 2009 heading the"ESA Stability Handbook" Working Group
- **since 2009 with Carbon Composites e.V. (mechanical engineering) and CU Bau (carbon concrete)**
- 2019-2023 "Life-Work Cuntze a compilation" (about 850 pages)