Which are the Fundamentals and Requirements Strength Failure Conditions should capture ?

Results of a time-consuming „hobby“

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Some well-known Developers which formulated isotropic 3D Strength Failure Conditions (SFCs)

Willam-Warnke, Ottosen etc.

Hencky-
Mises-
Huber

Richard von Mises Eugenio Beltrami Otto Mohr Charles de
Coulomb

1883-1953 1835-1900 1835-1918 1736-1806

Mathematician *Mathematician* *Civil Engineer* *Physician*

'Onset of Yielding' 'Onset of Cracking'

Hence again, a civil engineer may proceed

Motivation for my non-funded Investigations

Existing Links in the Mechanical Behaviour show up: ***Different structural materials***

- ***can possess similar material behaviour or***
- ***can belong to the same class of material symmetry***

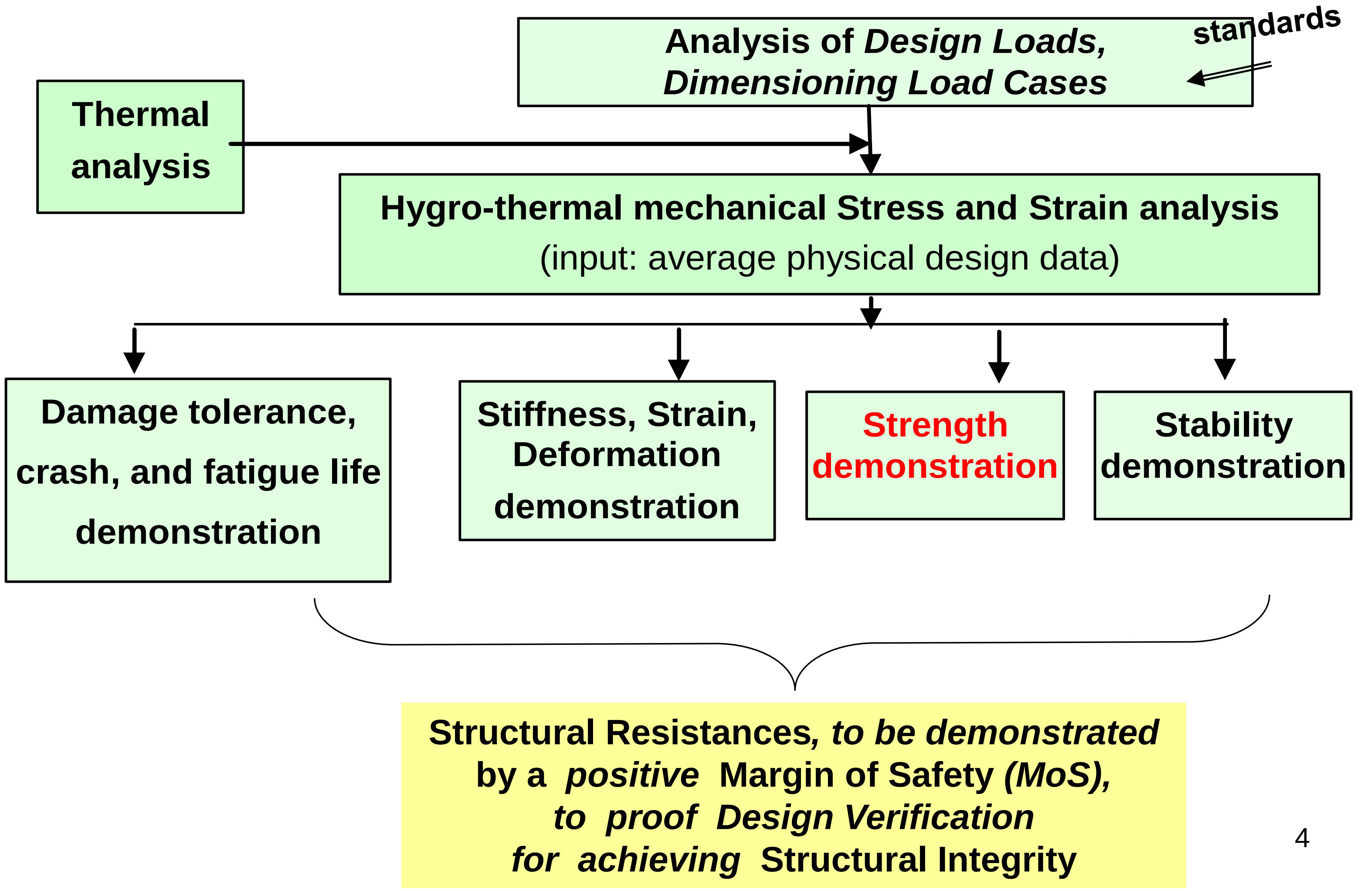
> **similarity aspect**

Welcomed Consequence:

- **The same strength failure function F can be used for different materials**
- **More information is available for pre-dimensioning + modelling**
  from experimental results of a similarly <u>behaving</u> material.

Background: ***Author's experience with structural material applications, range 4 K - 2000 K***

Which Design Verifications are mandatory in Structural Design ?



What do we speak about ?

Material: homogenized (macro-)model of the envisaged solid

Failure: structural part does not fulfil its functional requirements such as onset of yielding, brittle fracture, Fiber-Failure FF, Inter-Fiber-Failure IFF, leakage, deformation limit, delamination size limit, frequency bound

= project-fixed Limit State with *F* = Limit State Function

Failure Criterion: **F** >=< 1 , Failure Condition : *F* = 1= 100%

Failure Theory: general tool to predic failure of a structural part

Strength Failure Condition: subset of a strength failure theory

tool for the assessment of a

'multi-axial failure stress state ' in a critical location of the material.

→ **Stresses** are to be judged by **Strengths !**

Design Verification = Achievement of a Reserve against a Limit State

For each distinct Load Case with its single Failure Modes must be computed:

Reserve Factor (is load-defined) : ***RF = Failure Load / applied Design Load***

Material Reserve Factor : f_{Res} ***= Strength / Applied Stress***

if linear analysis: $f_{Res} = RF = 1 / Eff$

Material Stressing Effort : ***Eff = 100% if RF = 1*** **(Anstrengung)**

(Werkstoff-Anstrengung)

is applicable in linear and non-linear analysis.

Test Data Mapping versus Design Verification

• **<u>Validation of SFCs with Failure Test Data</u>** by

mapping their course by an average Failure Curve (surface)

•**<u>Delivery of a reliable Design Verification</u>** by

calculation of a Margin of Safety or a (load) Reserve Factor

MoS > 0 oder RF = MoS + 1 > 1

on basis of a statistically reduced failure curve (surface) .

Strength Failure Conditions are for homogenized materials

Prediction of ***Onset of Yielding*** + ***Onset of Fracture*** for non-cracked materials

Assessment of multi-axial stress states in a critical material location,

by **utilizing the uniaxial strength values R** and an
equivalent stress σ_{eq}, representing a distinct actual multi-axial stress state.

for *** dense & porous,**
*** ductile & brittle behaving materials,**

ductile : $R_{p0.2} \cong R_{c0.2}$ brittle, dense : $R_m^c \geq 3R_m^t$

for *** isotropic material**
*** transversally-isotropic material (UD := uni-directional material)**
*** rhombically-anisotropic material (fabrics) + ‘higher‘ textiles etc.**

Shall allow for inserting stresses from the utilized various coordinate systems into stress-formulated failure conditions, -and if possible- invariant-based.

WWFE Assumptions for UD Modelling

- **The UD-lamina is macroscopically homogeneous.**

  It can be treated as a homogenized ('smeared') material

- **The UD-lamina is transversely-isotropic:**

  On planes, parallel to the fiber direction it behaves orthotropic and on planes transverse to fiber direction isotropic (quasi-isotropic plane)

- **Uniform stress state about the critical stress 'point' (location)**

Global and Modal Strength Failure Conditions, General View

Drucker-Prager, Tsai-Wu

1 Global ***strength*** **failure condition** : F ({σ}, {R}) = 1 (usual formulation)

Set of Modal **strength failure conditions**: F ({σ}, R^{mode}) = 1 (addressed in FMC)

Mises, Puck, Cuntze

Example: UD

vector of 6 stresses (general)

$$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$$

vector of 5 strengths

$$\{R\} = (R_{||}^t, R_{||}^c, R_{\perp}^t, R_{\perp}^c, R_{\perp ||})^T$$

needs an Interaction of Failure Modes: performed by a

probabilistic-based 'rounding-off' approach (series failure system model)

directly delivering the (material) reserve factor in linear analysis

By-the-way, experience with Failure Prediction shows

Strength Failure Condition (SFC) is a necessary but not a sufficient condition to predict Strength Failure (i.e. thin-layer problem).

Facts of so-called Global SFCs

Global SFCs (one failure surface)

- **Regard all failure modes of the material by one single mathematical formulation. This might even capture a (simplified view) * 2-fold acting failure mode (such as $\sigma_I = \sigma_{II}$: *is a joint failure probability*) or a * 3-fold acting failure mode (such as $p_{hyd} = \sigma_I = \sigma_{II} = \sigma_{III}$)**

- **Requires a re-calculation of all model parameters in the case that a test data change must be performed in a distinct failure mode domain of the multi-fold failure surface (body). Consequence: A change in one failure domain deforms the failure surface in all other – physically independent – failure domains. There is a big chance that a Reserve Factor, to be determined in the independent domain, might be not on the conservative side**

- **There are global SFCs that just use basic strengths as model parameters. This is physically not permitted because Mohr-Coulomb friction acts in the case of brittle behaving materials.**

Note: a distinct failure mode can cause different failure "planes" , is maximum flaw driven

Lode angle J3

Facts of so-called Modal SFCs

Modal SFCs (multi-suface domains)

- **Describe one single failure mode in one single mathematical formulation (= one part of the failure surface)** *

  determine all mode model parameters in the respective failure mode domain *

  capture a twofold acting failure mode separately, such as $\sigma_I = \sigma_{III}$ (isotropic) or $\sigma_2 = \sigma_3$ (transversely-isotropic UD material), mode-wise by the well-known Ansatz f (J_2, J_3)

- **Re-calculation of the model parameters just in that failure mode domain where the test data must be replaced. One RF_{mode} must be freshly determined.**

Material Symmetry Requirements Aspects *(helpful, when generating SFCs*

1 If a material element can be homogenized to an <u>ideal</u> (= frictionless) crystal,

then, material symmetry demands for the transversely-isotropic UD-material

- 5 *elastic 'constants' , 5 strengths, 5 fracture toughnesses* (CF-lamellen) and

- *2 physical parameters (such as CTE, CME, material friction, etc.)*

(for isotropic materials the respective numbers are 2 and 1)

2 Mohr-Coulomb requires for the <u>real</u> crystal another inherent parameter,

- the *physical parameter 'material friction' : UD* $\mu_{\perp\parallel}$, $\mu_{\perp\perp}$ *, Isotropic* μ

3 Fracture morphology witnesses:

- Each strength corresponds to a distinct *failure mode*

and to a *fracture type* as Normal Fracture (NF) or Shear Fracture (SF).

Above Facts and Knowledge gave reason

why the FMC strictly employs single *<u>independent</u>* failure modes

by its <u>failure mode–wise concept</u>.

Interaction of Single Strength Failure Modes in the modal FMC

Interaction of adjacent Failure Modes by a *series failure system* model

= 'Accumulation' of interacting *failure danger portions* Eff^{mode}

$$Eff = \sqrt[m]{(Eff^{\text{mode 1}})^m + (Eff^{\text{mode 2}})^m + \ldots} \quad = 1 = 100\%\text{, if failure}$$

with mode-interaction exponent *2.5 < m < 3 from mapping experience*

***as modal* material stressing effort * (in German Werkstoffanstrengung)**

and

$$Eff^{\text{mode}} = \sigma_{eq}^{\text{mode}} / \overline{R}^{\text{mode}}$$

equivalent mode stress

mode associated average strength

later → example

* artificial technical term created together with QinetiQ

Cuntze's Pre-design Input for 3D UD SFCs

Test Data Mapping | **Design Verification**

- **5 strengths :** $\{\overline{R}\} = (\overline{R}_{||}^{t}, \overline{R}_{||}^{c}, \overline{R}_{\perp}^{t}, \overline{R}_{\perp}^{c}, \overline{R}_{\perp||})^{T}$ $\{R\} = (R_{||}^{t}, R_{||}^{c}, R_{\perp}^{t}, R_{\perp}^{c}, R_{\perp||})^{T}$

  average (typical) values | **strength design allowables**

- **2 friction values :** for 2D $\mu_{\perp||}$, for 3D $\mu_{\perp||}, \ \mu_{\perp\perp}$

  $\mu_{\perp||} = 0.1$ $\mu_{\perp\perp} = 0.1$

  values, recommended for pre-design

- **1 mode-interaction exponent :** ***m* = 2.6 .**

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by its <u>failure mode–wise concept</u>.

Which failure types (brittle or ductile) are observed ?

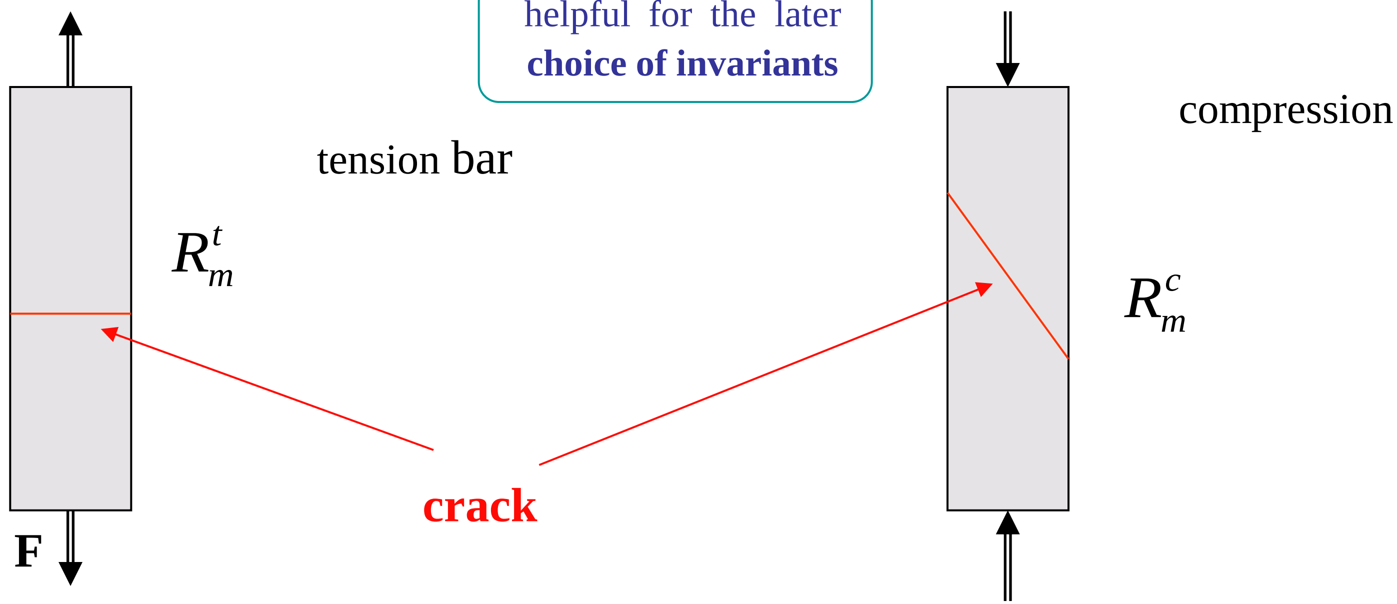
Cleavage fracture (NF) (Spaltbruch, Trennbruch) :

- poor deformation before fracture
- 'smooth' fracture surface

Shear fracture (SF) :

- shear deformation before fracture

knowledge is helpful for the later **choice of invariants**



conclusion: ▶ **2 strengths** to be measured

if brittle: failure = fracture failure

Isotropic Material *brittle*, *porous* *for UD-material*

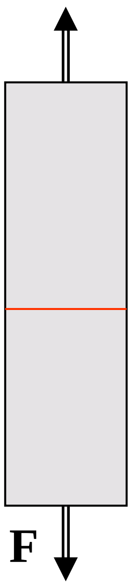
Normal Fracture (NF) (Spaltbruch, Trennbruch) :

- poor deformation before fracture
- rough fracture surface

Crushing Fracture (CrF): ⇐ **SF**

- volumetric deformation before fracture

helpful for the l
choice of invariants



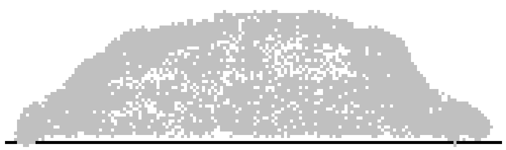
Tension

R_m^t

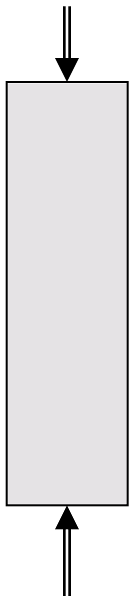
Compression

result of the compression test

= *hill of fragments* (crumbs)



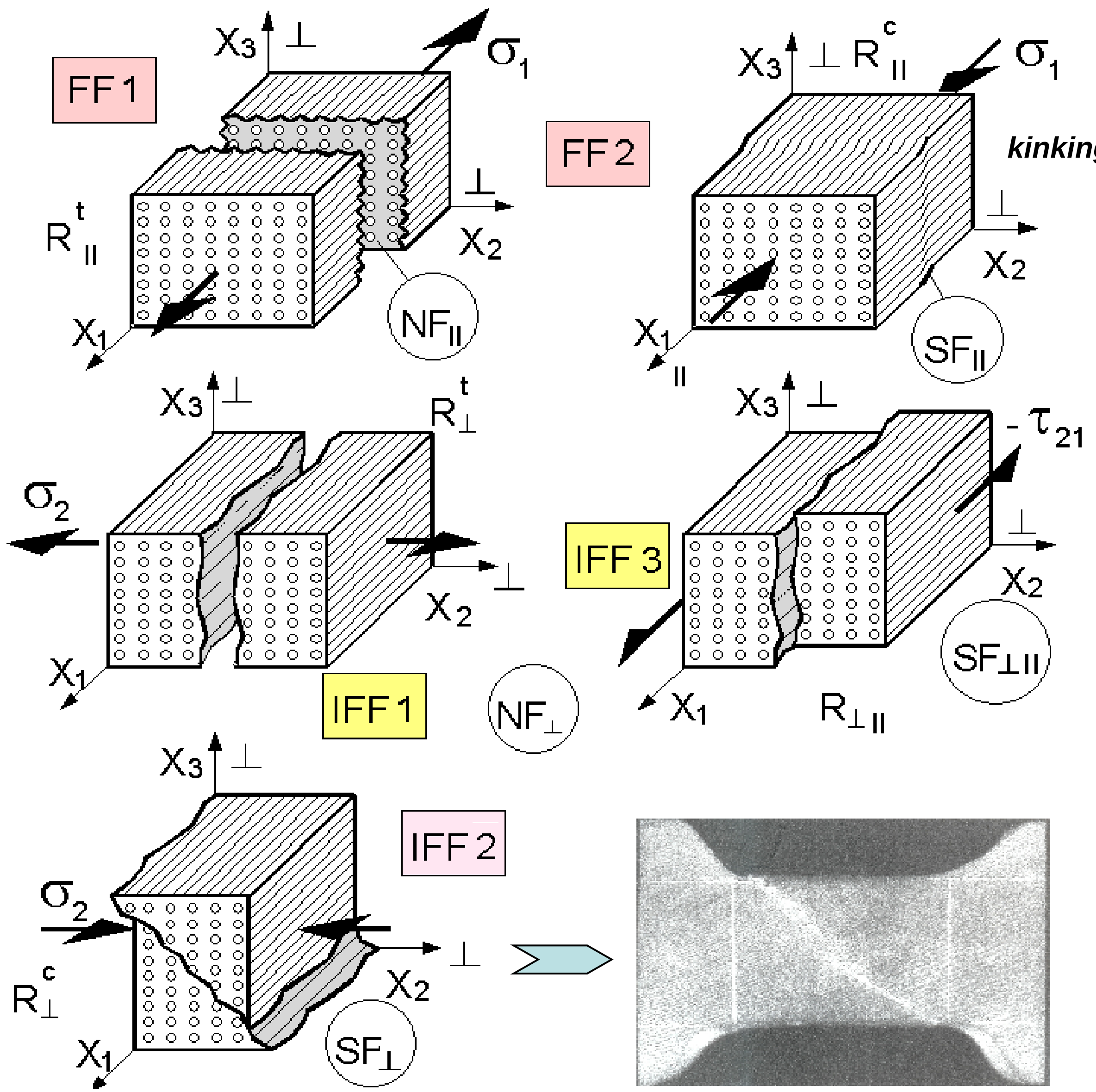
= decomposition of texture



R_m^c

► **2 strengths** to be measured

Observed Strength Failure Modes with Strengths of brittle UD Materials



wedge failure type

t = tension
c = compression

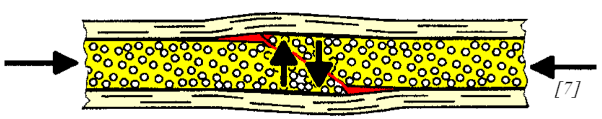
- **5 Fracture modes exist**

= 2 FF (Fibre Failure)
+ 3 IFF (Inter Fibre Failure)

Fracture Types:

NF := Normal Fracture

SF := Shear Fracture



Physically-based Choice of Invariants when generating invariant-based Strength Failure Conditions

* **Beltrami : "At 'Onset of Yielding' the material possesses a distinct *strain energy* composed of *dilatational energy* (I_1^2) and *distortional energy* ($J_2 \equiv Mises$) ".**

*** So, from Beltrami, Mises (HMH), and Mohr / Coulomb (friction) can be concluded:**

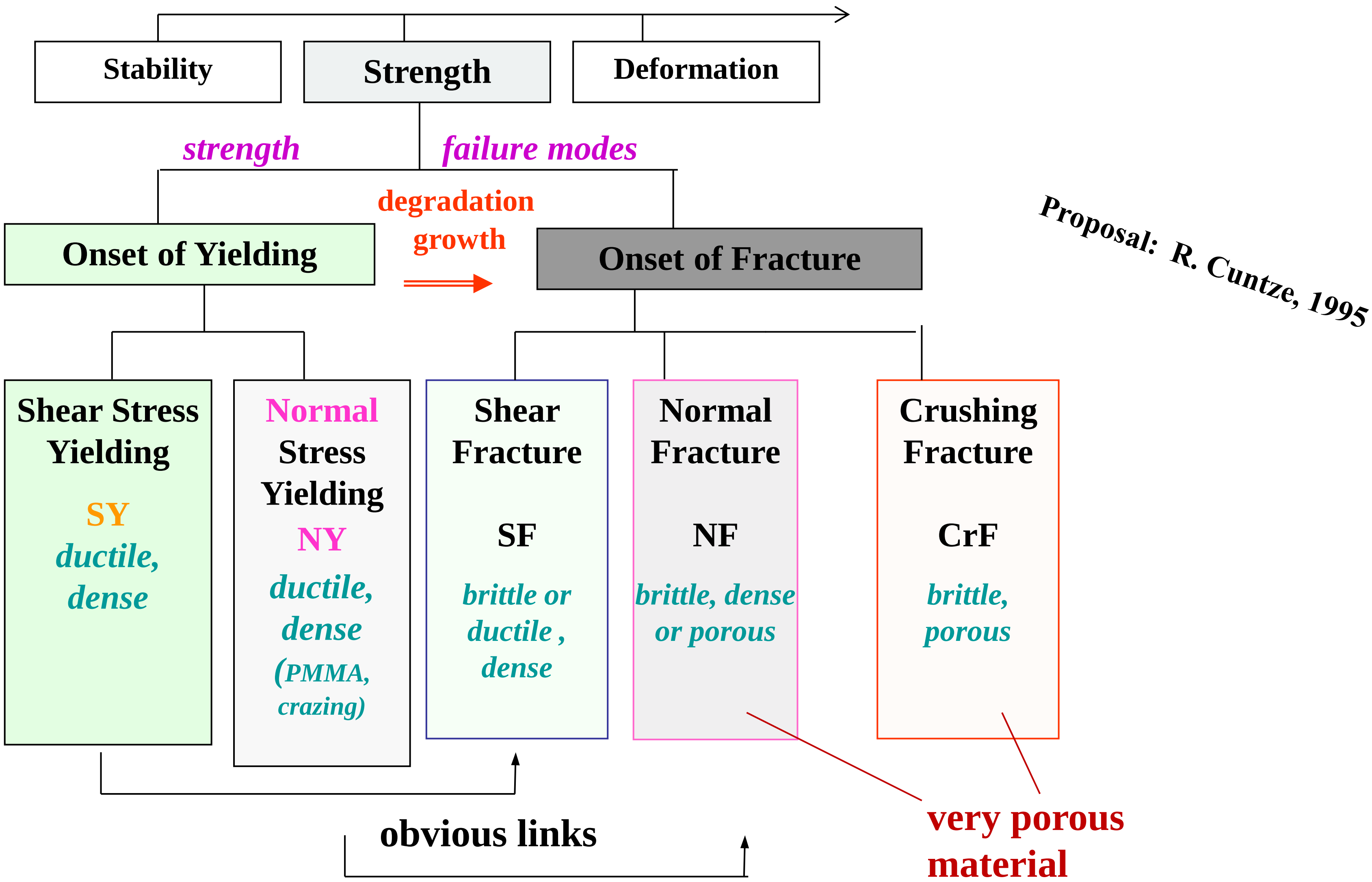
Each invariant term in the *failure function F* may be dedicated to one physical mechanism in the solid = cubic material element:

- volume change : I_1^2 … *(dilatational energy)* — relevant if porous

- shape change : J_2 (Mises) … *(distortional energy)* — relevant if brittle behaving; relevant if material element shape changes

and - friction : I_1 … *(friction energy)* — Mohr-Coulomb

Scheme of Strength Failures Types for *isotropic materials*



Note: The growing yield body (SY or NY) is confined by the fracture surface (SF or NF)!

Material Homogenizing (smearing) + Modelling

Investigation of the tensorial stress-strain relationships of materials

6x6 stress tensor and 3x3 physical properties respecting tensor **results in**

Material symmetry says and test evidence supports:

Number of strengths ≡ number of elasticity properties !

Application of material symmetry knowledge:

- ***Requires that homogeneity is a valid assessment for the <u>task-determined</u> model ,***

but, if applicable

- A ***minimum number of properties must be measured, only*** **(cost + time benefits) !**

For isotropic brittle behaving material, this means:

*** 2 material parameters of the ideal elastic material determining orthogonal stress plane (= π- or hoop plane of the fracture failure body)**

*** 1 material friction parameter μ of the non-ideal material due to friction inherent to brittle behav. material determining the slope of the meridians (axial shape of the fracture failure body)**

Material Homogenizing (smearing) + **Modelling**

Material symmetry shows:

Number of strengths ≡ number of elasticity properties !

Application of material symmetry knowledge:

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- A *minimum number of properties has to be measured, only* (cost + time benefits) !

4 Short Derivation of the Failure Mode Concept (FMC)

Failure Theory and Failure Conditions

A **3D Failure Theory** has to include:

1. Failure Conditions to ***assess multi-axial states of stress***
2. Non-linear Stress-strain Curves of a material as input
3. Non-linear Coding for structural analysis

A Failure Condition is the mathematical formulation of the failure surface !

Pre-requisites for the establishment of failure conditions are:

- **simply formulated, numerically robust,**
- **physically-based, and therefore, need only few information for pre-dimensioning**
- **shall allow for a simple determination of the design driving reserve factor.**

Basic Features of the author‘s Failure-Mode-Concept (FMC)

- **Each failure mode represents 1 independent failure mechanism and thereby 1 piece of the complete *failure surface***
- **Each failure mechanism is governed by 1 basic strength (is observed !)**
- **Each failure *mode* can be represented by 1 failure *condition.***

  Therefore, equivalent stresses can be computed for each mode !!

- **In consequence, this separation requires :**

  An interaction of the Modal Failure Modes !

Fundamentals of the FMC (*example*: UD material)

Remember:

- **Each of the observed fracture failure modes was linked to one strength**
- **Symmetry of a material showed :** ***Number of strengths =*** $R_{||}^{t}, R_{||}^{c}, R_{\perp||}, R_{\perp}^{t}, R_{\perp}^{c}$

  number of elasticity properties ! $E_{||}, E_{\perp}, G_{||\perp}, \nu_{\perp||}, \nu_{\perp\perp}$

Due to the facts above the

FMC postulates in its '*Phenomenological Engineering Approach*' :

► Number of failure modes = number of strengths, too !

e.g.: isotropic = 2 or above transversely-isotropic (UD) = 5

t:= tensile, c: = compression, || : = parallel to fibre, := transversal to fibre

Interaction of Single Strength Failure Modes in the FMC

Interaction of adjacent Failure Modes by a *series failure system* model

= 'Accumulation' of interacting *failure danger portions* Eff^{mode}

$$Eff = \sqrt[m]{(Eff^{\text{mode 1}})^m + (Eff^{\text{mode 2}})^m + \ldots.}$$ **= *1 = 100% , if failure***

with mode-interaction exponent *m from mapping experience*

and

$$Eff^{\text{mode}} = \sigma_{eq}^{\text{mode}} / \overline{R}^{\text{mode}}$$

***modal* material stressing effort**

(Werkstoffanstrengung)

equivalent mode stress

mode associated average strength

Formulation of Failure-Mode-Concept (FMC)-based Modal SFCs by Using

- **Invariants**
- **Hypotheses of**

  Beltrami = dedication of invariants to the deformation of the material element, whether it is a shape change (Mises) or a volume change and
  Mohr-Coulomb = internal friction of a brittle behaving solid material
- **Application of the Reqirements of Material Symmetry** = for isotropic brittle behaving materials the characteristic number of quantities is 2 (2 strengths, 2 strength fracture failure modes, 2 basic invariants)
- advantegeous **equivalent stresses** σ_{eq} and of the physically plausible **material stressing effort** (Werkstoffanstrengung) *Eff*

Consequence for needed number of parameters:

***Tension*: 1 strength parameter. *Compression*: 1 strength + 1 friction parameter. *Interaction*: exponent *m*.**

*** The "requirements" of material symmetry are backed by test observation.**

*** The bi-axial dents in the hoop plane are the consequence of a 2-fold occurring failuremode. The depth of the dent can be either calculated by an effortful probabilistic analysis or by elegantly using J3 as a good shape-giving third invariant to capture the bi-axial additional failure danger.**

*** Explanation of a multifold failure mode of a dense brittle behaving material :**

Uni-axial compression creates one failure mode *but* there are multiple fracture planes possible activated by the spatial flaw distribution with the critical maximum local flaw

<u>Note: C</u>haracteristic number of quantities for the transversely-isotropic unidirectional material UD is 5

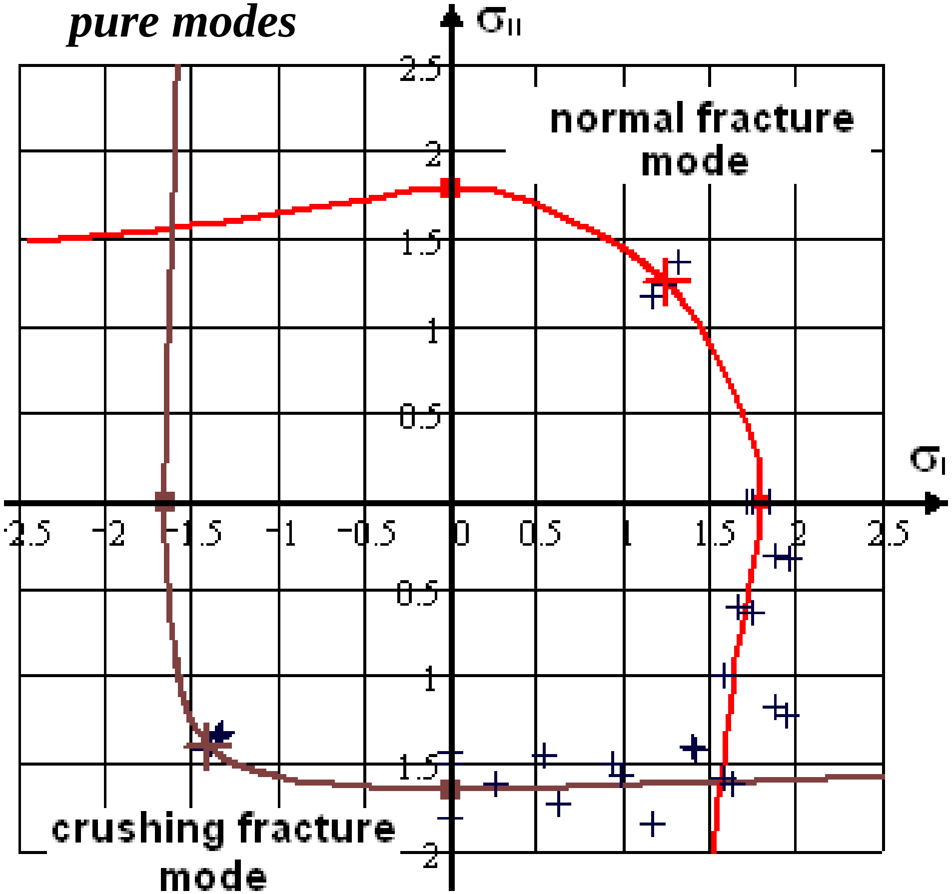
2D - Test Data Set and Mapping in the Principal Stress Plane

Rohacell 71 IG

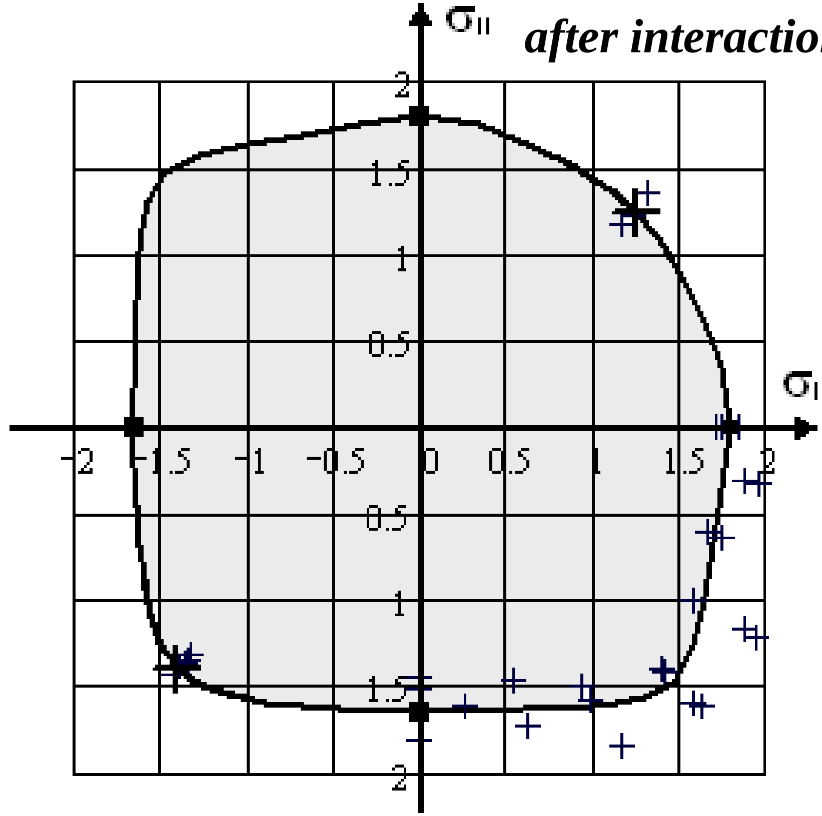
Principal Plane Cross-section of the Fracture Body (oblique cut)

as similarly behaving material

pure modes



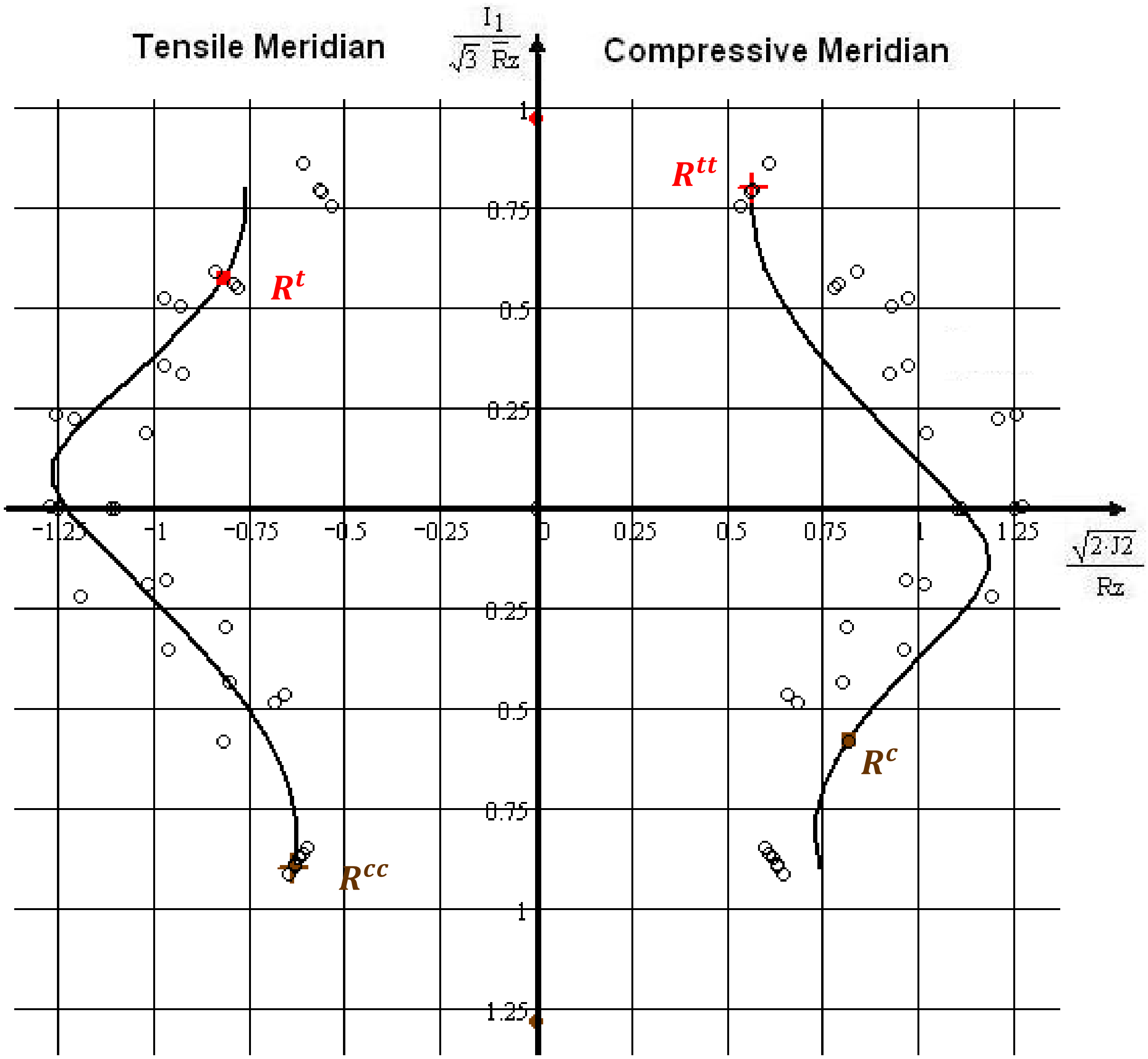
after interaction



- **Mapping <u>must</u> be performed in the 2D-plane because fracture data set is given there**
- **The 2D-mapping uses the 2D-subsolution of the 3D-strength failure conditions**
- **The 3D-fracture failure surface (body) is based on the 2D-derived model parameters.**

Courtesy: LBF-Darmstadt, Dr. Kolupaev

Generic Lines of Tensile and of Compressive Meridian



Meridional cross-sections of the Fracture Body

in Lode-Haigh-Westergaard coordinates

bi-axial = +

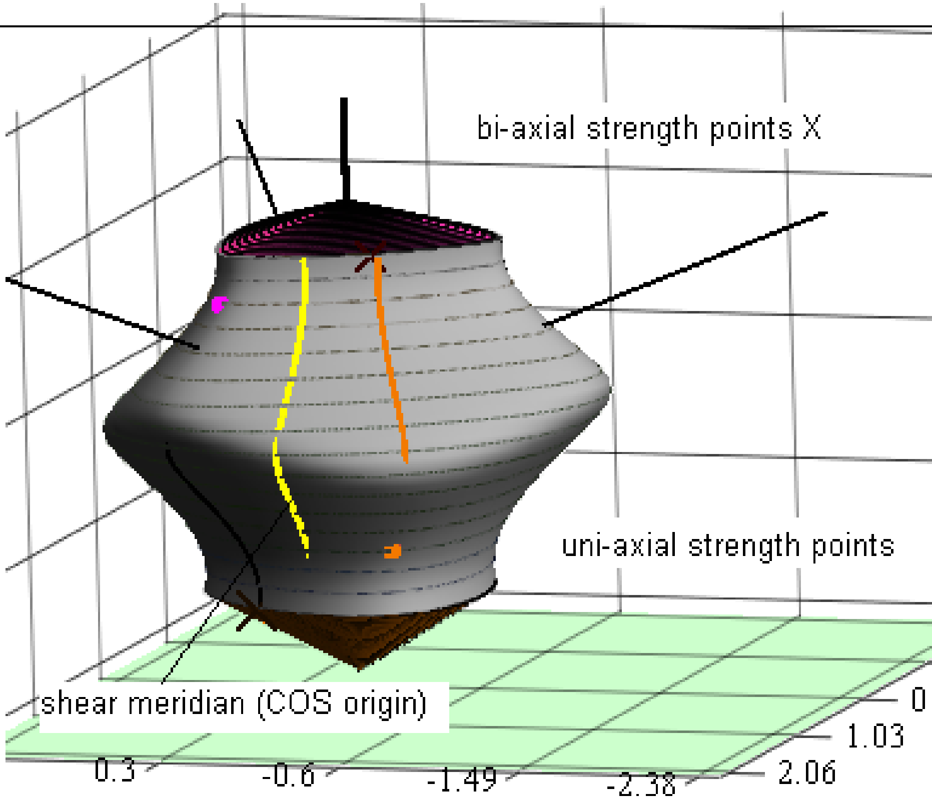
z = tensile, d = compressive

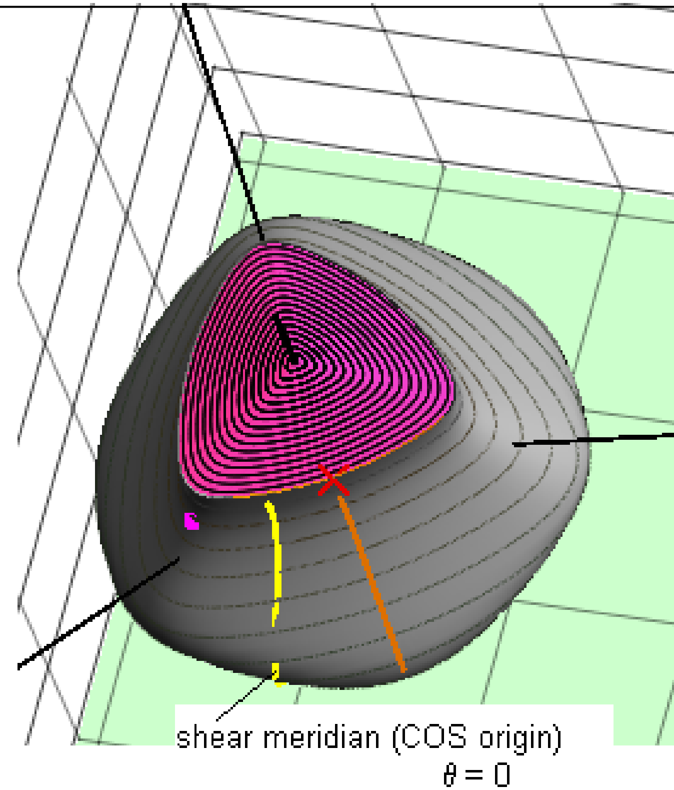
The fracture test data are located at a distinct Lode angle of its associated ring o, 120°-symmetry of the isotropic failure surface (body) .

Cap and bottom are closed by a cone-ansatz, a shape being on the conservative side.

Fracture Failure Surface of *Rohacell 71 IG*

The dent turns !

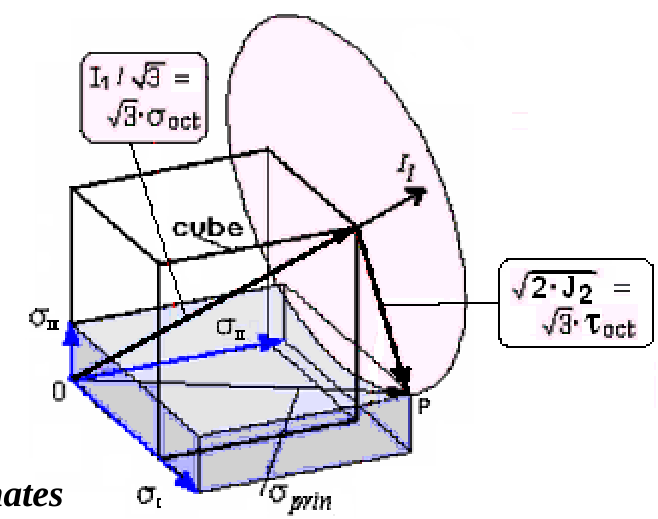




compressive meridian with tensile meridian form one cross-section shape

The 3D-strength failure condition enables to predict the 120°-symmetric failure body and to judge a 3D- stress state

visualization of the Lode-Haigh-Westergaard coordinates

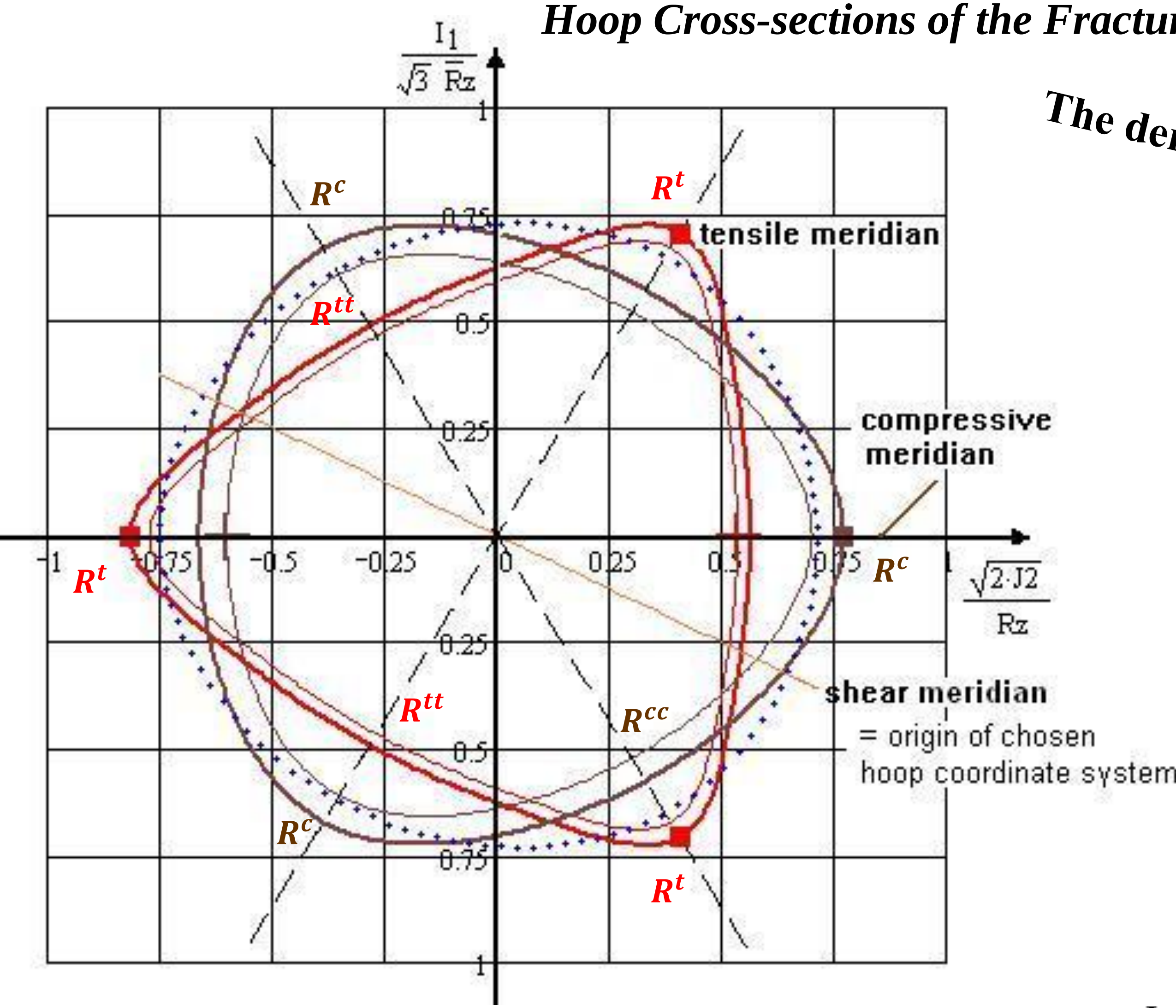


2D Test Data and Mapping in the Octahedral Stress Plane

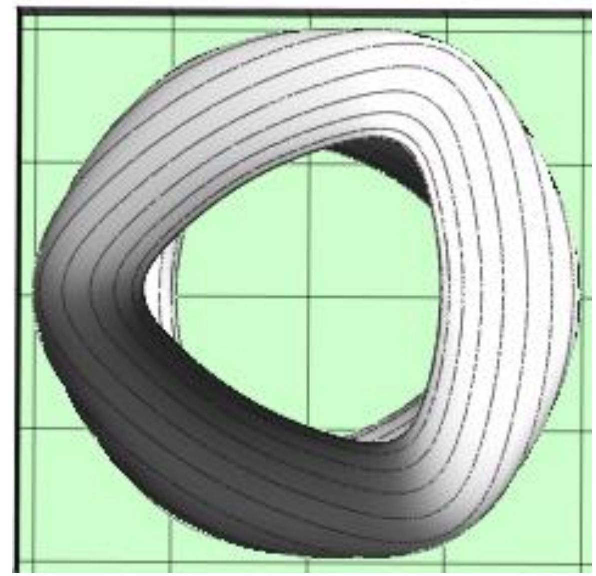
Rohacell 71 IG

as similarly behaving material

Hoop Cross-sections of the Fracture Body



The dent turns !



Caps: No test data, cone was chosen.

Lode-angle, here set as $\sin(3\,\theta)$:

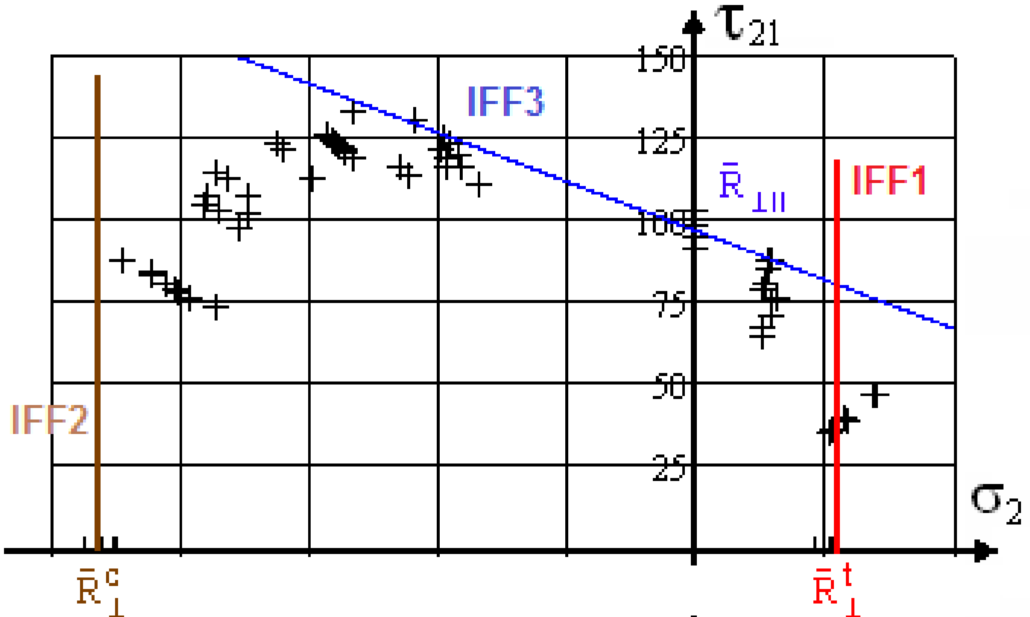
shear meridian angle = 0°

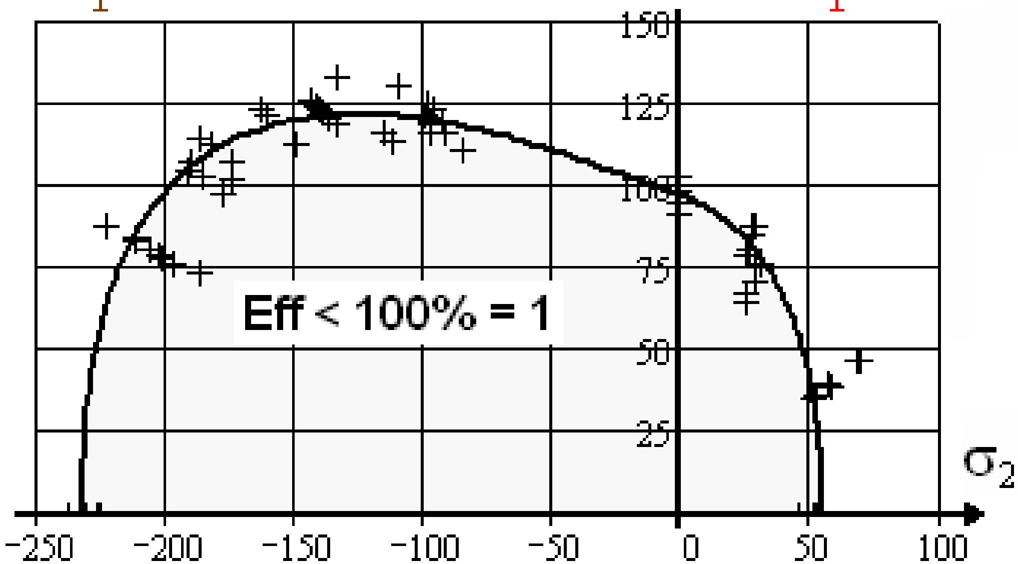
tensile meridian +30° +

compressive meridian -30° +

I1 = 0, is interaction domain: Is about a circle.

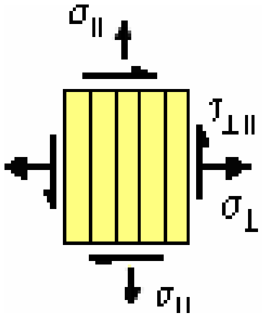
Interaction Visualization of UD Failure Modes $\tau_{21}(\sigma_2)$ $\breve{\sigma}_1 = 0$





Mapping of course of IFF test data in a pure mode domain by the *single Mode Failure Condition.*

3 IFF pure modes = straight lines !.



IFF 1 : $\frac{\sigma_2}{\bar{R}^t_\perp} = 1$

IFF 2 : $\frac{-\sigma_2}{\bar{R}^c_\perp} = 1$

IFF 3 (2D simplified) : $\frac{|\tau_{21}|}{\bar{R}_{\perp||} - \mu_{\perp||} \cdot \sigma_2} = 1$

Mapping of course of test data by *Interaction Model*

$$(Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp||})^m = 1$$

$$m = 2.5, \ \mu_{\perp||} = 0.3$$

Determination of the <u>Load-defined</u> Reserve Factor RF for a foam

Linear elastic problem for this brittle behaving material

Residual stresses = 0

$$\mathbf{RF} = f_{Res} \text{ (material reserve factor)} = \boldsymbol{Eff^{-1}}$$

<u>Stress state:</u>

$\sigma I := 0.9 \qquad \sigma II := -0.4 \qquad \sigma III := 0.5$

<u>Statistically reduced Strengths:</u>

$Rz := 0.9 \cdot \bar{R}z \qquad Rd := 0.85 \cdot \bar{R}d$

<u>Shape parameters:</u> $D\sigma = -0.71 \qquad D\tau = 0.21 \qquad c1\Theta\sigma = 1.15 \qquad c1\Theta\tau = 1.03$

$$I1 := \sigma I + \sigma II + \sigma III \qquad J2 := \frac{\left[(\sigma I - \sigma II)^2 + (\sigma II - \sigma III)^2 + (\sigma III - \sigma I)^2\right]}{6} \qquad J3 := \frac{\left[(2\cdot\sigma I - \sigma II - \sigma III)\cdot(2\cdot\sigma II - \sigma I - \sigma III)\cdot(2\cdot\sigma III - \sigma II - \sigma I)\right]}{27}$$

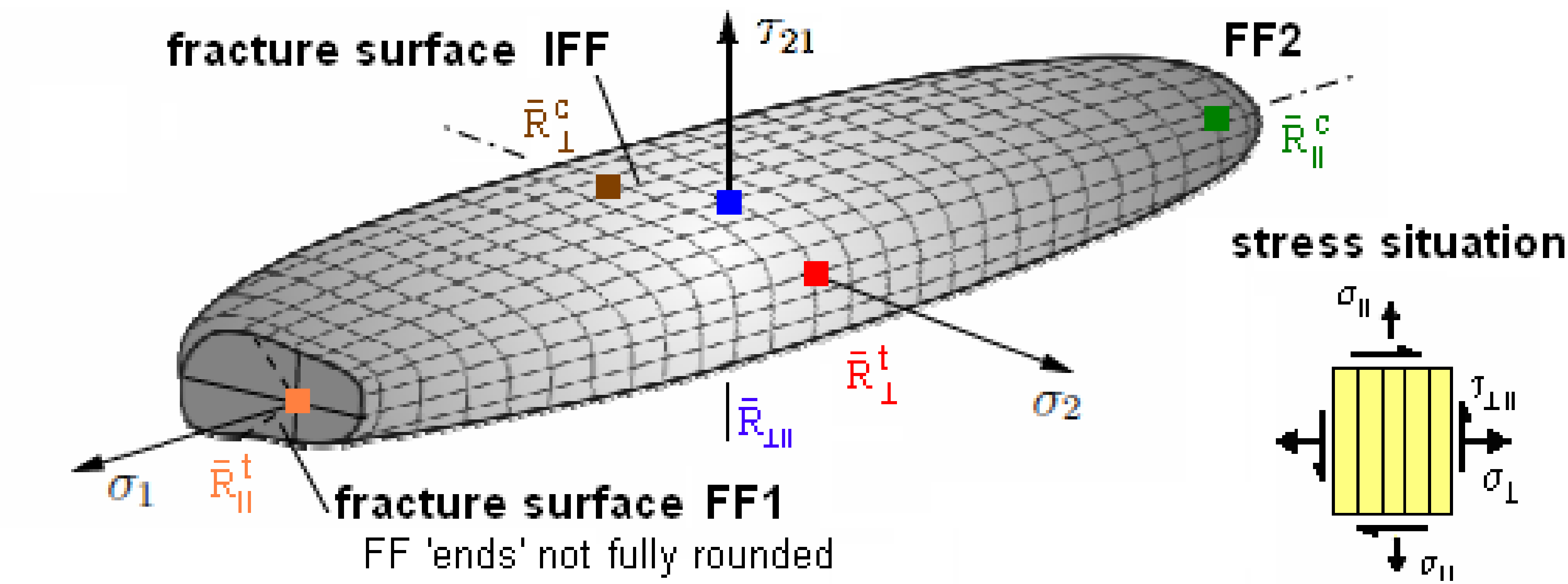
$I1 = 1 \qquad J2 = 0.44 \qquad J3 = -0.07$

$$Eff\Theta\sigma := c1\Theta\sigma \cdot \frac{\sqrt{4\cdot J2 \cdot \sqrt[3]{1 + D\sigma \cdot 1.5 \cdot 3^{0.5} \cdot J3 \cdot J2^{-1.5}} - \frac{1}{3}\cdot I1^2} + I1}{2Rz} \qquad Eff\Theta\tau := c1\Theta\tau \cdot \frac{\sqrt{4\cdot J2 \cdot \sqrt[3]{1 + D\tau \cdot \left(1.5\cdot 3^{0.5}\cdot J3 \cdot J2^{-1.5}\right)} - \frac{1}{3}\cdot I1^2} - I1}{2\cdot Rd}$$

$$Eff := \sqrt[9]{Eff\Theta\sigma^{m_{int}} + Eff\Theta\tau^{m_{int}}} \qquad Eff = 0.802 \qquad RF := \frac{1}{Eff} \qquad RF = 1.25$$

The loading may be monotonically increased by the factor RF !

Visualization of 2D UD SFCs as Fracture Failure Surface (Body)



$$\{\sigma\}=(\sigma_1,\sigma_2,0,0,0,\ \tau_{21})^T$$

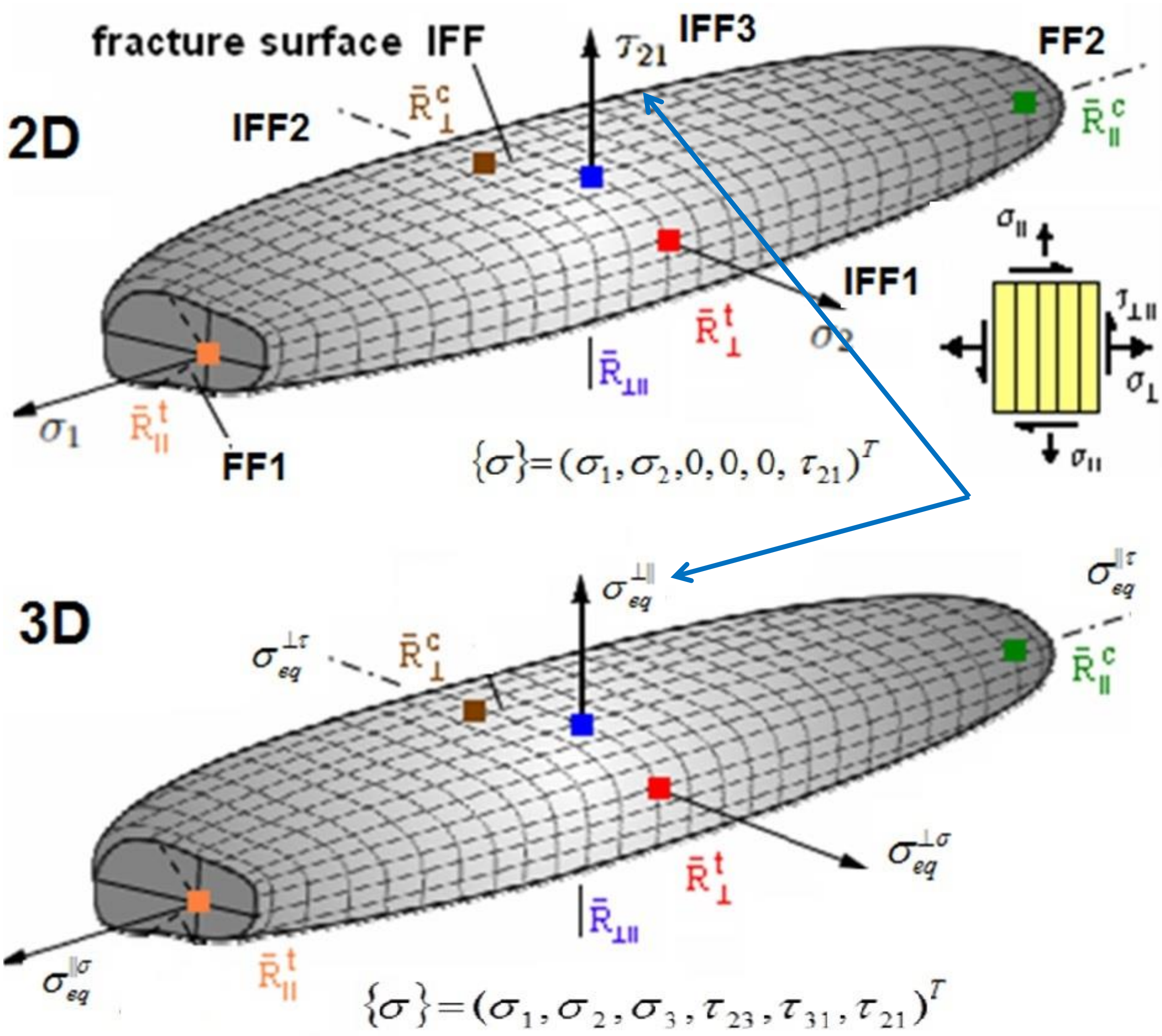
Mode interaction fracture failure surface *of FRP UD lamina*

$$Eff^m = (Eff^{\parallel\tau})^m + (Eff^{\parallel\sigma})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m = 1$$

(courtesy W. Becker) .

Mapping: Average strengths indicated

2D = 3D Fracture surface by replacing the stress by the equiv. stress



WWFE-II Set of <u>Modal</u> 3D UD Strength Failure Conditions (criteria)

Invariants replaced by their stress formulations

strains from FEA **[Cun04, Cun11]**

FF1 $Eff^{||\sigma} = \breve{\sigma}_1 / \bar{R}_{||}^t = \sigma_{eq}^{||\sigma} / \bar{R}_{||}^t ,\quad \breve{\sigma}_1 \cong \varepsilon_1^t \cdot E_{||}$ *

FF2 $Eff^{||\tau} = -\breve{\sigma}_1 / \bar{R}_{||}^c = +\sigma_{eq}^{||\tau} / \bar{R}_{||}^c ,\quad \breve{\sigma}_1 \cong \varepsilon_1^c \cdot E_{||}$

2 filament modes

IFF1 $Eff^{\perp\sigma} = [(\sigma_2 + \sigma_3) + \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}{}^2}\,] / 2\bar{R}_{\perp}^t = \sigma_{eq}^{\perp\sigma} / \bar{R}_{\perp}^t$

IFF2 $Eff^{\perp\tau} = [(\frac{\mu_{\perp\perp}}{1-\mu_{\perp\perp}}) \cdot (\sigma_2 + \sigma_3) + \frac{1}{1-\mu_{\perp\perp}} \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}{}^2}\,] / \bar{R}_{\perp}^c = +\sigma_{eq}^{\perp\tau} / \bar{R}_{\perp}^c$

3 matrix modes

IFF3 $Eff^{\perp||} = \{[\mu_{\perp||} \cdot I_{23-5} + (\sqrt{\mu_{\perp||}{}^2 \cdot I_{23-5}{}^2 + 4 \cdot \bar{R}_{\perp||}{}^2 \cdot (\tau_{31}^2 + \tau_{21}^2)^2}\,] / (2 \cdot \bar{R}_{\perp||}{}^3)\}^{0.5} = \sigma_{eq}^{\perp||} / \bar{R}_{\perp||}$

with $I_{23-5} = 2\sigma_2 \cdot \tau_{21}^2 + 2\sigma_3 \cdot \tau_{31}^2 + 4\tau_{23}\tau_{31}\tau_{21}$

Modes-Interaction :

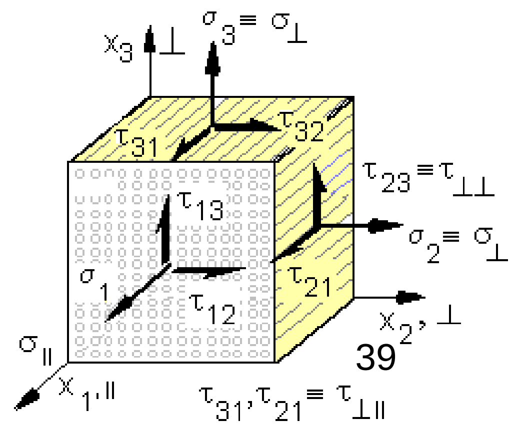
$$Eff^m = (Eff^{||\tau})^m + (Eff^{||\sigma})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp||})^m = 1$$

with mode-interaction exponent $2.5 < m < 3$ **from mapping tests data**

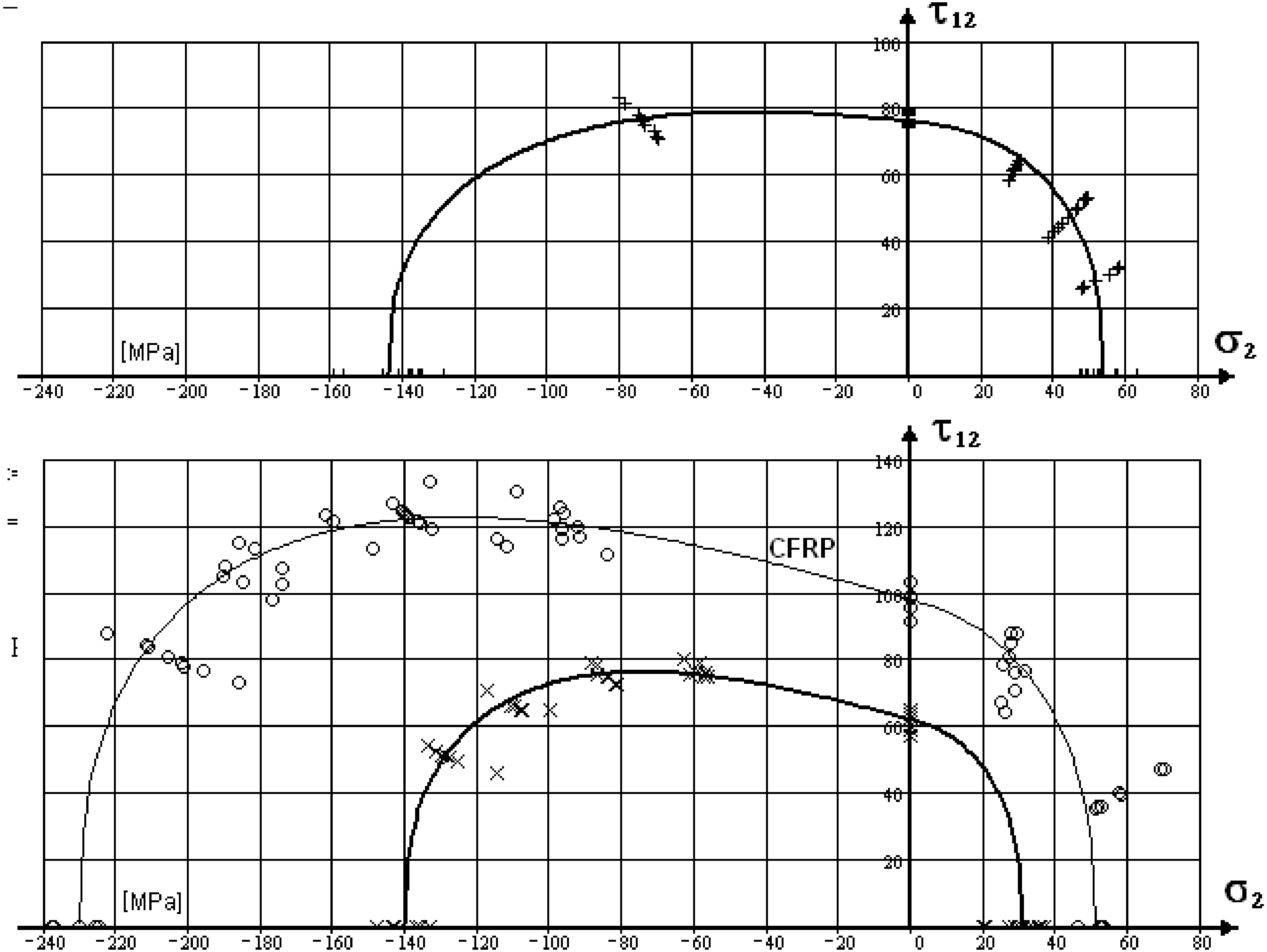
Typical friction value data range: $0.05 < \mu_{\perp||} < 0.3, \quad 0.05 < \mu_{\perp\perp} < 0.2$

Poisson effect * : bi-axial compression strains the filament without any σ_1

t:= tensile, c: = compression, || : = parallel to fibre, ⊥ := transversal to fibre



Determination of the Load-defined Reserve Factor RF



I1 :=

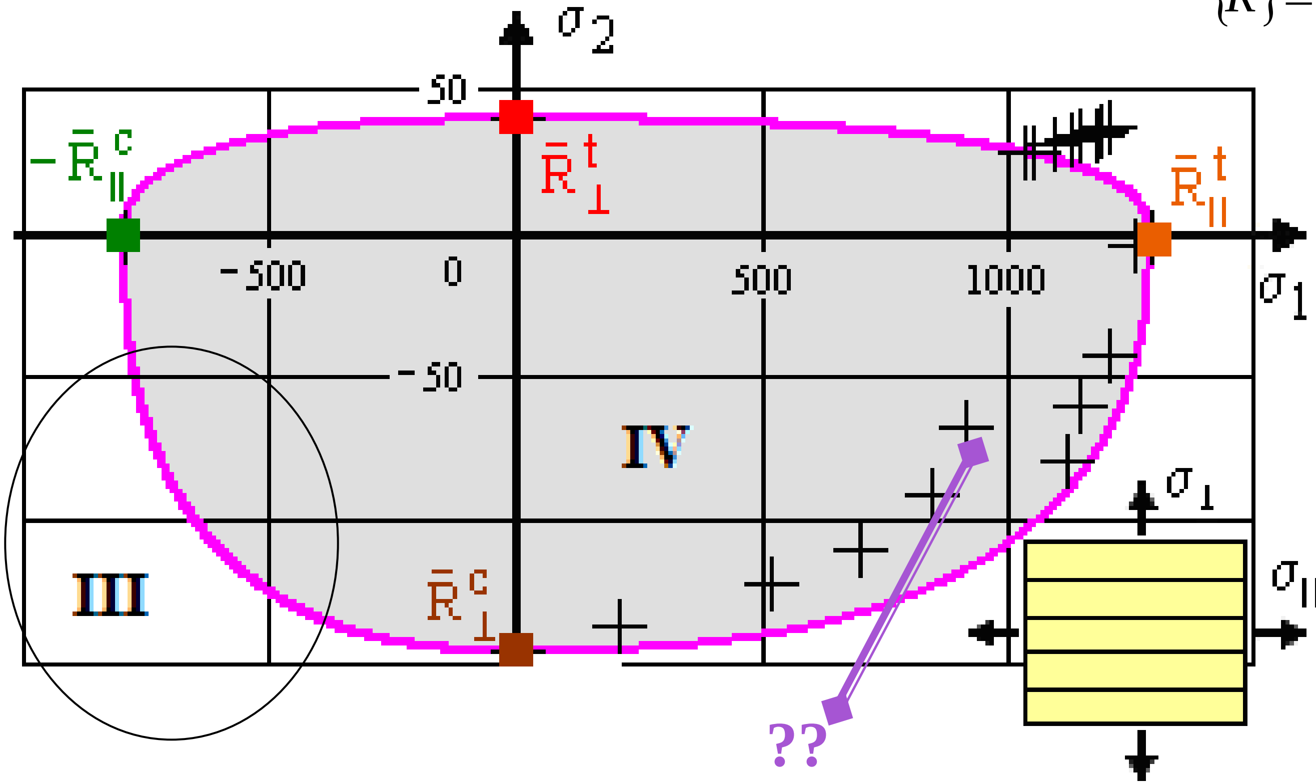
I1 =

from given

ge value

Test Case 3, WWFE-I $\sigma_2(\breve{\sigma}_1 \equiv \sigma_1)$

$\{\bar{R}\} = (1280,\ 800,\ 40,\ 145,\ 73)^T$



Hoop wound tube UD-lamina. E-glass/MY750epoxy +

$\sigma_1 = \sigma_{hoop}$

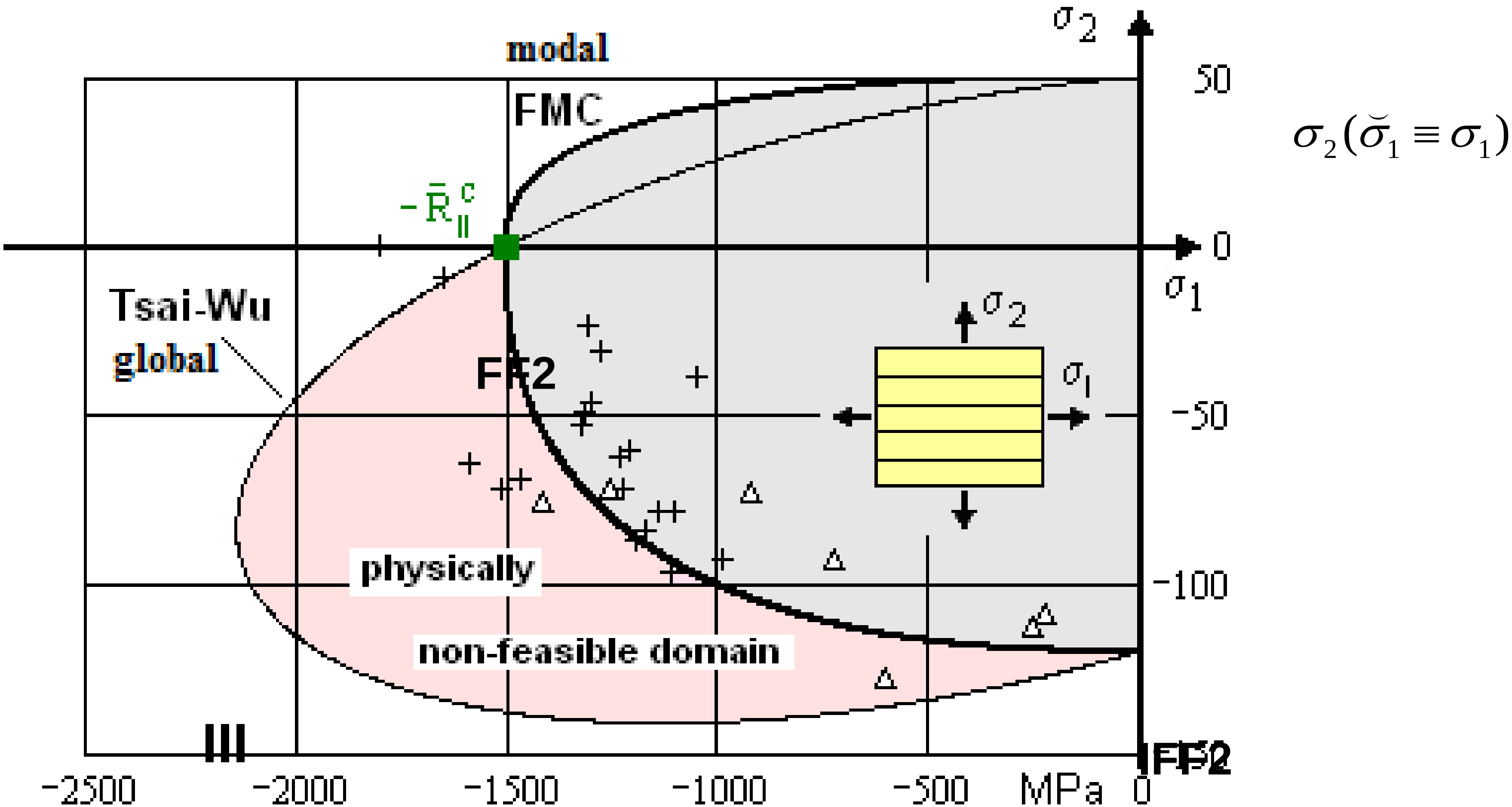
$\sigma_2 = \sigma_{axial}$

Part A: Data of strength points were provided, only

Part B: Test data in quadrant IV show discrepancy , testing?

No data for quadrants II, III was provided ! But, ..

Mapping in the ‘Tsai-Wu non-feasible domain‘ (quadrant III)



Data: courtesy IKV Aachen, Knops

Lesson Learnt: The modal FMC maps correctly, the *global* Tsai-Wu formulation predicts a non-feasible domain !

Conclusions

- **The FMC is an efficient concept,**

  that improves prediction + simplifies design verification

  is applicable to brittle and ductile, dense and porous, isotropic, transversely-isotropic and orthotropic materials

  if clear failure modes can be identified and the material element homogenized.

  Formulation basis is whether the material element experiences a *volume* change, a *shape change* and *friction* .

  Builds not on the ***material*** but on ***material behaviour !***

- **Delivers a <u>combined formulation</u> of *independent modal failure modes*,**

  without the well-known drawbacks of <u>global</u> SFC formulations

  (which *mathematically combine in-dependent failure modes)* .

- **The FMC-based Failure Conditions are simple but describe physics of each single failure mechanism pretty well.**

- **Mapping of the brittle behaving porous foam was successful and with new findings !**

Conclusions wrt. Beltrami-based *Failure Mode Concept*

- **The FMC – applied to UD material - is an efficient concept,**

  that improves prediction + simplifies design verification.

  Formulation basis is whether the material element experiences a *volume* change, a *shape change* and *friction* .

- **Delivers a combined formulation of *independent modal failure modes,***

  without the well-known drawbacks of global SFC formulations

  (which *mathematically combine in-dependent failure modes)* .

- **The FMC-based 3D UD Strength Failure Conditions are simple but describe physics of each single failure mechanism pretty well.**

Conclusions wrt Failure Mode Concept

- The FMC is an efficient concept,

  that improves prediction + simplifies design verification

  is applicable to brittle and ductile, dense and porous,
  isotropic, transversely-isotropic and orthotropic materials

  if clear failure modes can be identified and if the material element can be homogenized.

  Formulation basis is whether the material element experiences
  a *volume* change, a *shape change* and *friction* .

  Builds not on the *material* but on *material behaviour !*

- Delivers a <u>combined formulation</u> of *independent modal failure modes*,

  without the well-known drawbacks of <u>global</u> SFC formulations

  (which *mathematically combine in-dependent failure modes)* .

- The FMC-based Failure Conditions are simple but
  describe physics of each single failure mechanism pretty well.

- Mapping of brittle behaving concretes was successful, thereby validating the SFC models . Some new findings were provided !

Conclusions wrt SFCs

- **A modal SFC shall and can only describe a <u>1-fold</u> occurrence of a failure mode.**

- **A multi-fold occurrence is considered in the formulas:**

  <u>2-fold</u> $\sigma_{II} = \sigma_I$ **(probabilistic effect) is elegantly solved with** J_3

  <u>3-fold</u> $\sigma_{II} = \sigma_I = \sigma_{III}$ **(prob. effect) hydrost. compression, by closing-ansatz**

- **Dents in the *I1<0* – domain are oppositely located to those in the *I1>0* - domain**

- **The Poisson effect, generated by a Poisson ratio *v,* may cause tensile failure under bi-axially stressing (dense concrete)**

  (analogous to UD material, where filament tensile fracture may occur without any external tension loading)

- **Hoop Planes = deviatoric planes =** $\pi -$ **planes: *convex***

- **Meridian Planes : *not convex !***

Some Lessons Learnt

- **Prediction of shear fracture failure of brittle behaving materials is not possible, if the physically necessary friction value μ , being the 3rd model parameter is not known or cannot be determined by a test data fit.**
  Some global SFCs do not consider friction and therefore have a bottleneck due to this reduced applicability.
- **Validation of SFCs requires a uniform stress field at the failure-critical location**
- **Determination of modal SFC-parameters must be performed in the respective pure mode domain**
- **The 120°-dents are the probabilistic result of a 2-fold acting of the same failure mode. This shape is usually described by replacing J_2 through $J_2 \cdot \Theta(J_3, J_2)$**
- **In order to exploit the knowledge from other similar behaving materials watch the material behaviour and not that the observed material is a different one.**

Keep in mind: Failure is generated locally !

Some Literature

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Theory is the Quintessence of all Practical Experience

A. Föppl

“ Scientists would rather use
someone else's toothbrush
than someone else's terminology! “

… or *theory*

(Nobel laureate Murray Gell-Mann)

ANHANG

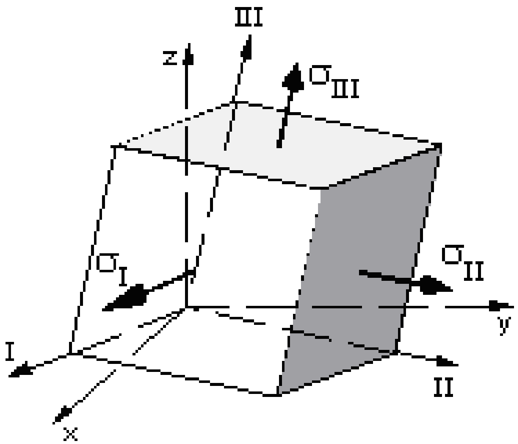
Self-explaining Notations for Strength Properties (homogenised material) neu !!!!

		Fracture Strength Properties									required by material symmetry
	loading	**tension**			**compression**			**shear**			
	direction or **plane**	1	2	3	1	2	3	12	23	13	
9	**general orthotropic**	R_1^t	R_2^t	R_3^t	R_1^c	R_2^c	R_3^c	R_{12}	R_{23}	R_{13}	**comments**
5	**UD, ≅ non-crimp fabrics**	$R_\parallel^t$ NF	$R_\perp^t$ NF	$R_\perp^t$ NF	$R_\parallel^c$ SF	$R_\perp^c$ SF	$R_\perp^c$ SF	$R_{\parallel\perp}$ SF	$R_{\perp\perp}$ NF	$R_{\parallel\perp}$ SF	$R_{\perp\perp} = R_\perp^t / \sqrt{2}$ (compare Puck's modelling)
6	**fabrics**	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	*Warp = Fill*
9	**fabrics general**	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	*Warp ≠ Fill*
5	**mat**	R_{1M}^t	R_{1M}^t	R_{3M}^t	R_M^c	R_{1M}^c	R_{3M}^c	R_M^τ	R_M^τ	R_M^τ	$R_M^\tau (R_M^t)$
2	**isotropic**	R_m SF	R_m SF	R_m SF	*deformation-limited*			R_M^τ	R_M^τ	R_M^τ	*ductile, dense* $R_M^\tau = R_m / \sqrt{2}$
		R_m NF	R_m NF	R_m NF	R_m^c SF	R_m^c SF	R_m^c SF	R_m^σ NF	R_m^σ NF	R_m^σ NF	*brittle, dense* $R_M^\sigma = R_m^t / \sqrt{2}$

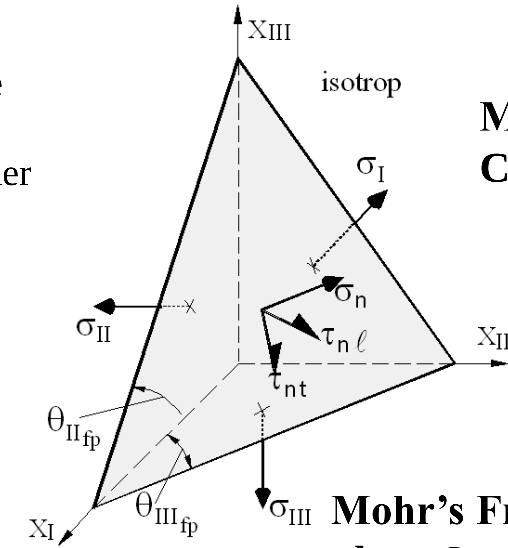
NOTE: *As a consequence to isotropic materials (European standardisation) the letter R has to be used for strength. US notations for UD material with letters X (direction 1) and Y (direction 2) confuse with the structure axes' descriptions X and Y . *Effect of curing-based residual stresses and environment dependent on hygro-thermal stresses. *Effect of the difference of stress-strain curves of e.g. the usually isolated UD test specimen and the embedded (redundancy) UD laminae. R_m := 'resistance maximale' (French) = tensile fracture strength (superscript t here usually skipped), R:= basic strength. Composites are most often brittle and dense, not porous! SF = shear fracture

Fundamentals when generating Strength Failure Conditions

Isotropic Material (3D stress state), Stresses & Invariants



The stress states in the various COS can be transferred into each other



Mohr's COS

Mohr's Fracture plane Stresses

Principal Stresses	**Structural Component Stresses**	
$\{\sigma\}_{principal} = (\sigma_I,\ \sigma_{II},\ \sigma_{III})^T$	$\{\sigma\}_{comp} = (\sigma_x,\ \sigma_y,\ \sigma_z,\ \tau_{yz},\ \tau_{xz},\ \tau_{xy})^T$	$\{\sigma\}_{Mohr} = (\sigma_\ell,\ \sigma_n,\ \sigma_t,\ \tau_{nt},\ \tau_{t\ell},\ \tau_{\ell n})^T$
$I_1 = (\sigma_I + \sigma_{II} + \sigma_{III})^T = 3\sigma_{oct} \equiv f(\sigma),$	**'isotropic' invariants !** $I_1 = (\sigma_x + \sigma_x + \sigma_z)^T$	$I_1 = (\sigma_\ell + \sigma_n + \sigma_t)^T$
$6J_2 = (\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2$ $= 4(\tau_{III}^2 + \tau_{II}^2 + \tau_I^2) = 9\tau_{oct}^2 \equiv f(\tau)$	$6J_2 = (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + (\sigma_x - \sigma_y)^2$ $+6(\tau_{yz}^2 + \tau_{zx}^2 + \tau_{xy}^2)\ \ (Mises, HMH)$	$6J_2 = (\sigma_n - \sigma_t)^2 + (\sigma_t - \sigma_\ell)^2 + (\sigma_\ell - \sigma_n)^2$ $+6(\tau_{nt}^2 + \tau_{t\ell}^2 + \tau_{n\ell}^2)$

$$27J_3 = (2\sigma_I - \sigma_{II} - \sigma_{III})(2\sigma_{II} - \sigma_I - \sigma_{III})(2\sigma_{III} - \sigma_I - \sigma_{II}),\quad I_\sigma = 4J_2 - I_1^2/3,\quad \sigma_{mean} = I_1/3$$

Invariant := Combination of stresses –powered or not powered- the value of which does not change when altering the coordinate system.

Literature

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