Transition Continuummechanics-Fracture Mechanics

2.9.1 Validity limits for SFC applications

The next table lists comments about and experiences with SFC applicability limits.

Table 2-5: Comments on Validity Limits for the Applicability of a SFC (mainly given for UD)

- As a SFC is a necessary but not a sufficient condition to predict failure *[Leguillon, W. Becker]* a fracture mechanics-based energy condition may be to fulfill, too. Even in plain (smooth) stress regions a SFC can be only a necessary condition which may be not sufficient for the prediction of 'onset of fracture', i.e. the in-situ lateral strength in an embedded lamina. Example: thick layers fail earlier than thin ones under the same 2D stress state see e.g. [*Flaggs-Kural 1982*].
- Attempts to link 'onset-of-fracture' combined with 'onset-of-cracking prediction' methods for structural components are under-gone, see e.g. *[Leg02*]. In his finite fracture mechanics Leguillon assumes a crack as one more unknown but one can solve the equation system by one more equation from fracture mechanics (see later sub-chapter)
- In case of discontinuities such as notches with steep stress decays only a *toughness* + *characteristic length-based energy balance condition* may form a sufficient set of two fracture conditions
- When applying test data from 'isolated lamina' test specimens (*like tensile coupons*) to an embedded lamina of a laminate one should consider that coupon test deliver tests results of 'weakest link' type. An embedded or even an only one-sided constrained lamina, however, possesses redundant behavior, see *Fig.2-11*
- A SFC usually describes only a one-fold occurrence of a mode or of a failure mechanism, respectively! A multiple occurrence of a mode, such as for $\sigma_I = \sigma_{II}$ or for $\sigma_2 = \sigma_{3,}$, is to map by an additional term in $Eff = 1$, see for instance in *Table 2-6*
- Each failure stress state belongs to *Eff* = 100% and represents one point on the surface of the failure body. This is valid for 1D- (these are the strength values), for 2D- and for 3D-stress states. In the case of a multiaxial compressive stress state the strength does not increase but the risk to fracture becomes smaller, indicated by *Eff* which then becomes lower than 100 % !
- Each failure mechanism is affected by an associated typical state of stress. The failure mechanism with the highest material stressing effort will dominate the UD failure. The mode effort has to become zero if the mode driving stress is zero
- Not design-driving stresses of a mode might increase or decrease the material stressing effort *Eff* which is basically sized by the design driving one. This influence is considered in the equivalent mode stress $\sigma_{eq}^{\text{mode}}$. Note that the Mises equivalent stress for the mode yielding is not the only equivalent stress
- The 5 strength and 2 friction parameters can be measured and therefore fulfil a basic design verification requirement: Strength properties shall be statistically-based, material friction properties μ are so-called physical quantities which shall be average (typical) values in order to best meet the optimum being the maximum expectation value of 50% probability
- The FFs are special fiber strain failure equations to capture filament fracture under bi-axial compression
- . The NF-function chosen for isotropic and UD materials enables to map a straight line of test data in the principle stress plane

- If the failure body is fully rotational symmetric then c^{NF} ($\Theta^{NF} = 1$ or $d^{NF} = 0$) = 1. Above NF can manage inward and outward dents by c^{NF} (Θ^{NF}) < 1 which renders the 120°-rotational symmetry
- The friction effect decreases with increasing porosity. Ideal dense materials possess no porosity. A fully porous material may be defined by $R^{cc} \cong R^{c}$. This case is modelled like the porous foam material [*Cun16a*]
- When mapping, then $\overline{R}$ must be used, because the average behavior or value is required
- Only Eff = 100% is equal to the SFC F = 1
- If any plane in a 3D stress state is a plane of maximum danger, being possible to become a fracture plane, then that plane with the most unfavorable flaw situation becomes the fracture plane
- Due to IFF the curing stresses decay in parallel to the degradation.

4.3.4 Finite Fracture Mechanics linking Continuum Mechanics - Fracture Mechanics

When applying SFCs ideal solids are viewed which are assumed to be free of essential micro-voids or microcrack-like flaws. When applying FM the solid is considered to contain macro-voids or macro-cracks. There are three approaches available to deal with the occurring stress situations: Strength conditions (SFC), Continuum Damage mechanics (CDM) conditions and Conditions of crack fracture mechanics (FM) which employ crack growth models.

As still mentioned before:

A SFC is a necessary condition but might not be a sufficient condition for the prediction of 'initiation of cracking' (Onset-of-Failure).

A bridge must be built from strength failure conditions to fracture mechanics failure conditions. Neglecting CDM, attempts to link SFC-described 'onset of fracture' prediction methods and FM prediction methods for structural components are actually undergone. Best known is the Hypothesis of Leguillon [*Leg02*]:

"*A crack is critical when and only when both the released energy and the local stress reach critical values along an assumed finite crack".*

While the application to combined stresses is very common in the instance of SFCs it is not at all common to apply FM conditions to 3D states of stress. Based on physical reasons, a bridge should provide a more unified perspective, because - ahead of the crack tip - both approaches are linked together. Such a bridging approach should respect a 3D-farfield state of stress and a full mixed-mode crack situation.

For the stress concentration examples above an energy-based FM failure condition delivers the second condition to check whether failure occurs or whether failure does not occur. After Leguillon the Linear Elastic Fracture Mechanics (LEFM) delivers the missing failure condition which is an *energetic failure condition* formulated by the above still used

total energy release rate $\mathcal{G} = \Sigma\ \mathcal{G}_i$ to be inserted into $\mathcal{G} / \mathcal{G}_{feasible} = 1$.

with $\mathcal{G}$ being the crack-driving force to create more fracture surface and the crack resistance $\mathcal{G}_{cr}$ *(most often a feasible value)* a fracture energy.

This so-called coupled criterion with $Eff = 1$ together with $\mathcal{G} / \mathcal{G}_{feasible} = 1$ has a pro-termced macroscopic nature. In the case of delamination between plies under normal and/or shear stresses, the determination of the critical energy release rate $\mathcal{G}_{cr}$ is crucial to analyze designs.
Typical methods of determining the critical energy release rate are through experiments, such as the double-cantilever beam (DCB) for mode I, end-notched flexure (ENF) for mode II, and mixed-mode flexure (MMF).
In the case that Linear Fracture Mechanics LEFM is permitted for the given material $\mathcal{G}_{cr}$ is $\mathcal{G}_{cr}(K_{cr})$.

Finite Fracture Mechanics (FFM) fills a gap in FM, since it is an approach to offer a condition to predict the crack initiation in brittle isotropic and UD materials. Within FFM assumed cracks of finite length are considered.

Examples are the locations of stress singularities like the stress concentrations of notched isotropic components or UD layer-composed laminates:

- at notches, where the notch stress causes failure even for relatively low far-field stresses
- the influence of the size of a bore has an effect
- the thickness of the in-situ behavior of an embedded layer in a laminate has an effect.

These are typical deformation-controlled stress situations. FFM can be used to predict the open-hole tensile strength of composite laminates. Failure is occurs when both stress-based and energy-based conditions are satisfied. Material properties required by the concept are the elastic properties of the lamina and the laminate unnotched strength and its fracture toughness.

Reminder:

The FMC is originally energy-based, however, the modified forms may not have sufficiently well kept this previously intended characteristic.

[Leg02] Leguillon D: *Strength or Toughness? –A criterion for crack onset at a notch.* Europ. J. of Mechanics A/Solids 21 (2002), 61 – 72 end. Ist. D. sci. Lett., Cl. Mat. Nat.18, 705-714 (1885
[Wei15] Weißgräber P, Leguillon D and Becker W: *A review of Finite Fracture Mechanics: crack initiation at singular and non-singular stress raisers*. Arch. Appl. Mech. DOI 10.1007/s00419-015-1091-7, Springer-Verlag Berlin Heidelberg 2015

Hier greift natürlich der Übergang der Rißgröße

15.6.2 On Short Cracks (SC) and Long Cracks (LC)

The coalescence of trans-granular micro-cracks in the numerical simulation of crack initiation is performed on the basis of the modified Tanaka-Mura equation (*see next paragraph*). Namely, if two micro-cracks meet each other at the same grain boundary (red solid line in *Fig.15-16*) and if the average stress in between their tips surpasses the elastic limit Re of the material's new micro-crack is created on this grain boundary line, uniting the two trans-granular micro-cracks into a single one. In the example case of pure iron Re is 260 MPa, $R_e \equiv R_{\text{p (proportional)}} \approx R_{0.01}$ structurally practical. Indicator is a temperature increase during tensile testing.

Already nucleated crack segments tend to extend along the whole grain, causing local stress relaxation as well as concentrations at their tips and by that amplifying the likelihood for new crack formation in the vicinity. In the course of further sequences of simulating, micro-cracks form along these slip bands grow and link together.

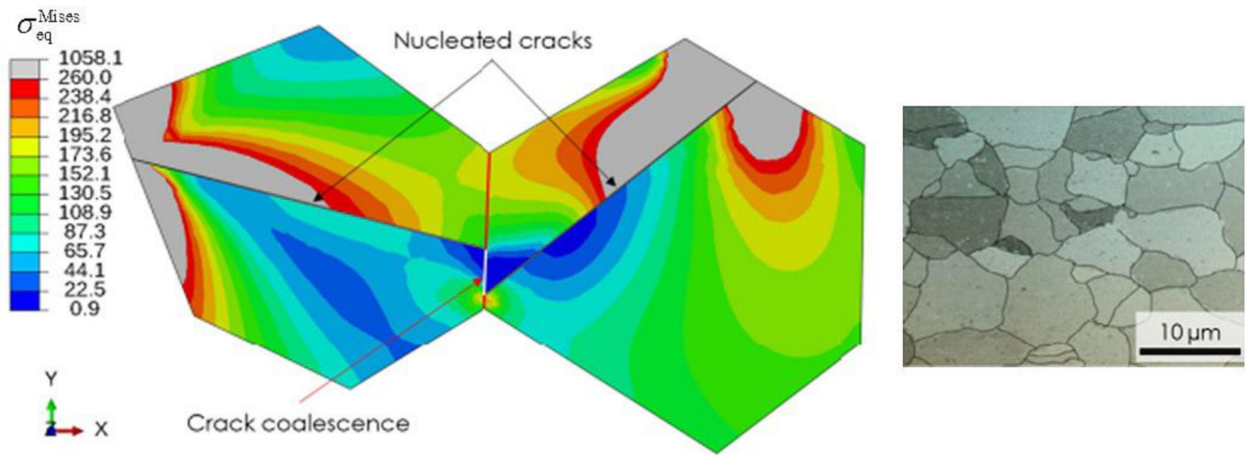

Fig.15-16: Simulation of AA micro-crack coalescence (Lorenzino, P.; Navarro, A. & Krupp, U. (2013), 'Naked eye observations of microstructurally short fatigue cracks', Int. J. of Fatigue 56(0), 8-16.

The change of the crack plane from active crystallographic plane to a non-crystallographic plane perpendicular to the external stress axis is called the transition from Stage I (*crystallographic growth*) to Stage II (*non-crystallographic growth*) or transition from the micro-crack initiation to micro-crack growth stage resulting in a short crack, as depicted in *Fig.15-17*.

However, the dominant short crack does not always continue propagating similar to a long crack. Namely, in the case of a lower stress level, the short crack may stop growing, i.e. it retards. Such situations are typically known as run-outs, see again *Fig.15-17*, which indicates that at very low stress levels an infinite life may be obtained. Run-out below the endurance limit means crack-retardation.

In the long-crack regime the fatigue crack growth rate da/dn can be characterized by the stress intensity factor range ΔK as a dominant driving parameter. A typical fatigue crack growth rate curve da/dn(ΔK) for the long crack is illustrated in *Fig.15-17*, too. If in a double logarithmic scale the LC propagation rate follows a straight line in region II, in sufficient distance from the threshold ΔK_{th} .

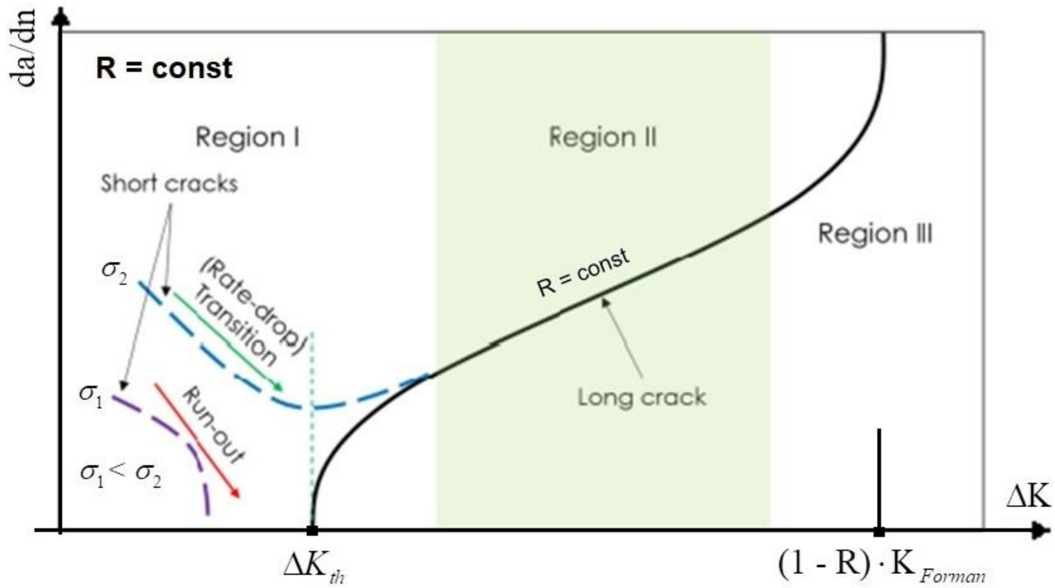

Fig.15-17: Fatigue growth rates of micro-cracks (short) and long cracksin dependence of Δ stress intensity factor. Schematic reperesentation of the loading level- dependent transition from region I into region II. (Newman, J.; Phillips, E. & Swain, M. (1999), 'Fatigue-life prediction methodology using small-crack theory', Int. Journal of Fatigue 21(2), 109-119)

In the LC fatigue crack growth rate domain in Region II most engineering alloys under a certain stress level can be well described by the so-called Paris law:

$$\text{Paris:} \quad da/dn = C_{Paris} \cdot \Delta K^{n_{Paris}} \;, \quad \text{Forman:} \quad da/dn = \frac{C_{Forman} \cdot \Delta K^{n_{Forman}}}{(1\text{-R}) \cdot \text{K}_{Forman} - \Delta K} \quad [HSB\ 63205\text{-}01]$$

In the formula above da/dn is the crack growth increment per cycle, ΔK is the range of stress intensity factor (ΔK = maxK – minK), and C (intercept with the y-axis) and n (slope) are material curve parameters that are deducted by fitting the course of experimental data. K_{Ic} is the so-called fracture toughness.

In order to map all 3 regimes by one function Forman's formula above is to mention. As former contributor to the HSB I like to mention the mathematically excellent procedure of the IASB colleague J. Broede in [*HSB 62211-05*].

For above two 'laws', the curve parameters for many metallic materials are provided in the respective HSB sheets, too, (*however just HSB sheets-contributing IASB members have access*).

By combining the modelling approaches on the different scales, the final numerical result of lifetime is obtained. Eventually, Damage Tolerance demonstrate in the Regions II and III by fracture-mechanical methods using deterministic methods for crack growth and for residual strength or more refined, probabilistic-based methods where risk analysis is required.

<u>LL,</u> viewing structural metals

Metals with different crystallographic structures, namely body-centered cubic (ferrous metals) and face-centered cubic ones show different behavior [Mli19]. Ferrous materials (including steels) generally show a sharp "knee" in the SN curve at about 10^6 cycles, after which the curve increasingly flattens and an endurance limit seems to be obtained. Most of *the materials – like the non-ferrous ones - exhibit a gradual flattening between 10^7 and 10^8 cycles. The so-called Haibach approach by 'halving the angle' is then often applied.*