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21. Münchner Leichtbauseminar, 2024, Keynote Lecture, 50 min + 10 min

Comparison of four UD Strength Criteria – including a 'Numerical Review'

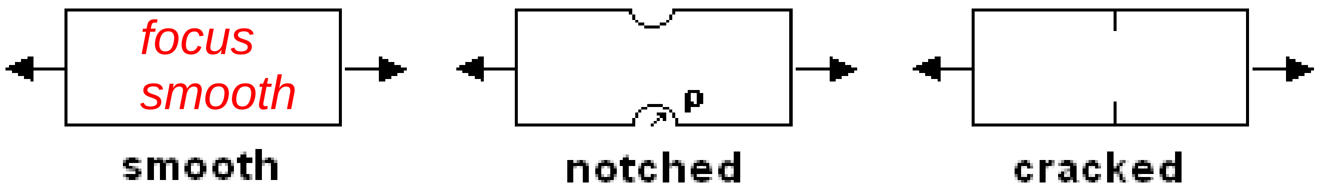
Prof. Dr.-Ing. habil. Ralf Cuntze VDI, formerly MAN-Technologie, linked to Composites United

1 **Concerns when Generating Strength Failure Criteria (*SFC*)**
2 **Terminology, Laminate Description, Material Stressing Effort *Eff***
3 **'Global' SFCs versus 'Modal' SFCs**
4 **Background of Cuntze's Failure-Mode-Concept (*FMC*)**
5 **3D /2D SFCs** (*harmonized due to VDI 2014*)
  5.1 **SFC Cuntze** with **3 Examples from the UD World-Wide-Failure-Exercise**
  5.2 **SFC Hashin**
  5.3 **SFC Puck**
  5.4 **SFC Tsai-Wu**
6 **Comparison of the different SFC Failure Envelopes** $\tau_{21}(\sigma_2)$, $\sigma_2(\sigma_1)$, $\tau_{21}(\sigma_1)$
7 **Computation of a SFC-linked Reserve Factor**
  Conclusions, Lessons Learned

NOTE, please: Results of a non-supported, non-funded "hobby" !

Streamlining the presentation:
Which structural component surfaces are faced by the Designing Engineer?

Strength theory | **Notch theory** | **Fracture Mechanics theory**



stresses | stress concentrations | stress intensities

stress singularities

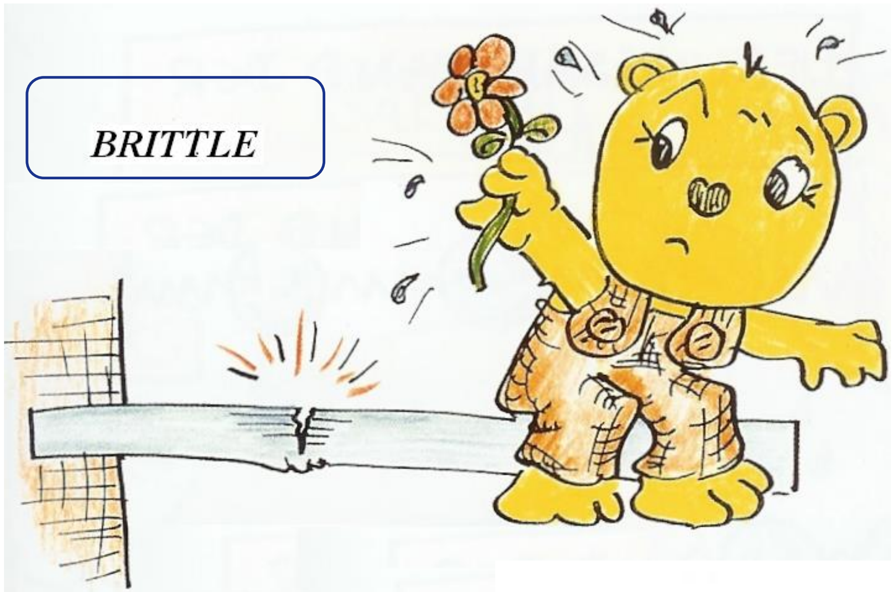
Strength Failure Criteria (SFC)
to assess stress states
in non-singularity (smooth) domains

Depending on stress state and environment a brittle material may behave brittle or ductile .

A UD-material might be defined brittle for a strength ratio

$$R_\perp^c \,/\, R_\perp^t \geq 2.5$$

Streamlining the presentation:
Which is the Material Behaviour to be Discriminated?

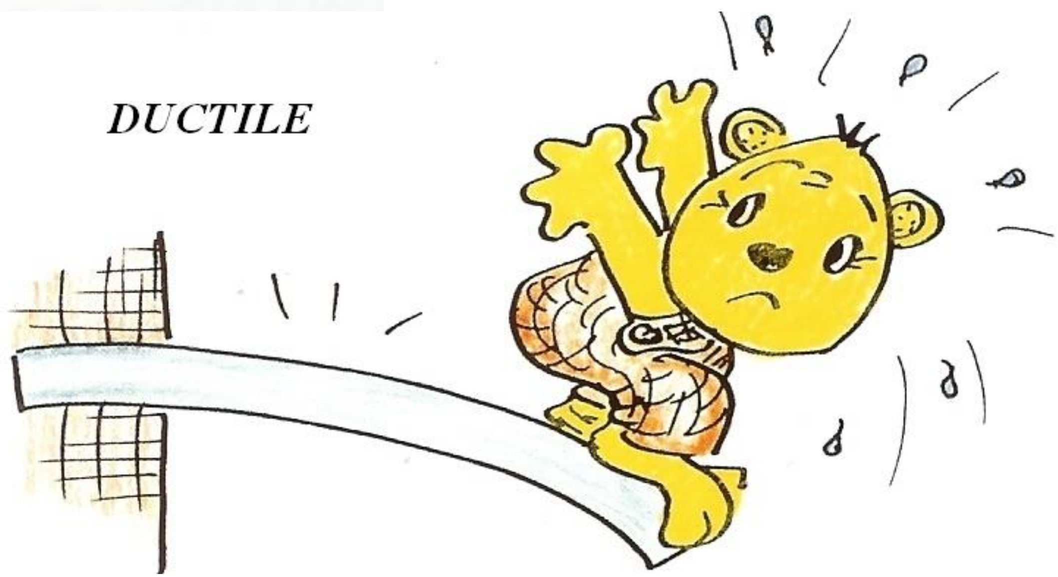


focus brittle

One feels good until sudden fracture occurs

Courtesy: Prof. C. Mattheck

Ductile Fracture = type of a failure mode in a material or structure generally preceded by a large amount of plastic deformation



Basis for the Performance of a Reliable Design Verification (Nachweis)

Same Terminology against misinterpretations

"A general system of signs and symbols is of high importance for a logically consistent universal language for scientific use !"

Gottfried Wilhelm Leibniz (about 1800)

Validated Theoretical Model

"Theory is the Quintessence of all Practical Experience"

A. Föppl

Daher benötigt man gute Werkstoffmodelle, wozu die Festigkeitsbedingungen gehören.

ähnlich der versuchsbegeisterte Kreiseltheoretiker Karl Magnus bei einer Projektbesprechung 1972 zur Urananreicherungs-Zentrifuge.

Für die Version mit **CFK**-Rohr noch einen Geschwindigkeits-Weltrekord gefahren. Allerdings dann das Aussteigen der MAN aus diesem Geschäft.

Terms used in the presentation

Objective = Product Certification by achieving Design Verification (here strength)

Failure function F : mathematical formulation of the failure event by $F = 1$, characterizing the Limit State

Tool Bricks:

Model with Modelling: Structure and Material

Model: Theoretical conception of a real process

(Strength) Failure Criterion SFC (mathematically accurate: ***condition***): Condition on which a failure becomes effective, meaning $F = 1$ for one limit state.

Analysis: Computation that uses fixed model parameters (*e.g. Design Verification of the final design*)

Simulation: Process, that consists of several analysis loops and lasts until the system is imitated in the Design Dimensioning process. Model parameters are adjusted hereby to the 'real world' parameter set.

'Generic' number: Witnessed material symmetry knowledge seems to tell: There might exist a 'generic' (*term was chosen by the author*) material inherent number for the UD material family,, namely 5 for this transversely-isotropic material, where the plane 2-3 is quasi-isotropic and due to that UD is termed transversely-isotropic

Validation of a model: 'qualification' of a created model by well mapping physical test results with the derived model (here the material failure model SFC)

Design Verification: fulfillment of a design requirement data set (for a deformation, a frequency, design load, etc) in the final design

Material Stressing Effort *Eff* (corresponds to Puck's stress exposure): see definition later.

Terminology Basis: VDI 2014 guideline (*Author was editor 2006 and co-author*)

Properties: symbolic indices are dedicated to measurable properties in order to bypass misinterpretation, $R_{\perp}^{t}$.
Theoretical or model parameters, running variables: numbers are dedicated according to mechanics
Model parameters (basically the focus here) are average values and marked by a bar over.

Some Notes on Application & Terminology of the Tsai-Wu 3D-UD SFC

*Such a 'global" formulation is mathematically elegant

*Prediction of a non-feasible domain in quadrant III of $\sigma_1(\sigma_2)$, whereas the 'modal' SFCs of Puck and the FMC-based one of Cuntze map the test data

*Treatment of σ_3 like σ_2 , which is not accurate but model-inevitably

*Cannot map for instance the humb in $\tau_{21}(\sigma_2^{\,c})$, because the material inherent internal friction cannot be directly considered in the global SFC. Hence, the computed Reserve Factor RF may not be on the safe side in this domain

*Difficult determination of the model parameters in the 3D-formulation.The stress interaction term $F_{12} = F_{13}$ (if UD) needs additional bi-axial (σ_1, σ_2)-tests. The bi-axial material parameter F_{12} is 'principally' obtained by bi-axial compression tests. Usually it is applied $F_{12} = -0.5$.

*For application just strength values are necessary, but this is not sufficient!

*No information on the prevailing failure mode FF or IFF is received

*Tsai's Strength Ratio **R** corresponds to 1 / Eff = f_{RF} (Altair also uses Strength Ratio, marked by the letters SR , corresponding to f_{RF})

(However: Since many decades, in mechanics the letter R is dedicated in Standards to Strength (Resistance) R. Further it reads valid the Strength Ratio R = compressive strength /tensile strength (better term is SR).

In fatigue is forever practice (straight letter) R = minσ / maxσ, termed Stress Ratio.

* On top, unfortunately in manuals, using Tsai-Wu, his computed 1/R value is called Safety Factor but a safety factor j is given and a reserve factor RF is to compute!

- Question: The differently termed out-of-plane shear strength $S_{23},\ \tau_{23}^{*},\ R_{23} = R_{\perp\perp}$ is how to measure??

Dear Ralf, July 3,

Thank you for your very important point. Too many undefined, or duplicate terms. Will have to clean them up. ***Thanks. Steve***

Since the author is looking at all 3 material families at the same time,

(*Which author has done this before*?)

he used a self-explanatory, symbolic indexing,

as he sensibly defined it as Editor and Co-author

of the VDI 2014, Sheet 3 'Analysis‘ 2006,

on the basis of already well-known applied designations in mechanics

together with his working group colleagues, such as A. Puck.

This only will make an understanding over the material & discipline fences possible

and was the

“Conditio sine qua non” for the elaboration of this comparison!

Example: Strength terms ►

Strength Terms, self-explaining by the chosen indexing

On basis of investigations for the VDI-2014 and the formerly planned novel ESA Materials Handbook, Cuntze proposed internationally not confusing designations for the strength properties (+physical properties). t = tension, c = compression

<u>Notes on some designations</u>:

* As a consequence to isotropic materials (*European standardization*) the letter *R* has to be used for strength !! US notations for UD material with letters X (*direction 1*,) and Y (*direction 2*,) confuse with the structural axes' descriptions X and Y.
* R_m := 'resistance maximale' (French) = tensile fracture strength (superscript ^{t} is usually skipped because design runs in tensile domain), *R* is basic strength. Composites are most often brittle and only slightly porous! SF is shear fracture, NF Normal Fracture.

number

	Fracture Strength Properties										
	loading	tension			compression			shear			
	direction or plane	1	2	3	1	2	3	12	23	13	
9	general orthotropic	R_1^t	R_2^t	R_3^t	R_1^c	R_2^c	R_3^c	R_{12}	R_{23}	R_{13}	friction properties
5	UD	$R_{\parallel}^t$ NF	$R_{\perp}^t$ NF	$R_{\perp}^t$ NF	$R_{\parallel}^c$ SF	$R_{\perp}^c$ SF	$R_{\perp}^c$ SF	$R_{\parallel\perp}$ SF	$R_{\perp\perp}$ NF	$R_{\parallel\perp}$ SF	$\mu_{\perp\perp}$, $\mu_{\perp\parallel}$
6	fabrics	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	*Warp = Fill*
9	fabrics general	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	μ_{W3}, μ_{F3}, μ_{WF}
5	mat	R^t	R^t	R_3^t	R^c	R^c	R_3^c	R^τ	R^τ	R^τ	(≡*UD material with turned direction*)
2	isotropic matrix	R^t SF	R^t SF	R^t SF	*deformation-limited, cylindrical test specimen bulges: What is then R^c?*			R^τ	R^τ	R^τ	μ
		R^t NF	R^t NF	R^t NF	R^c SF	R^c SF	R^c SF	R^τ NF !	R^τ NF	R^τ NF	μ

*R*23

number of independent properties due to material symmetry of isotropic materials

Clear Laminate Descriptions for the Designer

Modeling laminates is a challenge. In this context, essential for the interpretation of the failures faced after testing, is the knowledge about the lay-up (stack) of the envisaged laminate, because crimped fabrics and Non-Crimped Fabric (NCF) -materials behave differently. It is further extremely necessary to provide the material-modeling design engineer and his colleague in production (for the Ply Book) with a clear, distinguishing description of UD-lay-ups being Non Crimp Fabrics NCFs (stitched multi-UD-layer) or Fabric layers (crimped).

One could distinguish the various types by a clear optical designation, a square bracket [..] and a wavy bracket {..}, in order to enable a realistic material modelling in the case of ply-by-ply analyses, that optically helps to distinguish NCF **{**stitched UD-stack**}** from woven fabrics, where one practically cannot mechanically separate the single woven layers within one fabric layer as in the case of *plain weave* binding, which therefore is ‘globally’ symmetric in itself. Applied this means:

* Single UD-layers-*deposited* stack $[0/90]_S = [0/90/90/0]$-lay-up, prepregs.

* Semi-finished product, *stitched* NCF: $\{0/90\} + \{90/0\}$ symmetrically stacked, dry;

deliverable 'building blocks' are $\{0/45/\text{-}45/90\}$,

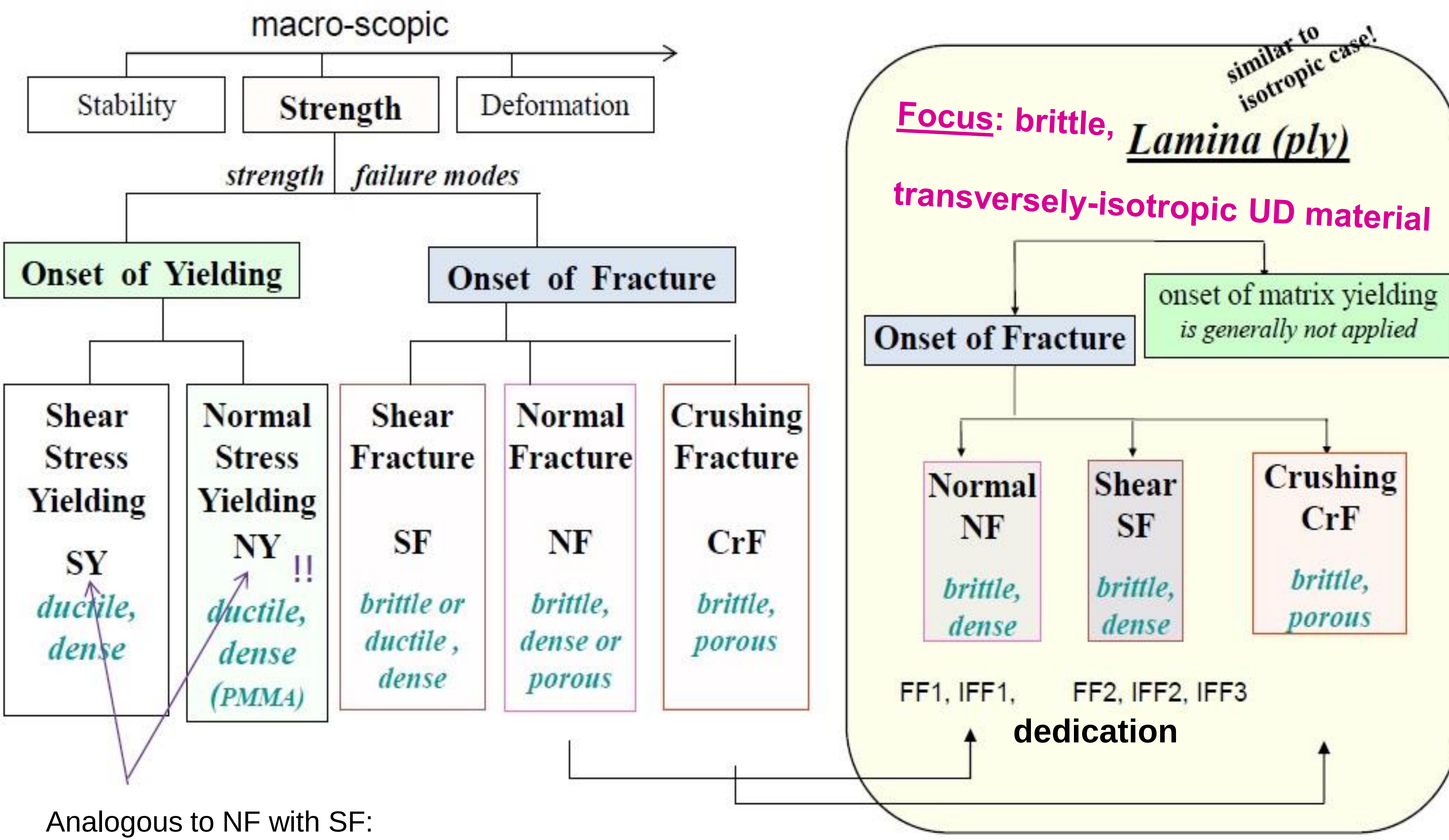
and the novel 'doble-double' C-plies $\{\varphi/\text{-}\psi/\text{-}\varphi/\psi\}$,

as DD building block and sub-laminate i.e. $\{75/-75/-15/15\}_r$ with r = repetitions.

* Semi-finished product, *woven fabric* $\begin{bmatrix} 0 \\ 90 \end{bmatrix}$, symmetric in itself.

Due to unclear descriptions unfortunately one can often not use the seldom available valuable test results of fiber-reinforced materials.

Cuntze's System (1990 proposed) of Macro-scopic Fracture Failure Modes NF , SF



Analogous to NF with SF:
Does NY exist beside SY ??
Yes, found by the author

SF comes from compression loading.
If porous → CrF replaces SF.

+ **delamination failure of *laminate***

FF Fiber Failure
IFF Inter Fiber Failure

Pre-requisites, when Generating FMC-based Strength Failure Criteria (SFC)

Pre-requisites for the establishment of the ***F***ailure function ***F*** are:

- **simply formulated, numerically robust**,
- **physically-based,** and therefore, need only few information for pre-dimensioning
- shall allow for a simple determination of the design driving mode
- all **model parameters shall be measurable** **(**is not standard in SFC theories**)**
- a SFC must become zero if its driving stress becomes zero.

A SFC, ***F* = 1,** is the mathematical description of the failure surface !
F is Failure function .

Pre-requisites, especially required for UD Material Modelling and **Model Validation**

- **The UD-lamina is homogenized to a macroscopically homogeneous solid** or the lamina can be seen as a ‘smeared‘ material
- The UD-lamina is transversely-isotropic:
  On planes transverse to the fiber direction it behaves quasi-isotropically
- **For validation** of the strength material model **a uniform stress state** about the critical stress ‘point‘ (location**) in the test specimen is mandatory**.

The presentation shall give an overarching comparative understanding

on the linear analysis level

of the 4 present Strength Failure Criteria from

Tsai-Wu, Hashin, Puck and Cuntze.

(LaRC from NASA was not included)

Persönlicher Bezug zu den Erzeugern von Festigkeitsbedingungen Tsai, Hashin, Puck

Steve Tsai:

* Zusammen mit Steve war ich 1986 Chairman einer ICCM-session. "*Mach Du den chair, ich mache das Licht an und aus, was damals sehr wichtig war.*"
* Seit Jahren bin ich mit Steve im Rahmen seiner neuen Ideen verbunden. Meine Übertragung in übliche Bezeichnungen sind in einem Beitrag enthalten, den ich mit Erik Kappel im Frühjahr 2024 veröffentlicht habe *" Benefits, applying Tsai's Ideas 'Trace', 'Double-Double' and 'Omni Failure Envelope' to UD-plies composed Laminates?*" Für letztere praktische Vor-Auslegungsidee habe ich eine formelmäßige Lösung anstatt der bisherigen numerischen Lösung gefunden, die ich Steve letztes Jahr als Weihnachtsgeschenk überreichen konnte.

Zvi Hashin:

Hashin saß vor mir in einer traditionsreichen Konferenz in Brüssel und sagte in etwa:

* ***Wir werden kaum in der Lage sein, mit Bruchkriterien jemals Nachweise für UD-Bauteile führen zu können"***

Protest: Als Industriemann darf man diese Aussage nicht tolerieren, weil wir Nachweise führen müssen, um Strukturintegrität für unser Produkt belegen zu können, um es verkaufen zu können.

(Sir) Alfred Puck:

* Zusammen mit Puck suchten Michael Gädke, DLR und Cuntze MAN in vielen Besprechungen seit etwa 1985 kontinuierlich nach einer Verbesserung der UD-SFCs, was dann ja auch gelungen ist.

Basics of the Mohr-linked IFF-SFCs of Hashin, Puck and Cuntze

Mohr's Statement for isotropic materials:

" The strengths of a material are determined by the stresses σ_n , τ_{nt} on the fracture plane" (the fracture plane is usually inclined with respect to the action of the external stresses)

Paul's modification of the Mohr-Coulomb Hypothesis:

" Brittle (behaving) material will fracture in either that plane where the shear stress τ_{nt} reaches a critical value which is given by the shear resistance of a fiber-parallel plane increased by a certain amount of friction caused by the simultaneously acting compressive stress σ_n on that plane. Or, it will fracture in that plane, where the maximum principal (tensile) stress reaches the transverse tensile strength $R_\perp^t$ (in the quasi-isotropic plane)".

* Hashin (1980):

Proposed a modified Mohr-Coulomb IFF approach but did not pursue this idea due to numerical difficulties (*Puck succeeded on this way*). Also into this paper he included an invariant-based global quadratic approach (*Cuntze's invariant way*)

* Puck's Action Plane IFF Conditions (1990):

Based his IFF conditions on Mohr-Coulomb and Hashin, Puck interacts the 3 Mohr stresses σ_n , τ_{nt} , τ_{n1} on the IFF fracture plane. He uses simple polynomials (*parabolic or elliptic*) to formulate a so-called master fracture body in the (σ_n , τ_{nt} , τ_{n1}) space. A compressive σ_n cannot cause fracture on its action plane

* Cuntze Failure-Mode-Concept – based IFF conditions (1993):

Used 3 different invariant IFF conditions, based on his idea that each fracture condition is governed by 1 strength.

History of Cuntze's Failure Mode Concept (FMC), Collaboration with Puck

* Since 1985 <u>common</u> search *with Prof. A. Puck (Uni Gh Kassel) and Dr. M. Gädke (DLR Braunschweig)* of improved fracture criteria to more reliably dimension UD lamina-composed laminates.

Results of this personal collaboration:

* Puck originally presented his stress-angle relationships by an excellent wording. For a discussion with the DLR in 1991 Cuntze recommended to use the present matrix formulation.
* Puck delivered in the author's WWFE-I PartA thankfully a ***Comparison of Puck's and Cuntze's failure theories,*** *considering the first and more complicate version of Cuntze's FMC-based SFC. His there used 'addition theorems' to combine invariant scalar formulation with Puck's – Mohr's vector formulation later enabled Cuntze to replace fictitious friction parameters of his scalar SFC model by 2 directly measurable friction values, good to estimate input values.*

* From 1992-1997, investigation of the '*Hashin-Puck Action Plane Strength Criterion*' (*see VDI Fortschrittbericht 1997, project leader R. Cuntze, MAN Technologie*).

* Since 1993, in parallel the elaboration of the FMC began.

This is based on von Mises invariant idea, who describes by his criterion (just) 1 failure mode, namely yielding. As describing function an isotropic invariant J_2 he used. It should be possible to transfer this idea from the yield mode of ductile isotropic materials to fracture modes of brittle materials. Of course, the invariants (which reflect material symmetry) to be applied for the transversely–isotropic UD materials are different.

** *Note on the VDI-Guideline (1980-2006): Puck's Hashin-based SFC-model was at the finalization-time of the VDI 2014,sheet 3, the best validated SFC and therefore Puck was kindly invited to include his SFC into the Guideline!*

Primary Objective in *Structural Design Verification* of the Structural Part
is a Reserve Factor *RF* > 1 against a Limit State
in order to achieve Certification for the Production

For each designed structural part it is to compute

for each distinct 'Load Case' with its various Failure Modes

Reserve **F**actor (load-defined) **:** ***RF = Failure Load / applied Design Load***

Material Reserve factor : f_{RF} ***= Strength / Applied Stress***

if linear analysis: $f_{RF} = RF = 1 / Eff$

Material Stressing Effort *: $Eff = \sigma / R = 100\%$ ***if*** $RF = 1$

** in German: Werkstoff-Anstrengung, a very expressive term.*

** equivalent artificial English term , being created in 2003 together with QinetiQ as organizer of the World-Wide-Failure-Exercises on UD-SFCs.*

stress exposure:	*körperliche Stressbelastung und Beanspruchungslimit (Belastbarkeit).*
material stressing effort:	*Werkstoffanstrengung, Beanspruchungsniveau*
effort:	*Anstrengung.*

Advantageous Use of the Equivalent Stress-linked Material Stressing Effort

Brittle materials possess a set of fracture failure modes

≡ *'Modal'* material stressing effort *

$$Eff^{mode} = \sigma_{eq}^{mode} \,/\, \overline{R}^{\,mode}$$

equivalent mode stress

mode associated average strength

$$Eff^{fracture\ mode} = \sigma_{eq}^{fracture\ mode} / R_m$$

$$Eff^{Mises} = \sigma_{eq}^{Mises} / R_{p0.2}$$

Onset-of-Yielding $\sigma_{eq}^{Mises} = \sqrt{3J_2}$

$Eff \rightarrow 100\%$ means $\sigma \rightarrow R$

Advantages using *Eff* with 'Modal' SFCs

- *Eff* and σ_{eq} are always clearly to define and thereby fix the driving failure mode
- *Eff* is linearly and non-linearly applicable !
- Just *Eff* can multi-axial strength capacity simply explain !

***Eff* will be necessary for the interaction of the mode failure portions !**

„Which SFC-Types are used?" So-called 'Modal' and 'Global' (pauschal) SFCs

Cuntze's 'Play on Words'

All modes are married in the Global formulation.
Any change hits all mode domains NF and SF of the fracture body surface

Drucker-Prager, Ottosen, Willam-Warnke, Tsai-Wu, Altenbach/Bolchun/ Kulupaev, Yu , etc.

1 Global SFC :	$F(\{\sigma\},\{R\}) = 1$	global formulation, usually
Set of Modal SFCs :	$F(\{\sigma\},\{R^{\text{mode}}\}) = 1$	model formulation in the FMC

Mises,Puck,Cuntze

All modes are separately formulated.
Any change hits only the relevant domain of the fracture body surface

$F(\{\sigma\},\{R^{\text{mode}};\ \mu^{\text{mode}}\}) = 1$ more precise formulation **Novel**

by direct introduction of the friction value
considering Mohr-Coulomb for brittle materials under compression

$$UD: \quad \{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T, \quad \{\bar{R}\} = (\bar{R}_{\parallel}^t, \bar{R}_{\parallel}^c, \bar{R}_{\perp}^t, \bar{R}_{\perp}^c, \bar{R}_{\perp\parallel};\ \mu_{\perp\parallel}, \mu_{\perp\perp})^T$$

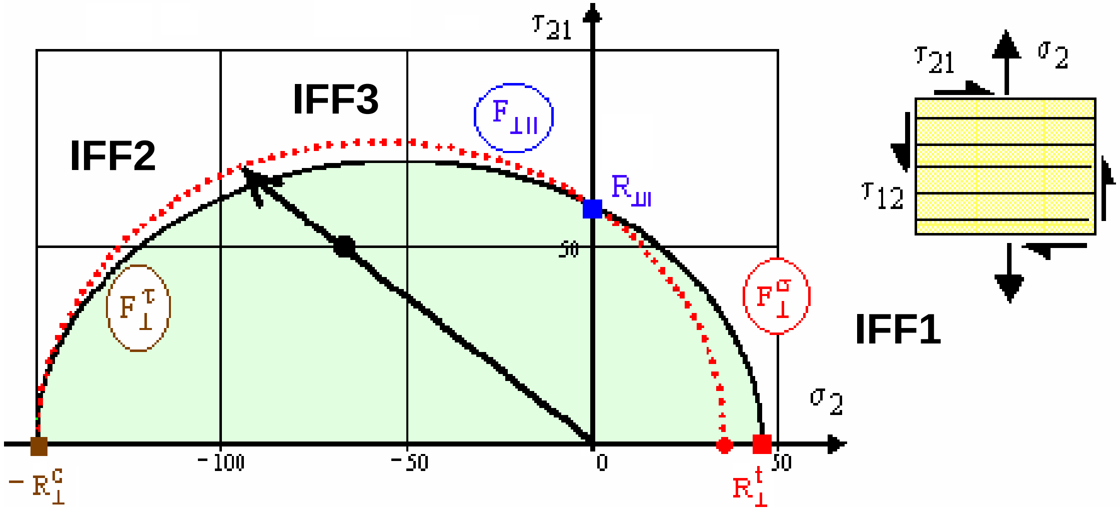
$$Isotrop: \{\sigma\} = (\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{zx}, \tau_{xy})^T = (\sigma_I, \sigma_{II}, \sigma_{III})^T, \quad \{\bar{R}\} = (\bar{R}^t, \bar{R}^c;\ \mu)^T$$

Consequences of mathematically forcibly-married 'Global' SFCs

'Global':

A change in one failure domain affects a physically fully independent other mode!

Here, a change in IFF1 affects IFF3 and IFF2 !!



ZTL: global

$$\frac{\sigma_2^{\,2}}{\overline{R}_\perp^t \cdot \overline{R}_\perp^c} + \sigma_2 \cdot \left(\frac{1}{\overline{R}_\perp^t} - \frac{1}{\overline{R}_\perp^c}\right) + \frac{\tau_{21}^{\,2}}{\overline{R}_{\perp\parallel}} = 1$$

FMC: modal

$$\left(\frac{\sigma_2}{\overline{R}_\perp^t}\right)^m + \left(\frac{-\sigma_2}{\overline{R}_\perp^c}\right)^m + \left(\frac{|\tau_{21}|}{\overline{R}_{\perp\parallel} - \mu_{\perp\parallel} \cdot \sigma_2)}\right)^m = 1$$

'Modal':

The figure visualizes for a distinct global SFC, termed ZTL (Zukunft Technik Luftfahrt), still used in the Airbus-linked structural HSB Handbook, how dramatically a change of the tensile strength affects the failure curve in the compression domain, although no physical impact can be! In the figure the word initially refers to the originally ZTL-mapped curve and finally to the ZTL-mapped curve considering the reduced tensile strength.

A change in IFF1 affects just IFF1 and not the two other modes as a global SFC (red dotted) !!

Cuntze's FMC-based Generation of SFCs involves :

performed by the author analogously to **:**

➢ **failure mode-wise** *(shear yielding failure, etc.)*	**Mises, Hashin, Puck etc.**
➢ **stress invariant-based** *(J_2 etc.)* using *physical content of the distinct Invariant*	**Mises, Tsai, Hashin** (also), **Christensen, etc.**
➢ **use of material symmetry** demands	**Christensen**
➢ **application of equivalent stresses**	**Mises for shear yielding, Rankine for fracture**

How can the Driving Ideas be realized?
Details of the first 3 points above ▶

➢ Failure mode-wise based **Features of the FMC** (1995)

It could be found:

- **Each failure mode represents 1 independent failure mechanism**

  and thereby 1 piece of the complete *failure surface*

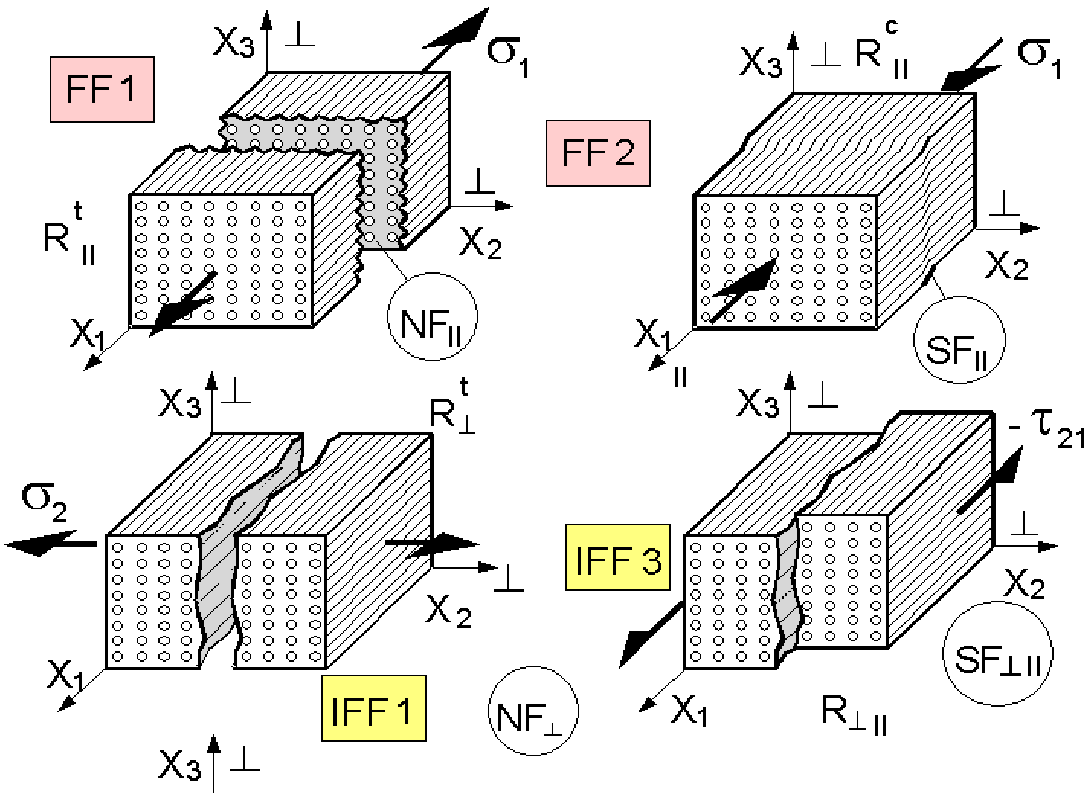
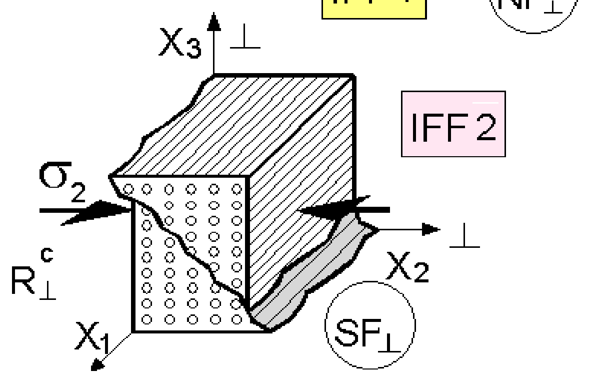
- **Each failure mechanism is governed by 1 basic strength**

Observation →

- **Each failure *mode* can be represented by 1 strength failure *criterion* *(SFC).***

  Therefore, equivalent stresses can be computed for each mode !!

Physical Observation: Which UD Strength Fracture Failure Modes are given ?

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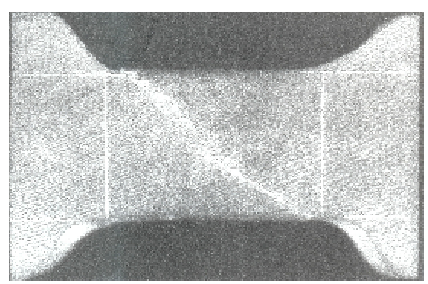


wedge failure type

t = tension
c = compression

▶ **5 Fracture modes exist**

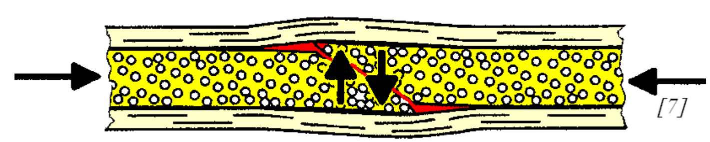
= 2 FF (Fibre Failure)
+ 3 IFF (Inter Fibre Failure)

Fracture Types:

NF := Normal Fracture

SF := Shear Fracture

dangerous IFF mode leading to delamination



➢ Stress Invariants-based

* Invariants remain unchanged under coordinate transformations

* Invariants (see Mises) can be dedicated to a physical mechanism of the deforming solid !

Following Beltrami, Mises (HMH) and Mohr-Coulomb (isotropic)

- volume change : I_1^2 … *(dilatational energy)* relevant if porous
- shape change : J_2 (Mises) … *(distortional energy)* relevant if material element shape changes
- friction : I_1 … *(friction energy)* relevant if brittle

*The well known **Isotropic invariants** are*

$$I_1 = \sigma_I + \sigma_{II} + \sigma_{III} = f(\sigma)\ ,\quad 6J_2 = (\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2 = f(\tau) \ \text{'Mises'}$$

*and for the **Transversely-isotropic** UD material (chosen by author) analogously to use*

- volume change : I_1^2 … *(dilatational energy)* relevant if porous
- shape change : J_2 ('Mises') … *(distortional energy)* relevant if ductile
- friction : I_1 … *(friction energy)* relevant if brittle

Mohr-Coulomb

*stress invariants: **isotropic materials** and **UD materials***

UD materials: I_1^2 , I_2^2 ; I_3 , I_4 ; I_2 [Böhler, 1985]

These I_1 are different !

$$I_1 = I_1^{UD} = \sigma_1,\ I_2 = \sigma_2 + \sigma_3,\quad I_3 = \tau_{31}{}^2 + \tau_{21}{}^2,\quad I_4 = (\sigma_2 - \sigma_3)^2 + 4 \cdot \tau_{23}{}^2,$$

$$I_5 = (\sigma_2 - \sigma_3) \cdot (\tau_{31}{}^2 - \tau_{21}{}^2) - 4 \cdot \tau_{23} \cdot \tau_{31} \cdot \tau_{21} \quad \text{(obtained from A. Boehler)}$$

➢ **Use of material symmetry demands** (‘generic number’ as novel idea)

There seems to exist (after intensive investigations of the author)
a ‘generic’ (term was chosen by the author) material inherent number
for the envisaged 3 Material Families:

Isotropic Material: 2

Transversely-Isotropic UD Material: 5

- 5 *elastic ‘constants’ E,v; 5 strengths R ; 5 strength failure modes* (NFs with SFs); *5 fracture mechanics modes K*

Orthotropic Material: 9

The existence of a ‘generic‘ number
***** **will significantly simplify the *Structural Mechanics Building***
***** **determines the *necessary Test Amount !***

and **Achieving** **above still mentioned** **Equivalent Stresses** σ_{eq} **!**

→ *This involves 2 aspects for the author:*

(1) σ_{eq} **captures the common action *Eff* (Werkstoffanstrengung)**

of a multi-axial stress state, active in a distinct failure mode

is equal to an <u>action</u>: a multi-axial stress state ***as in***

**** Mises*** σ_{eq} ***: ductile, Mode 'Shear stress <u>Yielding</u>',***

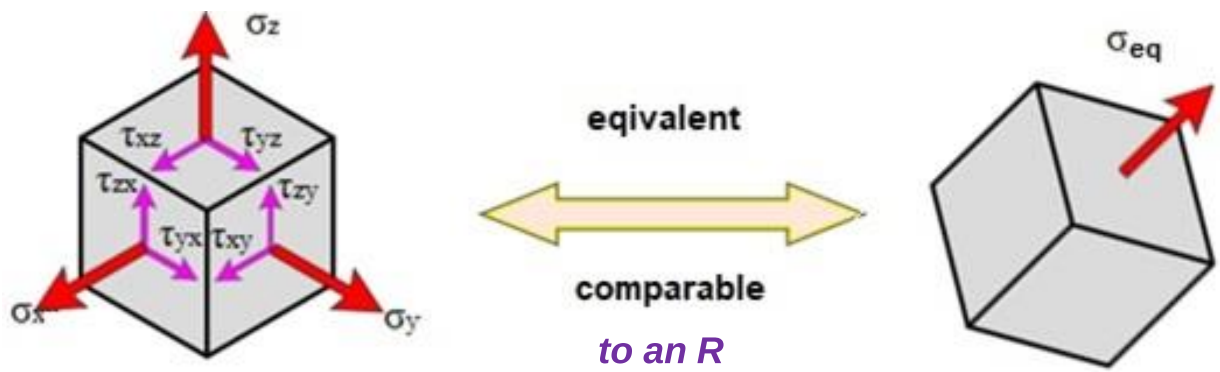
**** Maximum*** σ_{eq} ***: brittle, Mode 'Normal <u>Fracture'</u> etc.***

(2) The value of σ_{eq} **is**

comparable to a <u>resistance</u>: a strength value R

belonging to the activated failure mode.

Visualization for an isotropic material experiencing structural stresses:



Choice of Modal Concept → requires an interaction formula for the Modal SFC set

Multi-axial stress states usually activate more than one failure mode → interaction is to apply.

This **Interaction in the 'mode transition zones'** of

adjacent Failure Modes *is captured by a* ***series failure system*** **model**

= 'Accumulation' of interacting *failure danger portions* Eff^{mode}

$$Eff = \sqrt[m]{(Eff^{\text{mode 1}})^m + (Eff^{\text{mode 2}})^m + \ ....} = 1 = 100\,\%\ ,\ if\ \ failure$$

with a mode-interaction exponent *2.5 < m < 2.9* **,** *from a long mapping experience*

It is assumed engineering-like: *m* takes the same value for all mode transition zones !

Note:
Modal SFCs need an interaction of Failure Modes.
This is performed by a probabilistic approach (was a so-called *series failure system*)
in the transition zone between neighboring modes NF and SF.

Applying an interaction equation to consider all micro-damage causing portions of all activated modes makes to move from the absolute value of the Failure Function |*F*| to *Eff* !

For a simplified displaying 'isotropic' is taken:

* For a mathematically homogeneous Failure Function F using

$$Eff = \sigma / R \qquad \text{it reads}$$

$$F^{\text{Mises}}(\text{uniaxial}) = \sqrt{3J_2} / R = \sqrt{3 \cdot 2\sigma^2 / 6} / R = \frac{\sigma}{R} = Eff = 1 \quad \Rightarrow F \equiv Eff$$

* For a mathematically non-homogeneous F such as

$$F = c_1 \cdot \frac{\sigma^2}{R^2} + c_2 \cdot \frac{\sigma}{R} \quad \text{or} \quad F = c_1 \cdot Eff^2 + c_2 \cdot Eff \qquad \Rightarrow \text{F} \neq Eff.$$

Keep in mind, please:
The difference of the 'old' Failure Index (*FI*) and the material stressing effort *Eff* is essential, when not addressing a failure envelope, where *FI* =|F| = *Eff* = 100% = 1.

Assessing Multiple Stress states

- A SFC can only describe a <u>1-fold</u> occurring failure mode

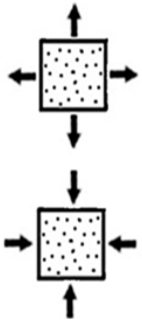
[figure Cuntze-Sukarie, issued in 1997]

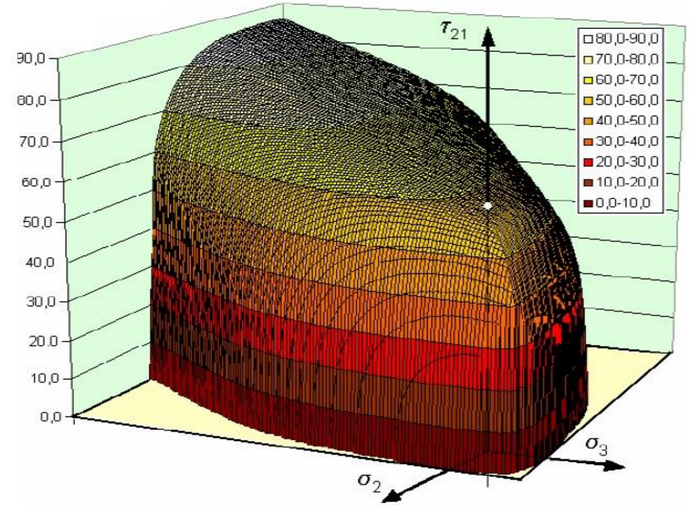
- **The different failure effect of σ_2 and σ_3 must be captured**
- **A multi-fold mode occurrence must be additionally considered in <u>all</u> available SFCs: *<u>2-fold</u>* :** $\sigma_2 = \sigma_3$

  = *probabilistic effect rounding the edge at* $\sigma_2 = \sigma_3$

  Cuntze does it with an additional term in his SFC

  Puck does it with a reduction $\{\sigma_{\mathbf{red}}\}$ *[Puc06. p. 183]*





- **What von Mises reached by the invariant description of the single failure mode yielding shall be performed for the set of 5 fracture failure modes faced with UD materials**
- **Applying Poisson ratio *v* directly in the SFC formula**
- **Enabling the necessary interaction of all activated single FF and IFF modes,** **when *'Modal'***
- **A UD Strength Failure Criterion should directly capture the fracture of the fiber, the matrix, fiber-matrix interface and of the delamination of a layer as a subpart of the laminate**
- **WWFE-II <u>requirements</u>:**

  - To consider that Poisson's ratio *v* may cause micro-mechanically axial tensile failure of the constituent filament under bi- axially compressive stressing without any external tension loading σ_1 (considered in Cuntze's FMC)

  - To capture weakening of the matrix under pressures > 200 bar. (effortfully considered in Cuntze's FMC-based Mathcad-program, however then was not interrogated by the organizer, who was familiar with this topic !)

Invariant Formulations of Cuntze's SFC set

$$FF1: \quad F_{\parallel}^{\sigma} = \frac{I_1}{\overline{R}_{\parallel}^{t}} \quad , \qquad FF2: \quad F_{\parallel}^{\tau} = \frac{-I_1}{\overline{R}_{\parallel}^{c}} \quad ,$$

$$IFF1: F_{\perp}^{\sigma} = \frac{I_2 + \sqrt{I_4}}{2\overline{R}_{\perp}^{t}}, \quad IFF2: F_{\perp}^{\tau} = a_{\perp\perp} \frac{I_2}{\overline{R}_{\perp}^{c}} + \frac{b_{\perp\perp}\sqrt{I_4}}{\overline{R}_{\perp}^{c}}, \quad IFF3: F_{\perp\parallel} = \frac{I_3^{3/2}}{\overline{R}_{\perp\parallel}^{\ 3}} + b_{\perp\parallel} \frac{I_2 \cdot I_3 - I_5}{\overline{R}_{\perp\parallel}^{\ 3}} \ ,$$

$$\text{with} \quad a_{\perp\perp}(\mu_{\perp\perp}), \ \mathrm{b}_{\perp\perp}(a_{\perp\perp}), \ \mathrm{b}_{\perp\parallel}(\mu_{\perp\parallel}) \quad \text{and}$$

$$I_1 = \sigma_1, \quad I_2 = \sigma_2 + \sigma_3, \quad I_3 = \tau_{31}^{\ 2} + \tau_{21}^{\ 2}, \quad I_4 = (\sigma_2 - \sigma_3)^2 + 4 \cdot \tau_{23}^{\ 2},$$

$$I_5 = (\sigma_2 - \sigma_3) \cdot (\tau_{31}^{\ 2} - \tau_{21}^{\ 2}) - 4\tau_{23}\tau_{31}\tau_{21},$$

$$I_2 \cdot I_3 - I_5 \ \rightarrow I_{23-5} = 2\sigma_2 \cdot \tau_{21}^{2} + 2\sigma_3 \cdot \tau_{31}^{2} + 4\tau_{23}\tau_{31}\tau_{21}.$$

(courtesy: obtained from Boehler, 1995)

The set after insertion of the respective stresses and the definitions of the mode equivalent stresses ⇨

Invariants can be formulated in structural stresses and in Mohr stresses as the author had to execute.

Cuntze's 3D SFC: 5 Mode Formulations for FF1, FF2 and IFF1, IFF2, IFF3

Invariant SFC-formulas now replaced by their stress formulations.

FF1: $Eff^{\|\sigma} = \sigma_1 / \bar{R}_{\|}^{t} = \sigma_{eq}^{\|\sigma} / \bar{R}_{\|}^{t}$

FF2: $Eff^{\|\tau} = -\sigma_1 / \bar{R}_{\|}^{c} = +\sigma_{eq}^{\|\tau} / \bar{R}_{\|}^{c}$

IFF1: $Eff^{\perp\sigma} = 0.5 \cdot [(\sigma_2 + \sigma_3) + \sqrt{\sigma_2^{\ 2} - 2\sigma_2 \cdot \sigma_3 + \sigma_3^{\ 2} + 4\tau_{23}^{\ 2}}\,] / \bar{R}_{\perp}^{t} = \sigma_{eq}^{\perp\sigma} / \bar{R}_{\perp}^{t}$

IFF2: $Eff^{\perp\tau} = [a_{\perp\perp} \cdot (\sigma_2 + \sigma_3) + a_{\perp\perp} + 1 \cdot \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^{\ 2}}\,] / \bar{R}_{\perp}^{c} = +\sigma_{eq}^{\perp\tau} / \bar{R}_{\perp}^{c}$

IFF3: $Eff^{\perp\|} = \{0.5 \cdot [b_{\perp\|} \cdot I_{23-5} + (\sqrt{b_{\perp\|}^{\ 2} \cdot I_{23-5}^{\ 2} + 4 \cdot \bar{R}_{\perp\|}^{\ 2} \cdot (\tau_{31}^2 + \tau_{21}^2)^2}\,] / \bar{R}_{\perp\|}^{\ 3}\}^{0.5} = \sigma_{eq}^{\perp\|} / \bar{R}_{\perp\|}$.

Insertion: Compressive strength point $(0, -\bar{R}_{\perp}^{c})$ with the effortfully derived friction μ relation

delivers $b_{\perp\perp} = a_{\perp\perp} + 1 \rightarrow 1/(1-\mu_{\perp\perp})$, $a_{\perp\perp} \cong \mu_{\perp\perp} / (1-\mu_{\perp\perp})$, $b_{\perp\|} \cong 2 \cdot \mu_{\perp\|}$.

From mapping experience obtained typical FRP-ranges: $0 < \mu_{\perp\|} < 0.3$, $0 < \mu_{\perp\perp} < 0.2$.

Two-fold failure danger in the σ_2-σ_3-domain (quadrant I) is modelled by (bi-axial fracture stress $\bar{R}_{\perp}^{tt}$)

$Eff_{\perp}^{\mathrm{MfFd}} = (\sigma_2^t + \sigma_3^t) / 2\bar{R}_{\perp}^{tt}$, and $\bar{R}_{\perp}^{tt} \approx \bar{R}_{\perp}^{t} / \sqrt[m]{2}$ after [*Awa*78] considering

$\sigma_2^t \cong \sigma_3^t$ with $\bar{R}_{\perp}^{tt} \le \bar{R}_{\perp}^{t}$ and $\sigma_2^c \cong \sigma_3^c$ with $\bar{R}_{\perp}^{cc} \ge \bar{R}_{\perp}^{c}$ (dense), $\bar{R}_{\perp}^{cc} \le \bar{R}_{\perp}^{c}$ (porous)

From mapping experience obtained typical range of interaction exponent $2.5 < m < 2.9$.

What is really required for the Pre-design using Cuntze's 3D UD SFCs ?

$F(\sigma, R, \mu) = 1$ Test Data Mapping Design Verification

(statistical mean to use, indicated by a bar over)

- 5 strengths : $\{\overline{R}\} = (\vec{R}_{||}^{t}, \overline{R}_{||}^{c}, \overline{R}_{\perp}^{t}, \overline{R}_{\perp}^{c}, \overline{R}_{\perp||})^T$ $\{R\} = (R_{||}^{t}, R_{||}^{c}, R_{\perp}^{t}, R_{\perp}^{c}, R_{\perp||})^T$

  average (typical) values strength design allowables

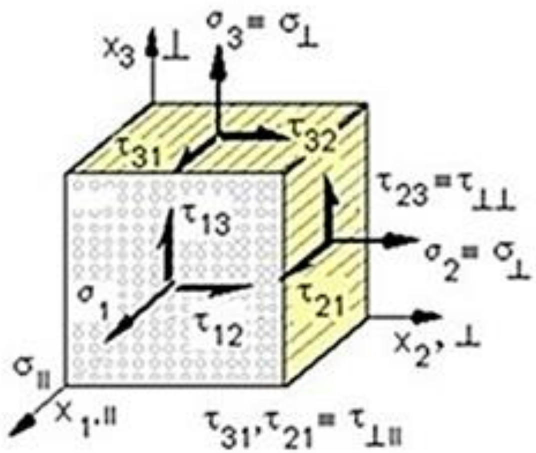
- 2 friction values : for 2D $\mu_{\perp||} = 0.15$ for 3D $\mu_{\perp\perp} = 0.2$

  friction values, recommended for pre-design

  for all mode transition zones taken

- 1 mode-interaction exponent : *m* = 2.6 .

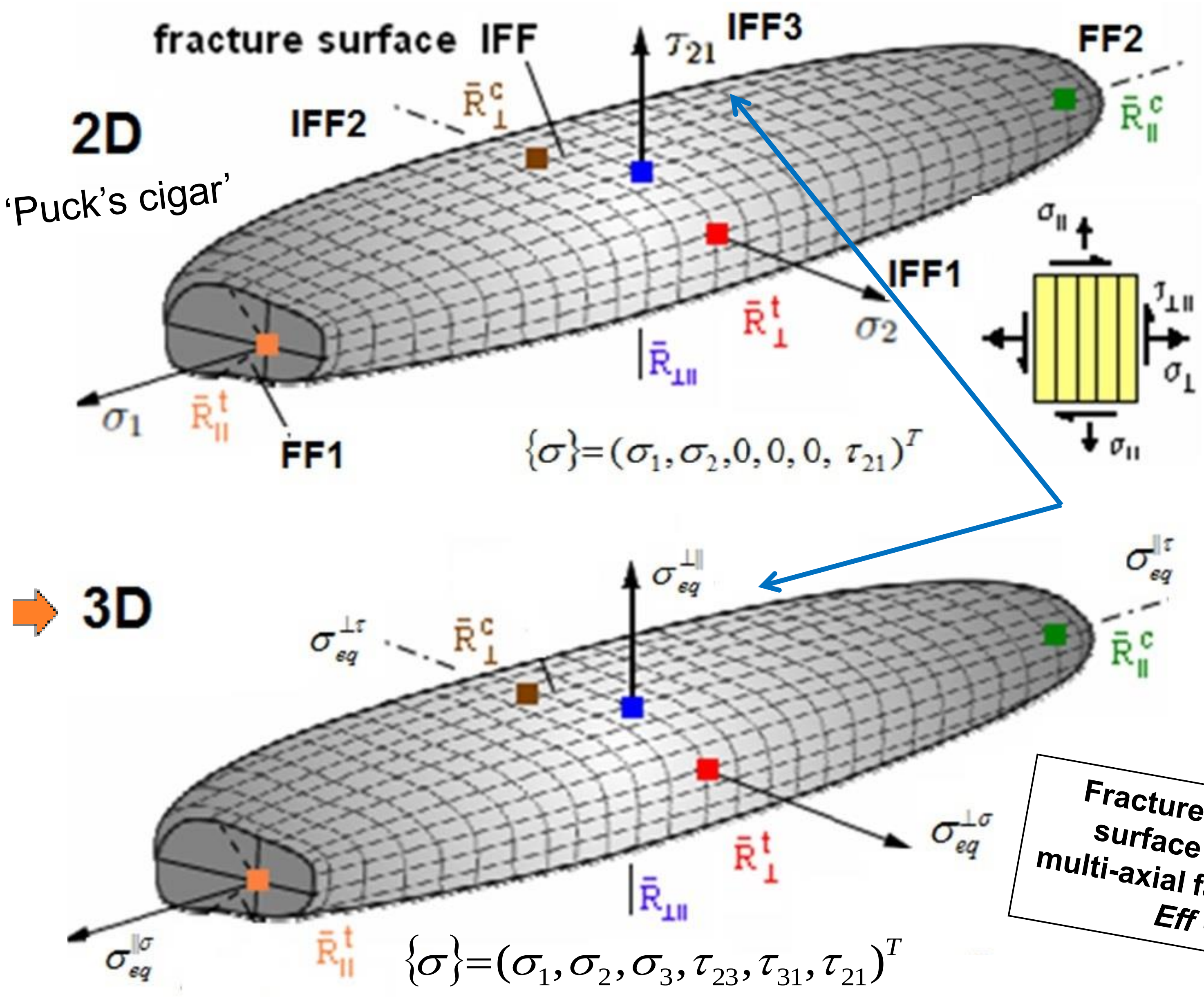
  recommended for pre-design



5 measurable parameters !
Not 75 Parameters
as **falsely** cited in the WWFE-II conclusions !
This looks like '***Damage to my reputation***',
but sorted out by which organizer person ??

Note : It reads $R_{\perp||}$, because τ_{21} is failure-responsible and not τ_{12} !

Eff = 1 : 2D and 3D Fracture Body (after Replacement of σ, τ by σ_{eq}^{mode})

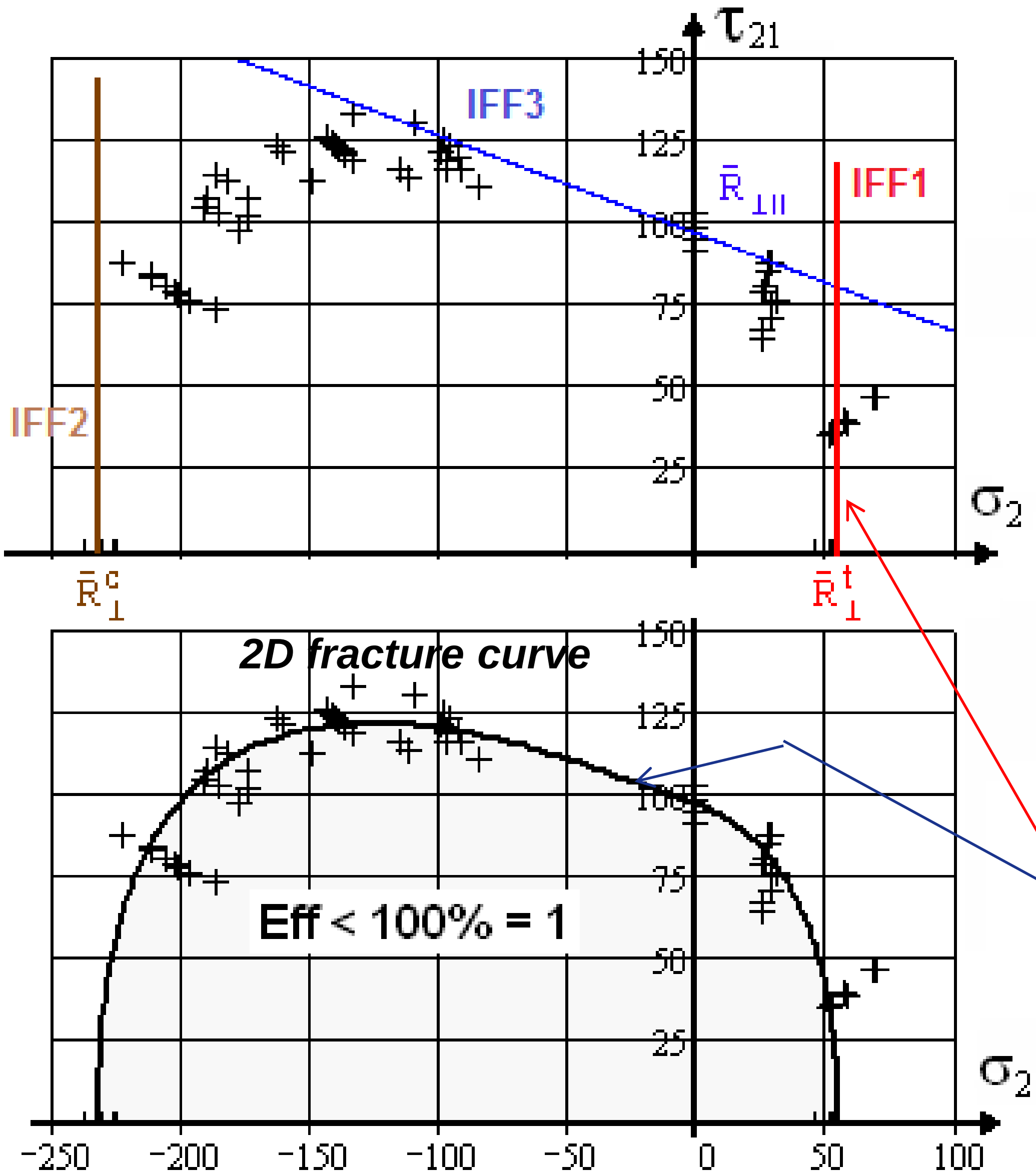


$\nu_{\perp\perp} = 0.4,$
$\nu_{\perp||} = 0.3$

Delamination onset predictable

Fracture Body = smoothed surface of the ends of the multi-axial failure stress vectors.
Eff = 1 = 100%.

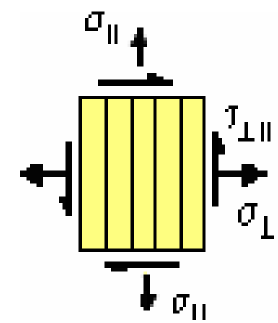
Interaction of UD Failure Modes = Outsmoothing in the Mode Transition Zones



$$\tau_{21}(\sigma_2) \text{ or } \{\sigma\}=(0,\sigma_2,0,0,0,\tau_{21})^T$$

Mapping of course of IFF test data in a pure mode domain by the *single Mode Failure Conditions.* *m = 2.7*

3 IFF pure modes = 3 piecewise straight lines !



Mapping of course of test data by *the Interaction Model*

$$(Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\|})^m = 1$$

$$\left(\frac{\sigma_2^{\,t}}{\bar{R}_2^t}\right), \quad \left(\frac{-\sigma_2^{\,c}}{\bar{R}_2^c}\right), \quad \left(\frac{|\tau_{12}|}{\bar{R}_{\perp\|} - \mu_{\perp\|}\cdot\sigma_2^c}\right)$$

in-plane Mohr-simplification (2004)

World-Wide-Failure-Exercises-I and –II on UD laminas (1991-2013)

‘*Mapping Round Robin Test*‘: Comparing theoretical UD-SFC-predictions with test results

Organizer : ***QinetiQ , UK*** *(Hinton, Kaddour, Soden, Smith, Shuguang Li)*

Aim: ***“Testing Predictive UD Failure Theories***

= SFC of the UD lamina material including programming, however

+ non-linearity treatment + programming of the structure **laminate**

of Fiber–Reinforced Polymer Composites to the full !“

Procedure of the WWFE-I (2D test data**) and WWFE-II** (**3D test data):**

Part A : ***Blind Predictions*** **with average strength data $\bar{R}$, only.**
(a necessary friction value information μ was not provided !)

Part B : ***Comparison Theory-Test*** **with Test data sets,** which were not applicable or even involved false failure points. More than 50% could not be initially used without specific care!

Author’s WWFE-contribution results :

Cuntze‘s invariant-based strength criteria mapped the provided *accurate* test data sets best. One third of the provided test data sets were finally not usable !

→The UD-examples serve for SFC Model Validation.

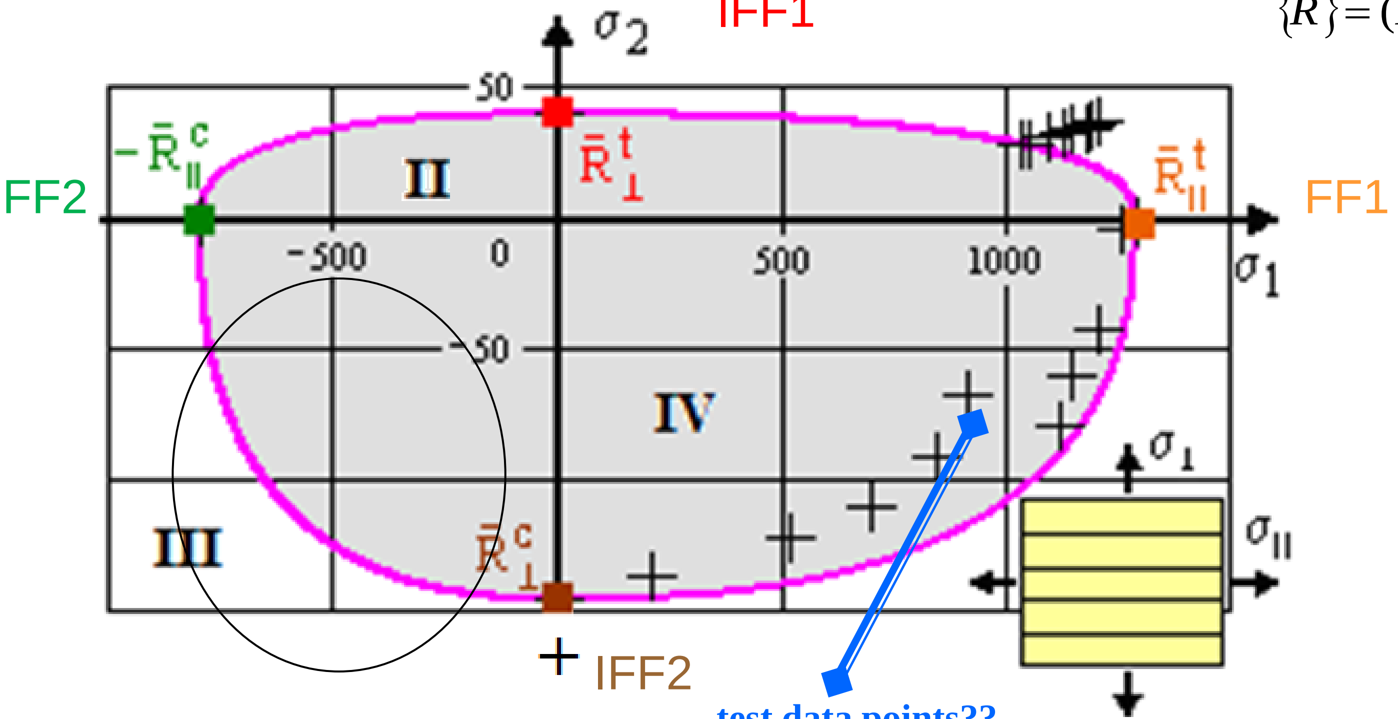
→ The WWFE-II laminate examples serve as Design Verification of the Laminate.

In WWFE-I the author was officially the winner *and*

In WWFE-II he ranked at the top !

1st Example: Mapping of UD WWFE-I,Test Case 3, data $\sigma_2(\sigma_1)$

$\{\bar{R}\} = (1280,\ 800,\ 40,\ 145,\ 73)^T$



Hoop wound tube
UD-lamina.
E-glass/MY750epoxy

$\sigma_1 = \sigma_{hoop}$

$\sigma_2 = \sigma_{axial}$

The author tried to ask the Russian originator whether the two different data sets might belong to different tests. Unfortunately he had passed way.

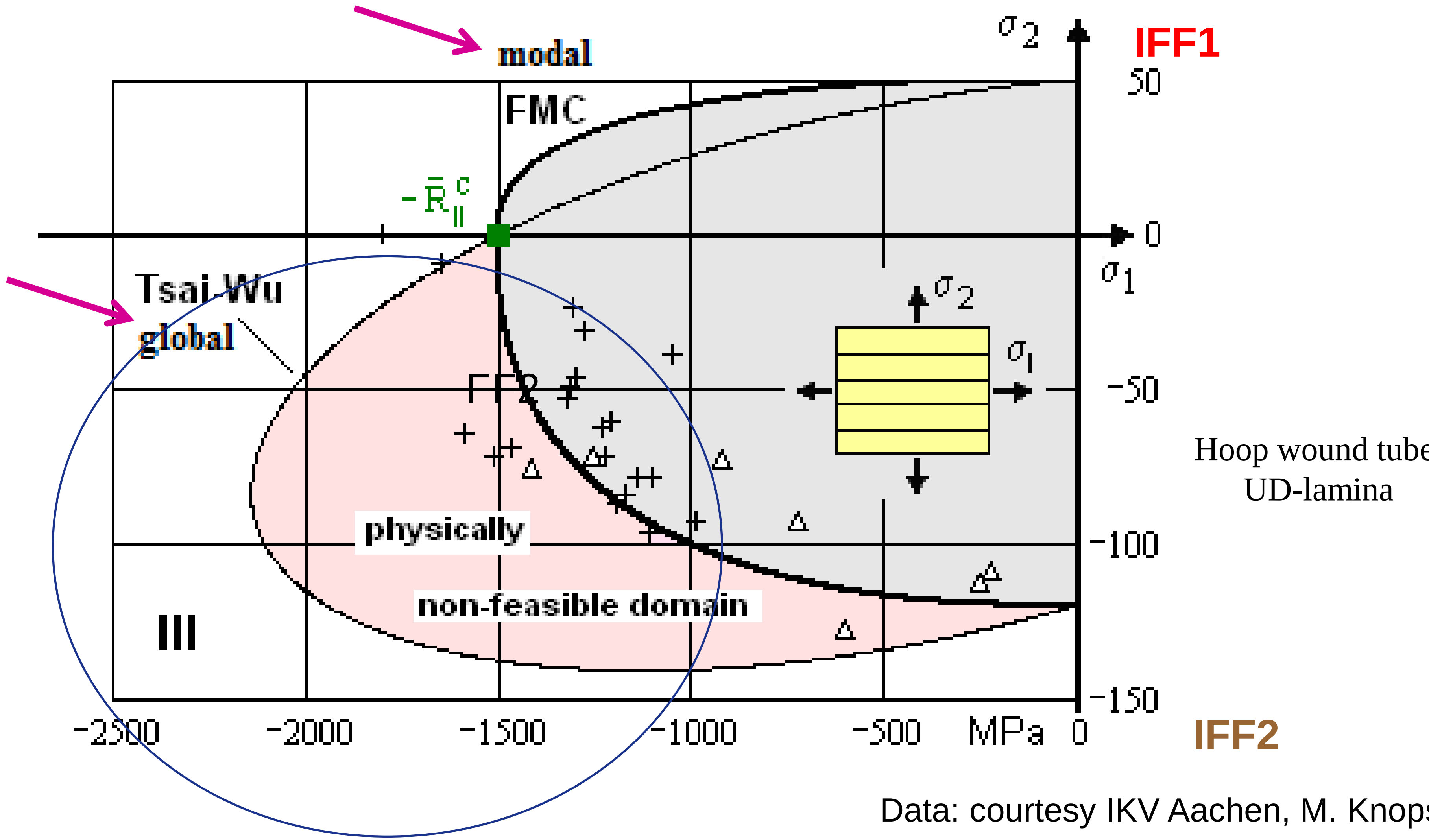
Part A: Data of 4 strength points were provided, only

Part B: Test data in quadrant IV show discrepancy, testing?

No data for quadrants II, III was provided !

Further Test Cases are assessed in [*CUN 22 Life compilation*]

Mapping in the 'Tsai-Wu non-feasible domain', quadrant III $\sigma_2(\sigma_1)$



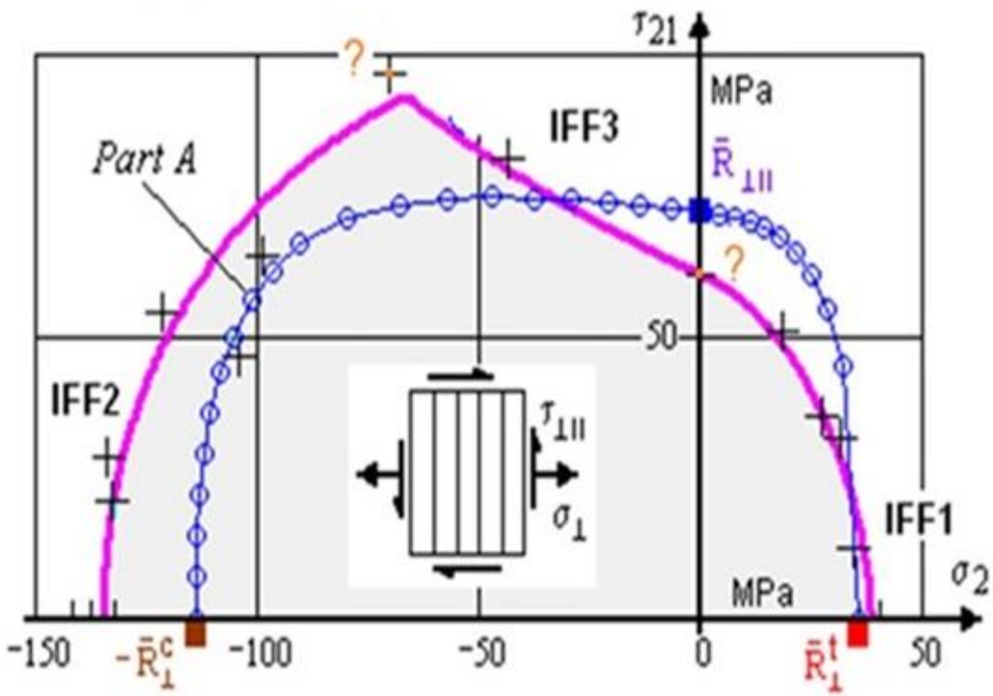
Data: courtesy IKV Aachen, M. Knops

<u>Lesson Learnt</u>:
The modal FMC maps correctly,
the fully global Tsai-Wu formulation predicts a non-feasible domain !

2nd Example WWFE-1, Test Case 1: $\tau_{21}(\sigma_2)$

Task: Determination of fracture failure curve, capturing IFF3, IFF1 and IFF2 by performing a so-called 'tension-compression-torsion tube test' campaign

originally given just 3 strengths



Tube, 90°-filament hoop wound

$\sigma_1 = \hat{\sigma}_y = \hat{\sigma}_{hoop} = p \cdot r_{int} / t$

$\sigma_2 = \hat{\sigma}_{ax} = p \cdot r_{int} / (2 \cdot t) + F_{ax} / (2\pi \cdot r_{int} \cdot t)$

$\tau_{21} = M_t$ /critical cross section area

*test evaluation based on un-deformed geometry

*monotonic loading performed in test

$\{\bar{R}\} = (1140,\ 570,\ 35,\ 135,\ 72)^T$ PartB-change

$b_{\perp\parallel} = 0.13;\ \mu_{\perp\parallel} = 0.25 \quad m = 3.1$

$\mu_{\perp\parallel} = 0.6$ compared to usual

$\Delta T = -68°C$ (after curing)

Part A, prediction: Data of elasticity properties, UD strengths $\bar{R}$ provided, only. No friction value μ for IFF3 slope provided! Lowest value for μ in Part A assumed (safe side)

Part B:*the 3 strength points were altered!*Two doubtful (?) failure stress points were provided. Nevertheless, QinetiQ asked to map the course of PartB-test data (magenta curve) despite of the fact that a Poissons ratio $\mu_{\parallel\perp}$ had to be taken which is much too large, compared to values of own and other test campaigns !

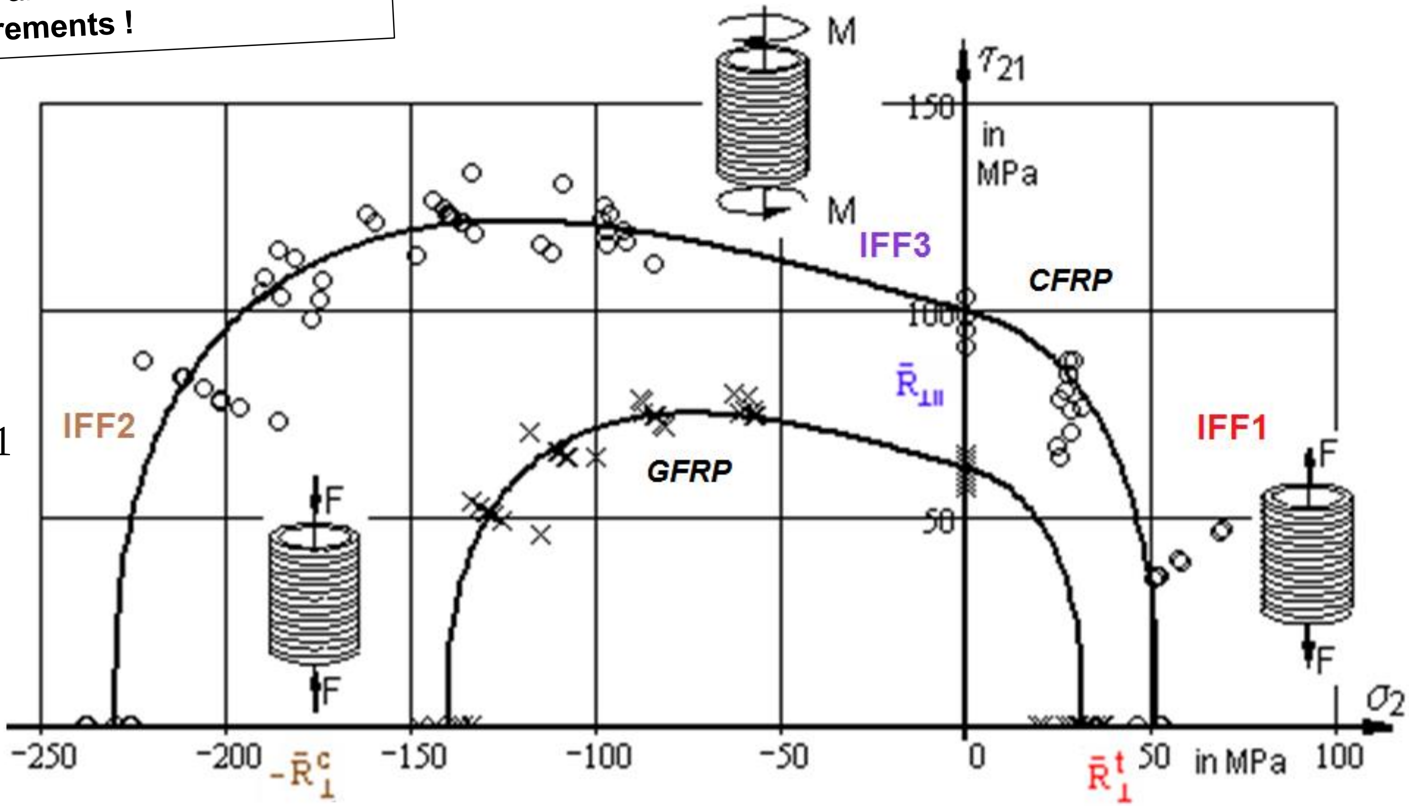
* *The author knew one originator (R. Aoki) of the experiments at the DLR Stuttgart and asked him whether the question-mark-indicated, unusually high 2 test points might stem from a false evaluation. Unfortunately, a check of the data base was not possible anymore. With the provided PartA inforation no reasonable Part A-mapping was possible!*

IFF Cross–section $\tau_{21}(\sigma_2)$ of the Fracture Failure Body

To prove Cuntze UD SFC model-validation a mapping of test data CFRP and GFRP.
Own measurements !

The respective examples in WWFE-I and-II include false test input !! ►►►

$$Eff = \left(\frac{\sigma_2^{\ t}}{\bar{R}_2^t}\right)^m + \left(\frac{-\sigma_2^{\ c}}{\bar{R}_2^c}\right)^m + \left(\frac{|\tau_{12}|}{\bar{R}_{\perp||} - \mu_{\perp||} \cdot \sigma_2^c}\right)^m = 1$$



"Test results can be far away from the reality like an inaccurate theoretical model, like the 'global' SFC, partly.

Theory creates a model of the reality, one experiment shows one realization of the reality".

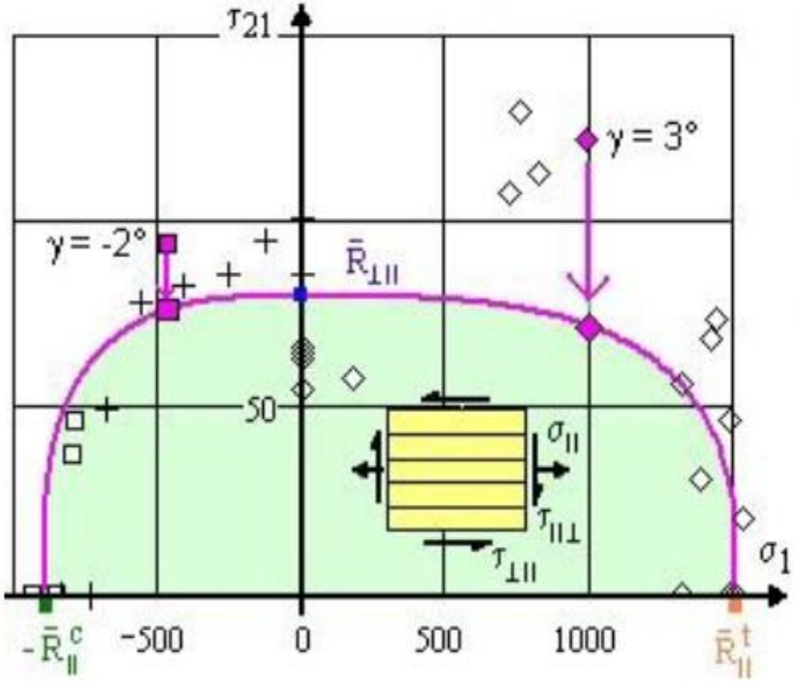
<u>Mind</u>: Being an *embedded* layer reduces the scatter in comparison to the measured technical strength results from the standard *isolated* specimens.

3rd Example **WWFE-I,** Test Case 2: CFRP, T300/BSL914C Ep $\tau_{21}(\sigma_1)$

Provision of a mixture of 90°-tube test data and not accurately evaluated 0°-tube data.

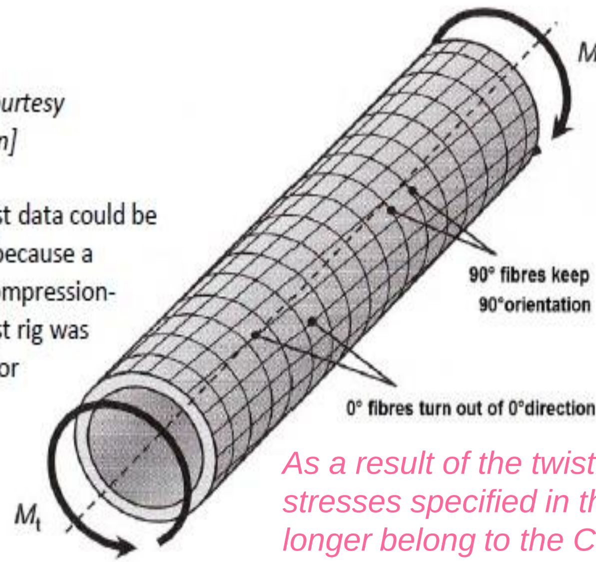
The author corrected the 0°-tube data such that these could be also advantageuosly used together with the 0°-ones !

Task: Determination of IFF curve, capturing IFF3, FF1 and FF2



[Figure: courtesy IKV Aachen]

Bi-axial test data could be obtained because a tension-compression-torsion test rig was available for tubes



just a copy of the author's contribution

As a result of the twisting, the fracture stresses specified in the 0° tube test no longer belong to the CoS of the ply!

Tube (90°-hoop wound) $\sigma_1 = \sigma_{hoop}, \sigma_2 = \sigma_{axial}$,

$\{\bar{R}\} = (1500,\ 900,\ 27,\ 200,\ 80)^T$,

Part B: $b_{\perp\parallel} = 0.13;\ m = 3,\ \Delta T = -125°C$ (after curing)

Part A, prediction: Strength data only provided, no friction value (slope) μ.

Part B, comparison: Strength data sets were provided, partly from 0°-test specimens (axial fiber direction) and partly from the traditional 90°- tube test specimen! After transformation, the two chosen ■◆ , by executing a *non-linearly CLT*-computed shear strain analysis, these two 0°-points exemplarily could be shifted onto the magenta envelope. The shear strength point (blue) had to be adjusted according to new B information.

- Data from 0°- tube test specimens ◊ cannot be used like hoop-wound 90°-tube test specimens, physically non-sense.
- The coordinate system of the 0°-test specimen lamina twists under torsion by γ. Hence, 0°-test data must be transferred into the twisted material coordinate system.

▶ TC2 test data set could serve for material model validation after correction by the author and choosing just 90°- wound tubes.

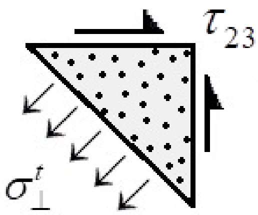
Accurate 3D-analyses <u>require</u> *to Unlock* the '*Mystery*' behind

Quasi-isotropic UD-plane

Tsai , Hashin/Puck : $S_{23} = R_{23} = \tau^{*}_{23}$; $R^{A}_{\perp\perp}$ (better R^{A}_{23} due to not being a measurable property)

In future we will have to analyze 3D stress states !

- The strength quantity R_{23} is 'formally' linked to the associated ply stress τ_{23}



$$max\,\tau_{23} = max\,\sigma^t_\perp = R_{23} = R^t_\perp$$

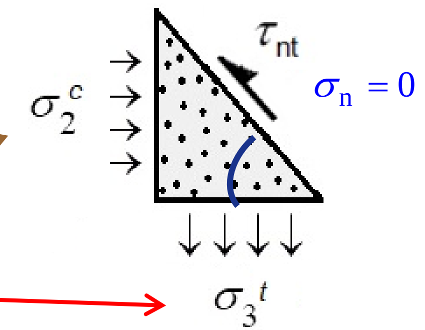
The following transfer relationship connects between the structural ply stresses and the associated Mohr stresses ($_{fp}$ = failure plane)

$$\begin{Bmatrix} \sigma_n(\theta_{fp}) \\ \tau_n(\theta_{fp}) \\ \tau_{n\ell}(\theta_{fp}) \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc & 0 & 0 \\ -sc & sc & c^2 - s^2 & 0 & 0 \\ 0 & 0 & 0 & s & c \end{bmatrix} \cdot \begin{Bmatrix} \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{21} \end{Bmatrix}, \quad c = \cos\theta_{fp} \ \text{ and } \ s = \sin\theta_{fp} \, .$$

the determination of the failure plane angle is now the challenge

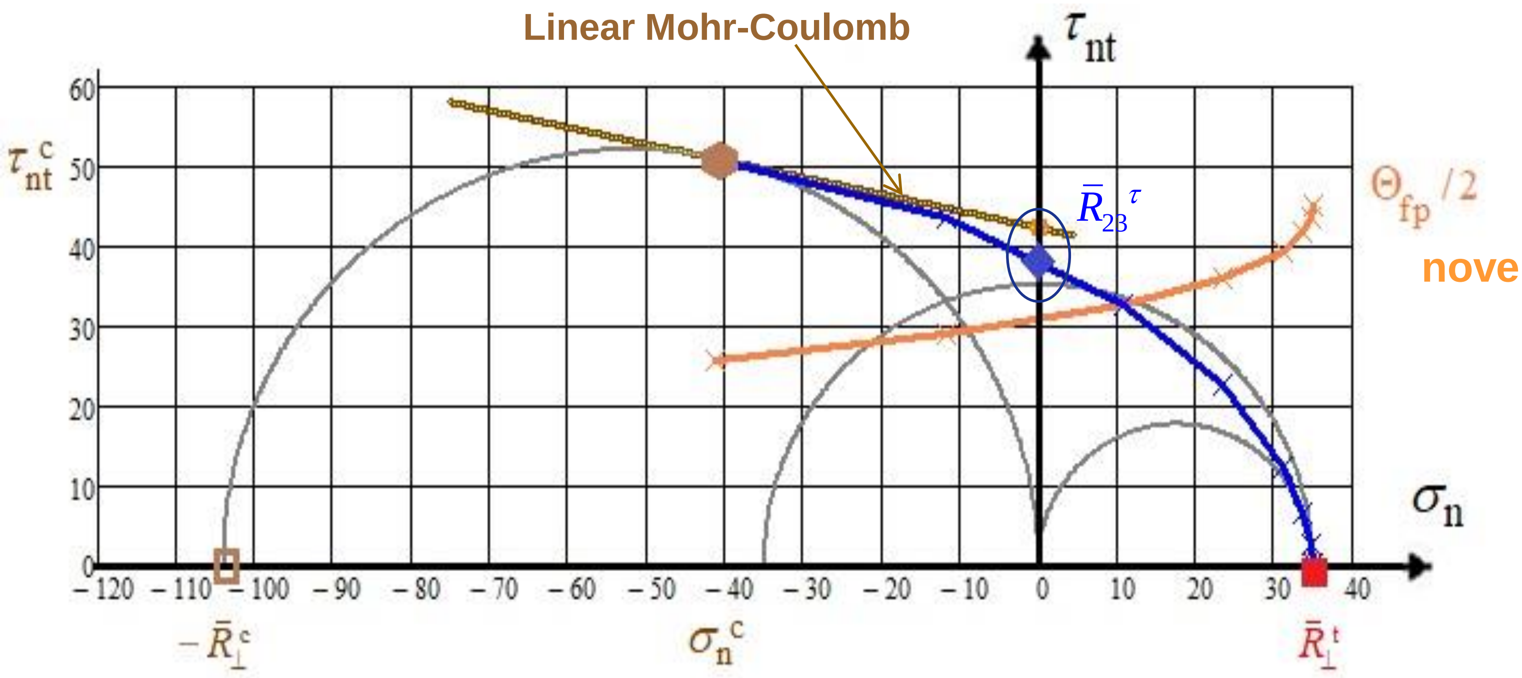
- The strength quantity Cohesive shear Strength defined by $\tau^{\text{fail}}_{\text{nt}}(\sigma_n = 0) = R_{23}^{\ \tau}$

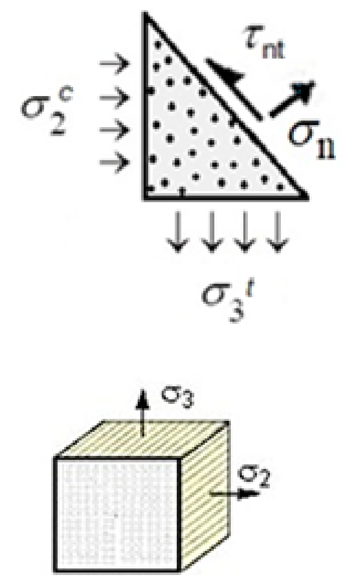
belongs to a point in the <u>transition zone b</u>etween IFF2 and IFF1.



Quasi-isotropic plane: Mohr Stresses

Display of two Cohesive Strength Determinations with fracture angle growth [Cun23a]





Extrapolation using just the SF (IFF2). **Accurate derivation using SF and NF (IFF!)**

Usual determination of a cohesive strength value (in the *transition zone between IFF2 and IFF1*) means an extrapolation from the compression strength point in the mode domain IFF2.
Puck's '*Action plane resistance'* corresponds to a quantity termed '*Cohesive Shear Strength*'
Such a quantity is linked to the chosen linear or non-linear 'Mohr model' !

Mohr Half Circles included for a better visualization

$\bar{R}_{\perp}^{t} = 35$ MPa, $\bar{R}_{\perp}^{c} = 104$ MPa, $\Theta_{fp}^{c}{}^{\circ} = 51°$, $C_{fp}^{c} = -0.21, \mu_{\perp\perp} \cong 0.21$, $\tau_{nt}^{c} = 50.9$ MPa, $\sigma_n^{c} = -41.3$ MPa, (2 modes)

* Linear extrapolation: $\bar{R}_{23}^{\tau} = 42$ MPa, $C = C_{fp}^{c} = const = -0.21$, $\Theta_{fp}{}^{\circ} = \Theta_{fp}^{c}{}^{\circ} = 51°$, $\sigma_2 = -52$ MPa, $\sigma_3 = 34$ MPa

* Improved by f_{corr}: $\bar{R}_{23}^{\tau} = 37.5$ MPa, $C = C_{fp}^{\tau} = -0.59$, $\Theta_{fp}{}^{\circ} = 63°$, $\sigma_2 = -73$ MPa, $\sigma_3 = 19$ MPa.

Using the Mohr stress-Layer stress relations $\sigma_n = 0.5 \cdot \left[\sigma_2 \cdot (C+1) + \sigma_3 \cdot (1-C)\right]$, $\tau_{nt} = -0.5 \cdot (1-C)^2 \cdot (\sigma_2 - \sigma_3)$.

Hashin's four invariant-based 3D-SFCs Set (4 modes, shear mode IFF3 is missing)

$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^{\mathrm{T}}$, $\{\bar{R}\} = (\bar{R}_{\|}^{t}, \bar{R}_{\|}^{c}, \bar{R}_{\perp}^{t}, \bar{R}_{\perp}^{c}, \bar{R}_{\perp\|}; \bar{R}_{23})^{\mathrm{T}}$; 6 strengths principally employed

Hypothesis 1: $F\left(\{\sigma^{A}\}, \{\bar{R}^{A}\}, \theta_{\mathrm{fp}}\right) = 1$, Puck's way $\Leftrightarrow$ Hypothesis 2: $F\left(\{\sigma\}, \{\bar{R}\}\right) = 1$, Cuntze's way

in Mohr stresses on the Action plane θ, failure if $\theta = \theta_{fp}$ $\Leftrightarrow$ use of invariants

Hashin does not formulate the (shear) IFF3,

however considers τ in the FF1-model !

Hyp. 1 : *"In the event that a failure plane under a distinct fracture angle can be identified, the failure is produced by the normal and shear stresses on that plane".*

Hashin herewith proposed a modified Mohr-Coulomb IFF approach but did not pursue this idea due to numerical difficulties (A. Puck succeeded on the Hypothesis 1 way).

Hyp.2: *"For UD – material the SFCs should be invariant under any rotation around the fiber direction."*

Hashin used the 5 stress invariants $I_1 = \sigma_1$, $I_2 = \sigma_2 + \sigma_3$, $I_3 = \tau_{31}^{2} + \tau_{21}^{2}$, $I_4^{Has} = \tau_{23}^{2} - \sigma_2 \cdot \sigma_3 \neq I_4^{\mathrm{Boehler}}$,

$I_5^{\mathrm{Hashin}} = 4\tau_{23}\tau_{31}\tau_{21} - \sigma_2 \cdot \tau_{31}^{2} - \sigma_3 \cdot \tau_{21}^{2}$. Invariants now replaced in the generated SFCs by their stress relationships

3D-SFCs:

FF1, $\sigma_1 > 0$: $\left(\dfrac{\sigma_1}{\bar{R}_{\|}^{t}}\right)^2 + \dfrac{\tau_{31}^2 + \tau_{21}^2}{\bar{R}_{\perp\|}^{2}}$ directly interacted ! ; FF2, $\sigma_1 < 0$: $\left(\dfrac{-\sigma_1}{\bar{R}_{\|}^{c}}\right)^2 = 1$,

IFF1, $\sigma_2 + \sigma_3 > 0$: $\dfrac{(\sigma_2 + \sigma_3)^2}{\bar{R}_{\perp}^{t\,2}} + \dfrac{(\tau_{23}^{2} - \sigma_2 \cdot \sigma_3)}{\bar{R}_{23}^{2}} + \dfrac{(\tau_{31}^{2} + \tau_{21}^{2})}{\bar{R}_{\perp\|}^{2}} = 1$, $\left(\bar{R}_{23} \equiv \bar{R}_{\perp\perp}\text{ sometimes}\right)$

IFF2, $\sigma_2 + \sigma_3 < 0$: $\left(\dfrac{\bar{R}_{\perp}^{c\,2}}{4 \cdot \bar{R}_{\perp\perp}^{2}} - 1\right) \cdot \dfrac{(\sigma_2 + \sigma_3)}{\bar{R}_{\perp}^{c}} + \dfrac{(\sigma_2 + \sigma_3)^2}{4 \cdot \bar{R}_{23}^{2}} + \dfrac{(\tau_{23}^{2} - \sigma_2 \cdot \sigma_3)}{\bar{R}_{23}^{2}} + \dfrac{(\tau_{31}^{2} + \tau_{21}^{2})}{\bar{R}_{\perp\|}^{2}} = 1$,

'Formalistically' used as τ23/R23. Hence, it is not a 'Mohr formulation' as also proposed by Hashin and then realized by Puck!

Interlaminar failure: $\sigma_3 > 0$: $\left(\dfrac{\sigma_3}{R_3^{t}}\right)^2 = 1$; $\sigma_3 < 0$: $\left(\dfrac{-\sigma_3}{R_3^{c}}\right)^2 = 1$.

► Hashin's four 2D-SFCs Set (*shear mode IFF3 is missing*)

2D – SFC : Equations without the suffix $_3$ remain. ⇒ 5 strengths + R_{23}, not real definition found

Questions : (1) The strength $\bar{R}_{\perp\perp}$ is seen to equal the failure shear stress of τ_{23}. No further detail.

(2) How does the mandatory interaction of the 4 modes to determine FPF read ??

$$\text{FF1, } \sigma_1 > 0: \left(\frac{\sigma_1}{\bar{R}_{\|}^{t}}\right)^2 + \left(\frac{\tau_{21}}{\bar{R}_{\perp\|}}\right)^2; \qquad \text{FF2, } \sigma_1 < 0: \left(\frac{-\sigma_1}{\bar{R}_{\|}^{c}}\right)^2 = 1,$$

$$\text{IFF1, } \sigma_2 > 0: \frac{(\sigma_2)^2}{\bar{R}_{\perp}^{t\,2}} + \frac{(\tau_{21}^2)}{\bar{R}_{\perp\|}^{\;2}} = 1, \qquad \text{IFF2, } \sigma_2 < 0: \left(\frac{\bar{R}_{\perp}^{c\,2}}{4\cdot \bar{R}_{23}^{\;2}} - 1\right)\cdot\frac{(\sigma_2)}{\bar{R}_{\perp}^{c}} + \frac{(\sigma_2)^2}{4\cdot \bar{R}_{23}^{\;2}} + \frac{(\tau_{21}^2)}{\bar{R}_{\perp\|}^{\;2}} = 1,$$

with a recommendation to use for $\bar{R}_{23}$ the value from Christensen's formula below

$$\text{Strength Ratio } SR = R_{\perp}^{c} \,/\, R_{\perp}^{t} \rightarrow R_{23} = R_{\perp}^{c}\cdot\sqrt{\frac{1+1/SR}{3+5/SR}\cdot\frac{1}{SR}}\;.$$

In order to remain compatible in the SFC-comparison the interaction of the two IFF- with the two FF-modes, will be performed like with Cuntze, by applying '*proportional stressing*'..

$$Eff^{\,m} = (Eff^{\,\|\tau})^m + (Eff^{\,\|\sigma})^m + (Eff^{\,\perp\sigma})^m + (Eff^{\,\perp\tau})^m + (0)^m = 1 \qquad \text{with}$$

$$\text{with } Eff^{\,\|\sigma} = \sqrt{\left(\frac{\sigma_1}{\bar{R}_{\|}^{t}}\right)^2 + \left(\frac{\tau_{21}}{\bar{R}_{\perp\|}}\right)^2}\;, \qquad Eff^{\,\|\tau} = \sqrt{\left(\frac{-\sigma_1}{\bar{R}_{\|}^{c}}\right)^2}$$

$$Eff^{\,\perp\sigma} = \sqrt{\frac{(\sigma_2)^2}{\bar{R}_{\perp}^{t\,2}} + \frac{(\tau_{21}^2)}{\bar{R}_{\perp\|}^{\;2}}}\;, \qquad Eff^{\,\perp\tau} = F = \left(\frac{\bar{R}_{\perp}^{c\,2}}{4\cdot \bar{R}_{23}^{\;2}} - 1\right)\cdot\frac{(\sigma_2)}{\bar{R}_{\perp}^{c}} + \frac{(\sigma_2)^2}{4\cdot \bar{R}_{23}^{\;2}} + \frac{(\tau_{21}^2)}{\bar{R}_{\perp\|}^{\;2}}$$

FF1 and FF2 : Taken are the usual *Maximum Stress Criteria*

IFF: Due to the Mohr IFF approach, two different eqations are provided [*Puc* 96, *p*.118] ,
using Mohr's 'Action plane' stresses and 'Action plane' resistances, superscript A.

In the tensile σ_n -domain the IFF formulations read: [*Puc*96, *p*.143]

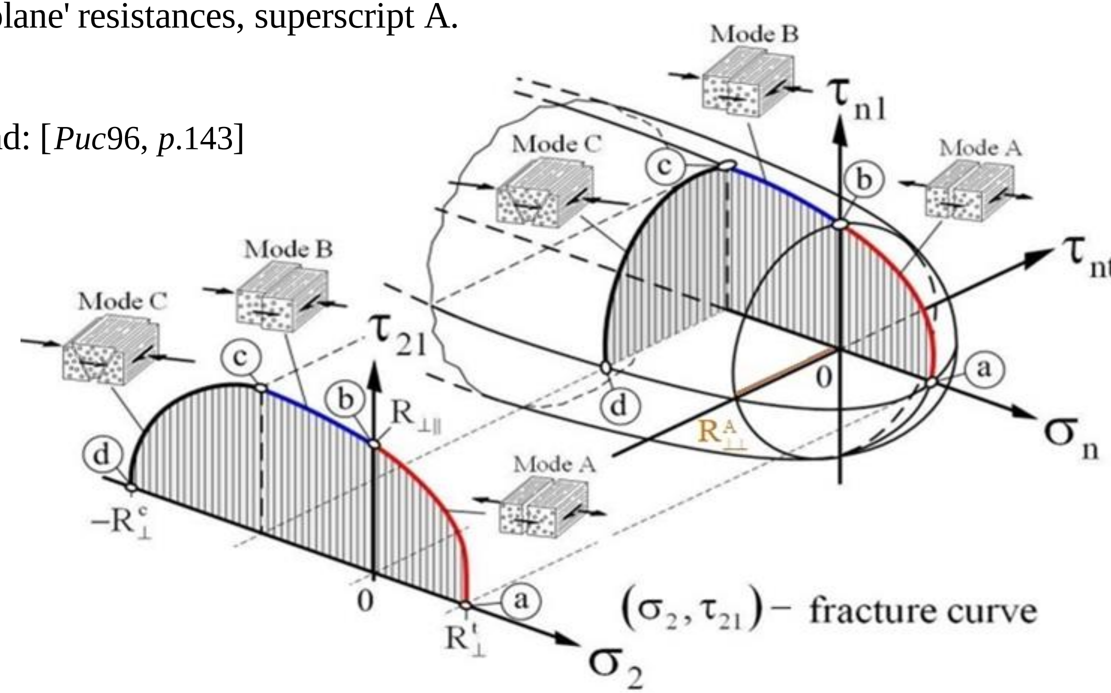
IFF: $\sigma_n > 0$:

$$\left(\frac{\tau_{nt}}{\bar{R}^A_{\perp\perp}}\right)^2 + \left(\frac{\tau_{n1}}{\bar{R}^A_{\perp||}}\right)^2 + \left(\frac{\sigma_n}{\bar{R}^A_{\perp}}\right) = 1,$$

In the compressive σ_n -domain friction is to consider

IFF: $\sigma_n < 0$:

$$\left(\frac{\tau_{nt}}{\bar{R}^A_{\perp\perp} - p_{\perp\perp}{}^c \cdot \sigma_n}\right)^2 + \left(\frac{\tau_{n1}}{\bar{R}^A_{\perp||} - p_{\perp||}{}^c \cdot \sigma_n}\right)^2 + \left(\frac{\sigma_n}{\infty}\right) = 1,$$



Master fracture body with Puck's IFF modes
and action plane stresses (σ_n, τ_{nt}, τ_{n1}).

[courtesy H. Schürmann]

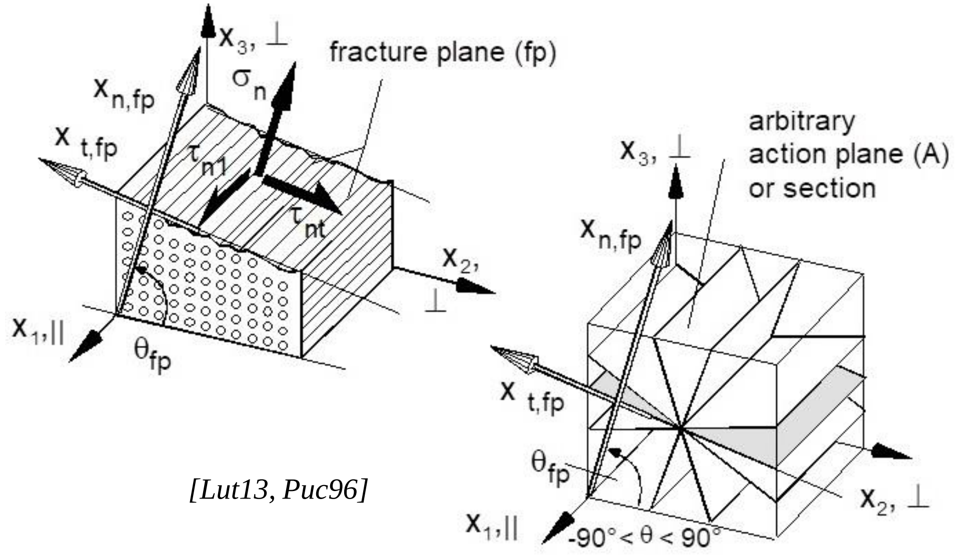
The Mohr approach-dependent action plane resistance $\bar{R}^A_{\perp\perp}$ is model-fixed.

→ Practically 5 technical strengths are addressed only, which supports Cuntze's 'generic' number 5 for UD materials.

3D-Search of the Fracture Plane Angle $\boldsymbol{\theta_{fp}}$ by variation of θ in a program with determining the minimum failure danger.

$R_{\perp\perp}^{A}$ would represent due to the Mohrstresses-based Approach the 'Cohesive strength' R_{23}^{τ}, a result of two acting modes, namely IFF2 with IFF1 causing a range $R_{\perp}^{c} > R_{\tau} > R_{\perp}^{t}$.
Due to the spatially linked parameters in the approach Puck can derive the *general* formulation :

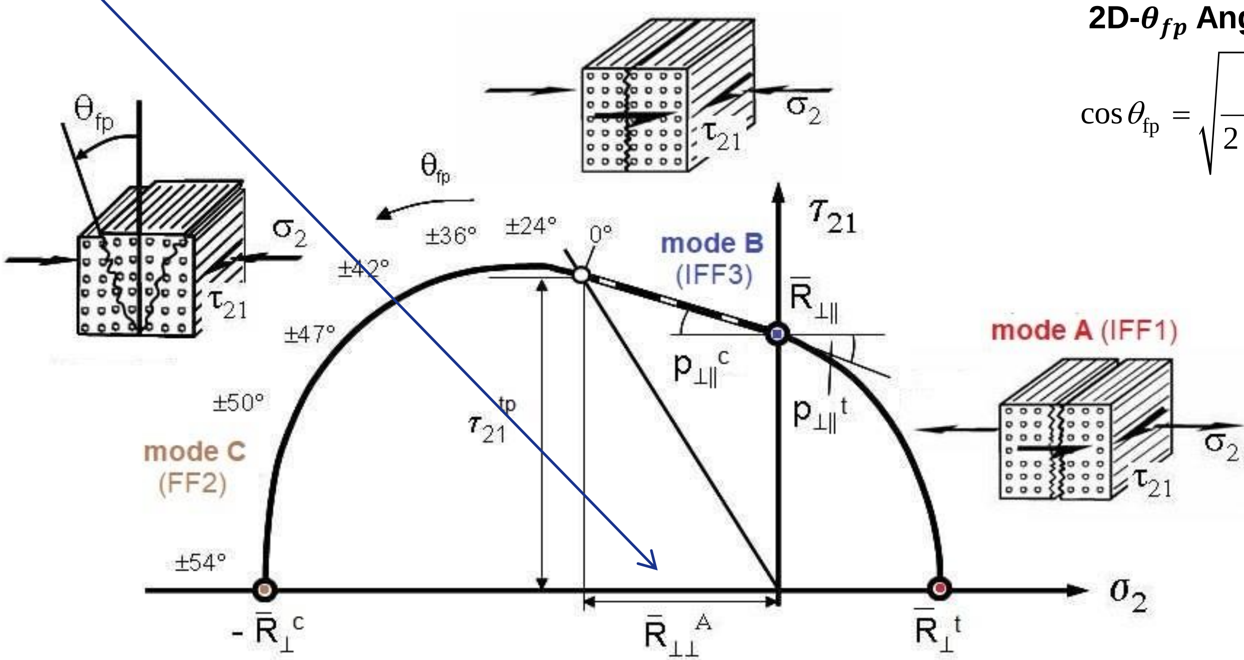
$$R_{\perp\perp}^{A} = \frac{R_{\perp\|}}{2 \cdot p_{21c}} \cdot \left(\sqrt{1 + 2 \cdot p_{21c} \cdot \frac{R_{\perp c}}{R_{\perp\|}}} - 1 \right)$$ [Puc96, eq. 5.8].



[Lut13, Puc96]

2D-$\boldsymbol{\theta_{fp}}$ Angle, Alteration: formula available

$$\cos\theta_{fp} = \sqrt{\frac{1}{2 + 2 \cdot p_{\perp\perp}^{c}} \cdot \left[\left(\frac{R_{\perp\perp}^{A}}{R_{\perp\|}} \right)^2 \cdot \left(\frac{\tau_{21}}{\sigma_2} \right)^2 + 1 \right]}.$$



Lamina stresses (σ_2, τ_{21}), Main IFF cross-section of the fracture body.
(average model → bar $\bar{R}_{\perp\perp}^{A}$

3D-'Global' UD-SFC of Tsai-Wu (there is also an orthotropic version)

$$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^{\mathrm{T}}, \quad \{\bar{R}\} = (\bar{R}^t_{||}, \bar{R}^c_{||}, \bar{R}^t_{\perp}, \bar{R}^c_{\perp}, \bar{R}_{\perp||};\ \bar{R}_{\perp\perp})^{\mathrm{T}} = (\mathrm{X}, X', Y, Y', S;\ \mathrm{S}_{23})^{\mathrm{T}}$$

$$F\left(\{\sigma\}, \{\bar{R}\}\right) = 1, \quad 6 \text{ strengths principally}$$

A general anisotropic tensor polynomial expression of Zakharov and Goldenblat-Kopnov with the parameters F_i, F_{ij} as strength model parameters was the basis of the Tsai-Wu SFC, see [14],

$$\sum_{i=1}^{6}(F_i \cdot \sigma_i) + \sum_{j=1}^{6}\sum_{i=1}^{6}(F_{ij} \cdot \sigma_i \cdot \sigma_j) = 1 \quad F_i \cdot \sigma_i + F_{ij} \cdot \sigma_i \cdot \sigma_j = 1 \quad \text{with} \quad (\mathrm{i,j} = 1,2,\ldots,6)$$

$$F_i \cdot \sigma_i + F_{ij} \cdot \sigma_i \cdot \sigma_j = 1 \quad \text{with} \quad (\mathrm{i,j} = 1,2..6) \quad \text{or executed}$$

σ_3 treated like σ_2

$$F_{11} \cdot \sigma_1^2 + F_1 \cdot \sigma_1 + 2F_{12} \cdot \sigma_1 \cdot \sigma_2 + 2F_{13} \cdot \sigma_1 \cdot \sigma_3 + F_{22} \cdot \sigma_2^2 + F_2 \cdot \sigma_2 +$$

$$+ 2F_{23} \cdot \sigma_2 \cdot \sigma_3 + F_{33} \cdot \sigma_3^2 + F_{33} \cdot \sigma_3^2 + F_3 \cdot \sigma_3 + F_{44} \cdot \tau_{23}^2 + F_{55} \cdot \tau_{13}^2 + F_{66} \cdot \tau_{12}^2 = 1$$

with the strength model parameters = coefficients of the criterion

$$F_1 = 1/\bar{R}^t_{||} - 1/\bar{R}^c_{||}, \quad F_{11} = 1/(\bar{R}^t_{||} \cdot \bar{R}^c_{||}), \quad F_2 = 1/\bar{R}^t_{\perp} - 1/\bar{R}^c_{\perp}, \quad F_{22} = 1/(\bar{R}_{\perp} \cdot \bar{R}^c_{\perp}) = F_{33},$$

$$F_{13} = F_{12}, \quad F_{55} = F_{66} = 1/\bar{R}^2_{\perp||}, \quad 2F_{23} = 2F_{22} - 1/\bar{R}^2_{23}, \quad F_{44} = 2 \cdot (F_{22} + F_{23}), \quad F_3 = F_2 (\text{UD-model})$$

and - *in order to avoid an open failure surface* - the so-called interaction term

$$F_{12} = \bar{F}_{12} \cdot \sqrt{F_{11} \cdot F_{22}} \quad \text{with} \quad -1 \le \bar{F}_{12} \le 1; \quad \text{usually it is applied } F_{12} = -0.5.$$

- The bi-axial material parameter F_{12} is 'principally' to obtain from aequi-biaxial compression tests to close the fracture body. This would mean, that F_{12} cannot consider the stress signs (question: effect in the domain σ_1^t, σ_2 ?
- For details considering F_{12}, see [*Li17*].

Transformation of the 2D-SFCs into an Eff-formulated Interaction-capable shape

$$F\left(\{\sigma\},\{\bar{R}\},\mu\right)=1: \;\Rightarrow\; Eff^{\,m} = [(Eff^{\,||\sigma})^m + (Eff^{\,||\tau})^m + (Eff^{\,\perp\sigma})^m + (Eff^{\,\perp||})^m + (Eff^{\,\perp\tau})^m] = 1 = 100\%$$

Template

directly including the friction value μ, with mode portions, formulated to avoid physically senseless negative $Eff^{\,modes}$

$$Eff^{\,||\sigma} = \frac{(\sigma_1 + |\sigma_1|)}{2\cdot \bar{R}^t_{||}},\; Eff^{\,||\tau} = \frac{(-\sigma_1 + |\sigma_1|)}{2\cdot \bar{R}^c_{||}},\; Eff^{\,\perp\sigma} = \frac{\sigma_2 - |\sigma_2|}{2\cdot \bar{R}^t_{\perp}},\; Eff^{\,\perp\tau} = \frac{-\sigma_2 + |\sigma_2|}{2\cdot \bar{R}^c_{\perp}},\; Eff^{\,\perp||} = \frac{|\tau_{21}|}{\bar{R}_{\perp||} - \mu_{\perp||}\cdot(\sigma^c_2)}.$$

Cuntze: 2 FF + 3 IFF

Hashin: 2 FF + 2 IFF

Puck: 2 FF with 3 Mohr-coupled IFFs

Tsai-Wu: Modes still globally interacted.

Computation code: Mathcad 15

2D (in-plane)- Formulations of the four envisaged SFCs (model mapping, bar)

$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{12})^T \rightarrow \{\sigma\} = (\sigma_1, \sigma_2, \tau_{21})^T$ requiring all 5 technical strengths $\{\bar{R}\} = (\bar{R}_{||}^t, \bar{R}_{||}^c, \bar{R}_{\perp}^t, \bar{R}_{\perp}^c, \bar{R}_{\perp||})^T$

Cuntze's FMC-based Set of Modal SFCs: FF1, FF2, IFF1, IFF2, IFF3 $\{\bar{R}\} = (\bar{R}_{||}^t, \bar{R}_{||}^c, \bar{R}_{\perp}^t, \bar{R}_{\perp}^c, \bar{R}_{\perp||})^T, \ \mu_{\perp||}, \ m \cong 2.6$

$$F(\{\sigma\}, \{\bar{R}\}, \mu) = 1: \ \Rightarrow \ Eff^{\,m} = [(Eff^{\,||\sigma})^m + (Eff^{\,||\tau})^m + (Eff^{\,\perp\sigma})^m + (Eff^{\,\perp||})^m + (Eff^{\,\perp\tau})^m] = 1 = 100\%$$

directly including the friction value μ, with mode portions, formulated to avoid physically senseless negative $Eff^{\,\mathrm{modes}}$

$$Eff^{\,||\sigma} = \frac{(\sigma_1 + |\sigma_1|)}{2 \cdot \bar{R}_{||}^t}, \quad Eff^{\,||\tau} = \frac{(-\sigma_1 + |\sigma_1|)}{2 \cdot \bar{R}_{||}^c}, \quad Eff^{\,\perp\sigma} = \frac{\sigma_2 + |\sigma_2|}{2 \cdot \bar{R}_{\perp}^t}, \quad Eff^{\,\perp\tau} = \frac{-\sigma_2 + |\sigma_2|}{2 \cdot \bar{R}_{\perp}^c}, \quad Eff^{\,\perp||} = \frac{|\tau_{21}|}{\bar{R}_{\perp||} - \mu_{\perp||} \cdot (\sigma_2^c)}$$

Tsai-Wu, global SFC *(interaction inherent)* $\{\bar{R}\} = (\bar{R}_{||}^t, \bar{R}_{||}^c, \bar{R}_{\perp}^t, \bar{R}_{\perp}^c, \bar{R}_{\perp||}; \ \bar{R}_{\perp\perp})^T = (\bar{X}, \ \bar{X}', \bar{Y}, \bar{Y}', \bar{S}_{12}; \bar{S}_{23})^T$

$$F(\{\sigma\}, \{\bar{R}\}, \bar{S}_{23}, F_{12}) = 1: F = Eff = \frac{\sigma_1^2}{\bar{R}_{||}^t \cdot \bar{R}_{||}^c} + \sigma_1 \cdot \left(\frac{1}{\bar{R}_{||}^t} - \frac{1}{\bar{R}_{||}^c}\right) + 2\frac{\bar{F}_{12} \cdot \sigma_1 \cdot \sigma_2}{\sqrt{\bar{R}_{||}^t \cdot \bar{R}_{||}^c \cdot \bar{R}_{\perp}^t \cdot \bar{R}_{\perp}^c}} + \frac{\sigma_2^2}{\bar{R}_{\perp}^t \cdot \bar{R}_{\perp}^c} + \sigma_2 \cdot \left(\frac{1}{\bar{R}_{\perp}^t} - \frac{1}{\bar{R}_{\perp}^c}\right) + \frac{\tau_{12}^2}{\bar{R}_{\perp||}^2} = 1$$

Hashin, modal SFCs, FF1, FF2, IFF1, IFF2, no IFF3 $\{\bar{R}\} = (\bar{R}_{||}^t, \bar{R}_{||}^c, \bar{R}_{\perp}^t, \bar{R}_{\perp}^c, \bar{R}_{\perp||}, \bar{R}_{23})^T$, determination of $\bar{R}_{23}$ is missing

Hyp.2: $F(\{\sigma\}, \{\bar{R}\}, \bar{S}_{23}) = 1$: invariant way (like Cuntze), modal-wise, but how is the interaction to perform ?

$\bar{R}_{23} \equiv \bar{S}_{23}$ Interaction will be performed as with Cuntze's *Effs*

Puck's Action Plane IFF SFCs, (*Mohr-based globally* combining the ***3 IFF***-domains) with ***the 2 modal FF1, FF2***

$F(\{\sigma^A\}, \{\bar{R}^A\}, \theta_{fp}) = 1$ with $\{\bar{R}\} = (\bar{R}_{||}^t, \bar{R}_{||}^c, \bar{R}_{\perp}^A = \bar{R}_{\perp}^t, \bar{R}_{\perp\perp}^A, \bar{R}_{\perp||}^A = \bar{R}_{\perp||})^T$ including $\bar{R}_{\perp\perp}^A \neq \bar{R}_{23}$

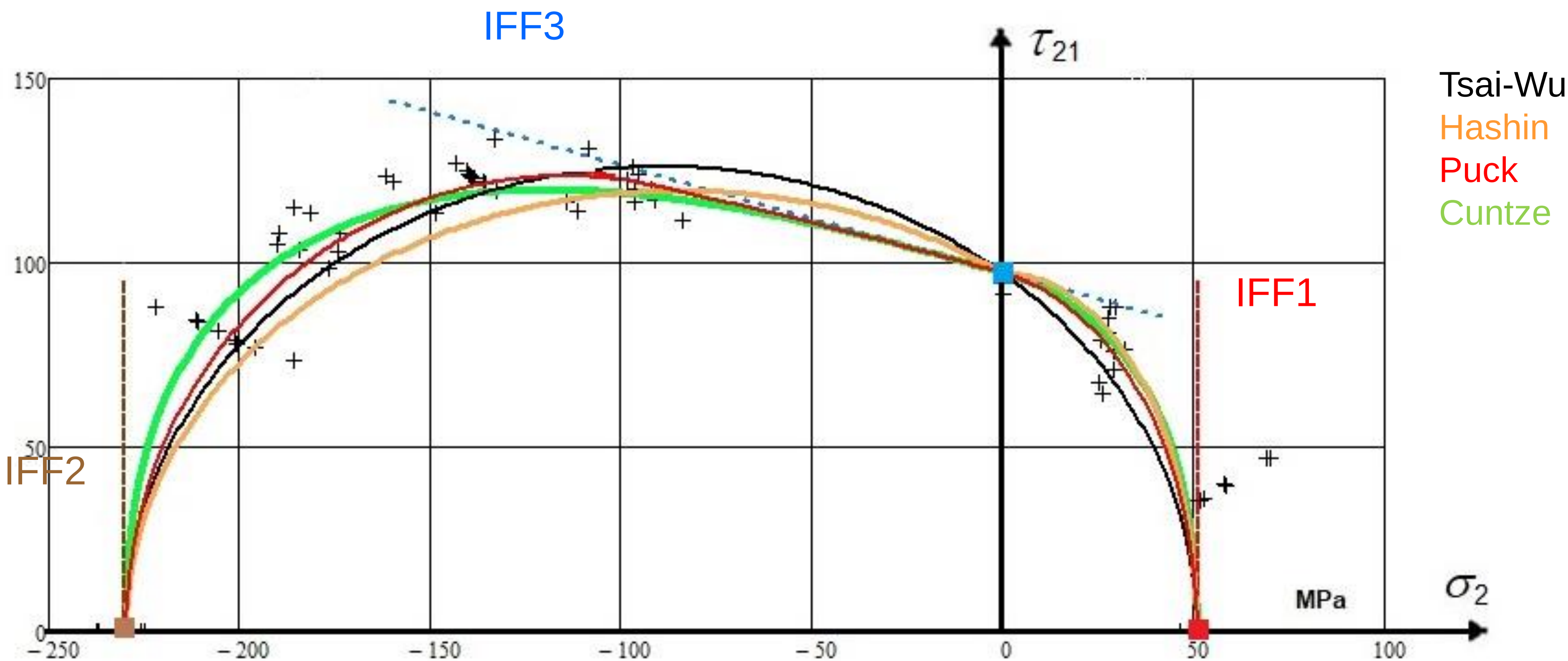
IFF-interaction is Mohr-model (IFF-globally($\bar{R}_{\perp}^c$, μ)) given $\Rightarrow$ *FF*-modes are to interact with the given *IFF*-mode formulations, domain-wise

$$Eff^{\,||\sigma} = \frac{(\sigma_1 + |\sigma_1|)}{2 \cdot R_{||}^t}, \quad Eff^{\,||\tau} = \frac{(-\sigma_1 + |\sigma_1|)}{2 \cdot R_{||}^c}, \quad \text{IFF1: } F^{\perp\sigma} = Eff^{\,\perp\sigma} = F = \frac{1}{\bar{R}_{\perp||}} \cdot \left(\sqrt{\left(\frac{\bar{R}_{\perp||}}{\bar{R}_{\perp}^t} - p_{\perp||}^{\ t}\right)^2 \cdot \sigma_2^{\ 2} + \tau_{21}^2} + p_{\perp||}^{\ t} \cdot \sigma_2\right), \ \text{IFF2: } Eff^{\,\perp\tau}, \ \text{IFF3: } Eff^{\,\perp||}.$$

Inclination parameter input: from physics and statistics Cuntze follows $p_{\perp||}^{\ t} = p_{\perp||}^{\ c}$ (no kink will be faced). and $p_{\perp||}^{\ c}$ corresponds to $\mu_{\perp||}$

Comparison of the Single Failure Envelopes $\tau_{21}(\sigma_2)$

Figure, after taking away numerically produced other curve branches of the envelope

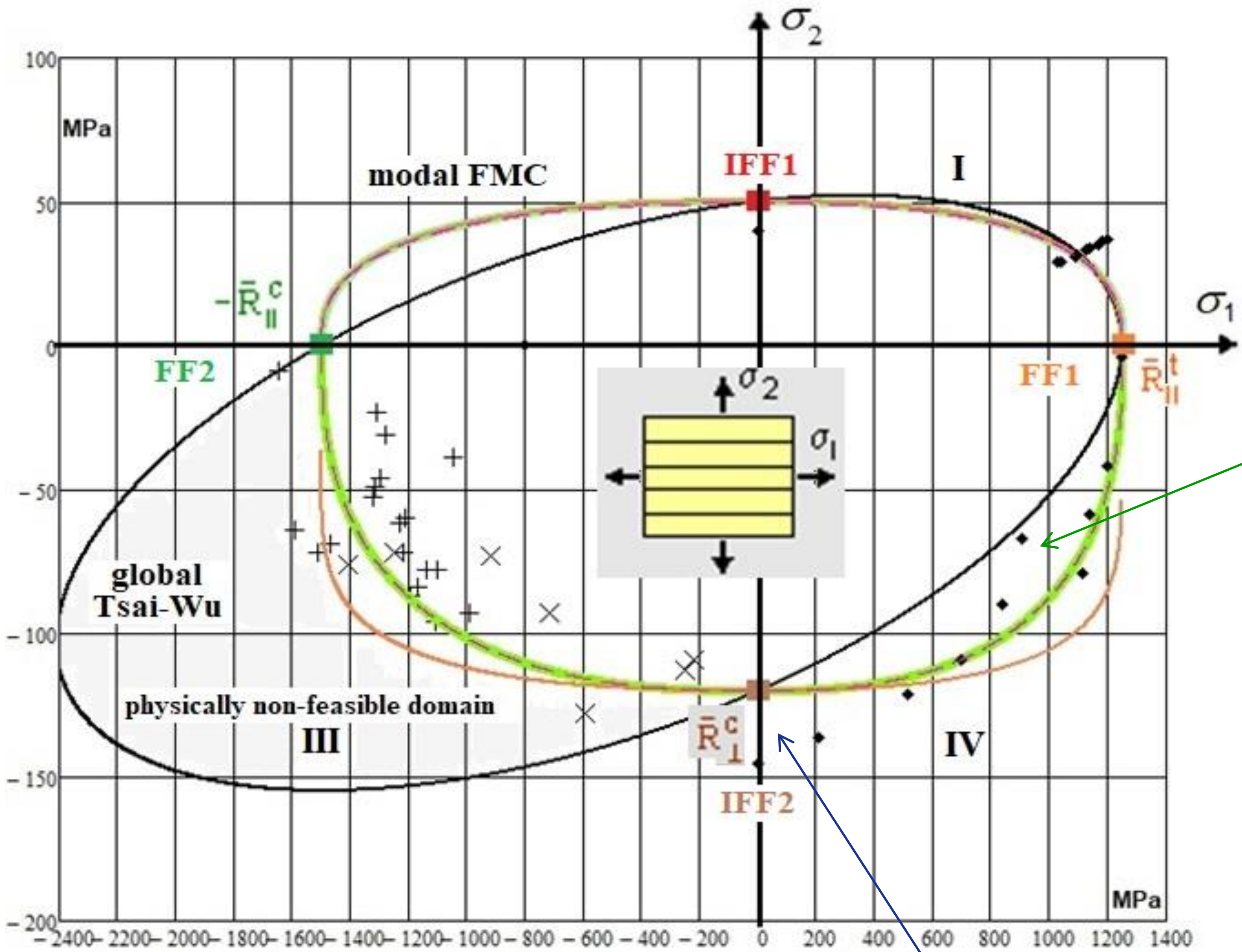


Notes:

- The differences of the four models are obvious and the reader is asked for an assessment
- CFRP test results (MAN Technologie research project with A. Puck, IKV Aachen et al.) [VDI 97] $\{\bar{R}\} = (1280,\ 800,\ 51,\ 230,\ 97)^T\,\text{MPa},\ \mu_{\perp\|} = 0.3$

The reader is asked to assess the resulting failure envelopes

Comparison of the Failure Envelopes $\sigma_2\ (\sigma_1)$



Tsai-Wu
Hashin
Puck
Cuntze

Test points ???:

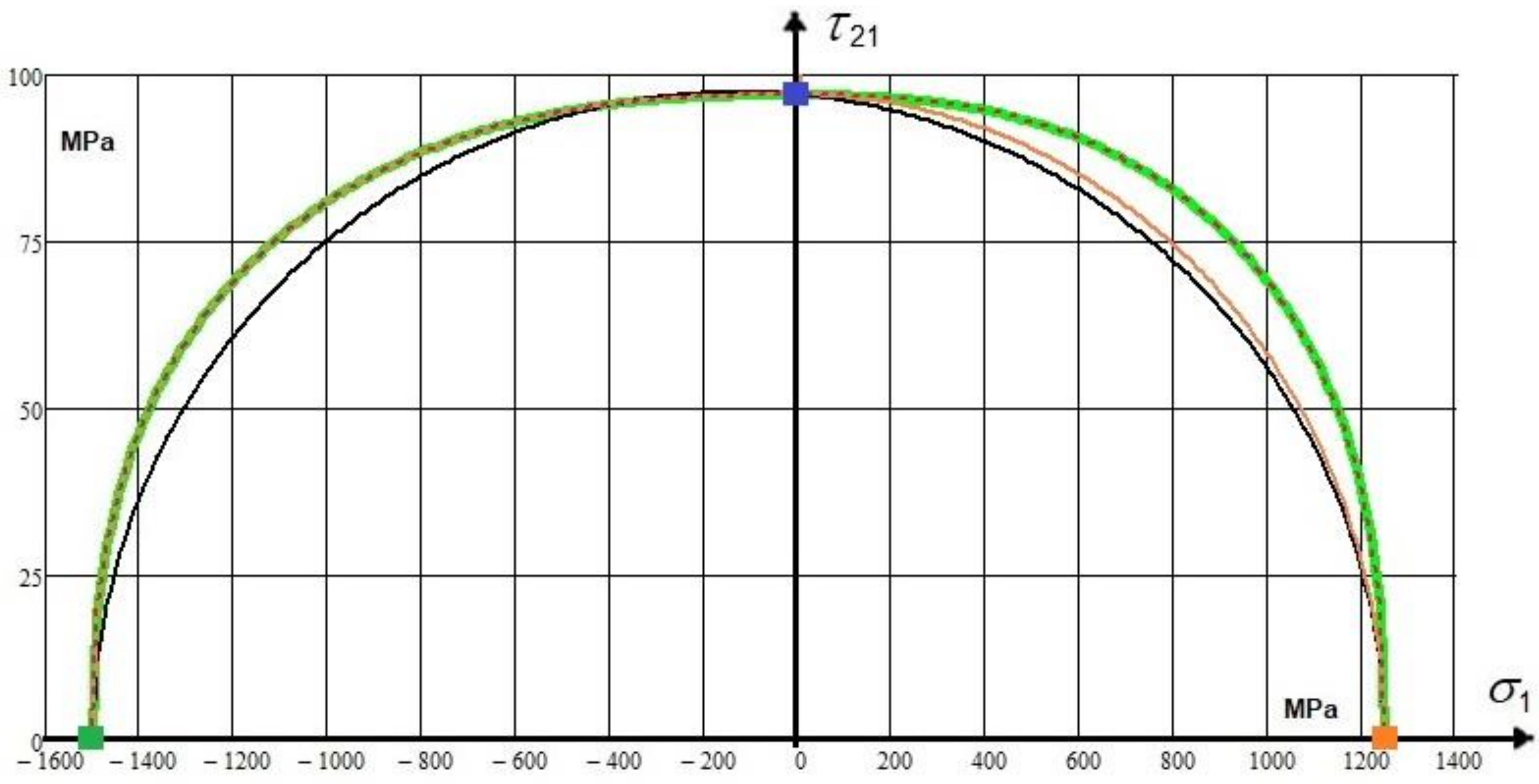
Cuntze tried to contact the Russian originator in 2004. Unfortunately he had passed away.

Notes, on this WWFE-I, TC3 example **:**

* The differences of the four models are clearly visualized and the reader asked for assessment.

* In the WWFE the 4 strength values were provided together with the not $R_{\perp}^{c}$ -matching test data in the fourth quadrant
However, the tendency of the two different (*to assume*) test sets can be used for validation.

Comparison of the Failure Envelopes $\tau_{21}(\sigma_1)$



Tsai-Wu
Hashin
Puck
Cuntze

Data:

Notes on this theoretical example:

- The differences of the four models are obvious.
- See Example 3, please. Another data set.

Computation of a Material Reserve Factor f_{RF} (linear analysis permitted)

~

Task of the designer: To prove that a reserve factor *RF* > 1 could be obtained for the structural component.

Linear analysis is sufficient (presumption): $\sigma \sim \text{load} \quad \Rightarrow RF \equiv f_{RF} = 1 / Eff$

Material Reserve Factor $$f_{RF,\,ult} = \frac{\text{Strength Design Allowable } R}{\text{Stress at } j_{ult} \cdot \text{Design Limit Load}} > 1,$$

Non-linear analysis required: σ not proportional to load

Reserve Factor (load-defined) $$RF_{ult} = \frac{\text{Predicted Failure Load at } Eff = 100\%}{j_{ult} \cdot \text{Design Limit Load}} > 1.$$

A very simple example for a Design Verification of an applied stress state in a critical UD lamina location of a distinct laminate wall design shall depict the *RF*-calculation as most essential task in design which streamlines every procedure when generating a design tool in the following chapters:

A very simple example shall depict the *RF*-calculation in design which streamlines the procedure when generating a design tool which is further to be applied such as in the following chapters.

→ A stress state assessment is usually based on an agreement to apply the so-called 'Proportional Loading (stressing) Concept'. If linear, all stresses alter proportionally. Margin-of-Safety *MoS* = *RF* -1.

<u>Design Load cases</u>:
design Limit Load = dLL ,
Design Ultimate Load = DUL

.

Numerical example UD Design Verification by a material f_{RF} > 1

2D-Design Verification of a critical UD lamina in a distinct laminate wall design

INPUT: Asssumption 'Linear analysis permitted' = good enough

* Design loading (action): $\{\sigma\}_{\text{design}} = \{\sigma\} \cdot j_{\text{ult}}$, design FoS $j_{\text{ult}} = 1.25$

* 2D-stress state: $\{\sigma\}_{\text{L}} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T \cdot j_{\text{ult}} = (500,\ -76,\ 0,\ 0,\ 0,\ 52)^T$ MPa

* Residual stresses: 0 $\left(\textit{effect vanishes with increasing micro-cracking} \right)$

* Strengths (resistance) : $\{\bar{R}\} = (1378,\ 950,\ 40,\ 125,\ 97)^T$ MPa average from mesurement,

statistically reduced $\{R\} = (R_{||}^t, R_{||}^c, R_{\perp}^t, R_{\perp}^c, R_{\perp||})^T = (1050,\ 725,\ 32,\ 112,\ 79)^T$ MPa.

* Friction value(s) : $\mu_{\perp||} = 0.3,\ (\mu_{\perp\perp} = 0.35)$, Mode interaction exponent: $m = 2.7$

$$\{Eff^{\text{ modes}}\} = \left(Eff^{||\sigma},\ Eff^{||\tau},\ Eff^{\perp\sigma},\ Eff^{\perp\tau},\ Eff^{\perp||}\right)^T = \left(0.88,\ 0,\ 0,\ 0.21,\ 0.20\right)^T$$

$$Eff = [(Eff^{||\sigma})^m + (Eff^{||\tau})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp||})^m]^{1/m}.$$

The results deliver the following failure danger portions *Eff*

* $Eff^{\perp\sigma} = \dfrac{\sigma_2 + |\sigma_2|}{2 \cdot \bar{R}_{\perp}^t} = 0,\quad Eff^{\perp\tau} = \dfrac{-\sigma_2 + |\sigma_2|}{2 \cdot \bar{R}_{\perp}^c} = 0.60,$

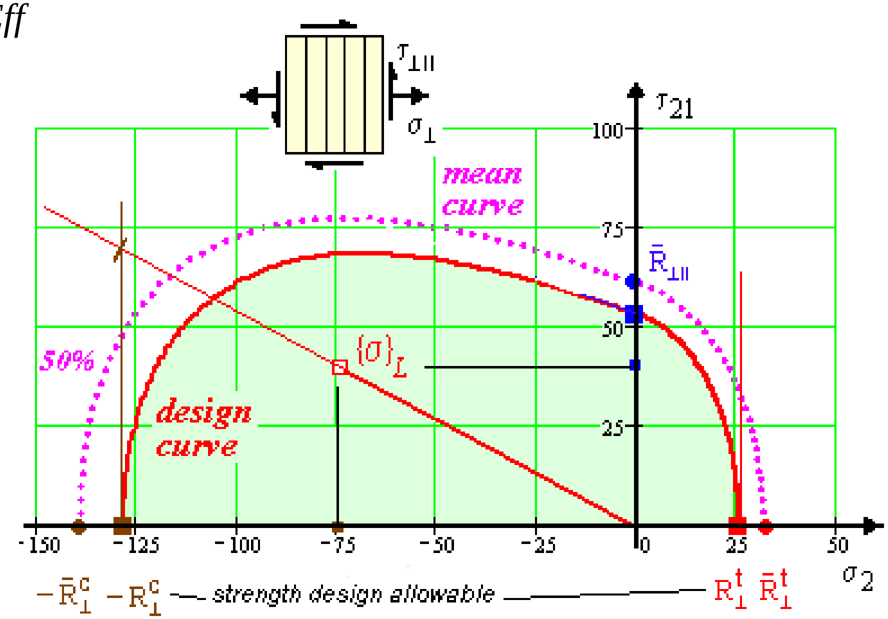
$$Eff^{\perp||} = \frac{|\tau_{21}|}{\bar{R}_{\perp||} - \mu_{\perp||} \cdot \sigma_2} = 0.55$$

$$Eff = [(Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp||})^m]^{1/m} = 0.80.$$

and the material reserve factor $f_{\text{RF}} = 1/Eff$

$\Rightarrow f_{\text{RF}} = 1/Eff = 1.25 \rightarrow$

RF = $f_{\text{RF}} \rightarrow$ MoS = $RF - 1 = 0.25 > 0$!



Conclusions regarding the Comparison of the 4 SFCs

- The 4 SFCs differently well map the sparsely available **reliable** test data
- ‘Modal‘ SFCs are less dangerous in application than the forcibly modes-marrying ‘global‘ ones
- The full capacity of the fracture conditions cannot be fully verified. Derivation of more representative experimental test data is necessary to really make a better judgement even of the 2D UD failure conditions possible
- Learning from WWFEs: Careful test data evaluation is of highest priority
- Cuntze and Puck seem to map the course of the reliable test points best. This is essential, when computing for Design Verification the Reserve Factor of the Final Design
- Each SFC can only describe a one-fold occurring failure mode. Multi-fold failure, such if $\sigma 2 = \sigma 3$, is additionally to consider
- For multi-directionally reinforced laminates, well-designed by netting theory, linear analysis is a good approximation on the safe side.
  - For 2D-analyses, after carefully looking at the comparison results, the designer is now enabled:

  “ *To not take the worst result to achieve a relatively conservative design” !*

* The full capacity of the fracture conditions could not be fully verified. Derivation of more representative experimental test data is necessary to really make a judgement of the 2D UD failure conditions possible.

- UD-SFCs are presently not 3D-failure stress states-validated? We use *the SFCs* without any questioning !

  Therefore, the future challenge will be 3D-analyses of joints etc.

Conclusions, Findings regarding Cuntze's SFC set

- In the frame of his material symmetry-driven thoughts the author could derived an 'Engineering-practical strength criteria set' SFC on basis of <u>measurable</u> parameters, only.
- First direct use of the measurable friction value μ in a SFC formulation (*after effortful investigations possible*)
- Clear notations to identify material properties and the observed laminate stack
- 'Generic' numbers found, simplify theoretical and test tasks: Isotropic (2), transversely-isotropic UD (5), Orthotr. (9)
- Mapping (fitting) of the courses of provided and of own test data is very good.
- Simple determination of the lowest mode reserve factor = design driving mode. Reliable computation of the material Reserve Factor f_{RF} . This requires *Eff* in the case of a non-homogeneous Failure function F or SFC
- At 1D, or 2D and 3D failure states *Eff* (*Werkstoff-Anstrengung)* can maximally achieve 100% ! This explains, that an axial failure stress under 3D-compression is higher than the Standard-fixed technical strength value (axial) *R*
- For laminates, well-designed by netting theory, linear analysis is a good approximation on the safe side
- Cuntze does not require the non-measurable cohesive shear strength design value as a sixth strength quantity
- The physical different action of laminate-embedded (more benihn) and isolated layers in test specimens is to consider.

Unfortunately, structural engineers believe in what is presented in the FE-code Manuals without questioning the quality of the given SFC or of any provided test results !?

Lessons to Learn from Bronstein-Semendjajev today again: he wrote in 1960

A short arithmetic stick (slide rule, Rechenschieber) and a longer one for instance for static dimensioning in constructions and a more accurate output. A stick made parametric dimensioning possible!
The book Bronstein-Semendjajew retained his excellent value over all the decades.

***About the purpose of the arithmetic stick**

“*The simplest calculations, in which multiplications, division, extraction of square and cube roots, exponentiation and operations with trigonometric functions occur, can be carried out approximately with the help of the arithmetic stick. The accuracy of the calculations varies from case to case, but it can be said, that on average the arithmetic stick of 25 cm length gives results on three decimals with an error of less than 1%”.*

*__*About the calculation accuracy*__:

„*When calculating, it is always important to consider the accuracy that you either have to achieve or can achieve.*

- *It is quite inadmissible to calculate with great accuracy if the nature of the task either does not allow it or does not require it*
- *If an approximate value contains superfluous decimal numbers, it must be rounded. In some cases, more realistic but more complicated models can be replaced by simpler ones that give a result with an acceptable error*.“

FEA-Tools for practicing engineering at 1960-1968

* I remember that I could not finish my dissertation in 1967 because o and 0 where not clearly marked in the coding which cost two months filled with despair.
* Later in the sixties Finite Element Analysis came up, however, there was no commercial tool available. The author had to beg for the building blocks of his intended program for a radial impeller at the DLR at his old university in Hannover. To punch cards was a must.

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Ich freue mich, dass Prof. D. Sabine Pfeiffer,
Lehrstuhl für Soziologie der Uni Erlangen-Nürnberg,
- als gelernte Werkzeugmacherin -
Sprecherin des DFG-Schwerpunktprogrammns
‘Digitalisierung der Arbeitswelten’
bekundet:

***“KI kann nicht alles leisten,
wir müssen uns stärker auf bewährtes Ingenieurwissen
besinnen und mehr vom Ziel her denken.***

Dies bedeutet zum Beispiel auch:

***Welche Tools sind wirklich vom ‘Niveau her’ notwendig
zum Erreichen des Ziels und vor allem robust?”***

Der Gebrauch des Rechenschiebers lehrt es uns.

Besten Dank für‘s Zuhören und Zusehen.

Ihr Ralf Cuntze

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- 1968, Dr.-Ing. Structural Dynamics (TU Hannover)
- 1968 - 1970, DLR FEA-programming
- **1970** - **2004, MAN-Technologie: Head 'Structural and Thermal Analysis'**
  ARIANE 1-5, GROWIAN, Uranium Enrichment centrifuges, Solar Plants, Pressure Vessels, etc.
- 1978, Dr.-Ing. habil. Mechanics of Lightweight Structures (TU Munich)
- 1980 – 2002 **Lecturer UniBw on Fracture Mechanics (construction), Lightweight**
- 1987, Full Professorship, *not started in favor of interesting industry tasks*
- 1998, Honorary Professorship at Universität der Bundeswehr München UniBw
- **1972 – 2018 contributor to the German** Aerospace **Hdbk HSB**
- **2006, VDI Guideline 2014 „Development of FRP-Components" (***editor sheet 3***)**
- 2019, GLOSSAR "Technical terms for composite parts". Springer
- 1972 - 2004 working on multiple ESA/ESTEC Standards and
  2004 - 2009 heading the„ESA Stability Handbook" Working Group
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Cuntze literature : download possible from **https://www.carbon-connected.de/Group/Prof.Ralf.Cuntze**